MULTIOBJECTIVE OPTIMIZATION OF A THERMOACOUSTIC REGENERATOR

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Abstract—. This paper presents a new mathematical approach for optimizing the geometry of a thermoacoustic regenerator, aimed at producing efficient thermoacoustic engines. Optimal set of parameters describing the device are computed for a chosen thermoacoustic couple to illustrate this approach. Hence, a non-linear multiobjective problem is formulated in GAMS and solved using Lindoglobal solver. Lexicographic optimization is presented as an alternative optimization technique to the common used weighting methods. This approach establishes a hierarchical order among all the optimization objectives instead of giving them a specific (and most of the time, arbitrary) weight. In this work, the optimization criteria are chosen as work output, viscous resistance as well as thermal losses that are typically disregarded when modeling the device. A practical example is given, in a hypothetical scenario, showing how the proposed optimization technique may help thermoacoustic regenerator designers to identify Pareto optimal solutions when dealing with geometric parameters.

Keywords: Thermoacoustics, multiobjective optimization, Gams, mathematical programming

I. Introduction

Thermoacoustic technologies are concerned with developing new concepts of engines, coolers and heat pumps which operate on the basis of a range of thermoacoustic effects. Thermoacoustic devices can potentially use high amplitude sound wave to serve a variety of purposes in fields such as cryogenics, cost-effective domestic refrigeration or electricity generation, without drawbacks such as expensive construction or maintenance costs, high part counts or adverse environmental impact associated with certain refrigerators. With greater media and scientific interest in the issues of climate change, thermoacoustics is also and increasingly popular field of study because of its potential advantages over conventional systems.

The basic mechanics behind thermoacoustics are already well understood. A detailed explanation of the way thermoacoustic coolers work is given by Swift [1] and Wheatly et al. [2]. Research is focusing on optimizing the method so that thermoacoustic coolers can compete with commercial refrigerators. The presence of a stack (Fig. 1) provides heat exchange with the sound field and the generation or absorption of acoustic power. With a suitable geometry substantial amounts of heat can be moved as demonstrated, for example, by Garrett and Hofler [3]. An interesting and important feature of such engines is that the performance depends on geometric factors and gas parameters [4].

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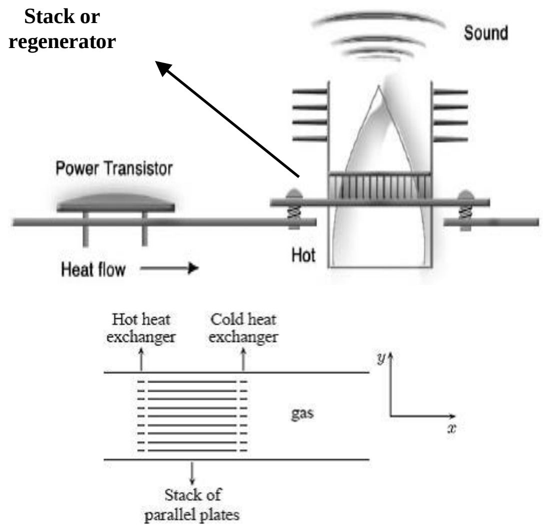

Figure 1. Example of prime mover interfaced with circuit

Optimization techniques as a design supplement are severely under-utilized, and previous efforts in the optimization of thermoacoustic devices are rare. Minner et al. [5], Wetzel [6], Besnoin [7] and Tijani et al. [8] utilized a linear approach while trying to optimize the device. Additionally, most studies (the exception being the Minner et al. study) vary only a single parameter, holding all else fixed and ignored thermal losses to the surroundings. These Parametric studies are unable to capture the nonlinear interactions inherent in thermoacoustic models with multiple variables, and can only guarantee locally optimal solutions.

Zink et al [9] and Trapp et al. [10] illustrate the optimization of thermoacoustic systems, while taking into account thermal losses to the surroundings that are typically disregarded. They use mathematical analysis and optimization and illustrate the conflicting nature of objective component considered in their modeling approach. Therefore since several conflicting objectives have been identified, an effort to effectively implement the epsilon constraint method for producing the Pareto optimal solutions in a multiobjective optimization mathematical programming method is carried out in our approach. This has been implemented in the widely used modeling language GAMS [11] (General Algebraic Modeling Language, www.gams.com). As a result, Gams codes are written to define, to analyze, and solve optimization problems to generate sets of Pareto optimal solutions unlike previous studies.

II. MODELING APPROACH

In this section, our modeling approach for the physical standing wave engine depicted in Fig. 2 is discussed; the development of our mathematical model and its corresponding optimization is included. The problem is reduced to a two dimensional domain, because of the symmetry present in the stack. Two constant temperature boundaries are considered namely one convective boundary and one adiabatic boundary, as shown in Fig. 2. For our model, only the regenerator geometry is considered; the model considers variation in operating condition and the interdependency of stack location and geometry.

Five different parameters namely the Stack length L, the stack height H, the stack placement Za (with Za=0 corresponding to the closed end of the resonator tube) and the number of channels N are considered to characterize the regenerator:

Those parameters have been allowed to vary simultaneously. Five different objectives as described by Trapp et al. [10] namely two acoustic objectives (Acoustic work W of the thermoacoustic engine and viscous resistance R_V through the regenerator [9]) and three thermal objectives (convective heat flow Q_{conv}, radiative heat flow Q_{rad}, and conductive heat flow Q_{cond}) are considered to measure the quality of a given set of variable value that satisfies all of the constraint. Ultimately, optimizing the resulting problem generates optimal objective function value $G^* = \left[W^*, R^*_V, Q^*_{conv}, Q^*_{rad}, Q^*_{cond}\right]$ and optimal solution $x^* = \left[L^*, H^*, d^*, Za^*, N^*\right]$.

Since the five objectives are conflicting in nature [10], a multiobjective optimization approach has been used. Each objective component has been given a weighting factor w_i to provide appropriate user-defined emphasis.

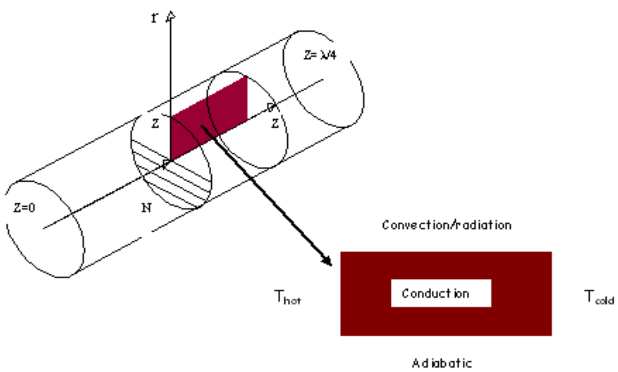

Figure 2. Computational domain

In our approach, we use the ε-constraint method for solving multiobjective mathematical programming problems. The basic step towards further penetration of the methods in our multiobjective mathematical problems is to provide appropriate codes in a Gams environment and produce efficient solutions.

III. ILLUSTRATION OF THE OPTIMIZATION PROCEDURE OF THE REGENERATOR

The five variables L, H, d, Za, N may only take values within the certain lower and upper bounds. The feasible domain for a thermoacoustic regenerator should be defined. Additionally, the total number of channels N of a given diameter d is limited by the cross-sectional radius of the resonance tube H. Therefore the following constraint relation can be determined:

$$N(d + t_w) \le 2H \quad (1)$$

where t_w represents the wall thickness around a single channel.

The acoustic power per channel has been derived by Swift [1]. The following equation can be derived for N channel:

$$W = \omega L N \left(\frac{\pi H^2}{2(d + t_w)} \right) \left[\delta_k \frac{(\gamma - 1)p^2}{\rho c^2 (1 + \varepsilon)} (\nabla T_{crit.} - 1) - \delta_v \rho u^2 \right] \quad (2)$$

Where δ_k represent the thermal penetration depth, δ_v the viscous penetration depth and $\nabla T_{crit.}$ the critical temperature. The viscous resistance can be derived as follow [1]:

$$R_V = \frac{\mu \prod L}{A_C^2 \delta_V N} = \frac{2\mu}{\delta_V} \frac{L}{(d + t_w)\pi H^2 N} \quad (3)$$

The total convective heat transfer across the cylindrical shell can be described by [10]:

$$Q_{conv} = 2\pi H L h \left[\frac{T_C - T_H}{\ln\left(\frac{T_C}{T_H}\right)} - T_\infty \right] \quad (4)$$

The radiation heat flux becomes increasingly important as T_H increases, as shown in the following equation:

$$Q_{rad} = 2\pi H L k_B \varepsilon \left[\frac{T_C^4 - T_H^4}{4 \ln\left(\frac{T_C}{T_H}\right)} - T_\infty^4 \right] \quad (5)$$

The conductive heat flux is representative of the heat loss across the cold end of the domain:

$$Q_{cond} = \frac{k_{zz}}{L}\pi H^2 T_C \ln\left(\frac{T_H}{T_C}\right) \qquad (6)$$

IV. EMPHASIZING ALL OBJECTIVE COMPONENTS

Most of the expressions involved in our mathematical model (MPF) have been presented in the previous section. Together with expressions in reference [10] and the following equation, they represent our non-linear mixed integer problem:

$$(\mathrm{MPF}) \min_{L,H,Z_a,d,N} \xi = w_1(-W) + w_2 R_V + w_3 Q_{conv} + w_4 Q_{rad} + w_5 Q_{cond} \qquad (7)$$

There is no single optimal solution that simultaneously optimizes all the two objectives functions. We apply the augmented ε-constraint method (AUGMECON) as proposed by Mavrotas [11] to compute the most preferred solutions. The mathematical details of computing payoff table for MMP problem can be found in [13]. To illustrate our approach, we consider the thermoacoustic couple as described in [14]. It consists of a parallel-plate stack placed in helium-filled resonator. All relevant parameters are given in Table I and Table II. The following constraints (upper and lower bounds) have been enforced on variables in other for the solver to carry out the search of the optimal solutions in those ranges:

$$\begin{aligned} &L.lo = 0.005; \quad L.up = 0.05; \\ &Za.lo = 0.005; \\ &H.lo = 0.005; \\ &d.lo > 2.\delta_k; \quad d.up < 4.\delta_k \end{aligned} \qquad (8)$$

The integer variable has been given values of 20 to 50. This process generates optimal solutions corresponding to some integer variable.

TABLE I. Specifications for Thermoacoustic couple

Parameter	Symbol	Value	Unit
Isentropic coefficient	ν	1.67	
Gas density	ρ	0.16674	kg/m^3
Specific heat capacity	c_p	5193.1	J/kg.K
Dynamic viscosity	μ	$1.9561.10^{-5}$	kg/m.s
Maximum velocity	u_{max}	670	m/s
Maximum pressure	p_{max}	114003	Pa
Speed of sound	c	1020	m/s
Thickness plate	t_w	1.91.10-4	M
Frequency	f	696	Hz
Thermal conductivity Helium	k_g	0.16	W/(m.K)
Thermal conductivity stainless steel	k_S	11.8	W/(m.K)
Isobaric specific heat capacity	c_p	5193.1	J/(kg.K)

TABLE II. Additionnal Parameters used for programming

Parameter	Symbol	Value	Unit
Temperature of the surrounding	T_∞	298	K
Constant cold side temperature	T_C	300	K
Constant hot side temperature	T_H	700	K
Wavelength	λ	1.466	m
Thermal expansion	β	$1/T_\infty$	1/K
Thermal diffusivity	α	2.1117E-5	m^{2}s-1

The following figure report only one set of Pareto solutions obtained:

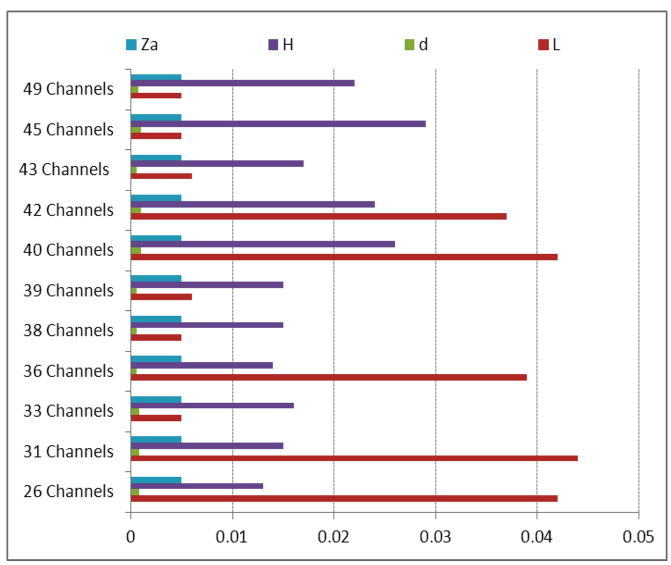

Figure 3. Optimal structural variables

The Pareto optimal solutions are represented graphically; These results shows that there is not only a single optimal solution that optimize the geometry of the regenerator and most importantly highlight the fact that the geometrical parameters are interdependent, which support the use of a multiobjective approach for optimization.

V. CONCLUSION

In order for a thermoacoustic engines to be competitive on the current market, they have to be optimized in order to improve their overall performance. This work target the geometry of the thermoacoustic regenerator and uses multiobjective optimization approach to find the optimal set of geometrical parameters that optimizes the device. Five different parameters describing the geometry of the device have been studied. Five different objectives have been identified; a weight has been given to each of them to allow the designer to place desired emphasis. A non-linear multiobjective programming approach for thermoacoustic regenerator has been implemented in GAMS. To illustrate our approach, one efficient point that optimizes the device has been computed the geometrical parameters of the regenerator have been found to be interdependent which support the use of our multiobjective approach for optimization of thermoacoustic engine's geometry.

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