Small World Network Based Dynamic Topology for Particle Swarm Optimization

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Abstract— **A new particle optimization algorithm with dynamic topology is proposed based on 'small world' network. The technique imitates the dissemination of information in a 'small world network' by dynamically updating the neighborhood topology of particle swarm optimization. The proposed dynamic neighborhood strategy can effectively coordinate the exploration and exploitation ability of particle swarm optimization. Simulations demonstrated that convergence of the swarms is guaranteed. Experiments demonstrated that the proposed method maintained the population diversity and enhanced the global search ability.**

Keywords-particle swarm; small world network; neighborhood topology; global model; local model

I. INTRODUCTION

Particle Swarm Optimization, inspired by the behavior of birds flocking and fish schooling, is one form of an artificial intelligence algorithms for optimizing hard numerical functions. In a particle swarm optimizer, each individual (called particle) which represents a potential solution to the optimization problem, flies to the optimal region of the multidimensional space by adjusting its flying trajectory [1][2][3][4][12]. Particle Swarm Optimization technique has been successfully applied in many science and engineering areas such as pattern recognition, signal processing, robot control, data clustering and so forth.

In the particle swarm optimization, every particle has some number of neighbors around itself, which has an effect on each other [7]. Neighborhood topology reflects the mode of sharing information among particles, so the topology plays a very important role in the performance of the algorithm. In the early stages of particle swarm optimization research, investigators only used one kind of topology which is called the 'global best version' ('gbest') [1][2]. In [5][6][7] [8] Kennedy and Mendes proposed the 'local best version' ('lbest') as a kind of topology to deal with more complex engineering problems. They also have investigated the effects of various neighborhoods topologies in the performance of particle swarm algorithms, such as a ring topology, a wheel topology, a von Neumann topology, and so on. Suganthan proposed a number of improvements such as gradually increasing the local neighborhood, time varying random walk and inertia weight values and two alternative schemes for determining the local optimal solution for every individual [9]. However, this method adds additional time consumption when calculating the distance, thus it increase the complexity of the algorithm.

In 1969, Travers and Milgram published 'Six Degrees of Separation' the phenomenon [14][21]. In [10], Watts and Strogatz had proposed the concept of classic 'small world' network model and its construction method. They have shown that the characteristics of the network connections can influence the velocity of the information flowing [6][11]. 'Small world' networks have since been observed in many real-world problems, such as data clustering, optimization of oil and gas field development planning, linear programming, reactive power optimization, computer science, networks of brain neurons, telecommunications, mechanics, and social influence networks [17][18][19][20]. Since the particle swarm optimization was inspired by nature, this paper postulates that it is possible to enhance optimization performance if the 'small world' network topology is used as part of the particle swarm optimization process. A Dynamic Topology Particle Swarm Optimization based on 'Small World' network (DTSWPSO), which imitates information dissemination 'small world' networks by dynamically adjusting the neighborhood topology, is proposed. Moreover, a varying neighborhood strategy can effectively coordinate the exploration and exploitation ability of the algorithm. The DTSWPSO is compared with the classic topology version, using the dynamic 'small world' neighborhood structure.

The rest of the paper is organized as follows: In Section 2 and Section 3, the neighborhood topology and 'small world' network are described. Section 4 discusses the method and in Section 5 the comparative numerical simulation results are given, which are also discussed and analyzed. Finally, the concluding remarks are given in Section 6.

II. NEIGHBORHOOD TOPOLOGY

In the particle swarm optimization system, each individual defines its trajectory according to its previous best solution and the optimal solution of some neighbors [6][7][13]. Each individual selects the success of its neighbors to be a source of influence and ignores the others. So the neighborhood topology structure of the particle swarm plays important role in the optimization performance, and the size of the neighborhood directly affects the performance of the algorithm.

At present, particle swarm optimization has been studied in two general types of neighborhood structures, one is global best (called 'gbest') , the other is local best (called 'lbest') [7][13]. In Fig.1(a), the 'gbest' neighborhood which is also known as 'all' topology, any two individuals in the entire community are connected, this neighborhood topology structure is equivalent to a fully connected topology in which each particle is attracted to the best position found by all the other members, so the all members of the whole can share information, and each individual is able to choose a new point to test according to the best success of the entire population [5]. However, in the local best network, each particle is allowed to be influenced by some smaller numbers of adjacent members of the topological members. Classic 'lbest' neighborhood topology is a ring lattice as shown in Fig.1(b). Kennedy and Mendes constructed and tested several typical neighborhood configurations, for instance, the 'Pyramid' neighborhood (Fig.1(c)) and the square neighborhood (Fig.1(d)) [6]. The authors discovered that particle swarm optimization with the square neighborhood topology performed better than other ones with other topologies structure including the standard particle swarm optimization on a suite of standard test functions which are the Sphere function, Rstrigin function, Griewank function, Rosenbrock function, and Shaffer's f6 function [7].

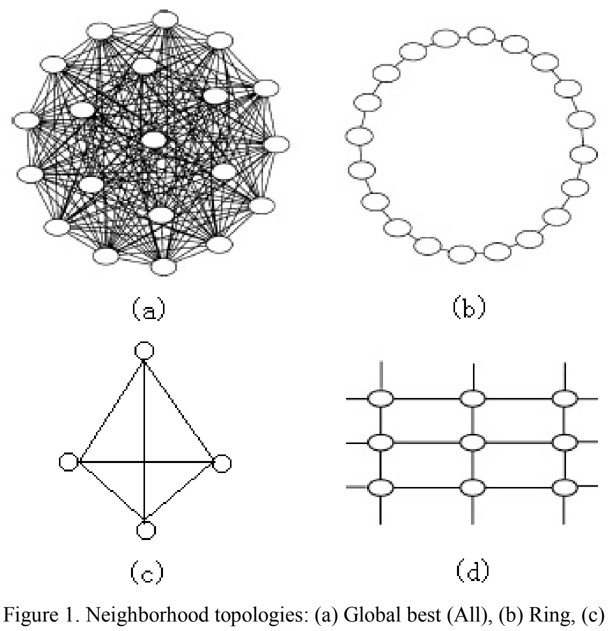

Figure 1. Neighborhood topologies: (a) Global best (All), (b) Ring, (c) Pyramid, (d) Square

III. SMALL WORLD NETWORK

In the 1960's, Milgram proposed the theory of 'six degrees of separation' [14]. 30 years later, after the study of the 'small world' network by Watts and Strogetz [10], the 'small world' phenomenon was gradually noticed and quickly became a hot research topic in complex system and complexity theory. 'Small world' network is based upon relationships in human society and is an intermediate network form between a regular and a random network. In [10], Watts and Strogetz drew a main conclusion that 'small world' network has small characteristic path length of the random lattice and relative highly clustering coefficient of the regular lattice. In limited cases, if $p = 0$, the original is regular network, if $p = 1$, it becomes the 'gbest' topology graph. Four realization processes of 'small world' networks are shown in Fig. 2 [10][11][14]. The construction of a 'small world' network is as follows [10][11]:

1) Start with a ring structure of n vertexes as shown in Fig. 2(a).

2) Each vertex is connected to its nearest k neighbors by k undirected edges, in Fig. 2(b), $k = 4$.

3) Choose a vertex in a clockwise direction. With probability p, connect this vertex to the other one chosen uniformly at random over the entire ring for k times. With duplicate edges forbidden, otherwise rechoose another one (Fig. 2(c)).

4) Repeat the process 3) by moving clockwise around the ring, considering each vertex in turn until one lap is completed.

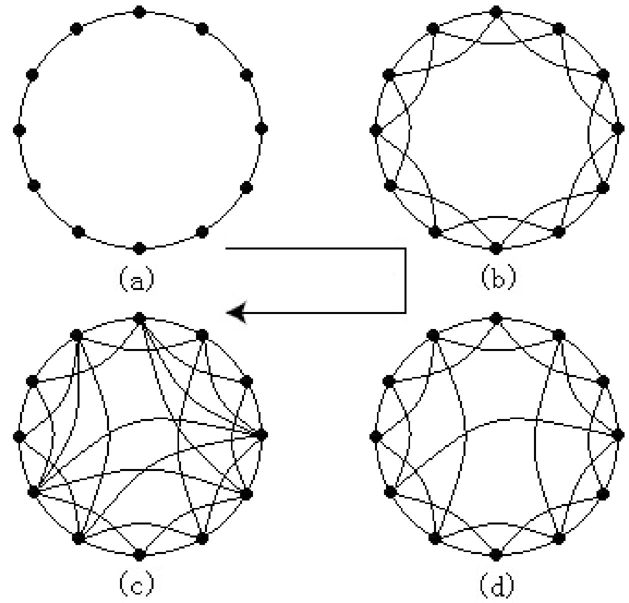

Figure 2. Construction process of 'small world' network with p probability

IV. METHOD

In particle swarm optimization, each particle will change its velocity and direction according to the nodes (called neighborhoods) connected to it and decide which particle of its neighborhood has got the optimal solution so far [5]. Therefore, the neighborhood topology structure of population determines the breadth and extent of influences on the member, and the quantity of the particle's neighbors determines the speed of information dissemination.

Kennedy and Mendes [6] suggested that the 'gbest' neighborhood topology converges fast because of the most immediate communication possible, and it has a strong global search ability, yet it is easy to fall into a local optimum. On the other hand, in the particle swarm optimization with the ‘lbest’ neighborhood topology, each particle explores its respective search space, so information disseminates slowly along the neighborhood topology, and the information of success takes a long time to spread throughout the entire population [13]. So it has a strong local search ability, it also maintains the diversity of the population to a certain extent, and it isn’t easy to trap in the local optima.

According to the characteristics of the ‘small world’ [5][10][11], the randomness of construction process ensures the diversity of population, connections in an otherwise orderly network should ensure the propagation of information throughout the entire neighbors.

From the above discussion, it is not hard to imagine that if we introduce some characteristics of ‘small world’ network to the particle swarm optimization as the neighborhood topology, that the algorithm will have improved global searching ability and fast convergence speed. In addition, due to the randomness of ‘small world’ network in the construction process, it keeps the population divers. In order to balance the exploration and exploitation abilities of the particle swarm optimization algorithm, a ‘Small World’ network based Dynamic Topology for PSO (DTSWPSO) is proposed, the neighborhood topology of this algorithm changes gradually by adjusting the probability *p* with every iteration. The specific adjustment procedure of this dynamic topology is shown in Table 1. From the data in Table 1, the parameter The parameter ‘*max_iteration’* represents the maximum number of iterations. The probability *p* was decreased linearly with each iteration increasing. In the early stage of iteration, the neighborhood topology is gradually decreased, so that at last, it becomes the ‘lgbest’ topology structure (i.e. a ‘ring’ topology).

Table 1. The value of *p* on different stage of evolution

Iteration numbers	**The value of *p***
[1, max_iteration/10]	0.9
[(max_iteration/10) + 1, 2(max_iteration/10)]	0.8
...	...
[8(max_iteration/10) + 1, 9(max_iteration/10)]	0.1
[9(max_iteration/10) + 1, 1]	0

In order to test the optimization performance of the proposed technique, we compared it with the other three typical topologies which are respectively ‘gbest’, ‘ring’, and ‘Von Neumann’. Five famous benchmark optimization problems were used in this research. These test functions were Sphere function, Rastrigin function, Griewank function, Rosebrock function and Schaffer’s f6 function. In these test functions, sphere function, Rastrigin function and Rosebrock function were run in 30 dimensions, 10 and 30 dimensions were considered for Griewank function, Shaffer’s f6 was a 2 dimensions function. The parameters and criteria of the five standard test functions are given in Table 2.

Since the three topologies are standard, we haven’t described these here [16]. The three typical topologies for particle swarm optimization algorithms and the proposed algorithm were applied with a population size of 20, and the maximum number of iterations was 1000, each algorithm was run 20 times. The position and the velocity of each particle was initialized with a random value, their iterative formulas are as follows:

$$V_{id}^{t+1} = \omega V_{id}^{t} + c_1 r_1 (p_{id} - X_{id}^{t}) + c_2 r_2 (p_{ld} - X_{id}^{t}) \quad (1)$$

$$X_{id}^{t+1} = X_{id}^{t} + V_{id}^{t+1} \quad (2)$$

Here, V_{id} and X_{id} represent the velocity vector and the position vector of particle *i* in *d* dimension. The index *t* is the iteration number. P_i is the best position of particle *i* found by itself in the search space. P_l refers to the best position of the particle found so far by any member of its neighborhood. The inertia weight [12] $\omega = 0.729$, the two positive constants $c_1 = c_2 = 1.49445$, r_1 and r_2 are two random weights in the range [0, 1].

Table 2. Parameters and criteria for the five test functions conditions

Function	**Dimensions**	**Initial range**	**Criterion**
Sphere	30	[-100, 100]	0.01
Rastrigin	30	[-5.12, 5.12]	100
Griewank	10	[-600, 600]	0.05
Griewank	30	[-600, 600]	0.05
Rosenbrock	30	[-30, 30]	100
Schaffer’s f6	2	[-100, 100]	0.00001

V. RESULTS AND DISCUSSION

Table 3 shows a comparison of algorithms on four neighborhood topologies. As we can see, the solutions obtained by the proposed method (DTSWPSO) were better than for the other three algorithms previously reported. However, the DTSWPSO is computationally more expensive than the other three algorithms as the adjacency matrix of population needs to be dynamically adjusted in the searching process. For Sphere function, Griewank function, and Schaffer’s f6 functions, all four algorithms found the optimal solution. The “ring” topology is better than the other three algorithms for Rosenbrock function. As Rosenbrock function is a typical sick function, there is a narrow valley between the local optimum and the global optimum. It is difficult to distinguish the global optimum for the algorithm which has larger neighborhood size. The “ring” and the “Von Neumann” topologies obviously show good performance because of their smaller neighborhood sizes. However, the DTSWPSO algorithm has a relatively large neighborhood in the early searching stage, its performance was unsatisfactory which confirms no free lunch theorem is correct to some extent. In order to further compare four algorithms, we draw their evolutionary curves respectively on each test function which was shown in Fig.3-8. From Fig.4-8, it can be easily seen that the DTSWPSO algorithm converges faster than the other three algorithms. Because the Sphere function is a unimodal function, from Fig.3, it is easy to find that the optimization solutions with four different algorithms, furthermore, the more simple a topology structure is, the faster it converges.

As can be seen from the parameter of 'Computer Time' in Table 3, there was a significant effect for the 'gbest' topologies. In fact, it is obvious the running time of the 'lbest' version is longer than for the 'gbest' version. With enough time, it can also find the optimal solution, furthermore has a strong global searching ability. In the earlier searching stage, the value of p is large, and the neighborhood population is relatively large, the characteristic approaches the 'all' topology, so our proposed algorithm has a high searching speed. Moreover, due to the randomness and the rapid information dissemination ability, the proposed method maintained the diversity of the population.

VI. CONCLUSION

This paper proposed a new dynamic neighborhood topology structure particle swarm optimization algorithms. According to the way a 'small world' network is generated, the local neighborhood topology decreased gradually by adjusting the probability p with increasing iterations. The simulation results of five typical test functions demonstrated that the new method can maintain the diversity of population, balance the exploration and exploitation ability, and guarantee the convergence of the particle swarm searching. Consequently, the proposed technique improved the practicality and effectiveness of PSO. Because of the randomness of the 'small world' network, the dynamic neighborhood topologies should have many adjustment modes, which will be studied in future work.

Table 3. Comparison of algorithms on four neighborhood topologies

Function (Dimensions)	Topology	Best	Mean	Std	Worst	Computation Time（s）
Sphere (30)	'gbest'(All)	**1.247314×10^{-13}**	1.258973×10^{-7}	5.445516×10^{-7}	2.499484×10^{-6}	**31.817000**
	Ring	5.617827×10^{-7}	3.015519×10^{-6}	2.298009×10^{-6}	1.018020×10^{-5}	46.306000
	Von Neumann	2.118575×10^{-9}	1.1710336×10^{-8}	9.959108×10^{-9}	3.336135×10^{-8}	51.519000
	DTSWPSO	1.092860×10^{-10}	**1.577930×10^{-9}**	**1.569352×10^{-9}**	**6.135994×10^{-9}**	55.813000
Rastrigin (30)	'gbest'(All)	4.875293×10^{1}	8.572606×10^{1}	2.441177×10^{1}	1.363797×10^{2}	**38.441000**
	Ring	3.588527×10^{1}	8.112883×10^{1}	2.448405×10^{1}	1.345413×10^{2}	53.541000
	Von Neumann	4.472569×10^{1}	7.779298×10^{1}	**1.714094×10^{1}**	**1.115588×10^{2}**	54.459000
	DTSWPSO	**3.084369×10^{1}**	**7.552781×10^{1}**	2.023088×10^{1}	1.284203×10^{2}	60.595000
Griewank (10)	'gbest'(All)	1.477978×10^{-2}	1.061901×10^{-1}	5.776070×10^{-2}	2.606520×10^{-1}	**36.274000**
	Ring	1.969000×10^{-2}	**4.859381×10^{-2}**	2.537146×10^{-2}	1.106998×10^{-1}	50.826000
	Von Neumann	1.477240×10^{-2}	5.699827×10^{-2}	3.245313×10^{-2}	1.278158×10^{-1}	58.610000
	DTSWPSO	**1.477240×10^{-2}**	6.814345×10^{-2}	**2.292590×10^{-2}**	**1.081724×10^{-1}**	61.516000
Griewank (30)	'gbest'(All)	**1.255592×10^{-10}**	5.876083×10^{-2}	8.177025×10^{-2}	2.917093×10^{-1}	**37.799000**
	Ring	1.114118×10^{-6}	1.004084×10^{-2}	1.638711×10^{-2}	5.399861×10^{-2}	47.679000
	Von Neumann	8.58439010^{-9}	1.460001×10^{-2}	1.801622×10^{-2}	7.306310×10^{-2}	60.199000
	DTSWPSO	1.348202×10^{-10}	**8.490838×10^{-3}**	**9.694268×10^{-3}**	**3.438406×10^{-2}**	64.667000
Rosenbrock (30)	'gbest'(All)	6.778356	9.229869×10^{3}	2.695696×10^{4}	9.007947×10^{4}	**41.552000**
	Ring	9.234692	**7.8886625×10^{1}**	**5.931295×10^{1}**	**2.944472×10^{2}**	56.582000
	Von Neumann	**5.568997**	3.376229×10^{2}	9.084359×10^{2}	3.082412×10^{3}	59.697000
	DTSWPSO	10.095813	1.181437×10^{2}	1.615559×10^{2}	5.762526×10^{2}	50.886000
Schaffer's f6 (2)	'gbest'(All)	0	6.801136×10^{-3}	4.452389×10^{-3}	9.715909×10^{-3}	**33.517000**
	Ring	0	6.814980×10^{-3}	**4.431603×10^{-3}**	9.715909×10^{-3}	40.121000
	Von Neumann	0	4.857954×10^{-3}	4.857954×10^{-3}	9.715909×10^{-3}	46.545000
	DTSWPSO	**0**	**4.114885**$\times10^{-3}$	4.642313×10^{-3}	**9.715909×10^{-3}**	47.450000

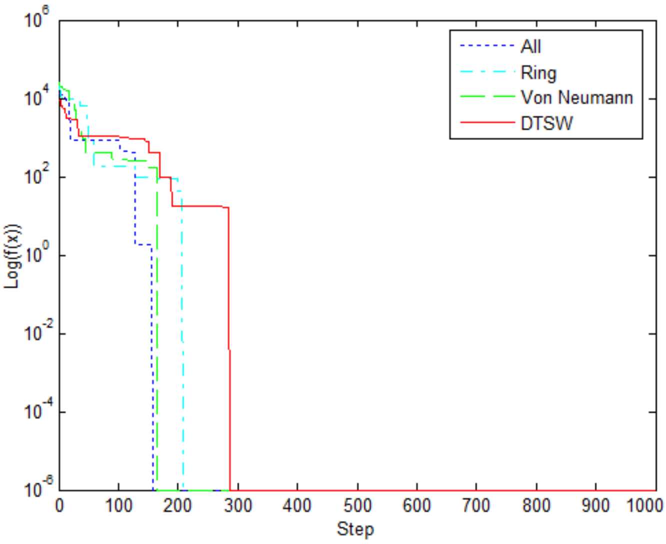

Figure 3. Time evaluation of Sphere function

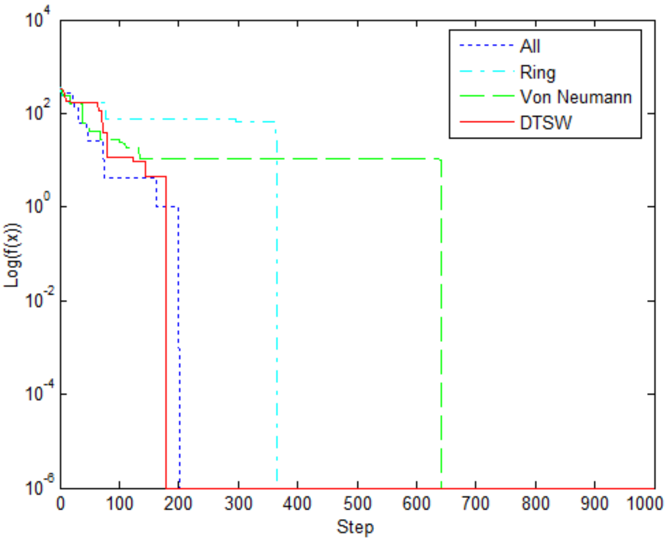

Figure 4. Time evaluation of Rastrigin function

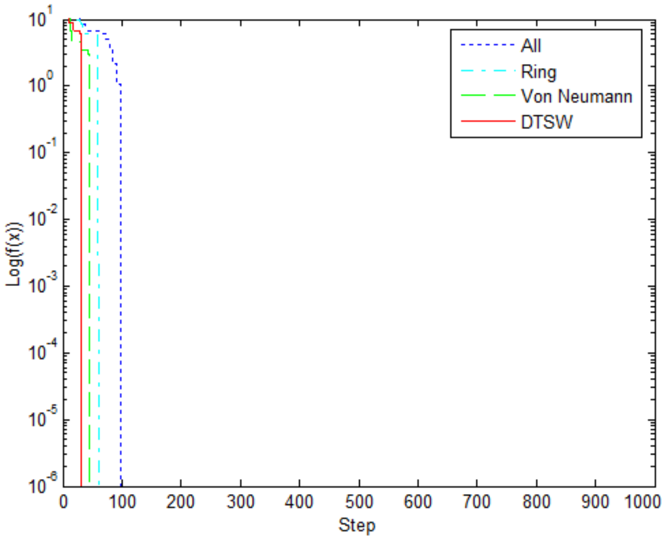

Figure 5. Time evaluation of Griewank function(10 demisions)

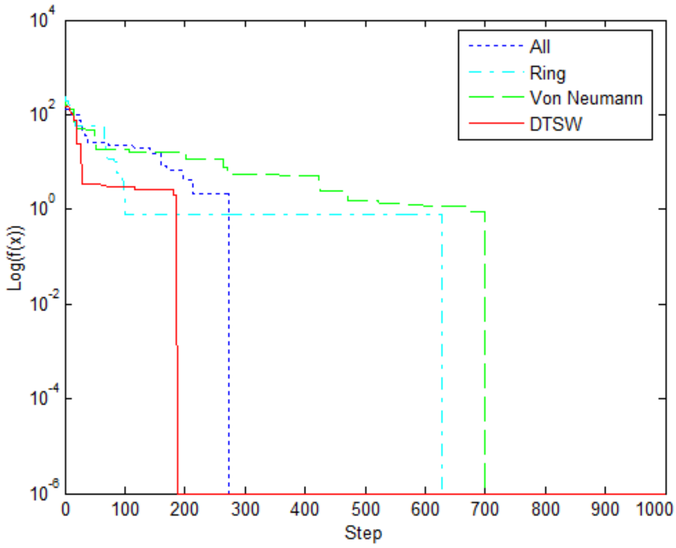

Figure 6. Time evaluation of Griewank function(30 demisions)

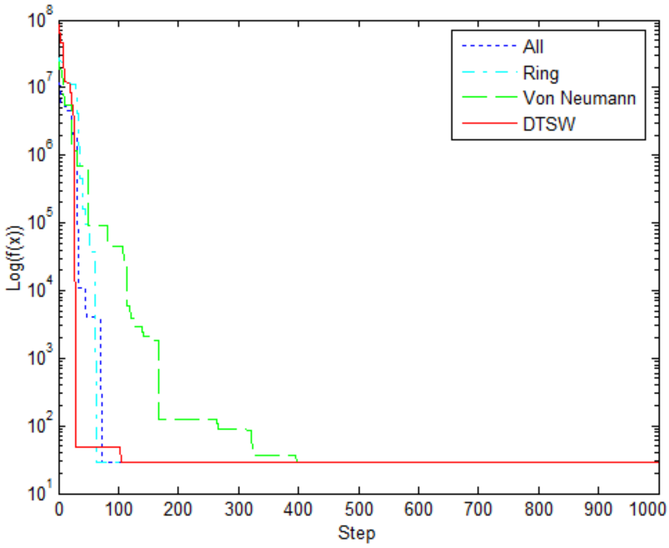

Figure 7. Time evaluation of Rosenbrock function

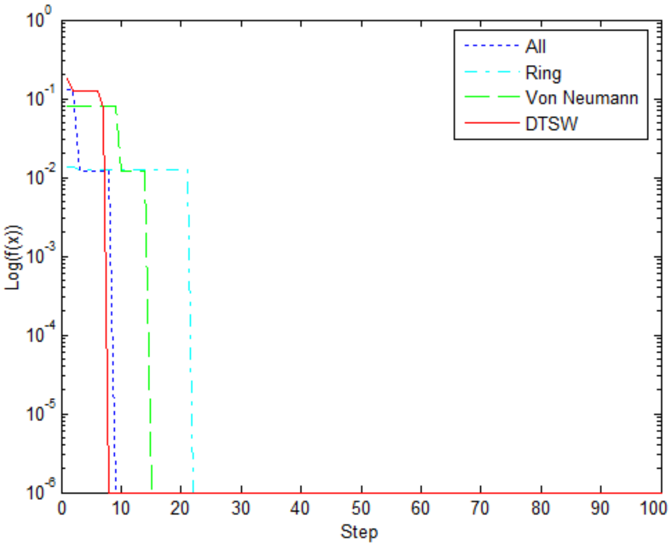

Figure 8. Time evaluation of Schaffer's f6 function

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