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Maximal Ratio Combining In N Dual-hop Nakagami-m Faded Relay Branches

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***Abstract*—In recent years relay based communication has gained attention of researchers and the scientific community. In this paper the source-to-destination statistics of a two hop amplify-forward relay branch, with the channel fading statistics of each hop being Nakagami-m distributed has been evaluated. The expression for the statistics of the signal envelope at the output of a maximal ratio combiner in the destination node of a N-path dual-hop relay braches is derived and compared with the simulation results. The statistics are helpful in evaluating the performance of systems using co-operative dual hop relaying and employing maximal ratio combining at the receiver terminal.**

I. Introduction

In modern wireless communication systems cooperative relaying is emerging as an important technique and has attracted the attention of researchers and scientists. In cooperative relaying the mobile terminals take part in the transmission of information, themselves not being the initial source or the final destination [1][2]. The reliability of the radio links are increased by implementing different diversity combining techniques at the receiver. In cooperative diversity schemes the relay nodes are used as virtual antennas to assist the communication between the source-destination pair. Cooperation diversity techniques are applied in mobile wireless ad-hoc networks, advanced cellular architectures, and hybrid networks in order to increase coverage, throughput, and capacity to transmit to the actual destination or to the next relay.

Cooperation diversity systems can be broadly categorized into two categories depending on the functionality of the relay, namely non-regenerative and regenerative [3]. In non-regenerative case, the relay just amplifies and forwards (A & F) the received signal, while for non-regenerative scenario the relay decodes, encodes, and forwards the received signal. A & F mode is often used when the issue of complexity and/or latency needs to be addressed as the processing burden on the relay is less in the A & F mode. However, in a non-regenerative system, because of the presence of relay nodes between the source and the destination, the statistics of the signal received at the destination node depends on the channel statistics the signal experiences in the individual links. For properly analyzing the relay link in design of a system a good understanding of the statistical behavior of the channels is required. Analysis of statistical behavior of relay channel has been a research area of immense interest and recently some papers dealing with the methods of determining the statistics of such relay have appeared in the literature [3].

The statistics of two-hop amplify forward type relay channels where the individual link experiences Nakagami-m fading is considered first [4][6]. The versatility of the Nakagami fading model lies in its flexibility of changing the individual link statistics by changing the Nakagami parameter m. The conventional Rayleigh fading model is obtained for while for , the channel is made to behave more like a Rician channel. The density function of the signal received in the destination through such a relay link is determined first. The relay is assumed to be an ideal noise free repeater with unity gain. Such assumptions set the upper bound on the system performance. In practical systems, the performance will degrade when noise is present at the relay. Next, maximal ratio combining of N such two-hop-amplify-forward relay links, where individual links are Nakagami-m faded has been considered. The analytical expression for the density function of the received signal at the output of the maximal ratio combiner has been derived. The density function obtained analytically is compared for some specific m (Nakagami parameter) values with those obtained from simulation studies.

This paper thus provides a mathematical analysis for determining the statistics of a two-hop-amplify-forward relay channels and the maximal ratio combining of N such relay channels when the individual links are Nakagami-m distributed.

The remainder of the paper is organized as follows. Section II presents the end-to-end channel statistics of a two hop relay branch, with individual hops experiencing Nakagami-m fading. In section III the probability density function of the signal amplitude at the output of the maximal ratio combiner for a N two-hop relay branches with individual links experiencing Nakiagami-m fading has been derived and compared with the simulation results. Finally, section IV summarizes the main results and concludes the paper.

II. Channel Statistics For A Two Hop Relay Branch

A typical relay based system without any diversity is shown in Fig 1. The signal reaches the destination node (D) from the source node (S) via a relay node (R). $h_1(t)$ and $h_2(t)$ are the channel statistics between the source-relay and relay-destination link respectively. $h_1(t)$ and $h_2(t)$ are assumed to be Nakagami-m distributed so that fading scenarios corresponding to different physical environment can be generated as particular cases of the generalized model. Being Nakagami-

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m distributed the probability density function (*pdf*) of the amplitudes of $h_1(t)$ and $h_2(t)$ can be written as,

$$f_{R_i}(r_i) = 2\left(\frac{m_i}{\Omega_i}\right)^{m_i} \frac{{r_i}^{2m_i-1}}{\Gamma(m_i)} \exp\left(-\frac{m_i}{\Omega_i}{r_i}^2\right) \quad (1)$$

where, $r_i > 0$ & $i = 1, 2$. $\Gamma(\cdot)$ is the gamma function, m_i denotes the m parameter of Nakagami-m distribution and $\Omega_i = E\left[r_i^2\right]$.

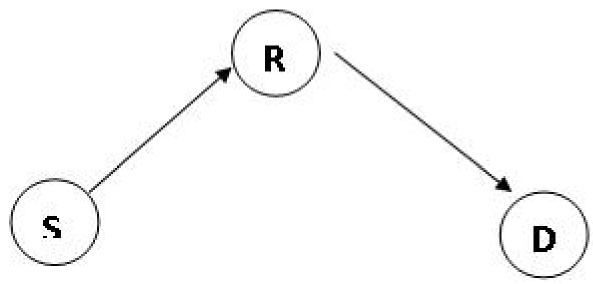

Fig. 1. A typical two hop relay based system without any diversity

R_i represent the random variables representing the amplitudes of the wireless links. $i = 1$ represents the S-R link whereas $i = 2$ represents the R-D link. m_1 and m_2 are the Nakagami-m parameters for the S-R and R-D channels respectively. In a typical 2-hop cooperative relaying environment with non-regenerative amplify and forward relays, the source transmits the information in a time slot $\frac{T}{2}$ and the relay amplifies and retransmits the same information in another time slot $\frac{T}{2}$. For the flat fading case the signal received by the destination node may be written as,

$$y(t) = A(t)\, h_1(t)\, h_2(t)\, x(t) + A(t)\, h_2(t)\, n_1(t) + n_2(t) \quad (2)$$

where, $x(t)$ is the transmitted signal, $A(t)$ is the gain of the relay, $n_1(t)$ and $n_2(t)$ are the additive noise at the relay and the destination nodes respectively.

It has been assumed that $A(t) = 1$ and $n_1(t) = 0$. Considering the assumption the received signal at the destination can be written as,

$$y(t) = h_1(t)\, h_2(t)\, x(t) + n_2(t) \quad (3)$$

The effective channel between the source and the destination node is basically the product of two random variables and can be written as $Z = R_1 \cdot R_2$. The density function of the random variable gives the channel statistics between the source and the destination via a relay. The *pdf* of can be written in terms of the *pdf*'s of R_1 and R_2 as [5][8][9].

$$f_Z(z) = \int_{-\infty}^{\infty} f_{R_1}(r_1) f_{R_2}\left(\frac{z}{r_1}\right) \frac{1}{|r_1|} dr_1 \quad (4)$$

If R_1 and R_2 are Nakagami-m distributed then the random variable Z can be written as,

$$f_Z(z) = \frac{4}{z\Gamma(m_1)\Gamma(m_2)} \left(\frac{z^2 m_1 m_2}{\Omega_1 \Omega_2}\right)^{\frac{m_1+m_2}{2}} \cdot K_{m_1-m_2}\left(2\sqrt{\frac{z^2 m_1 m_2}{\Omega_1\Omega_2}}\right) \quad (5)$$

Where, $K_\nu(\cdot)$ represents the modified Bessel function of second kind of order ν.

III. N-Branch Maximal Ratio Diversity Combining

Different combining techniques like selection combining, maximal ratio combining, equal gain combining, etc are applied at the destination node for improving the performance of the wireless link. In this paper maximal ratio combining of signals at the destination has been performed, using the statistics of the channels described in the earlier section. Fig. 3. shows the system model, containing N relays. Here it has been assumed that the destination receives the signal only through the relays. The source-destination link via the relays R_1, R_2,..., R_N are assumed to be independent. $f_{Z_1}(z_1), f_{Z_2}(z_2), ...f_{Z_N}(z_N)$ are independent and gives the statistics of the source-destination link via relay R_1, R_2,..., R_N respectively. The signal from the N relays are assumed to reach the destination in N orthogonal time slots, which are buffered for post processing.

The joint probability density function of N-independent relay channels is,

$$f_{Z_1 Z_2 .....Z_N}(z_1 z_2 .....z_N) = f_{Z_1}(z_1) \cdot f_{Z_2}(z_2) ....f_{Z_N}(z_N) \quad (6)$$

Combining eqn. 5 and 6 the probability density function of N-independent dual-hop Nakagami-m faded channel can be written as,

$$f_{Z_1 Z_2 .....Z_N}(z_1 z_2 .....z_N) = (4)^N \prod_{i=1}^{N} \frac{\left(\frac{{z_i}^2 m_{2i-1} m_{2i}}{\Omega_{2i-1}\Omega_{2i}}\right)^{\frac{m_{2i-1}+m_{2i}}{2}}}{z_i \Gamma(m_{2i-1})\Gamma(m_{2i})} \cdot K_{m_{2i-1}-m_{2i}}\left(2\sqrt{\frac{{z_i}^2 m_{2i-1} m_{2i}}{\Omega_{2i-1}\Omega_{2i}}}\right) \quad (7)$$

In a maximal ratio diversity system the output signal-to-noise ratio is equal to a weighted combination of the signal to noise ratio of all the channels. The voltage signal to noise ratio denoted by $SNR_{V_{MRC}}$ can be written in terms of the input signals of the diversity branches as,

$$SNR_{V_{MRC}} = \sqrt{z_1^2 + z_2^2 + ... + z_N^2} \quad (8)$$

In the above equation the noise powe has been considered to be unity.

The joint probability density function of independent relay channels is given in eqn. 7. It needs to be integrated to find

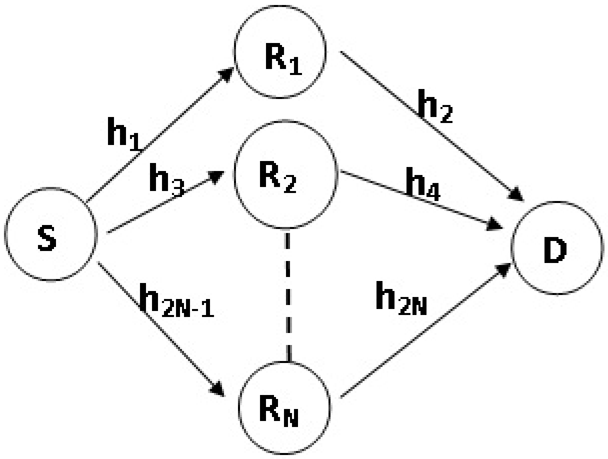

Fig. 2. System model containing N relays

the probability density function at the output of the maximal ratio combiner. The integral becomes too involved in the Cartesian co-ordinates. Changing to polar co-ordinates makes the computation more tractable. The transformation of the variables from the N dimensional cartesian to N dimensional polar co-ordinates can be written as,

$$\begin{aligned} & z_1 = m\cos(\phi_1) \\ & z_2 = m\sin(\phi_1)\cos(\phi_2) \\ & z_3 = m\sin(\phi_1)\sin(\phi_2)\cos(\phi_3) \\ & \vdots \\ & z_{N-1} = m\sin(\phi_1)\sin(\phi_2)\cdots\cos(\phi_{N-1}) \\ & z_N = m\sin(\phi_1)\sin(\phi_2)\cdots\sin(\phi_{N-2})\sin(\phi_{N-1}) \end{aligned} \tag{9}$$

As, Nakagami-m distribution is a positive distribution hence, $z_i > 0 \; for \; 1 \le i \le N \; so \; \phi_i \; for \, 1 \le i \le N-1$ lies between 0 and $\frac{\pi}{2}$.

The new probability density function can be deduced from eqn. 7, by change of variables and introduction of the Jacobian of the transformation. The new density function in the polar co-ordinates may be written as,

$$\begin{aligned} & f_{M\Phi_1\Phi_2\ldots\Phi_{N-1}}(m,\phi_1,\phi_2,\ldots\phi_{N-1}) = \\ & \qquad \left|\widetilde{J}\right| \cdot f_{Z_1Z_2\ldots Z_N}(z_1,z_2,\ldots z_N) \end{aligned} \tag{10}$$

Where, the Jacobian is given by,

$$\left|\widetilde{J}\right| = \begin{vmatrix} \frac{\partial z_1}{\partial m} & \frac{\partial z_1}{\partial \phi_1} & \cdots & \frac{\partial z_1}{\partial \phi_{N-1}} \\ \frac{\partial z_2}{\partial m} & \frac{\partial z_2}{\partial \phi_1} & \cdots & \frac{\partial z_2}{\partial \phi_{N-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial z_N}{\partial m} & \frac{\partial z_N}{\partial \phi_1} & \cdots & \frac{\partial z_N}{\partial \phi_{N-1}} \end{vmatrix} \tag{11}$$

Therefore eqn. 10 can be written as,

$$\begin{aligned} & f_{M\Phi_1\Phi_2\ldots\Phi_{N-1}}(m,\phi_1,\phi_2,\ldots\phi_{N-1}) = \\ & \left|\widetilde{J}\right| \cdot \left. f_{Z_1Z_2\ldots Z_N}(z_1,z_2,\ldots z_N) \right|_{\substack{z_1 = m\cos(\phi_1) \\ \vdots \\ z_N = m\sin(\phi_1)\cdots\sin(\phi_{N-1})}} \end{aligned} \tag{12}$$

As all the angles lie between 0 and $\frac{\pi}{2}$, so eqn. 12 can be written as,

$$\begin{aligned} f_M(m) = \int_0^{\pi/2} \cdots \int_0^{\pi/2} & f_{M\Phi_1\ldots\Phi_{N-1}}(m,\phi_1,\ldots\phi_{N-1}) \cdot \\ & d\phi_1 \ldots d\phi_{N-1} \end{aligned} \tag{13}$$

The above equation gives the density function at the output of the maximal ratio combiner.

A. *Example: A three branch relay diversity system*

For a three branch case the joint density function is given as,

$$f_{Z_1Z_2Z_3}(z_1z_2z_3) = f_{Z_1}(z_1)\, f_{Z_2}(z_2)\, f_{Z_3}(z_3) \tag{14}$$

Combining eqn. 7 and 10,

$$\begin{aligned} & f_{Z_1Z_2Z_3}(z_1z_2z_3) = \\ & \frac{4}{z_1\Gamma(m_1)\Gamma(m_2)}\left(\frac{{z_1}^2 m_1 m_2}{\Omega_1\Omega_2}\right)^{\frac{m_1+m_2}{2}} K_{m_1-m_2}\left(2\sqrt{\frac{{z_1}^2 m_1 m_2}{\Omega_1\Omega_2}}\right) \\ & \cdot \frac{4}{z_2\Gamma(m_3)\Gamma(m_4)}\left(\frac{{z_2}^2 m_3 m_4}{\Omega_3\Omega_4}\right)^{\frac{m_3+m_4}{2}} K_{m_3-m_4}\left(2\sqrt{\frac{{z_2}^2 m_3 m_4}{\Omega_3\Omega_4}}\right) \\ & \cdot \frac{4}{z_3\Gamma(m_5)\Gamma(m_6)}\left(\frac{{z_3}^2 m_5 m_6}{\Omega_5\Omega_6}\right)^{\frac{m_5+m_6}{2}} K_{m_5-m_6}\left(2\sqrt{\frac{{z_3}^2 m_5 m_6}{\Omega_5\Omega_6}}\right) \end{aligned} \tag{15}$$

Or,

$$\begin{aligned} f_{Z_1Z_2Z_3}(z_1z_2z_3) = 64 \prod_{i=1}^{3} & \frac{\left(\frac{{z_i}^2 m_{2i-1} m_{2i}}{\Omega_{2i-1}\Omega_{2i}}\right)^{\frac{m_{2i-1}+m_{2i}}{2}}}{z_i\Gamma(m_{2i-1})\Gamma(m_{2i})} \\ & \cdot K_{m_{2i-1}-m_{2i}}\left(2\sqrt{\frac{{z_i}^2 m_{2i-1} m_{2i}}{\Omega_{2i-1}\Omega_{2i}}}\right) \end{aligned} \tag{16}$$

From eqn. 9 and 12, the distribution of the envelope at the output of the maximal ratio combiner can be written as,

$$f_{M\Theta\Phi}(m,\theta,\phi) = \left|\tilde{J}\right| \cdot f_{Z_1Z_2Z_3}(z_1z_2z_3) \tag{17}$$

where,

$$\begin{aligned} & z_1 = m\cos(\phi)\cos(\theta) \\ & z_2 = m\sin(\phi) \\ & z_3 = m\cos(\phi)\sin(\theta) \end{aligned} \tag{18}$$

and,

$$\left|\widetilde{J}\right| = \begin{vmatrix} \frac{\partial z_1}{\partial m} & \frac{\partial z_1}{\partial \theta} & \frac{\partial z_1}{\partial \phi} \\ \frac{\partial z_2}{\partial m} & \frac{\partial z_2}{\partial \theta} & \frac{\partial z_2}{\partial \phi} \\ \frac{\partial z_3}{\partial m} & \frac{\partial z_3}{\partial \theta} & \frac{\partial z_3}{\partial \phi} \end{vmatrix} \tag{19}$$

Thus,

$$f_M(m) = \int_0^{\pi/2} \int_0^{\pi/2} f_{M\Theta\Phi}(m,\theta,\phi)\, d\theta\, d\phi \tag{20}$$

We know,

$$f_{M\Theta\Phi}(m,\theta,\phi) = \left|\tilde{J}\right| \cdot f_{Z_1 Z_2 Z_3}(z_1 z_2 z_3)\Big|_{\substack{z_1 = m\cos(\phi)\cos(\theta) \\ z_2 = m\sin(\phi) \\ z_3 = m\cos(\phi)\sin(\theta)}} \tag{21}$$

Inserting the value of the joint density function from eqn. 16,

$$f_M(m) = \left|\tilde{J}\right| \cdot \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \frac{64}{\prod_{i=1}^{3} z_i \Gamma(m_{2i-1})\Gamma(m_{2i})} \cdot \prod_{i=1}^{3} \left(\frac{z_i^2 m_{2i-1} m_{2i}}{\Omega_{2i-1}\Omega_{2i}} \right)^{\frac{m_{2i-1}+m_{2i}}{2}} \cdot K_{m_{2i-1}-m_{2i}} \left(2\sqrt{\frac{z_i^2 m_{2i-1} m_{2i}}{\Omega_{2i-1}\Omega_{2i}}} \right) \Bigg|_{\substack{z_1 = m\cos(\varphi_1) \\ z_2 = m\sin(\varphi_1)\cos(\varphi_2) \\ z_2 = m\sin(\varphi_1)\sin(\varphi_2)}} d\varphi_1\, d\varphi_2 \tag{22}$$

On simplification we obtain,

$$f_M(m) = m^2 \frac{64}{\prod_{i=1}^{3} \Gamma(m_{2i-1})\Gamma(m_{2i})} \prod_{i=1}^{3} \left(\frac{m_{2i-1} m_{2i}}{\Omega_{2i-1}\Omega_{2i}} \right)^{\frac{m_{2i-1}+m_{2i}}{2}} \cdot \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(\varphi_1) \prod_{i=1}^{3} (z_i)^{m_{2i-1}+m_{2i}-1} \cdot K_{m_{2i-1}-m_{2i}} \left(2\sqrt{\frac{z_i^2 m_{2i-1} m_{2i}}{\Omega_{2i-1}\Omega_{2i}}} \right) \Bigg|_{\substack{z_1 = m\cos(\varphi_1) \\ z_2 = m\sin(\varphi_1)\cos(\varphi_2) \\ z_2 = m\sin(\varphi_1)\sin(\varphi_2)}} d\varphi_1\, d\varphi_2 \tag{23}$$

The density function at the output of the maximal ratio combiner for a three branch diversity obtained through simulation and analytically from eqn. 23 has been plotted in Fig. 3 and Fig 4. For obtaining Fig. 3 the m-parameters of all the six links, of the three branches, were set to unity. $\Omega_1, \Omega_2, \Omega_3, \Omega_4, \Omega_5$ and Ω_6 were taken to be 2. For, Fig. 4 the m parameter of one of the link was changed to 2 whereas for the other links it was kept at unity. Ω for the link with m value of 2 was set at 4 whereas for the other links the value of Ω were kept at 2. The simulation studies were carried out in MATLAB. The Nakagami-m distributed random variables were generated following the procedure given in [7]. Maximal ratio combining was performed at the receiver. For Fig. 3 and Fig. 4 it can be observed that the analytical formulation matches well with the simulation results.

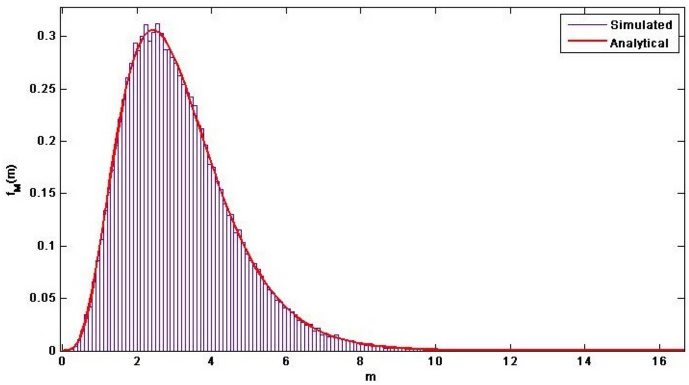

Fig. 3. Density function at the output of the maximal ratio combiner for a three diversity branch system with $m_i = 1$ and $\Omega_i = 2$ for i = 1 to 6

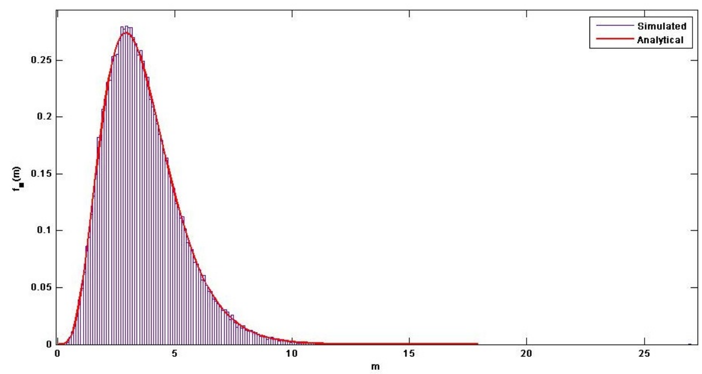

Fig. 4. Density function at the output of the maximal ratio combiner for a three diversity branch system with $m_1 = 2$, $\Omega_1 = 4$, $m_i = 1$ and $\Omega_i = 2$ for i = 2 to 6

IV. Conclusion

In this paper the signal envelope at the output of a maximal ratio combiner, having signals from N independent dual hop relay channels as inputs has been presented. The analytical results were compared with those obtained through simulation. The validity of the analysis were verified with a three branch diversity case for different values of the Nakagami-m parameter of the links.

References

[1] A. Sendonaris, E. Erkip and B. Aazhang, User cooperation diversity-part-I: system description, IEEE Transactions on Communications, vol. 51, no. 11, November 2003.

[2] A. Sendonaris, E. Erkip and B. Aazhang, User cooperation diversity-part-II: implementation aspects and performance analysis, IEEE Transactions on Communications, vol. 51, no. 11, November 2003.

[3] C. S. Patel, G. L. Stuber and T. G. Pratt, Statistical properties of amplify and forward relay fading channels, IEEE Transactions on Vehicular Technology, vol. 55, no. 1, January 2006.

[4] M. Nakagami, The m-distributiona general formula of intensity distribution of rapid fading, in Statistical Methods in Radio WavePropagation. Oxford, U.K.: Pergamon, 1960, pp. 336.

[5] I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrals,Series, and Products, 5thed., San Diego, CA: Academic, 1994.

[6] N. C. Beaulieu, Efficient nakagami-m fading channel simulation, IEEE Transactions on Vehicular Technology, vol. 54, no. 2, March 2005.

[7] K. Dietze, Analysis of a Two-Branch Maximal Ratio and Selection Diversity System with Unequal Branch Powers and Correlated Inputs for a Rayleigh Fading Channel ,MS Thesis, Virginia Polytechnic Institute and State University, 2001.

[8] B. S. Paul and R. Bhattacharjee, Selection combining of two amplify-forward relay branches with individual links experiencing Nakagami fading, IEEE APCC 2007, Bangkok

[9] B. S. Paul, Performance Analysis of Selection Combining In N Dual-hop Relay Branches with Individual Links Experiencing Nakagami-m Fading, CCN 2012, New Delhi, India.