# Heat transfer and pressure drop experimental correlations for air–water bubbly flow

Wenzhi Cui [a],*, Longjian Li [a], Qinghua Chen [b], Quan Liao [b], Tien-Chien Jen [b]

[a] *College of Power Engineering, Chongqing University, Chongqing 400044, China*
[b] *Department of Mechanical Engineering, University of Wisconsin-Milwaukee, Milwaukee, WI 53211, USA*

## Abstract

In this paper, a novel air–water bubbly flow heat transfer experiment is performed to investigate the characteristics of pressure drop of airflow and heat transfer between water and tubes for its potential application in evaporative cooling. The attempts to reduce the pressure drop while maintaining higher heat transfer coefficient have been achieved by decreasing the bubble layer thickness through the water pump circulation. Pressure drops of air passing through the sieve plate and the bubbling layer are measured for different height of bubble layer, hole–plate area ratio of the sieve plate and the superficial air velocity. Experimental data show that the increase of bubble layer height and air velocity both increase the pressure drop while the effect of the hole–plate area ratio of the sieve plate on the heat transfer coefficient is relatively sophisticated. A pressure drop correlation including the effects of all the tested parameters is proposed, which has a mean absolute deviation of 14.5% to that of the experimental data. Heat transfer coefficients of the water and the outside tube wall are measured and the effects of superficial air velocity, heat flux and bubble layer height are also examined. Through a dimensional analysis, a heat transfer correlation with a mean absolute deviation of 9.7% is obtained based on experimental data.
© 2006 Elsevier Ltd. All rights reserved.

*Keywords:* Bubbly flow; Gas–liquid flow; Heat transfer; Pressure drop

## 1. Introduction

Evaporative coolers or condensers have been extensively used in practical industries, such as refrigeration and chemical engineering. The conventional design for evaporative cooler or condenser employs water spraying onto the heat transfer tube bundle with air blowing over it. In most cases the convection thermal resistance of the water film outside the tubes is the dominant thermal resistance in the heat transfer process. To reduce the convection heat resistance, the present authors have proposed a uniquely designed air–water bubbly flow heat exchange mechanism with heat transfer tube bundle submerged into the bubble or foam layer.

Bubbly flows can be found in a number of technical and industrial processes, such as effluent treatment, flotation processes, and bubble column reactors [1–4]. Most researches, including experimental and modeling studies, such as [5–8], are focused on the hydrodynamic behaviors of the bubbly flow in bubble column. Only a few papers deal with the heat transfer performance utilizing bubbly flow, which will be reviewed here.

The use of air–water bubbly flow as gas–liquid contacting evaporative cooling process was first investigated by Mizushina and Miyashita [9] decades ago. The heat transfer tube bundle was submerged into a bubbling pool, heat and mass transfer were greatly enhanced compared with that in single-phase convection due to the local turbulence causes by the rising bubble. Their experimental data showed that heat transfer coefficient between cooling water and tubes was much higher than that of the falling film flow

* Corresponding author. Tel.: +86 23 6511 18677.
  *E-mail address:* wzcui@cqu.edu.cn (W. Cui).

**Nomenclature**

| | | | |
|---|---|---|---|
| $A_h$ | total area of the plate holes, $m^2$ | $W$ | flow rate, $kg\,h^{-1}$ |
| $\bar{t}_{wall}$ | average tube wall temperature, °C | $\Delta P$ | pressure drop, Pa |
| $\bar{t}_{water}$ | average water temperature, °C | | |
| $D$ | outer diameter of heating tube, m | *Greek symbols* | |
| $h$ | heat transfer coefficient, $kW/m^2\,K$ | $\beta$ | hole–plate area ratio |
| $H$ | height of the bubbling layer, mm | $\lambda$ | heat conductivity, $W\,m^{-1}\,K^{-1}$ |
| $Nu$ | Nusselt number, $hD/\lambda$ | $v$ | kinematic viscosity, $m^2\,s^{-1}$ |
| $q$ | heat flux, $kW/m^2$ | | |
| $Re$ | Reynolds number, $u_s d_p/v$ | *Subscripts* | |
| $S$ | sieve plate area, $m^2$ | A | dry air |
| $u_f$ | frontal air velocity of the sieve plate, $m\,s^{-1}$ | exp | experimental data |
| $u_s$ | superficial air velocity, $m\,s^{-1}$ | p | dry plate |
| $V$ | airflow rate, $m^3\,s^{-1}$ | pre | prediction by correlation |

on the horizontal tubes. Although the heat transfer enhancement is significant, the pressure drop of air through the bubbling pool is also quite high due to the considerable depth of the pool. This had made such kind of device not feasible or at least uneconomical in practical evaporative cooling application since additional pump work would be required.

Tong et al. [10] experimentally investigated the water-cooling processes using a similar setup with Mizushina and Miyashita. Their results showed that the circulating water could be cooled down to a temperature below the ambient temperature through bubbling means. The heating tubes used in their setup were merely to keep the initial temperature of the water. They only dealt with heat and mass transfer between water and air. The heat transfer between water and heating surfaces was not included in their studies.

Li et al. [11,12] studied the heat transfer between an immersed vertical and horizontal tube and the two-phase flow in a bubble column. Effects of the superficial gas velocity, liquid viscosity and surface tension on the heat transfer coefficient were examined. They found that the heat transfer of both horizontal and vertical tubes could be calculated by the same correlation, i.e., the orientation of the tube had no effect on the heat transfer, which is highly skeptical.

Although the purposes of the above works were different, they all dealt with the heat transfer surfaces submerged into a water pool. In order to keep relatively high gas fraction of the bubbly flow, high blowing fan energy should be supplied. So far, no research on the heat transfer of low liquid or bubbly layer is found in the open literatures. In this study a novel air–water bubbly flow heat transfer experiment is performed to investigate the characteristics of pressure drop of airflow and heat transfer between water and tubes for its potential application in evaporative cooling. The attempts to reduce the pressure drop while maintaining higher heat transfer coefficient has been achieved by decreasing the bubble layer thickness through

the water pump circulation and the height of weir. Correlations of pressure drop and heat transfer are obtained based on the experimental data.

## 2. Experimental setup

The schematic of the experimental setup is shown in Fig. 1. Air is pressurized by a centrifugal fan, and then enters the air–water bubbly flow evaporative cooling system from its bottom. There is an electrically heating tube bundle installed horizontally above the PVC sieve plate. The water is pumped onto the sieve plate while air passing through the holes of the sieve plate. The bubbly flow occurs due to the mixing flow of air and water, and it makes the heating coil submerged into the foam (bubble) layer. When the tube bundle is heated the evaporative heat transfer takes place on the tube wall. An over-flowing weir is used to adjust the height of the foam layer. Four different foam layer heights, i.e., 30, 60, 80 and 100 mm, which flood in succession the first through the fourth line of tubes, are applied in this experiment.

The thickness of the sieve plates is 2 mm. The effective bubbling area is 160 mm × 400 mm. The geometric para-
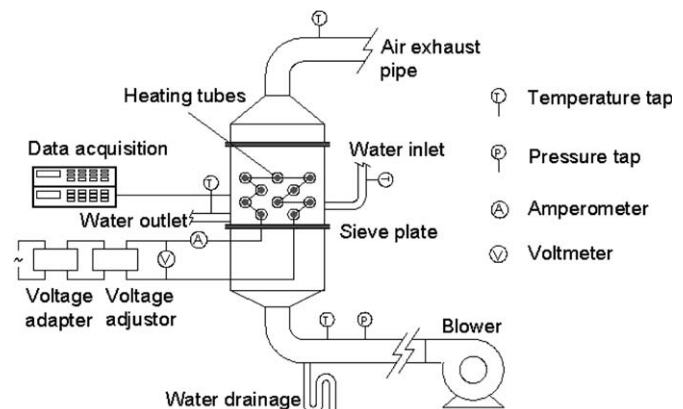


Fig. 1. Schematic of the experimental apparatus.

meters of the sieve plates used in the experiment are hole diameter, hole spacing, number of holes and the hole–plate area ratio, respectively. All the actual values of these four parameters used are shown in Table 1. The heating tube bundle is made of ten (10) 13.5 mm OD, 400 mm long copper tubes with embedded concentric electrical heaters, and the arrangement of the tubes is illustrated in Fig. 2. The installation of thermocouples is also included in Fig. 2, where the thermocouples are installed on several positions along the tube axis and each position has four thermocouples. The temperature of a specific tube wall is obtained by arithmetic averaging all the thermocouples' readings. A Pitot velocimeter is mounted into the air-supplying pipe to measure the air velocity in the pipe, and then the air volume flow rate can be calculated.

In the experiment, the outer surface temperatures of heating coil, the inlet and outlet temperatures of water, the air temperature at the entrance and exit, the pressure drop across the sieve plate and the air volume flow rate are all recorded by a data acquisition system (i.e., an HP3457A/HP3488A precise data acquisition system).

The tests are conducted by varying the hole–plate area ratios of the sieve plate, the heights of the weir, the airflow rates (or the superficial air velocities) and the heat flux inputs. The superficial air velocity, $u_s$, is defined as the air velocity passing through the sieve plate hole, i.e., it is the ratio of the airflow rate, $V$, and the total area of the plate holes, $A_h$. It also can be calculated using the frontal air velocity of the sieve plate, $u_f$, and the hole–plate area ratio, $\beta$, as follows:

$$u_s = V/A_h = u_f/\beta \tag{1}$$

The heat transfer coefficient can be obtained using the following equation:

$$h = q/(\bar{t}_{wall} - \bar{t}_{water}) \tag{2}$$

Table 1
The sieve plate geometric parameters

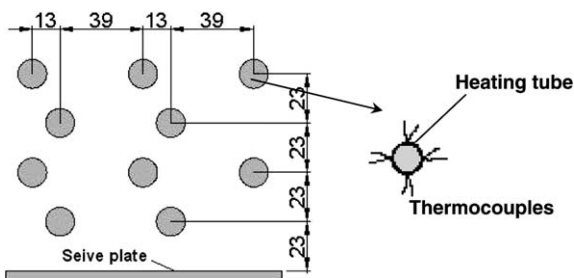| Plate No. | Hole diameter (mm) | Hole spacing (mm) | Number of holes | Hole–plate area ratio (%) |
|---|---|---|---|---|
| No. 1 | 4 | 14 | 363 | 7.13 |
| No. 2 | 4 | 20 | 184 | 3.61 |
| No. 3 | 3 | 14 | 396 | 4.37 |
| No. 4 | 6 | 20 | 184 | 8.13 |



Fig. 2. Side view of the heating tube arrangement (unit: mm).

where $\bar{t}_{wall}$ and $\bar{t}_{water}$ are the average tube wall temperature and the average water temperature of the bubbly flow, respectively.

## 3. Data analysis and uncertainty

The instrumental errors involved in this experiment are shown in Table 2, and the error analysis in this paper is based on the policy of reporting uncertainties in experimental measurements and results [13,14]. According to these references, the experimental uncertainty is defined as follows:

For the variable $R$, $R = R(x_1, x_2, x_3, \ldots, x_n)$, the uncertainty is defined as

$$U_R = \left\{ (B_R)^2 + (P_R)^2 \right\}^{1/2} \tag{3}$$

where the $B_R$ and $P_R$ are the bias limit and the precision limit of variable $R$, respectively. And they are defined as follows:

$$B_R = \left\{ \left(\frac{\partial R}{\partial x_1} Bx_1\right)^2 + \left(\frac{\partial R}{\partial x_2} Bx_2\right)^2 + \left(\frac{\partial R}{\partial x_3} Bx_3\right)^2 + \cdots + \left(\frac{\partial R}{\partial x_n} Bx_n\right)^2 \right\}^{1/2} \tag{4}$$

$$P_R = \left\{ \left(\frac{\partial R}{\partial x_1} Px_1\right)^2 + \left(\frac{\partial R}{\partial x_2} Px_2\right)^2 + \left(\frac{\partial R}{\partial x_3} Px_3\right)^2 + \cdots + \left(\frac{\partial R}{\partial x_n} Px_n\right)^2 \right\}^{1/2} \tag{5}$$

Generally, it is difficult to find the bias error for the specific experiment system because $B_R$ is an estimate of the magnitude of the fixed, constant error. Therefore, in this paper we just consider the precision limit, $P_R$, and regard this error as the experimental uncertainty of the specific variable:

$$U_R = P_R \tag{6}$$

Using the similar calculation procedure and the definition of superficial velocity in Eq. (1) and the heat transfer coefficient in Eq. (2), the uncertainties for these two parameters can be calculated, and the results are as follows: $u_s \pm 1.45\%$ and $h \pm 10.8\%$.

Table 2
Instrumental errors

| Test parameters | Sensors | Absolute uncertainties | Relative uncertainties (%) |
|---|---|---|---|
| Temperature | ∅0.2T-type Thermocouple | ±0.15 °C | 0.15 |
| Pressure difference | Pitot pressure gauge | 0.5 mm $H_2O$ | 1 |
| Pressure | U-type manometer | 1 mm $H_2O$ | 0.5 |
| Volt | Voltmeter | 0.5 V | 1.5 |
| Current | Amperometer | 0.2 A | 0.5 |

## 4. Results and discussion

### 4.1. Pressure drop

Fig. 3 shows the pressure drops, $\Delta P$, of the four sieve plates with different hole–plate area ratios, $\beta$, as listed in Table 1 at the condition of no water pumped onto the sieve plate (dry plate). Note that the symbol H denotes the height of the bubbling layer, and since this is a dry plate, there will be no bubbling layer (i.e., $H = 0$). It is shown in the figure that the pressure drop increases as the superficial air velocity, $u_s$, increases. It is also worth noting that the pressure drop is almost not affected by the hole–plate area ratio of the sieve plate for dry plate case.

When spraying water onto the sieve plates, the pressure drop, which is the sum of that through the sieve plate and the bubbly layer, have similar behavior as dry plate case (Fig. 3), as can be seen in Fig. 4, where the height of the foam layer is 60 mm. Again, it shows that the hole–plate area ratios do not have significant effect on the pressure drop.

When the height of the foam layer is increased to 100 mm, however, it is interesting to see from Fig. 5 that the hole–plate area ratios do have an effect on the pressure drop of bubbly flow especially at relatively low superficial air velocities. It should be noted that the diameters of different sieve plates are varied, i.e., the ratio of hole diameter to plate thickness is varied for different plates. It is also well known that the flow patterns of bubbly flow above the sieve plate are affected by the ratio of hole diameter to plate thickness. Therefore, the hole–plate area ratio of the sieve plate is not the only parameter to describe the flow characteristics of bubbly flow.

Fig. 6 shows the results of the pressure drop for sieve plate No. 3 at different height of bubbling layers, $H$, and
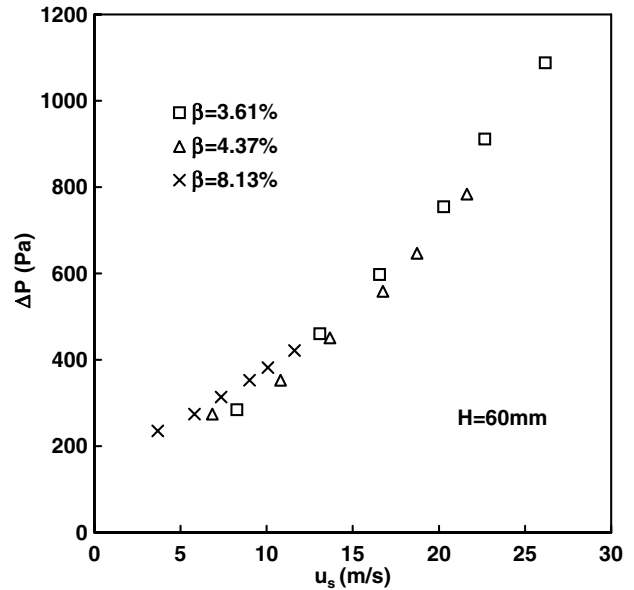


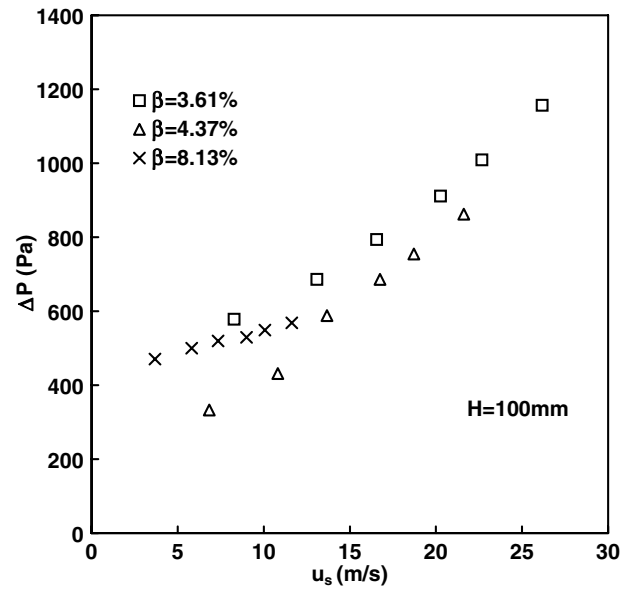Fig. 4. The pressure drop of bubbly flow at lower bubbling layer.



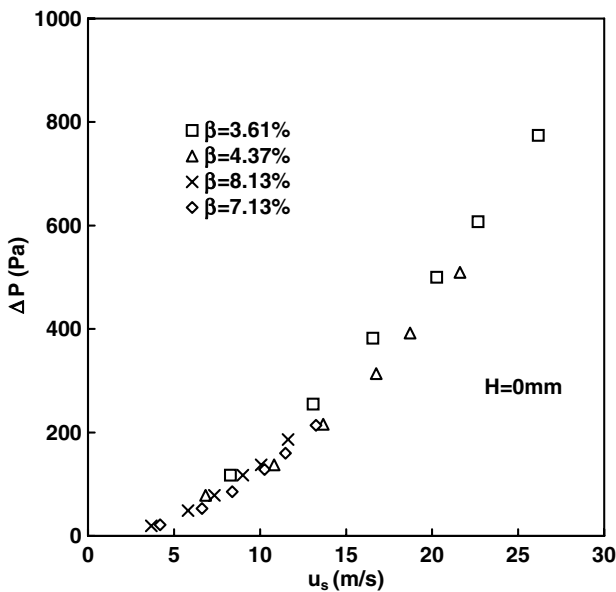Fig. 5. Pressure drop to the air velocity at higher bubbling layer.

the comparison with that of dry plate ($H = 0$). It can be seen clearly that with the increases of air velocity, the pressure drops also increase. It also shows that, as the height of bubbling layer increases, the increase of the pressure drop is not significant. This feature is particularly beneficial to the evaporative cooling application. For other sieve plates, i.e., the hole–plate area ratios of 3.61%, 7.13% and 8.13%, the effects of bubble layer height to pressure drop are very similar, and will not be repeated here.

It is worth noting that, in Mizushina and Miyashita [9], the pressure drop was simply considered as the height of the static liquid as follows:
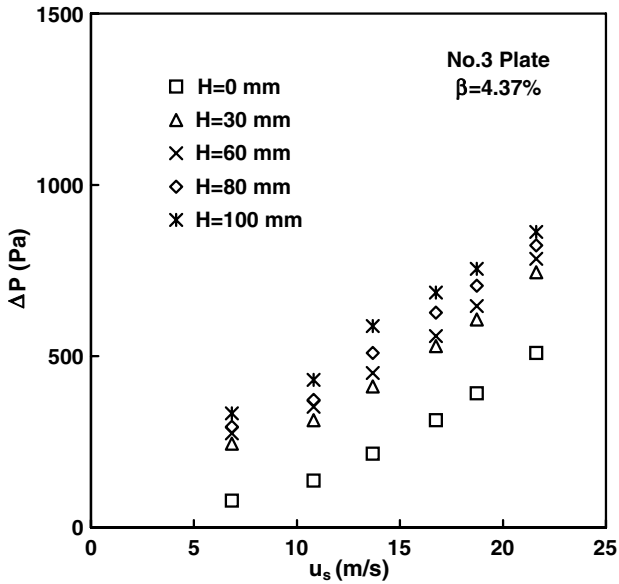


Fig. 3. The variation of dry plates pressure drop with air velocity and hole–plate area ratio.

Fig. 6. The variation of pressure drop at different height of bubbling layer.



Fig. 7. Comparison of predicted pressure drop of bubbly flow with experimental data.

$$\Delta P = \Delta P_p + 1000Z \tag{7}$$

In above formula, $Z$ is the height of weir and $\Delta P_p$ is the pressure drop of dry plate, which can be expressed as

$$\Delta P_p = 5.88 \times 10^{-7} \left( \frac{0.094}{S} \right)^2 \left( \frac{W_A}{\beta} \right)^2 = 5.2 \times 10^{-9} \left( \frac{W_A}{S \cdot \beta} \right)^2 \tag{8}$$

Eq. (8) is valid for $0.0317 < \beta < 0.108$. Here $W_A$ denotes the flow rate of dry air, and $S$ is the area of sieve plate. In fact, different superficial air velocities can cause different gas void fractions, so different gas–liquid mixture densities may result. Their results (i.e., Mizushina and Miyashita [9]) are only valid for the case that the liquid height is relatively high with static liquid, which is different from the conditions in the present study (i.e., the present study has much higher gas void fraction). If the correlation presented in Mizushina and Miyashita [9] is used, an effective height of the bubble layer should be introduced instead of the static liquid height.

A correlation of pressure drop for air passing through the sieve plate and bubbling layer has been developed based on regression analysis of the experimental data, as following:

$$\Delta P = 1.216 u_s^2 \left[ 1 + 20.36 (\beta H)^{0.058} e^{-0.18 u_f} \right] \tag{9}$$

which is valid in the range of $3.67 < u_s < 26.17 \text{ m s}^{-1}$, $3.61\% < \beta < 8.13\%$ and $0 < H < 100 \text{ mm}$. The second term in the square brackets of Eq. (9) can be viewed as the effective height of the bubbly layer, which is based on a pure experimental correlation.

The mean absolute deviation of the above-mentioned correlation is defined in Eq. (10):

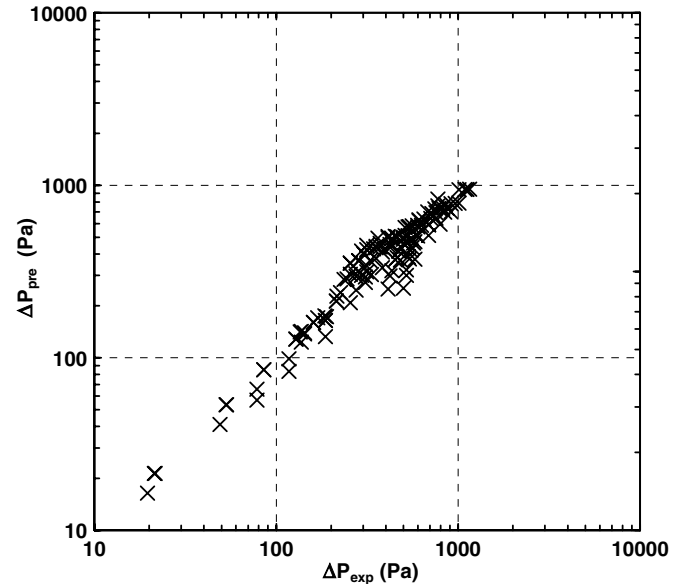$$\overline{D} = \frac{1}{n} \sum_{i=1}^{n} \frac{|\Delta P_{exp} - \Delta P_{pre}|}{\Delta P_{pre}} \tag{10}$$

where $n$ is number of test data, $\Delta P_{exp}$ and $\Delta P_{pre}$ are the experimental and predicted pressure drop, respectively. Based on the experimental data obtained from Fig. 7, this mean absolute deviation is found to be 14.5%, which is fairly reasonable.

### 4.2. Heat transfer

Heat transfer characteristics of the horizontally submerged heating tubes in the bubbly layer are tested for all the sieve plates at several different superficial air velocities, heat fluxes and bubbly layer heights. From the analysis to the data of all the tested sieve plates, the heat transfer performances of the four sieve plates are quite similar. Only the experimental results of the No. 3 plate, which has an intermediate hole–plate area ratio, will be shown in this paper as the representative of heat transfer results.

Fig. 8 shows the heat transfer coefficient with the air velocity at four different heat fluxes for the sieve plate No. 3. The results indicate that even at considerably lower height of bubbling layer (here it is 30 mm) the heat transfer coefficient can excess $5 \text{ kW/m}^2 \text{ K}$ when the superficial air velocity is more than $16 \text{ m s}^{-1}$.

Fig. 8 also shows that the heat transfer coefficient depends strongly on the air velocity and the value of heat transfer coefficients is much higher than that of the convection flow across single circular cylinders and tube bundles. However, it is worthy to notice that among the four different heat flux values in the experiments the heat transfer coefficients at the lowest heat flux are higher than that at the highest heat flux under the condition of same air velocity. And, overall, the increasing trend of heat transfer coefficients is falling when the heat flux value in the experimental range is increased. These phenomena, unlike
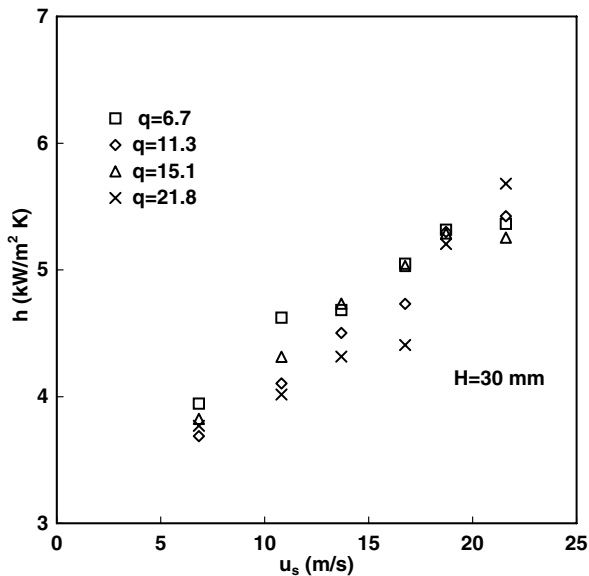
Fig. 8. Heat transfer coefficients of the bubbly flow at different heat flux.



Fig. 9. Effects of the height of bubbling layer to the heat transfer coefficients.

the ones in the conventional convection flow across single circular cylinders and tube bundles, give the evidence that the higher heat transfer coefficient is due to the existence of evaporation of thin water film formed on the tube surface during the period of air bubbles flowing across the tube. Certainly, the evaporation would be speeded up with the increasing of heat flux. Once the evaporation is fast enough to dryout part of tube surface before it rewet, the local heat transfer is weakened. The dryout on the tube surface, apparently, will cause the decreasing of heat transfer coefficients between the tube surface and water. Heat flux is not the only effect to the cause of dryout on the tube surface. Actually, the air velocity, bubbling layer thickness, bubble size and shape, inlet air dryness and tube diameter all have significant effects on the dryout on the tube surface. Therefore, there still exists some unclearness in the phenomena of this complicated process, which warrants further investigations.

The effect of the height of bubbling layer on the heat transfer coefficient is also investigated. Fig. 9 shows the results of the condition that the heat flux of the heating tube wall is fixed at 9.7 kW/m². It is interesting to see that the effect of the height of bubbling layer on the heat transfer coefficient is somewhat irregular compared with that of air velocity and heat flux. It can be seen from Fig. 9 that the heat transfer coefficients of 30 mm high bubbling layer are higher than that of other higher bubbling layers tested in this study. A sudden decrease in the heat transfer coefficient was observed when the bubbling layer was increased to 60 mm. When the height of bubbling layer exceeds 60 mm, the heat transfer coefficients increase stably with the bubbling layer height. This non-linear variation of heat transfer coefficients suggested that there could exist a minimum. As discussed in Section 4.1, the flow patterns above sieve plate are affected by many factors other than the bub-
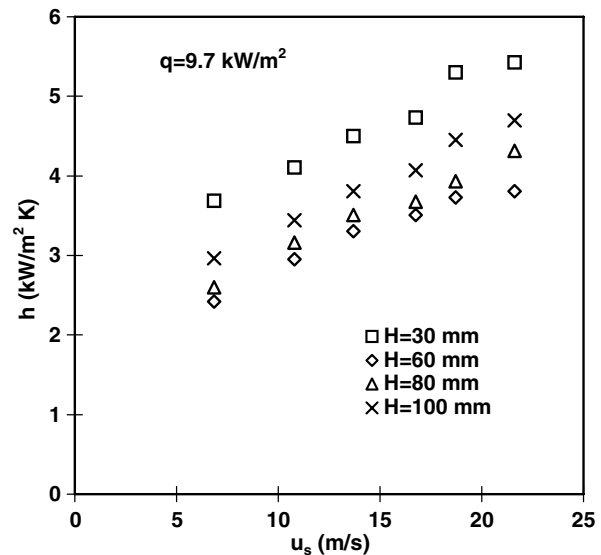
bling layer height or hole–plate area ratio. Different flow patterns lead to different heat transfer mechanisms. Considering the changes of flow pattern, the thin film evaporative heat transfer processes between bubbly flow and the heating tube are rather complicated. The flow and heat transfer characteristics of such condition deserve further experimental and theoretical research.

To obtain a heat transfer correlation of the present experiment, a dimensional analysis is carried out on the specific process of the bubbly flow and heat transfer. Four governing dimensionless groups are obtained to depict the heat transfer process. The first one is the Nusselt number, $Nu = hD/\lambda$, where $h$ is the heat transfer coefficient, $D$ is
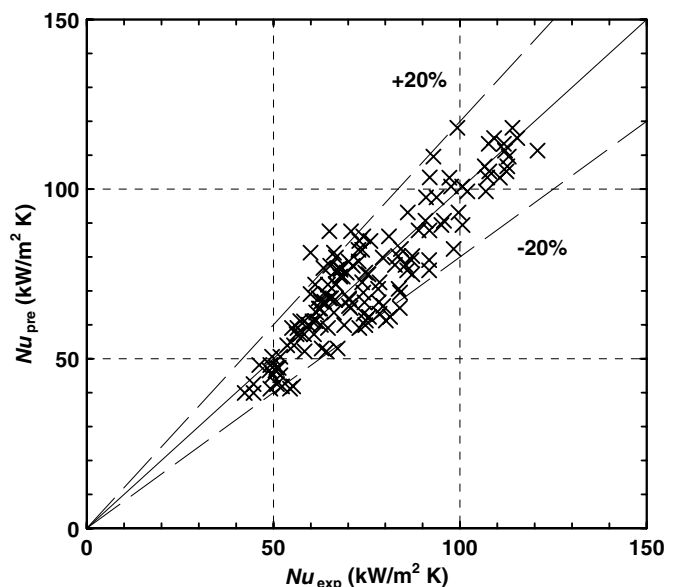


Fig. 10. Heat transfer coefficient prediction.

the outer diameter of heating tube, and $\lambda$ is the conductivity of water. The second parameter is Reynolds number, $Re$, defined as $Re = u_s d_p / v$, where $d_p$ is the diameter of the holes in the sieve plate, and $v$ is the kinematic viscosity of air. Another parameter obtained through dimensional analysis has the form as $q/rm$, where $q$ is the heat flux, $r$ is the vaporization latent heat and $m$ is the mass flow rate of air. And the effect of the weir height, $H$, could be included into a dimensionless parameter as $H/D$, where $D$ is the outer diameter of heating tube. Based on the regression analysis of all the test data, a correlation of heat transfer is obtained, as follows:

$$Nu = 2.845 Re^{0.47} \left(\frac{q}{rm}\right)^{-0.05} \left(\frac{H}{D}\right)^{-0.43} \quad (11)$$

The experimental correlation above has a mean absolute deviation of 9.44% to that of the experimental data. The comparison of the calculation values of the correlation and the test data is shown in Fig. 10.

## 5. Conclusions

(1) The pressure drop from sieve for dry plate increases as the air velocity increases and not effected by the hole–plate area ratio of sieve plate.

(2) At wet conditions, for lower weir height of the test range, the pressure drop is almost not affected by the hole–plate area ratio. When weir height increases higher, however, hole–plate area ratio affects the pressure drop under lower superficial air velocity.

(3) Higher superficial air velocity causes larger pressure drop across the sieve plate and bubbling layer. The increase of pressure drop is small as the height of bubbling layer increases.

(4) At the condition of considerably lower height of bubbling layer (i.e., $H = 30$ mm) the heat transfer coefficient can exceed 5 kW/m$^2$ K when the superficial air velocity higher than 16 m s$^{-1}$. The effect of the heat flux input is not significant and the heat transfer coefficient depends strongly on the air velocity.

(5) Correlations for pressure drop and heat transfer are developed based on the experimental data. They both have a good agreement with the test data.

## References

[1] Q. Wang, M.P. Dudukovic, A two phase recycle with cross-flow mixing model (TRCF) for churn turbulent bubble column reactors, the AIChE Annual Meeting, Chicago, 1996.

[2] M.P. Dudukovic, N. Devanathan, Bubble column reactors: some recent developments, in: H.I. DeLasa et al. (Eds.), Chemical Reactor Technology for Environmentally Safe Reactors, NATO ASI Series E: Appl. Sci., 225, 1993, pp. 379–408.

[3] S.B. Kumar, N. Devanathan, D. Moslemian, M.P. Dudukovic, Effect of scale on liquid recirculation in bubble columns, Chem. Eng. Sci. 49 (24B) (1994) 5637–5652.

[4] D.R. Adkins, K.A. Shollenberger, T.J. O'Hern, J.R. Torczynski, Pressure effects on bubble column flow characteristics, in: ANS Proceedings of the National Heat Transfer Conference, THD-Vol. 9, American Nuclear Society, LaGrange Park, IL, 1996, pp. 318–325.

[5] A. Kitagawa, K. Sugiyama, Y. Murai, Experimental detection of bubble–bubble interactions in a wall-sliding bubble swarm, Int. J. Multiphase Flow 30 (2004) 1213–1234.

[6] H. Chanson, Air–water flow measurements with intrusive, phase-detection probes: can we improve their interpretation? J. Hydraul. Eng. (March) (2002) 252–255.

[7] K. Sugiyama, S. Takagi, Y. Matsumoto, Multi-scale analysis of bubbly flows, Comput. Methods Appl. Mech. Eng. 191 (2001) 689–704.

[8] T. Hibiki, M. Ishii, Development of one-group interfacial area transport equation in bubbly flow systems, Int. J. Heat Mass Transfer 45 (2002) 2351–2372.

[9] T. Mizushina, H. Miyashita, Experimental study of an evaporative cooler, J. Chem. Eng. Jpn. 32 (10) (1968) 987–992.

[10] M. Tong, X. Liu, H. Zhang, Theoretical and experimental analysis of the bubbling cooling, Chin. J. Chongqing Univ. 17 (6) (1994) 15–18.

[11] Y. Li, C. Liu, Z. Shen, Heat transfer in bubble column and gas-lift loop reactors, Chin. J. Eng. Thermophys. 10 (1) (1989) 72–74.

[12] Y. Li, J. Gu, Heat transfer between an immersed horizontal tube and the two-phase flow in bubble column, Chin. J. Tianjin Inst. Technol. (2) (1993) 52–56.

[13] J.H. Kim, T.W. Simon, R. Viskanta, Journal of heat transfer policy on reporting uncertainties in experimental measurements and results (Editorial), ASME J. Heat Transfer 115 (1) (1993) 5–6.

[14] R.J. Moffat, Using uncertainty analysis in the planning of an experiment, ASME J. Fluids Eng. 107 (2) (1985) 173–178.