Error Correction of Frequency-Selective Fading Channels with Spectral Nulls Codes

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Abstract— In this paper, we investigate a prior error correction technique for frequency selective fading channels. Spectral nulls codes with nulls at submultiple frequencies are used to avoid fades at the corresponding frequencies. Coincide the spectrum null at the corresponding fading frequency and playing with its wideness, will minimize the bit errors caused by the channel.

I. INTRODUCTION

We will present in this paper a new error correction technique, called a prior error correction, because we generate codes with certain patterns to avoid some specific bad channels. Designing codes with certain spectral shapes is a technique to avoid channel fades in wireless and wire-line communication channels. Spectral Nulls codes, when transmitted serially, will have a spectrum with zeros occurring at $f = 1/kT$, where k is an integer or a ratio of relatively prime integers. Two classes of spectral nulls codes are presented, which are called binary and non-binary spectral nulls codes.

We furthermore investigate the performance of these codes in the presence of frequency selective fading channels.

The paper is organized as follows. In Section II we present the designing technique of Spectral Nulls Codebooks, whether they are binay or non binary.

In Section III we briefly describe the channel and received signal models.

The error correction of the fade and results of the technique are in section IV, and finally in section V a conclusion of the performed analysis will be drawn.

II. SPECTRAL NULLS CODES CONSTRUCTION

The technique of designing codes to have a spectrum with nulls occurring at certain frequencies started with Gorog [1], when he considered the vector $x = (x_1, x_2, ..., x_M)$, $x_i \in \{-1, +1\}$ with $1 \leq i \leq M$, to be an element of a set S, which is called codebook of codewords with elements in $\{-1, +1\}$, here we represent -1 as 0. And then we apply the Fourier transform to those codewords to get:

$$X(w) = \sum_{i=1}^{M} x_i e^{-jiw}, -\pi \leq w \leq \pi. \quad (1)$$

So the idea of having nulls at certain frequencies is the same as having the power spectral density function equal to zero at those frequencies. This means that $H(w) = 0$, where

$$H(w) = \frac{1}{Mn} \sum_{i=0}^{M-1} |X^{(i)}(w)|^2. \quad (2)$$

It can be seen from (2), that the power spectral density depends on the frequency value. So we can design a codebook to generate nulls at certain chosen frequencies. Usually for simplification we present the codeword length, M, as an integer multiple of k, where $f = r/k$ represents the spectral nulls at rational sub multiple r/k. The parameter k could be chosen either prime or not prime and divides M [2], i.e.

$$M = ks,$$

In the case where k is a prime number, we have to satisfy

$$A_1 = A_2 = \cdots = A_k, \quad (3)$$

where

$$A_i = \sum_{\lambda=0}^{s-1} x_{i+\lambda k}, \quad i = 1, 2, \ldots, k. \quad (4)$$

In the case where k is not prime we have to suppose that $k = cd$, where c and d are integer factors of k. The equation, which leads to nulls, is

$$\begin{aligned} &A_u = A_{u+vc} \\ &u = 1, 2, \ldots, c \\ &v = 1, 2, \ldots, d-1 \\ &k = cd \end{aligned} \quad (5)$$

where A_u is the same as in (4).

If all the codewords in a codebook satisfy these equations, the codebook will exhibit nulls at the required frequencies.

As can be seen from the introduction of the technique, the x_i are binary data and the resulting codes are binary spectral nulls codes (B-SNC). This technique is also applied to non binary data, with some modifications, which can be emphizised later, the obtained code is called non-binary spectral nulls codes (NB-SNC).

Example 1 Taking the case of $M = 8$ where $k = 4$ and $s = 2$, the equation (3) is written as follow:

$$\underbrace{\underbrace{x_1 + x_5}_{A_1} = \underbrace{x_2 + x_6}_{A_2} = \underbrace{x_3 + x_7}_{A_3} = \underbrace{x_4 + x_8}_{A_4}}_{k=4} \quad (6)$$

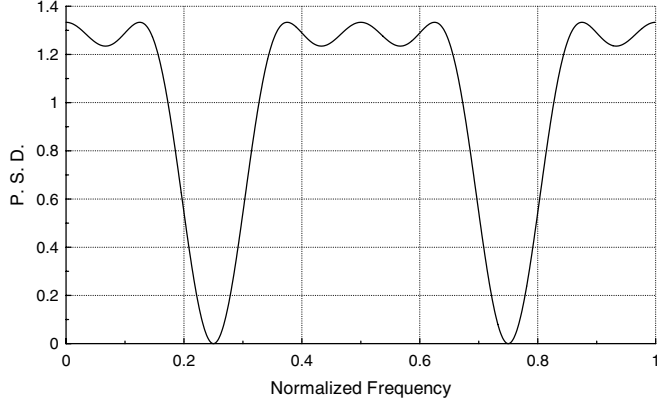Fig. 1. Power spectral density of B-SNC: $M = 8$, $k = 4$

As it can be seen from (6), the value of k represents the number of groupings A_i and also the corresponding nulls $1/k$, and s is the number of symbols in each groupping. The corresponding codebook has nulls at $1/4, 1/2, 3/4, 1$. To generate the codebook we need to start with all $000...000$ or all $111...111$, taking into consideration that 0 is mapped onto -1 and permuting the x_i in (6) with an exhaustive search to find the sequence that satisfys it.

The resultant codebook is presented as follow and its power spectral density is shown in Fig. 1.

$$C_B = \begin{Bmatrix} 00000000, 00000101, 00001010, 00001111, \\ 00010100, 00011110, 00101000, 00101101, \\ 00111100, 01000001, 01001011, 01010000, \\ 01010101, 01011010, 01011111, 01101001, \\ 01111000, 01111101, 10000010, 10000111, \\ 10010110, 10100000, 10100101, 10101010, \\ 10101111, 10110100, 10111110, 11000011, \\ 11010010, 11010111, 11100001, 11101011, \\ 11110000, 11110101, 11111010, 11111111 \end{Bmatrix}$$

$\square$

Example 2 The case of non binary sequences is a bit difficult to the binary ones because we are not dealing with two levels but with multilevel sequences. To generate the NB-SNC we have to start with a sequence that satisfy the spectral nulls equation in (3), we call this sequence a *SN sequence*. Each non binary symbol (NBS) in this sequence is mapped to a channel symbol (CS), which could represent the voltage level. In general, for odd M the symbol mapping is

$$\begin{array}{cccccccc} \text{NBS:} & 1 & 2 & \cdots & \frac{M+1}{2} & \cdots & M-1 & M \\ & \downarrow & \downarrow & & \downarrow & & \downarrow & \downarrow \\ \text{CS:} & -\frac{M-1}{2} & -\frac{M-3}{2} & \cdots & 0 & \cdots & +\frac{M-3}{2} & +\frac{M-1}{2} \end{array}$$

and for even M the symbol mapping is

$$\begin{array}{cccccccc} \text{NBS:} & 1 & 2 & \cdots & \frac{M}{2} & \frac{M+2}{2} & \cdots & M-1 & M \\ & \downarrow & \downarrow & & \downarrow & \downarrow & & \downarrow & \downarrow \\ \text{CS:} & -\frac{M}{2} & -\frac{M-2}{2} & \cdots & -1 & +1 & \cdots & +\frac{M-2}{2} & +\frac{M}{2} \end{array}$$

Then it is just a matter of placing the symbols in such a way that the SN equations are satisfied.

In this example we consider an SN sequence for $M = 8$ and $k = 4$, with channel symbols assigned as follows

$$\begin{array}{ccccccccc} \text{NBS:} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{CS:} & -4 & -3 & -2 & -1 & +1 & +2 & +3 & +4 \end{array}$$

From (3) and (4) we must satisfy

$$x_1 + x_5 = x_2 + x_6 = x_3 + x_7 = x_4 + x_8, \qquad (7)$$

with each grouping having two elements with indices differing by k. This means that if x_i is mapped to a negative channel symbol, the corresponding positive channel symbol must be mapped to x_{i+k}, as follows

$$\begin{array}{ccccccccc} \text{NS:} & 1 & 2 & 3 & 4 & 8 & 7 & 6 & 5 \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{CS:} & -4 & -3 & -2 & -1 & +4 & +3 & +2 & +1 \\ & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \end{array}$$

Clearly this satisfies (7) and the required SN sequence that generates nulls at frequencies $0, 1/4, 1/2, 3/4, 1$ is therefore $(1)(2)(3)(4)(8)(7)(6)(5)$.

The resultant codebook is presented as follow and its power spectral density is shown in Fig. 2.

$$C_{NB} = \begin{Bmatrix} 12348765, 14328567, 32146785, 34126587, \\ 12438756, 13428657, 42135786, 43125687, \\ 21347865, 24317568, 31246875, 34216578, \\ 21437856, 23417658, 41235876, 43215678, \\ 12358764, 15328467, 32156784, 35126487, \\ 12538746, 13528647, 52134786, 53124687, \\ 21357864, 25317468, 31256874, 35216478, \\ 21537846, 23517648, 51234876, 53214678 \end{Bmatrix}$$

$\square$

Table I presents selected SN sequences and the corresponding spectral nulls for different values of k and s.

III. CHANNEL MODELLING

As it is known that if the path from the transmitter to the receiver either has reflections or obstructions, we can get fading effects, which means in this case that the received signal from many differnt routes, each a copy from the original, has a slighltly defferent delay and different gain, and here we get some frequencies may be in deep fade.

Some mathematical/statistical models for multipath fading were introduced to present the caracteristics of these type of channels.

We take the Watterson model [3] as an example to understand the model of this type of channel. The Watterson model consists of a tapped delay line fed by the transmitted signal. A

TABLE I
SELECTED SPECTRAL NULLS SEQUENCES

M	k	s	Spectral nulls	SN sequence
4	2	2	0, 1/2, 1	(1)(2)(4)(3)
6	2	3	0, 1/2, 1	(1)(2)(4)(3)(5)(6)
6	3	2	0, 1/3, 2/3, 1	(1)(2)(3)(6)(4)(5)
8	4	2	0, 1/4, 1/2, 3/4, 1	(1)(2)(3)(4)(8)(7)(6)(5)
8	2	4	0, 1/2, 1	(1)(2)(4)(3)(5)(6)(8)(7)
9	3	3	0, 1/3, 2/3, 1	(1)(2)(3)(5)(6)(4)(9)(7)(8)
10	5	2	0, 1/5, 2/5, 3/5, 4/5, 1	(1)(2)(3)(4)(5)(10)(9)(7)(8)(6)
12	6	2	0, 1/6, 1/3, 1/2, 2/3, 5/6, 1	(1)(2)(3)(4)(5)(6)(12)(11)(10)(9)(8)(7)
16	4	4	0, 1/4, 1/2, 3/4, 1	(1)(2)(3)(4)(8)(7)(6)(5)(9)(10)(11)(12)(16)(15)(14)(13)
16	8	2	0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8, 1	(1)(2)(3)(4)(5)(6)(7)(8)(16)(15)(14)(13)(12)(11)(10)(9)
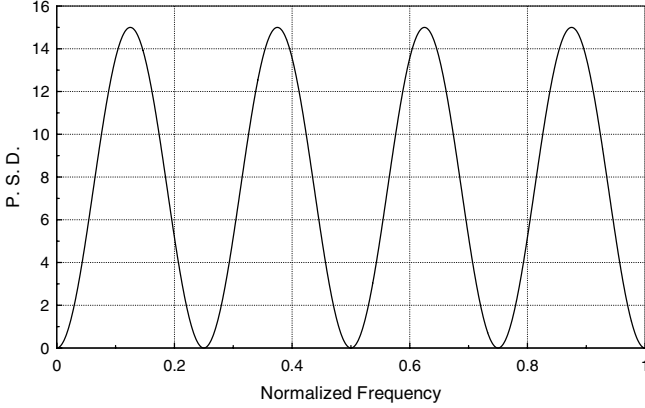
Fig. 2. Power spectral density of NB-SNC: $M = 8$, $k = 4$

IV. CORRECTION OF FREQUENCY SELECTIVE FADING CHANNEL WITH SPECTRAL SHAPING CODES

In this section we use a simplified model of the FSF channel represented by a FIR filter [4]-[5], where a deep notch is at the frequency $1/4$ in the normalized frequency scale. We investigate different spectral nulls codes with different properties. In order to see how the nulls could play an important role in minimizing the number of errors caused by a fade in the channel, we plot the spectrum of different types of data.

To present the idea clearly we have sent random data to a fading channel and other data from two different spectral nulls codes with different nulls at different frequencies. One of the codes has nulls corresponding with the frequency of the fading, while the other code has its nulls located differently.

Fig. 4 shows the spectrum of random data how it is effected badly by the channel reponse, while Fig. 5 shows when the null coincides with the fade, we garrantee less modification and damage to the data. Now we sent spectral nulls code

delayed versions of the transmitted signal are then modulated in amplitude and phase by a suitable tap-gain function, $G_i(t)$. The delayed and modulated signals are then summed to form the output or received signal. We have to mention that one tap is used for each of the resolvable multipath components.

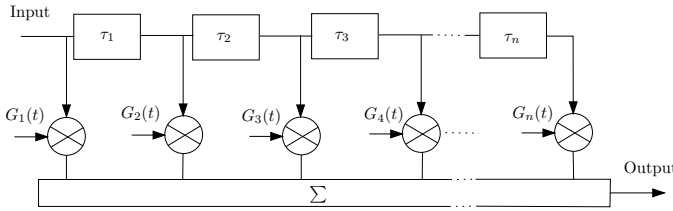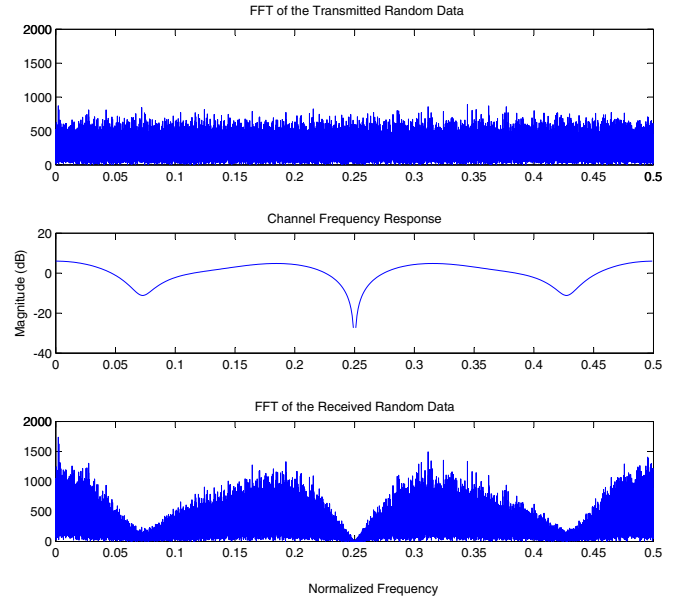Fig. 3 shows the general case of the block diagram of the Watterson model.

Fig. 3. Tap-delay model of multipath channel.

The impulse response of this model is presented by the following equation,

$$H(f,t) = \sum_{i=1}^{n} e^{(-j2\pi\tau_i f)} G_i(t), \qquad (8)$$

where τ_i is the ith path delay, G_i is the ith complex tap-gain function, and n is the number of paths.

Fig. 4. Spectrum of Random data in a FSF Channel.

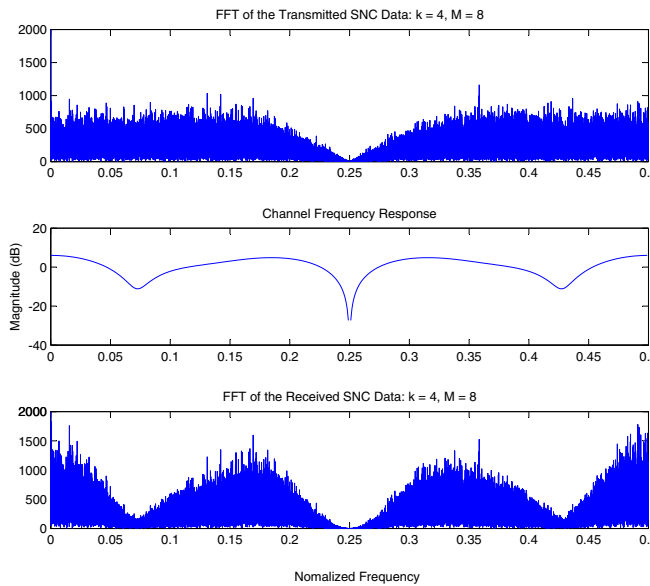Fig. 5. Spectrum of SNC data: k = 4, M = 8 in a FSF channel.

Fig. 7. Spectrum of NB-SNC data: k = 4, M = 8 in a FSF channel.

sequence over the channel but with nulls at different locations of the fade, to show the importance of the null to be at the corresponding frequency of the frequency selective fading of the channel. Fig. 6 shows that even we send data with nulls, but still have errors at the receiver when these nulls are not at the frequency of the fade as the case in Fig. 5.
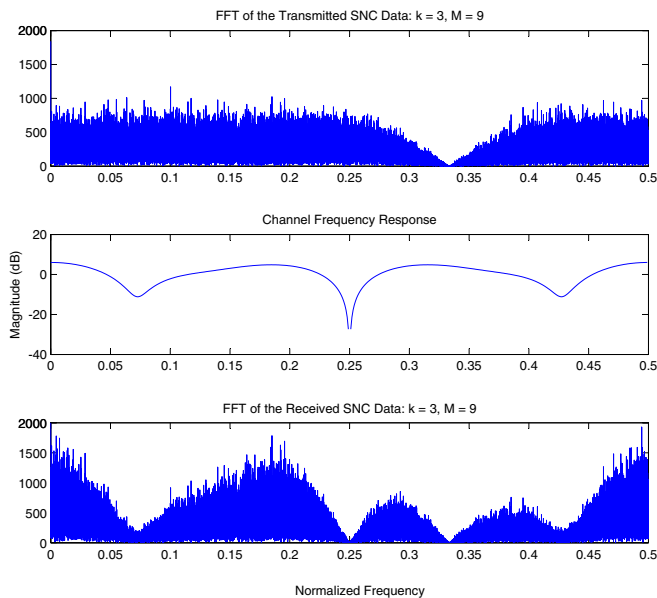

Fig. 6. Spectrum of SNC data: k = 3, M = 9 in a FSF channel.

Same experiment was done when sending non-binary sequence and here we can see the advantage of a code to be a DC-free code, the spectrum shown in Fig. 7 proves the inportance of a DC-free code.

After proving that the coincidance of the null of the trans-mitted data with the location of the fade will have good results

in minimizing the errors.

We can investigate further the from another angle. As it is known in the construction of spectral nulls codes, that two parameters are important in the construction of these codes, which are k reflecting the value of the null and s, which represents the number of symbols in each groupping. So the value of the null is fixed by the value k. We can design different codebooks with the same nulls but with different codewords lengths. Fig. 8 shows how the variation of the number of symbols in each group, which is effect the wideness of the null and this has an important result on the fade. As much as our null is wider as much as we avoid the fade and then less errors occure.
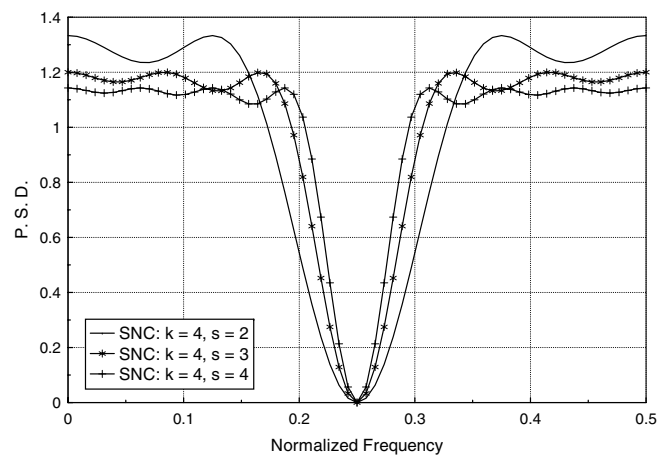

Fig. 8. Spectrum of SNC data: different values of s.

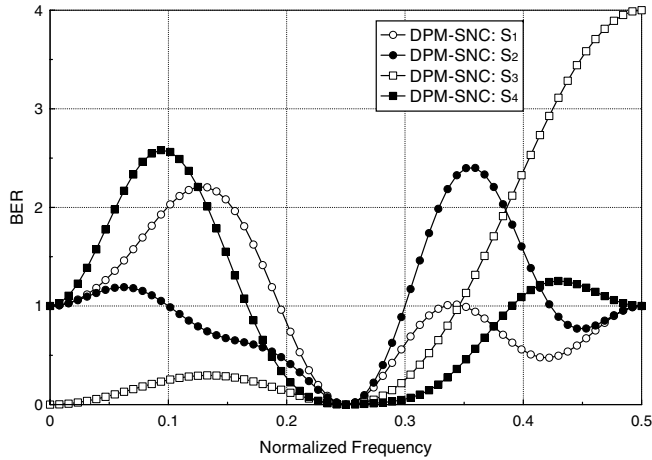As it is known, the parameter digital sum variation (DSV),

Fig. 9. Spectrum of SNC data: different different values of DSV.

Fig. 10. Spectrum of SNC data: Subset S_1.

plays an important role in the understanding and design of spectral properties of digital codes. The DSV is a measure of how steep the PSD-function is around the null at the frequency $1/k$. The smaller the DSV, the more suppressed the frequencies around $1/k$ [6].

We investigation by using subsets of the same spectral nulls code but with different DSV of the sequences, We have chosen four sets and plotted their spectrums,

$$S_1 = \{01111000, 01111101, 10000010, 10000111\},$$
$$S_2 = \{01011111, 01101001, 10010110, 10100000\},$$
$$S_3 = \{01010101, 01011010, 10100101, 10101010\},$$
$$S_4 = \{00001010, 00001111, 11110000, 11110101\}.$$

The results in Fig. 9 show the difference of the wideness of the nulls at the corresponding frequency, which is here $1/4$.

Figures 10, 11, 12 and 13 show different subsets taken from the same codebook but with different digital sum variation and how the wideness has an impact on the results.

Results are presented in Table II and more generally in Fig.14, which show the bit error rates (BER) [7] as a function of attenuation parameter values in the FIR filter. It is clear that we can benefit from a sequence of data generating nulls at the corresponding fading frequencies.

V. CONCLUSION

New spectral null codes called non-binary spectral nulls codes were obtained by using the same technique of the design of binary spectral nulls codes. Frequency slective fading channel reperesents a challange in the wireless and wireline communication and even in non-AWGN channels like power line channels [8]. And the spectral nulls presented a improvement in the reduction of error in such channels.
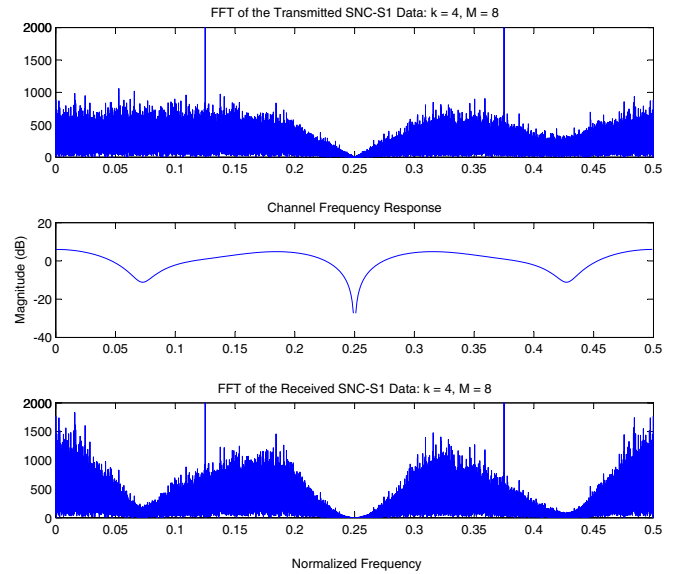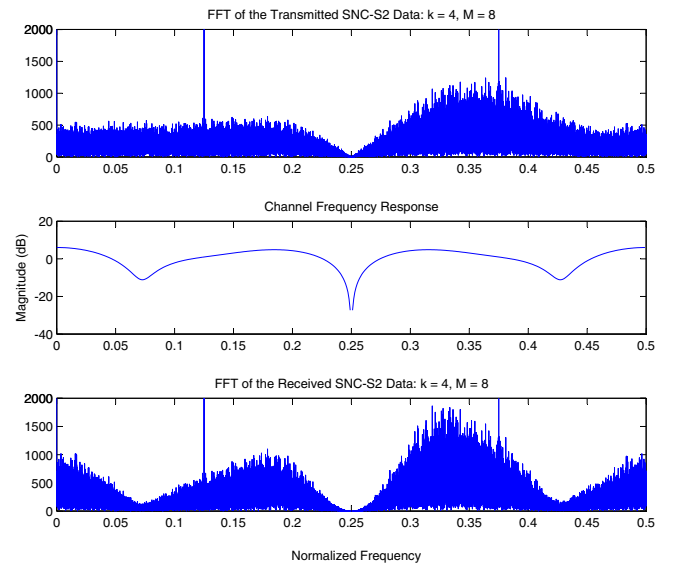

Fig. 11. Spectrum of SNC data: Subset S_2.

The technique of spectral shaping proved its utility not only in magnetic recording systems but in other communication channels as it is used in cancelling narrowband interferences and many other problems in the telecommunications field.

Our focus is to generalize our technique and combine our codes with modulations that proved to be good condidate for fading channels like the OFDM modulation [9].

We can add the technique of distance preserving mappings [10] to improve the error correction capabilities of our codes because of the existance of many other types of noises in the channel.
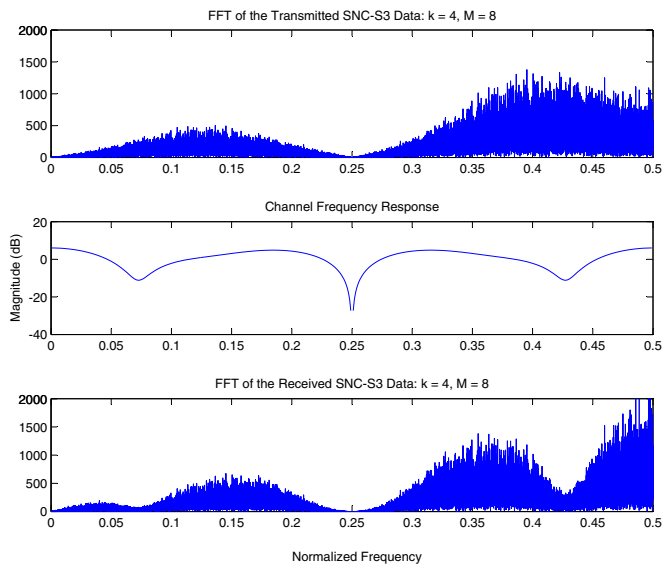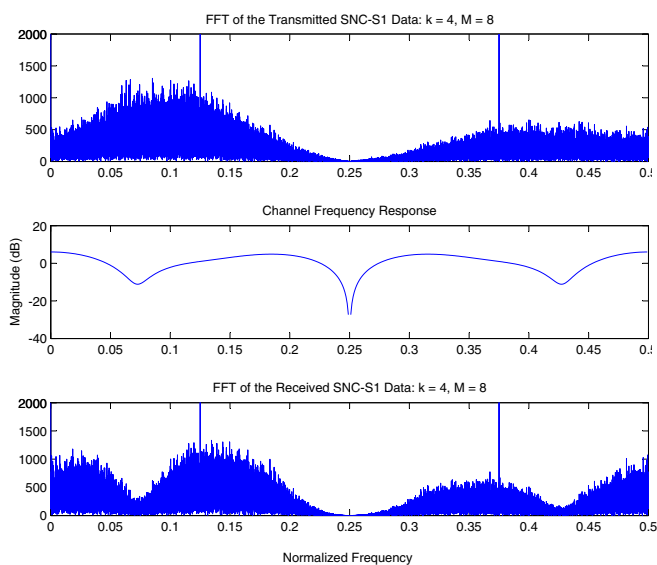
Fig. 12. Spectrum of SNC data: Subset S_3.

Fig. 13. Spectrum of SNC data: Subset S_4.

Acknowledgement

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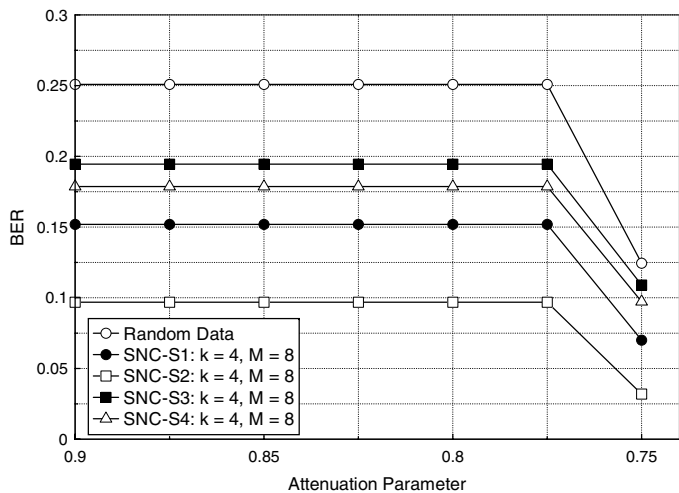| Sent Data	Number of Sent Bits	Number of Bit Errors	BER
Random	80000	20070	25.1%
Binary SNC–S1: $k = 4$, $M = 8$	80000	12151	15.2%
Binary SNC–S2: $k = 4$, $M = 8$	80000	7750	9.7%
Binary SNC–S3: $k = 4$, $M = 8$	80000	15560	19.5%
Binary SNC–S4: $k = 4$, $M = 8$	80000	14291	17.9%

Fig. 14. BER of different subsets S_i at different attenuation parameter values.

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