The Effects of Periodic Impulsive Noise on OFDM

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Abstract—The effect of periodic impulsive (short duration) noise on OFDM is investigated. We present results on the nature of periodic impulsive noise, showing that the PDF of periodic impulsive noise is not exactly Gaussian. We also present results showing that periodic impulsive noise can be more devastating to OFDM compared to random impulsive noise. This is because periodic impulsive noise energy is not spread by the FFT on the receiver side of the OFDM, instead it appears periodic in the frequency domain. Results showing the effect of nulling to mitigate periodic impulsive noise are presented. We suggest a simple short block code (as opposed to long block codes) that can effectively combat the effects of periodic impulsive noise.

Index Terms—Periodic impulse noise, OFDM, error correcting coding, block code.

I. INTRODUCTION

Impulsive noise is known to be one of the most devastating noise in power line communication (PLC). There are two types of impulsive noise observed in the PLC channel, random and periodic impulsive noise. Random impulsive noise has been extensively studied and utilised by many researchers as a Middleton Class A noise model ([1]–[8].) A general study of random impulsive noise models (including the Middleton Class A model) can be found in [9] and the references therein. The Middleton Class A noise model gives the probability density function (PDF) of a noise sample, say n_k as follows:

$$\mathcal{P}(n_k) = \sum_{m=0}^{\infty} P_m \mathcal{N}(n_k; 0, \sigma_m^2), \tag{1}$$

where

$\mathcal{N}(x_k; \mu, \sigma^2)$ represents a Gaussian PDF with mean μ and variance σ^2, from which the kth sample x_k is taken.

$$P_m = \frac{A^m e^{-A}}{m!} \tag{2}$$

and

$$\sigma_m^2 = \sigma_I^2 \frac{m}{A} + \sigma_g^2 = \sigma_g^2 \left(\frac{m}{A\Gamma} + 1 \right), \tag{3}$$

where A is the "impulsive index" (gives the probability of impulse noise), σ_I^2 is the variance of the impulsive noise and σ_g^2 is the variance of the background noise (AWGN). The parameter $\Gamma = \sigma_g^2/\sigma_I^2$ gives the Gaussian to impulsive noise power ratio.

In this paper we study the effects of periodic impulse noise on OFDM. We consider impulsive noise occurring periodically in the time domain with period T_p and amplitude $\sigma_I > 0$. Periodic and random impulsive noise is mentioned in [10] and [11]. A special treatment of periodic impulsive noise is addressed in [12] and [13], where models for periodic impulsive noise are proposed. However, in most cases addressing periodic impulsive noise, an analysis on the effect of the noise on OFDM systems is not given. In the article, we discuss the effect of periodic noise on OFDM and show that it can be more devastating than random impulsive noise. We therefore discuss periodic impulsive noise and its effect on OFDM transmission. Nulling and error correcting coding are proposed to combat the effects of the two types of impulsive noise, with more focus on periodic impulsive noise. The error correcting code proposed is a short block code. Such a code has benefits of reduced decoding complexity and yet still being very effective.

Nulling and interleaving for the PLC standard technologies employing OFDM (PRIME and PLC G3 systems) was investigated by Shongwe and Vinck in [14], where narrowband interference was considered. In the paper we consider nulling, in the time-domain, for impulsive noise on OFDM systems. The nulling procedure we employ searches for nulling thresholds that can be good for reducing the effects of both periodic and random impulsive noise. Consider a practical OFDM system which cannot distinguish between the two types of impulsive noise (random and periodic impulsive noise). For such a system, a nulling threshold that will reduce the effect of one type of impulsive noise without making the other types of noise worse will be required; if possible, a threshold that can reduce the effects of both random and periodic impulsive noise is desirable.

II. PERIODIC AND RANDOM IMPULSIVE NOISE

We assume a simplistic view of periodic impulsive noise, which is a discrete-time sequence with constant amplitude σ_I at periodic intervals T_p. Taking the two-state Middleton Class A case [9], the random impulsive noise will have a probability of occurrence A. For comparison purposes, the periodic impulse noise can have the same probability of occurrence $f_p = A$, which we term frequency of occurrence. Therefore the periodic impulsive noise will occur every $T_p = 1/f_p$

with constant amplitude σ_I. To compare the two types of noise, periodic and random impulsive noise we discuss at two interesting features which manifest in an OFDM system. These features are the spectrum and PDF of the noise amplitude in the frequency domain, which we discuss next.

From digital signal processing theory we know that the periodic impulsive noise has a periodic spectrum (a periodic delta train in time domain will also result in a periodic delta train in the frequency domain). The periodic spectrum obtained from the periodic impulsive noise has serious implications on OFDM performance. OFDM is known to have a spreading effect on random impulsive noise, i. e., it spreads the energy of random impulsive noise over all subcarriers. However, the energy of the periodic noise is not spread over the subcarriers, instead it appears on individual subcarriers. Therefore the problem of periodic impulsive noise needs to be addressed differently to that of random impulsive noise. Another interesting characteristic of periodic impulsive noise is that its power/amplitude spectrum (amplitude in the frequency domain) does not exhibit a Gaussian behaviour as assumed in random impulsive noise. This means that periodic impulsive noise cannot be analysed the same way random impulsive noise is analysed.

III. SIMULATION RESULTS AND DISCUSSION OF RESULTS

For the simulations, the parameters were as follows:

- QPSK-OFDM was used for the simulation, with FFT size $N = 256$ and $N = 2048$.
- For the random impulsive noise, the average impulsive noise power (in relation to the AWGN power) was given by $1/\Gamma = 1/0.01 = 100$, hence the average amplitude was $\sqrt{1/0.01} = 10$. The probability of the impulsive noise was $A = 0.1$ or $A = 0.01$.
- For the periodic impulsive noise, the amplitude was fixed to $\sigma_I = 10$. The frequency (or probability) of occurrence of the noise was $f_p = 0.1$ or $f_p = 0.01$.

The system model used for simulations is shown in Fig. 1 (a), and the corresponding frame is shown in Fig. 1 (b).
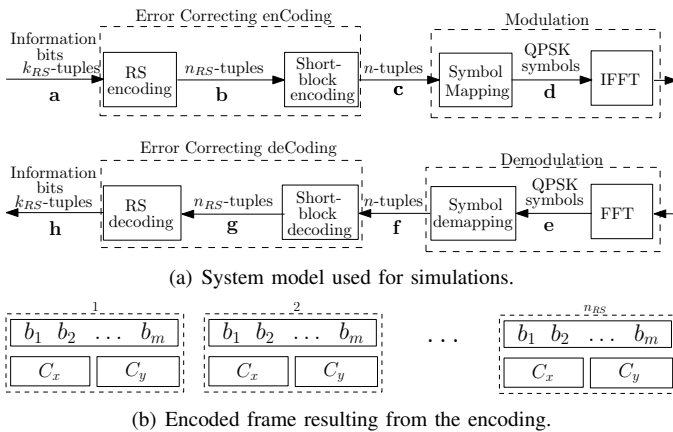

(a) System model used for simulations.

(b) Encoded frame resulting from the encoding.

Fig. 1. System model for communications with a frame after encoding with the RS code and the short block code.

For uncoded transmission, the "error correcting encoding/decoding" blocks in Fig. 1 (a) are ignored, hence the information bits are mapped directly to QPSK in the "modulation" blocks.

For coded transmission all the blocks in Fig. 1 (a) are taken into account and the system can be described as follows, with the help of the frame in Fig. 1 (b) : the RS code and short block code work together such that the encoded symbols fit in a single OFDM symbol ($N = n_{RS} \times n$). Therefore, the eight bits of an RS code symbol ($b_1 b_2 \ldots b_m$, where $b_i \in \{0, 1\}$) are passed on to be encoded by the short block code as follows: the first four bits of the RS code symbol result in a single codeword of the short block code and the other remaining four bits result in another codeword of the short block code. Since the codeword of the short block code contains $n = 4$ symbols, then the RS code symbol corresponds to two codewords of the short block code ($b_1 b_2 \ldots b_{m/2} \rightarrow C_x$ and $b_{m/2+1} b_{m/2+2} \ldots b_m \rightarrow C_y$), making up the eight symbols of the short block code. Note that the short block code is chosen carefully to operate over GF(4) so that its symbols $\{0, 1, \alpha, \alpha^2\}$ correspond directly to QPSK symbols $\{1 + j, 1 - j, -1 - j, -1 + j\}$. It is these short block code symbols which are passed on to the IFFT as complex QPSK symbols. The short block code is described by the generator matrix in (4).

We present bit error rate (BER) versus signal-to-noise ratio (SNR) simulation results, assuming that each single impulse occurs in only one of the N time-slots at a time in an OFDM symbol. We first give simulation results without error correcting coding applied in Figs. 2 – 7.

The results will show the effect of random and periodic impulse noise on a QPSK-OFDM system, with background noise modelled as AWGN. The periodic impulse noise frequency of occurring was f_p, which meant that it occurred every $T_p = 1/f_p$ samples of the N time domain samples of the OFDM. For the random impulsive noise, the two-state Middleton Class A noise model was employed with probability of impulsive noise A, and ratio the of the AWGN variance to impulsive noise variance given by Γ.

The results will also show the effect of time-domain nulling. In nulling, the received discrete-time OFDM signal is first pre-processed by identifying positions considered to have impulse noise according to a nulling threshold T_r. If a sample in the received OFDM signal has a magnitude greater than T_r, we set it to zero.

Fig. 2 shows the BER performance of the QPSK-OFDM system (with FFT size $N = 2048$) when random impulsive noise is present. The random impulsive noise had a probability of occurrence $A = 0.1$, and $\Gamma = 0.01$. It can be seen in Fig. 2 that lower nulling thresholds ($T_r = 2.3$ or $T_r = 6$) improve the system performance for low SNR, but can degrade the performance for high SNR. While higher nulling thresholds like $T_r = 15$ and $T_r = 20$, do not degrade the performance, they do not improve the performance.

Fig. 3 shows the BER performance of the QPSK-OFDM system (with FFT size $N = 2048$) when periodic impulsive noise is present. The periodic impulsive noise had a frequency
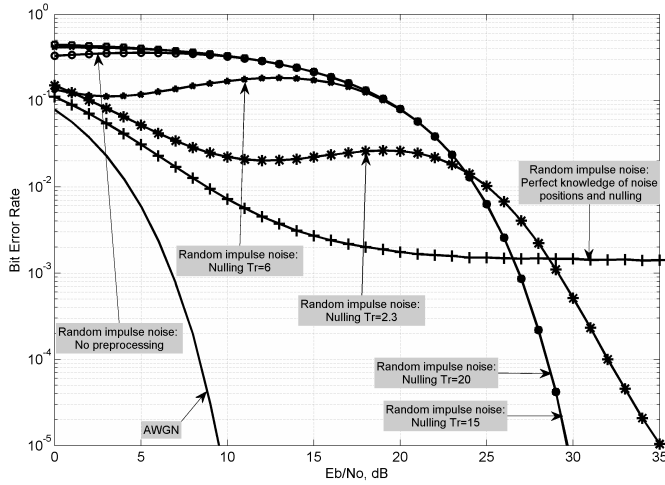
Fig. 2. Bit error rate curves for random impulsive noise. The random impulsive noise parameters were $A = 0.1$ and $\Gamma = 0.01$. The communication system was QPSK-OFDM with FFT size $N = 2048$.
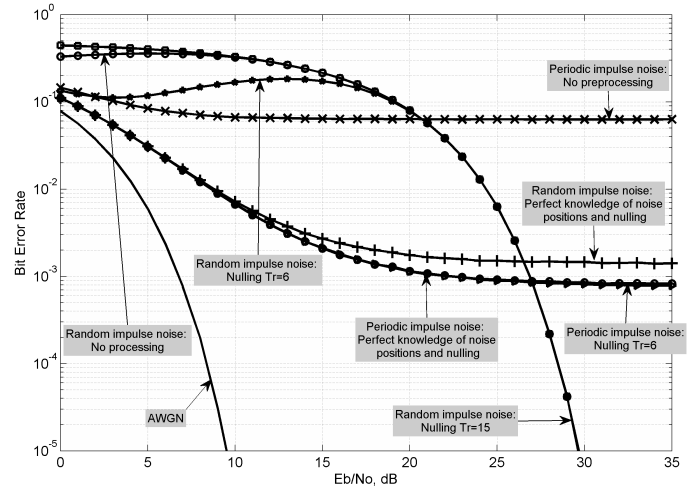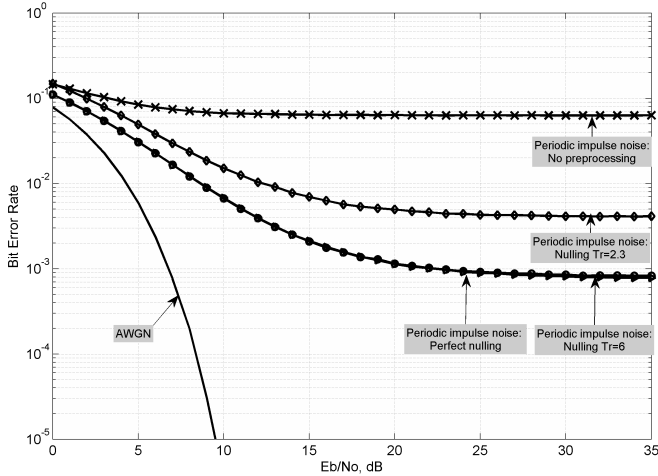

Fig. 3. Bit error rate curves for periodic impulsive noise. The periodic impulsive noise parameters were $f_p = 0.1$ and amplitude $\sigma_I = 10$. The communication system was QPSK-OFDM with FFT size $N = 2048$.

Fig. 4. Bit error rate curves for periodic and random impulsive noise. The periodic impulsive noise parameters were $f_p = 0.1$ and amplitude $\sigma_I = 10$, and the random impulsive noise parameters were $A = 0.1$ and $\Gamma = 0.01$. The communication system was QPSK-OFDM with FFT size $N = 2048$.
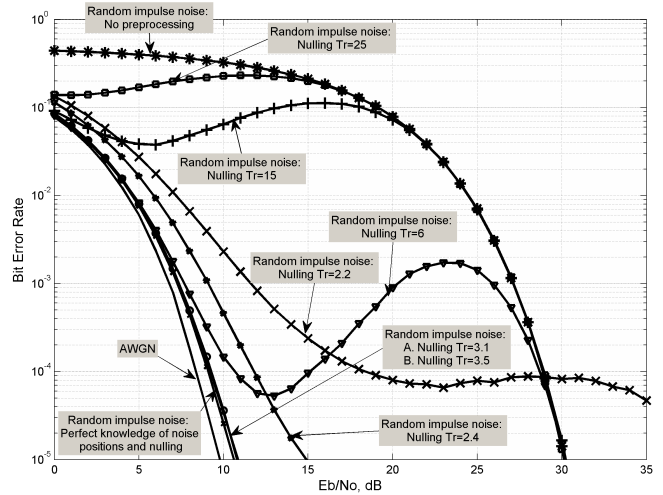
in Figs. 5, 6 and 7.

Fig. 5. Bit error rate curves for random impulsive noise. The random impulsive noise parameters were $A = 0.01$ and $\Gamma = 0.01$. The communication system was QPSK-OFDM with FFT size $N = 2048$.

of occurrence $f_p = 0.1$, and $\sigma_I = 10$. It can be seen in Fig. 3 that the nulling threshold $T_r = 6$ is a good threshold as it results in performance which matches that of when the receiver has perfect knowledge of the impulsive noise position.

In Fig. 4 we compare the random and periodic impulsive noise BER curves already shown in Figs. 2 and 3, respectively. The results show that the single best nulling threshold is $T_r = 6$ because: for periodic impulsive noise it gives the best performance (equivalent to when the receiver has perfect knowledge of the impulsive noise position), and for random impulsive noise it does not make the performance worse as did the nulling threshold of $T_r = 2.3$.

We now change only the probability of random and periodic impulsive noise to 0.01 and present BER versus SNR results, in the same format as Figs. 2, 3 and 4. Such results are shown

In Fig. 5, with random impulsive noise parameters being $A = 0.01$ and $\Gamma = 0.01$, it can be observed that nulling thresholds above $T_r = 2.3$ improve the performance of the system. Applying nulling threshold $T_r = 2.4$ significantly improves the performance over $T_r = 2.3$, and $T_r = 3.1$ results in the best performance (equivalent to when the receiver has perfect knowledge of the impulsive noise position). However, as the nulling threshold is increased above $T_r = 3.1$, the performance may begin to degrade, for example $T_r = 6$.

For the periodic impulsive noise, with noise parameters being $f_p = 0.01$ and $\sigma_I = 10$, nulling thresholds in the
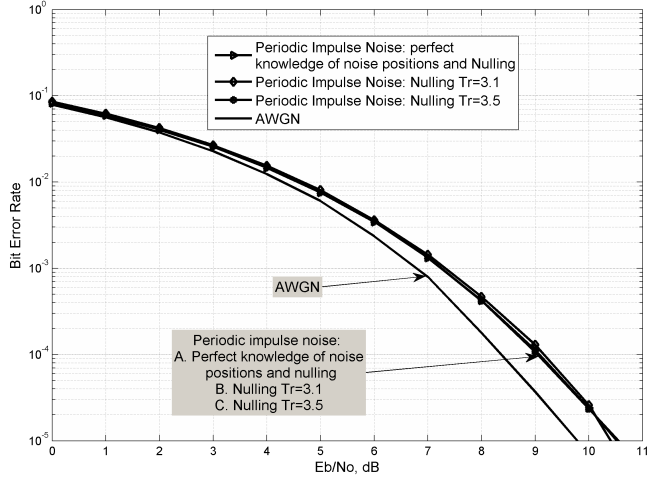
Fig. 6. Bit error rate curves for periodic impulsive noise. The periodic impulsive noise parameters were $f_p = 0.01$ and amplitude $\sigma_I = 10$. The communication system was QPSK-OFDM with FFT size $N = 2048$.
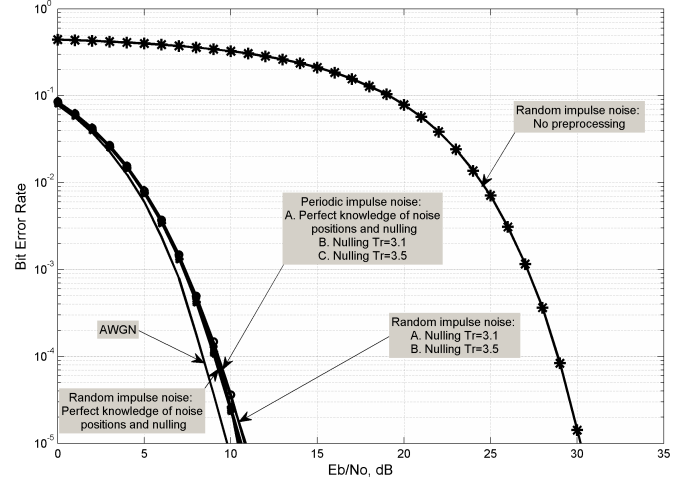

Fig. 7. Bit error rate curves for periodic and random impulsive noise. The periodic impulsive noise parameters were $f_p = 0.01$ and amplitude $\sigma_I = 10$, and the random impulsive noise parameters were $A = 0.01$ and $\Gamma = 0.01$. The communication system was QPSK-OFDM with FFT size $N = 2048$.

range $3.1 - 3.5$ resulted in the best performance (equivalent to when the receiver has perfect knowledge of the impulsive noise position) as shown in Fig. 6.

In Fig. 7, similarly to Fig. 4, we again show results of the effect of random and periodic impulse noise on a QPSK-OFDM system in one figure. However, the impulsive noise occurred less frequently compared to the results in Fig. 4. In Fig. 7 the periodic impulse noise frequency of occurring was $f_p = 0.01$, which meant that it occurred every $T_p = 100$ samples of the $N = 2048$ time domain samples of the OFDM. For the random impulsive noise, the two-state Middleton Class A noise model was employed with probability of impulsive noise $A = 0.01$, and ratio of AWGN variance to impulsive noise variance $\Gamma = 0.01$.

Fig. 7 shows that the best nulling threshold for both random and periodic impulsive noise is $Tr = 3.1$. Both Figs. 4 and 7 demonstrate a fact that the nulling threshold does not only depend on the amplitude of the impulsive noise, it also depends on the probability of impulsive noise in the OFDM system. Next we demonstrate the effect of error correcting coding on the two types of impulsive noise.

Fig. 8 shows bit error rate simulation results for periodic and random impulsive noise when nulling and error correcting coding are applied. For both periodic and random impulsive noise we kept the amplitudes constant ($\sigma_I = 10$ and $\sqrt{1/\Gamma} = \sqrt{1/0.01} = 10$) and varied the frequency or probability of impulsive noise occurrence (f_p or A). The error correcting code used is a short block code (SBC) over GF(4) with the following parameters: $n = 4$, $k = 2$, $d_{\min} = 3$, where n is the length, k is the dimension and $d_{\min}$ is the minimum Hamming distance of the code. The generator matrix for the SBC is

$$ G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & \alpha \end{bmatrix}. \tag{4} $$

The effect of the periodic impulsive noise on ODFM is similar to the effect of narrow-band interference on OFDM. Hence we can use the same analysis in [15] to give the bound on the decoding error probability of the short block code. For high SNR, where the dominant noise is the periodic impulsive noise we can estimate the decoding error probability of the short block code as

$$ P_e = \sum_{j=e+1}^{n} \binom{n}{j} (p)^j (1-p)^{n-j}, \tag{5} $$

where $e = \lfloor (d_{\min} - 1)/2 \rfloor = 1$ is the maximum number of correctable symbol errors; p is the probability of error resulting from nulling (the error floor). For nulling at known position of periodic impulse noise the bound on the error floor is

$$ p = Q\left(\sqrt{\frac{E_b}{(1 - f_p)\sigma_g^2 + f_p E_b}} \right), $$

where $Q(x)$ is the complementary error function, E_b is the bit energy, f_p is the frequency of occurrence of periodic impulsive noise and σ_g^2 is the AWGN variance.

It is observed in Fig. 8 that the short block code significantly reduces the error floor caused by periodic impulsive noise, after nulling. It can be concluded from Fig. 8 that the short block code is very effective in combating the effects of periodic impulsive noise.

While bit error rate curves are useful in judging the performance of noise combatting schemes, they may not be very useful in practical systems sending data in frames. Therefore, in Fig. 9 we show simulation results of frame error rate (FER), where a frame is an OFDM symbol. The error correcting coding used in the simulation results was a concatenation of the ($n = 4$, $k = 2$) short block code and an ($n_{RS} = 64$, $k_{RS} = 56$) Reed-Solomon code. The FFT size for the OFDM
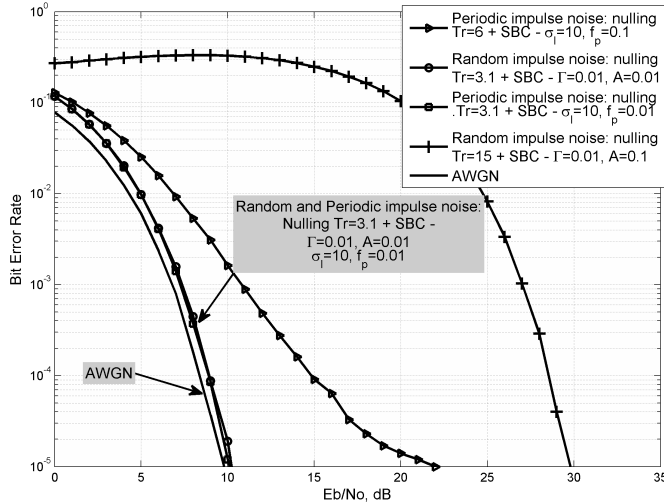
Fig. 8. Bit error rate curves for periodic and random impulsive noise. The periodic impulsive noise parameters were $f_p = 0.1$ and $f_p = 0.01$, both with amplitude $\sigma_I = 10$. The random impulsive noise parameters were $A = 0.1$ and $A = 0.01$, both with $\Gamma = 0.01$. The communication system was QPSK-OFDM with FFT size $N = 2048$, coded with a single error correcting block code described in (4).
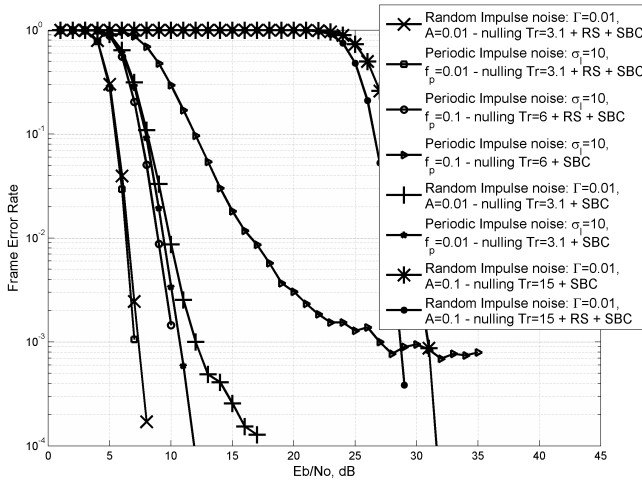

Fig. 9. Frame error rate curves for periodic and random impulsive noise. The periodic impulsive noise parameters were $f_p = 0.1$ and $f_p = 0.01$, both with amplitude $\sigma_I = 10$. The random impulsive noise parameters were $A = 0.1$ and $A = 0.01$, both with $\Gamma = 0.01$. The communication system was QPSK-OFDM with FFT size $N = 256$, coded with a concatenation of a (64, 56) RS code and the short block code in (4).

system used was $N = 256$. The Reed-Solomon (RS) code was chosen such that it can correct four errors like the RS codes used in PLC G3, and the FFT size of $N = 256$ is the same as the one used in the OFDM of the PLC G3 system (approved by the PLC standard). In the PLC G3 system an RS code is concatenated with a convolutional code. This requires the PLC G3 system to add an interleaver to assist the convolutional code. The advantage of our concatenation of an RS code with a short block is that an interleaver is not required. For high

SNR, and for a concatenated RS code and short block code, the FER can be approximated by the first term in

$$P_{\text{FE}} = \sum_{i=t+1}^{n_{RS}} \binom{n_{RS}}{i} P_e^i (1 - P_e)^{n_{RS}-i},$$

where $t = (n_{RS} - k_{RS})/2$ is the maximum number of correctable symbol errors for the RS code and P_e is the the probability of error of the short block code as defined in (5).

In Fig. 9 we can see the benefit of the RS code in reducing the frame error rate of the OFDM system. We see a huge improvement in the SNR when the noise is periodic, and that is because our main goal was to combat periodic impulsive noise. Even for the random impulsive noise scenario, the performance is good, and that is because of our careful choice of nulling threshold and error correcting short block code.

IV. CONCLUSION

We have studied the nature of periodic impulsive noise, showing its characteristics. We demonstrated the devastating effects of periodic impulsive noise on OFDM. Then suggested simple known methods to combat the effects of impulsive noise on OFDM. The methods we suggested were nulling (with a properly chosen threshold such that the OFDM system experiencing random noise is not corrupted further) and a simple short block code that can effectively combat effects of periodic impulsive noise. The simple short block code will obviously result in a system with far reduced complexity compared to one implementing a long block code. We successfully combatted the effects of impulsive noise (both random and periodic) on OFDM.

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