Another Case of Discrepancies when Evaluating Power Theories Using Real Data

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***Abstract*—Most non-sinusoidal power theories have been developed from theoretical techniques, as well as relying on hypothetical and experimental networks to highlight the advantages of each. The drawback of the power theories is that no single one has been universally accepted as a benchmark for other developments. This paper will however show this weakness by means of evaluating two power theories in the time domain, using real recorded data.**

***Index Terms*—α-β plane, .instantaneous, power theory, *p-q* theory**

I. Introduction

The authors [10], [11], [12], [14] states that there is still great uncertainty defining the power components under non-sinusoidal conditions. Attempts were made to apply the single phase definitions to three-phase [11] systems, but this also leads to some confusion if the set of definitions apply equally to three-wire or four-wire systems. The term apparent power (AP), according to Filipski [12], is widely used and different understandings of this exist within the power engineering profession. Several authors [2], [3], [7], [8] and [13], states that the Instantaneous Reactive Power (IRP***)*** ***p-q*** theory has become a "*very attractive theoretical tool not only for the active power filter control but also for analysis and identification of power properties of three-phase systems with non-sinusoidal voltages and current*". Due to the popularity the IRP ***p-q*** theory, this paper will evaluate two authors work [7] and [13] through the application of recorded data to each and the tabulation of the results. The main purpose of this paper, in contrast to propose a new power theory, will be to rather discuss differences found in the results and to discuss assumptions and potential utilization of the results in dynamic load modeling.

II. Basic Definitions

A. *Classic Instantaneous Reactive Power Theory*

In order to be able to mathematically calculate both the instantaneous real and reactive power the instantaneous space vectors for the voltages and currents is used [7]. The authors [7], [13] both use the three-phase to quadrature transformation, *a-b-c* co-ordinates to α-β co-ordinates, based on the Clarke Transform producing (1) and (2).

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \sqrt{\frac{3}{2}} & -\sqrt{\frac{3}{2}} \end{bmatrix} \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \sqrt{\frac{3}{2}} & -\sqrt{\frac{3}{2}} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (2)$$

From equation (1) and equation (2) it follows, that the instantaneous real power in the three-phase circuit is represented by (3), [13].

$$p = u_\alpha i_\alpha + u_\beta i_\beta \quad (3)$$

The conventional instantaneous power, which is equal to (3), is defined according to the authors [3], [7], [9] and [13],

as:

$$p = u_a i_a + u_b i_b + u_c i_c \quad (4)$$

and the conventional instantaneous reactive power [7] is defined as:

$$q = u_\alpha i_\beta + u_\beta i_\alpha \quad (5)$$

The authors in [2], [13] have defined the reactive power as (6), which seems to be a typing error.

$$q = u_\alpha i_\beta - u_\beta i_\alpha \quad (6)$$

The authors [7] and [13], develops the instantaneous currents on the α-β co-ordinates, namely i_α and i_β. These two instantaneous currents are subdivided into two components namely, active and reactive current. These currents are calculated as follow:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \quad (7)$$

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where:

$$i_{\alpha p} = \frac{u_\alpha}{u_\alpha^2+u_\beta^2} p \tag{8}$$

$$i_{\alpha q} = \frac{-u_\beta}{u_\alpha^2+u_\beta^2} q \tag{9}$$

$$i_{\beta p} = \frac{u_\beta}{u_\alpha^2+u_\beta^2} p \tag{10}$$

$$i_{\beta q} = \frac{u_\alpha}{u_\alpha^2+u_\beta^2} q \tag{11}$$

The α-β plane instantaneous active and reactive currents are transformed back to three-phase quantities [13] leading to:

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = [C]\begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + [C]\begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \tag{12}$$

with:

$$[C] = \sqrt{\frac{3}{2}}\begin{bmatrix} 1 & 0 \\ \frac{-1}{2} & \sqrt{\frac{3}{2}} \\ \frac{-1}{2} & -\sqrt{\frac{3}{2}} \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} i_{pa} \\ i_{pb} \\ i_{pc} \end{bmatrix} + \begin{bmatrix} i_{qa} \\ i_{qb} \\ i_{qc} \end{bmatrix} \tag{14}$$

The authors [7] introduced an instantaneous zero-phase sequence current and (14) becomes:

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} i_{a0} \\ i_{b0} \\ i_{c0} \end{bmatrix} + \begin{bmatrix} i_{pa} \\ i_{pb} \\ i_{pc} \end{bmatrix} + \begin{bmatrix} i_{qa} \\ i_{qb} \\ i_{qc} \end{bmatrix} \tag{15}$$

with $i_{a0} = i_{b0} = i_{c0} = \frac{i_0}{\sqrt{3}}$ (16)

Akagi [7], also defined the instantaneous powers in the a-axis and b-axis as follows:

$$\begin{bmatrix} P_\alpha \\ P_\beta \end{bmatrix} = \begin{bmatrix} u_\alpha i_\alpha \\ u_\beta i_\beta \end{bmatrix} = \begin{bmatrix} u_\alpha i_{\alpha p} \\ u_\beta i_{\beta p} \end{bmatrix} + \begin{bmatrix} u_\alpha i_{\alpha q} \\ u_\beta i_{\beta q} \end{bmatrix} \tag{17}$$

The authors [7] manipulate (7) and (17) to derive the instantaneous real power in the three-phase circuit namely:

$$p = p_\alpha + p_\beta \tag{18}$$

Further manipulation of (18) results in (19) and (20):

$$p = u_\alpha i_{\alpha p} + u_\beta i_{\beta p} \equiv p_{\alpha p} + p_{\beta p} \tag{19}$$

$$0 = u_\alpha i_{\alpha q} + u_\beta i_{\beta q} \equiv p_{\alpha q} + p_{\beta q} \tag{20}$$

The authors [13], assumed a balanced three-phase supply voltage both in amplitude and phase. The three-phase load currents are however defined as the respective sum of the fundamental frequency sinusoidal waveform and a harmonic of the fundamental frequency of order n. The transformation of the three-phase voltages and currents to α-β plane and then the calculation of the instantaneous real and reactive power lead to the following:

$$p = u_\alpha i_\alpha + u_\beta i_\beta \tag{21}$$

$$= \frac{3}{2}\hat{U}\hat{I} + \sum_n \frac{\hat{U}\hat{I}_n}{2}\{K_1 cos[(n+1)\omega t] + K_2 cos[(n-1)\omega t]\}$$

$$q = u_\alpha i_\alpha - u_\beta i_\beta \tag{22}$$

$$= \sum_n \frac{\hat{U}\hat{I}_n}{2}\{K_1 sin[(n+1)\omega t] + K_2 sin[(n-1)\omega t]\}$$

with:

$$K_1 = 1 + \cos(n+1)120° + \cos(n+1)240° \tag{23}$$

$$K_2 = 1 + \cos(n-1)120° + \cos(n-1)240° \tag{24}$$

III. PRESENTATION OF EXPERIMENTAL DATA

The data used in this paper were recorded on one of three supply feeders to a large Aluminium plant. The voltage and current trend applicable to the manipulation process is shown in Fig. 1. This represents a trend during a fault experienced on the 400 kV supply line to the step down transformer, supplying 132 kV Incomer number one at the smelter. Two more identical supplies are present at the medium voltage yard at the smelter. The authors used a Class A recording device to record the disturbance.

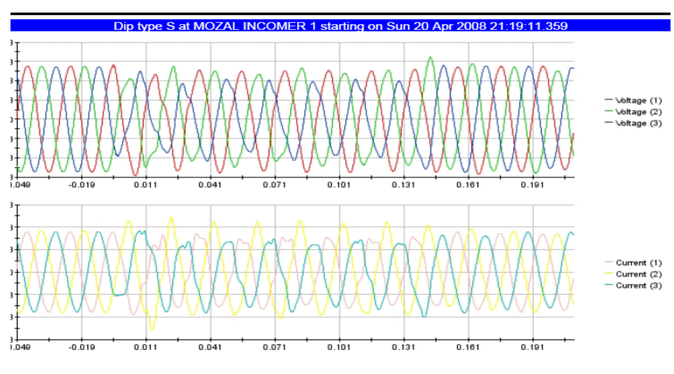

Fig. 1 – Voltage and Current Trend during fault on Line 1 red phase on 400 kV supply

A. Methodology followed in apply the power theories

The experimental manipulation of the data, before substituted into the power definitions of [7] and [13] were done in the Mathcad calculation aid environment.

The recorded data presented in Fig. 1 has a pre-fault recording as well as the actual recording during the fault condition. The time scale is from minus 0.049 seconds to plus 0.21 seconds. Due to the nature of the investigation, as well as that it is an experimental evaluation of the power theories through actual raw data, six increments of time were chosen for which the equivalent voltage and current values will be substituted into the power theories upon which the results will be evaluated. The authors chose to ignore the pre-fault values and used a value as close as possible to zero (0.000016) seconds. Following this an arbitrary value of 0.011 seconds were chosen and one full incremental cycle following this value namely 0.031, 0.051, 0.071 seconds and a final value of 0.15 seconds. The time increments can be seen in Table I.

B. Presentation and discussion of the power theories results

The following Table, Table I represents the results of the substitution of recorded data, during a fault conditions upstream from the recorder, into the two power theories [7] and [13]. As expected the equations (3) and (4) calculated exactly the same values due to the fact that except for the favoring of e above u for the voltage representation, the construction of the equations is exactly the same. The main difference is however in the calculation of the reactive instantaneous power q where the authors [7] equation (5) add the two space vector products where the authors of [2] and [13], equation (6) subtract these two space vectors. An interesting observation is that the difference between the results of the two equations is not constant or linear but more outlier or scattered results. This can in part be due to the non-sinusoidal nature of the waveforms during the fault condition. There is however a significant difference in the results obtained from equations (19) by [7] and equation (21) by [13]. The results of (19) correlates perfectly with that of (3), but (21) does not correlate with (3). The authors [13] state that (3) and (21) is equal by virtue of transformation of the three-phase voltages and currents to the α-β plane, calculating the instantaneous real and reactive power with the assumption of balanced voltage supply in both magnitude and phase. It can clearly be seen from Fig. 1, that the three-phase supply voltage is not balanced and the phase angles will not be balanced. Equation (3) and (21) can therefore not be treated as equal under non-sinusoidal supply voltage conditions. This situation is supported by the results of (3) and (21) in Table I.

IV CONCLUSION

This paper has done some experimental investigation into the behavior of two well-known power theories [7] and [13], under non-sinusoidal conditions and interpreted the power theory results. The authors do acknowledge that, in order to draw concise conclusions more data needs to be manipulated and results trended to study the results in more detail and for different faults conditions which normally leads to unbalanced supply voltages. It is also clear from Table I that the data from the α-β transformations (3) and (19) appears to be more useful than (21) in dynamic load modeling processes as the load component characteristics, frequency and phase angles is determined through different mathematical processes than used in power calculations. The intention of this paper has not been to go into lengthy theoretical discussions on the suitability of any of the power theories per say, but rather used as a practical application approach. The authors further acknowledges that any recorded data used in manipulation processes is only a small representation of a vast dynamic process and hence the resulting conclusions drawn from these mathematical processes is also representative of a discrete portion of a larger system.

TABLE I. POWER THEORIES RESULTS

	POWER THEORIES	
	Akagi *et el*	**Marshall *et el***
Eq. (3)@ t=0.00 sec	122.73 MVA	122.73 MVA
Eq. (3)@ t=0.011 sec	271.88 MVA	271.88 MVA
Eq. (3)@ t=0.031 sec	181.37 MVA	181.37 MVA
Eq. (3)@ t=0.051 sec	236.06 MVA	236.06 MVA
Eq. (3)@ t=0.071 sec	216.43 MVA	216.43 MVA
Eq. (3)@ t=0.15 sec	243.19 MVA	243.19 MVA
Eq. (4)@ t=0.00 sec	123.307 MW	123.307 MW
Eq. (4)@ t=0.011 sec	267.4 MW	267.4 MW
Eq. (4)@ t=0.031 sec	180.96 MW	180.96 MW
Eq. (4)@ t=0.051 sec	238.26 MW	238.26 MW
Eq. (4)@ t=0.071 sec	214.76 MW	214.76 MW
Eq. (4)@ t=0.15 sec	256.47 MW	256.47 MW
Eq. (5), (6)@ t=0.00 sec	12.16 MVAr	-19.4 MVAr
Eq. (5), (6)@ t=0.011 sec	-38.45 MVAr	-86.87 MVAr
Eq. (5), (6)@ t=0.031 sec	-51.92 MVAr	-58.65 MVAr
Eq. (5), (6)@ t=0.051 sec	-53.14 MVAr	-38.13 MVAr
Eq. (5), (6)@ t=0.071 sec	-45.85 MVAr	-65.16 MVAr
Eq. (5), (6)@ t=0.15 sec	111.8 MVAr	19.64 MVAr
Eq. (19), (21)@ t=0.00 sec	122.73 MVA	96.82 MVA
Eq. (19), (21)@ t=0.011 sec	271.88 MVA	261.07 MVA
Eq. (19), (21)@ t=0.031 sec	181.37 MVA	189.47 MVA
Eq. (19), (21)@ t=0.051 sec	236.06 MVA	237.13 MVA
Eq. (19), (21)@ t=0.071 sec	216.43 MVA	217.5 MVA
Eq. (19), (21)@ t=0.15 sec	243.19 MVA	111.64 MVA

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