STUDY OF THE LEPTONIC DECAYS OF THE Z^0 BOSON

DELPHI Collaboration

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Measurements are presented of the cross section ratios $R_\ell = \sigma_\ell(e^+e^- \to \ell^+\ell^-)/\sigma_h(e^+e^- \to \text{hadrons})$ for $\ell = e$, μ and τ using data taken from a scan around the Z^0. The results are $R_e = (5.09 \pm 0.32 \pm 0.18)\%$, $R_\mu = (4.96 \pm 0.35 \pm 0.17)\%$ and $R_\tau = (4.72 \pm 0.38 \pm 0.29)\%$ where, for the ratio R_e, the t-channel contribution has been subtracted. These results are consistent with the hypothesis of lepton universality and test this hypothesis at the energy scale $s \sim 8300\ \text{GeV}^2$. The absolute cross sections $\sigma_\ell(e^+e^- \to \ell^+\ell^-)$ have also been measured. From the cross sections the leptonic partial widths $\Gamma_e = (83.2 \pm 3.0 \pm 2.4)$ MeV, $(\Gamma_e\Gamma_\mu)^{1/2} = (84.6 \pm 3.0 \pm 2.4)$ MeV and $(\Gamma_e\Gamma_\tau)^{1/2} = (82.6 \pm 3.3 \pm 3.2)$ MeV have been extracted. Assuming lepton universality the ratio $\Gamma_\ell/\Gamma_h = (4.89 \pm 0.20 \pm 0.12) \times 10^{-2}$ was obtained, together with $\Gamma_\ell = (83.6 \pm 1.8 \pm 2.2)$ MeV. The number of light neutrino species is determined to be $N_\nu = 3.12 \pm 0.24 \pm 0.25$. Al the data are consistent with the predictions of the standard model.

1. Introduction

In this paper we present the results of a study of the leptonic decays $Z^0 \to \ell^+\ell^-$ ($\ell = e, \mu, \tau$), using data taken with the DELPHI detector at the LEP collider at CERN at ten centre of mass energies between 88.28 and 95.04 GeV.

The cross section for the s-channel production of a fermion pair $f\bar{f}$ near the Z^0 pole can be written as a modified Born approximation [1]

$$\sigma_f = \sigma(e^+e^- \to f\bar{f})$$
$$= \frac{s}{M_Z^2} \frac{12\pi\Gamma_e\Gamma_f}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} [1 + \delta(s)] ,$$

where M_Z and Γ_Z are the mass and total width of the Z^0 boson, Γ_e is the partial width for the decay $Z^0 \to e^+e^-$, Γ_f is the partial width for the decay $Z^0 \to f\bar{f}$, s is the square of the centre of mass energy and $\delta(s)$ accounts for the effects of radiative corrections. The above formula refers to the s-channel exchange of the Z^0 only. In addition there is an s-channel photon term as well as an interference term between the two amplitudes. Furthermore, for the channel $e^+e^- \to e^+e^-$ there are t-channel terms for the exchange of both the γ and Z^0. These additional terms, together with the effects of higher order terms which significantly distort the cross section line shape, are fully taken into account as discussed below. Results are presented on the cross section ratios

$$R_\ell = \frac{\sigma(e^+e^- \to \ell^+\ell^-)}{\sigma(e^+e^- \to \text{hadrons})} ,$$

and on the behaviour of the cross section σ_ℓ as a function of $\sqrt{s}$.

2. Apparatus and data collection

Details of the DELPHI detector can be found in ref. [2]. The analysis was restricted to the barrel region covering the angular range $50° < \theta < 130°$, where θ is the polar angle with respect to the incident e^- direction (the z-axis). The main components of DELPHI used were as follows:

- The inner detector (ID), the time projection chamber (TPC) and the outer detector (OD) for charged track reconstruction.
- The high density projection chamber (HPC) to distinguish electrons from minimum ionising particles.
- The barrel muon chambers (MUB) for muon identification.
- The time-of-flight counters (TOF) for triggering.
- The hadron calorimeter (HAC) to cross-check the muon identification.
- The small angle tagger (SAT) to determine the luminosity.

For leptonic events, the trigger was an OR of the following sub-triggers:

- Coincidences of back-to-back TOF sectors.
- Coincidences between ID and OD requiring back-to-back tracks.
- The HPC scintillator system set to a threshold of ~2 GeV.

The measured trigger efficiencies, with the statistical and systematic errors included, were 97 ± 2% for the $\mu^+\mu^-$ and $\tau^+\tau^-$ channels and >99.7% for the e^+e^- channel. These efficiencies were determined using the final event samples extracted as described below.

The luminosity determination using the SAT [2,3] was estimated to have a systematic error of ±2.4%.

The raw data, part of which (~30%) was taken with a magnetic field of 0.7 T and the remainder with the nominal field of 1.2 T, were processed by the DELPHI analysis package, DELANA [4]. Events with eight or less charged particles coming from the interaction region were initially selected as leptonic candidates. The interaction region was defined by $|\delta z| < 3.0$ cm and $\delta r < 1.5$ cm, where δz and δr are the distances of closest approach to the nominal interaction point in the longitudinal and radial directions respectively.

The additional selection criteria for the various lepton channels are described in the following sections. For each channel, the data sample is restricted to those experimental runs where the required detector components were fully functional. Consequently, the numbers of events in each channel are not directly comparable. The selection criteria described below were cross-checked by scanning a sample of events on graphical devices.

In order to determine the cross section ratios R_ℓ, the number of hadronic Z^0 decays at each energy was also determined as described in ref. [3]. The statistical errors on the numbers of hadronic events were small in comparison with those for the leptonic channels.

To evaluate the acceptance of the apparatus for a given channel and to compute the size of some of the potential backgrounds, a detailed simulation program DELSIM [5] was used. The event generators BABAMC [6], DYMU3 [7] and KORALZ [8] were used for the final states e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$ respectively. The simulated data were analysed in an identical manner to the real data.

3. $Z^0 \to e^+e^-$ channel

The selection of $e^+e^- \to e^+e^-$ events was made by requiring:

- At least one energy cluster in the HPC with $E > 25$ GeV ($E > 15$ GeV for the $B = 0.7$ T data).
- At least one HPC energy cluster with $E > 10$ GeV in the hemisphere opposite the most energetic one.
- These clusters to be compatible with extrapolated charged tracks reconstructed in the TPC and/or ID.
- The acollinearity angle to be less than 10°.
- The most energetic cluster to be within the angular region $50° < \theta < 130°$.
- A fiducial region cut, applied to the most energetic cluster, to avoid the border regions of the HPC modules.

This gave a total of 263 events. This number of events was corrected by the following factors:

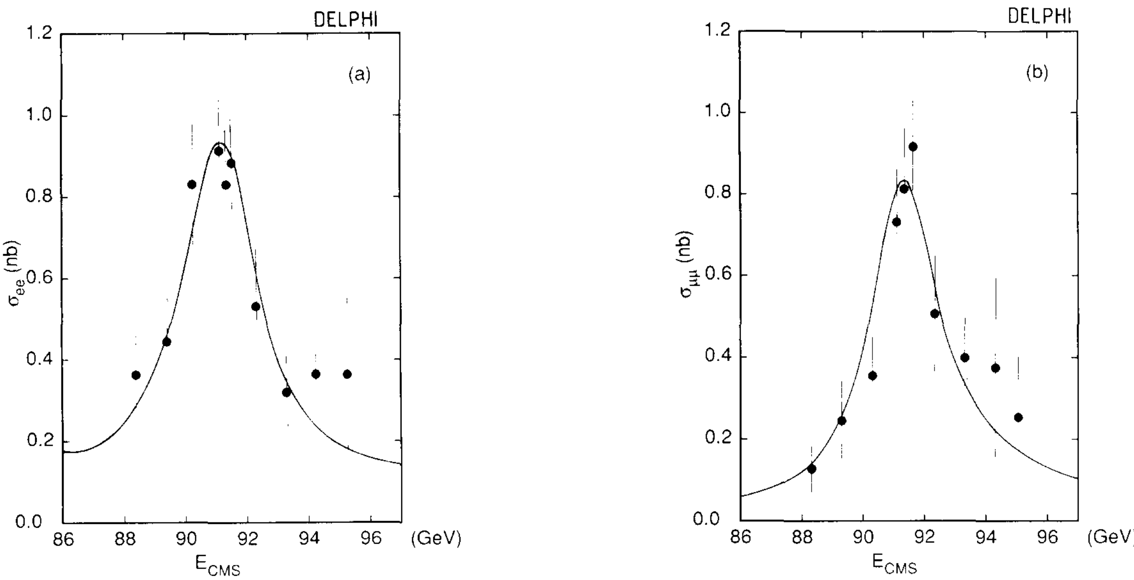
- 0.94±0.02 for the $\tau\tau$ background, estimated by using DELSIM/KORALZ and also from a sample of real $\tau\tau$ events (section 5). Contamination from hadronic and $\gamma\gamma$ events was negligible.
- 1.03±0.01 for the acollinearity cut. This correction, evaluated by simulation, changes slightly with the beam energy.
- 1.02±0.01 for losses due to cuts on energy deposition and track reconstruction, determined by simulation.
- 1.27 for geometric losses due to the fiducial region cut.

For each energy the uncorrected numbers of events within the selected angular region and the corresponding cross sections are given in table 1.

The cross section ratio R_e, corresponding to the s-channel alone, has been determined by correcting for

Table 1
The number of selected events and cross sections for the e^+e^- final state. The cross sections are for the angular interval $50° < \theta < 130°$ and the errors are statistical only. No t-channel subtraction has been made to the cross sections. The corresponding integrated luminosity is 510 nb^{-1}. The overall systematic error on these points is ±4.1%.

$\sqrt{s}$ [GeV]	Number of e^+e^- events	$\sigma(e^+e^-)$ [nb]
88.28	15	0.36±0.10
89.28	16	0.44±0.11
90.28	37	0.83±0.15
91.04	47	0.92±0.13
91.28	43	0.84±0.13
91.54	66	0.87±0.11
92.29	13	0.53±0.15
93.28	12	0.32±0.09
94.28	10	0.37±0.11
95.04	4	0.37±0.18



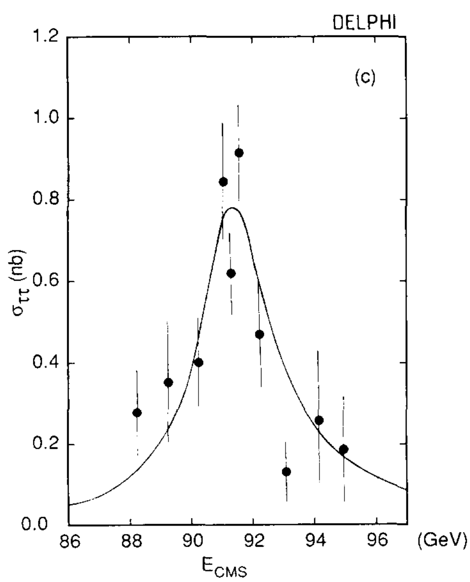


Fig. 1. Cross sections for (a) $e^+e^- \to e^+e^-$, (b) $e^+e^- \to \mu^+\mu^-$ and (c) $e^+e^- \to \tau^+\tau^-$ as a function of the centre of mass energy $\sqrt{s}$ around the Z^0 pole. The curves are fits to the data and are described in the text. All the cross sections are for the angular region $50° < \theta < 130°$. No t-channel subtraction has been made for the cross section for $e^+e^- \to e^+e^-$.

the t-channel exchange contribution using the analytic expression given in ref. [9]. The average correction factor was 0.85. The average cross section ratio, extrapolated to the full (4π) angular range, was determined to be

$$R_e = (5.09 \pm 0.32 \pm 0.18)\% ,$$

where the first error is the statistical error and the second is the estimated systematic error. The systematic error comes from both the uncertainties in the correction factors and from the uncertainty in the determination of the number of hadronic Z^0 decays.

The partial width Γ_e was extracted by fitting the cross section distribution with the same expression

[9]. The contribution of hard photon final states not included in this expression was evaluated by Monte Carlo methods [6]. The mass and the total width of the Z^0 were fixed to the values found by DELPHI [3] from a fit of the hadronic line shape, namely $M_Z = 91.171$ GeV and $\Gamma_Z = 2.511$ GeV. The fit, shown in fig. 1a, gave a value of

$$\Gamma_e = 83.2 \pm 3.0 \pm 2.4 \text{ MeV} .$$

Due to the small statistics at some energy points the fit was made using a maximum likelihood method and Poisson statistics. The systematic error includes the uncertainties in the values of M_Z (± 0.030 GeV statistical error and ± 0.030 GeV due to the beam energy uncertainty) and Γ_Z (± 0.065 GeV statistical error) given above.

In order to estimate the size of the theoretical uncertainties in extracting Γ_e, a fit was also made to the cross section data corrected for the t-channel contribution. This fit was performed in the same way as that for the $\mu\mu$ and $\tau\tau$ channels using the method of Bardin et al. and the program ZCUTCOS [10]. The value found for Γ_e is about 3 MeV higher than that quoted above.

4. $Z^0 \to \mu^+\mu^-$ channel

An initial selection of $e^+e^- \to \mu^+\mu^-$ candidates was made by requiring two charged tracks with:

- The momentum of each track greater than 15 GeV/c (10 GeV/c for the $B = 0.7$ T data).
- At least one track in the angular range $50° < \theta < 130°$.
- The acollinearity angle less than 10°.

Cosmic rays, identified as being out of time with respect to the beam cross over by timing measurements using the OD or TOF, were removed.

Two methods were used for muon identification:

(a) The association of at least one hit in the MUB with the extrapolated charged track.

(b) The association of a non-showering track in the HPC (less than 20 hits) with the extrapolated charged track.

For each of the two methods, the identification efficiency for a single muon was established from the data using event samples where both tracks had been identified as muons using the other method. This efficiency, after a small correction for the $\tau\tau$ background determined from simulation for each method, was found to be $95 \pm 2\%$ for method (a) (consistent with the MUB dead space) and $98 \pm 1\%$ for method (b).

The $\mu^+\mu^-$ events were selected by requiring each track to be identified by either method (a) or method (b). This gave a total sample of 195 events with an overall identification efficiency of $99.8 \pm 0.2\%$. A sample of these events was checked for consistency with two minimum ionising particles traversing the HAC.

The number of events was corrected by the following factors:

- 0.96 ± 0.01 for the $\tau^+\tau^-$ background, estimated using DELSIM/KORALZ. This was cross-checked using the HAC data. The e^+e^- background was negligible.
- 1.07 ± 0.02 for the loss of tracks in the dead space of the TPC.
- 1.03 ± 0.02 for the trigger efficiency.
- 1.03 ± 0.01 for the acollinearity and momentum cuts.

For each energy, the uncorrected numbers of events within the selected angular region and the corresponding cross sections are given in table 2. Taking into account the systematic errors discussed above, the cross section ratio corresponding to the full angular range was determined to be

Table 2
The number of selected events and cross sections for the $\mu^+\mu^-$ final state. The cross sections are for the angular interval $50° < \theta < 130°$ and the errors are statistical only. The corresponding integrated luminosity is 390 nb^{-1}. The overall systematic error on these points is $\pm 4.1\%$.

$\sqrt{s}$ [GeV]	Number of $\mu^+\mu^-$ events	$\sigma(\mu^+\mu^-)$ [nb]
88.28	5	0.13 ± 0.06
89.28	7	0.24 ± 0.09
90.28	14	0.35 ± 0.09
91.04	30	0.72 ± 0.13
91.28	33	0.80 ± 0.14
91.54	70	0.90 ± 0.11
92.29	13	0.50 ± 0.14
93.28	17	0.39 ± 0.09
94.28	3	0.37 ± 0.21
95.04	3	0.25 ± 0.15

$R_\mu = (4.96 \pm 0.35 \pm 0.17)\%$.

A fit, shown in fig. 1b, to the $\mu^+\mu^-$ cross section, using the method of Bardin et al. and the program ZCUTCOS [10] with values for M_Z and Γ_Z given above, gave the result

$(\Gamma_e \Gamma_\mu)^{1/2} = 84.6 \pm 3.0 \pm 2.4$ MeV .

The systematic error includes the uncertainties in the values of M_Z and Γ_Z given above.

5. $Z^0 \to \tau^+\tau^-$ channel

The analysis of the $e^+e^- \to \tau^+\tau^-$ channel was restricted to events where one of the taus decayed into a single isolated charged particle and the other decayed into n charged particles where n varied from 1 to 5 (referred to below as topology 1–n). The requirement of an isolated track was imposed to minimise the background from $q\bar{q}$ events.

The selection of candidates for these topologies was made requiring:
- The total visible energy was greater than 8 GeV.
- Only tracks with momentum greater than 1 GeV/c were considered.
- The isolated charged track was identified by requiring the angle to the nearest track to be greater than 150°.
- The polar angles of the isolated track and at least one track in the opposite hemisphere to be in the angular range $50° < \theta < 130°$.

For topology 1–1, expected to be the most abundant, events were selected if:
- The total charge was zero.
- The electromagnetic energy was less than 30 GeV on each side to reduce the background from e^+e^- events.
- Either the total electromagnetic energy deposition was greater than 3 GeV or there were no hits associated in the MUB with the extrapolated track(s) on at least one side. These selection criteria reduce the $\mu^+\mu^-$ background.

After this selection the background from e^+e^- and $\mu^+\mu^-$ events was estimated to be ~30%. This was reduced by two orders of magnitude using the following topological properties of $\tau^+\tau^-$ events:
- The acoplanarity angle > 1° or the acollinearity angle > 3°.
- The sum of the absolute values of momenta of the tracks was less than 60 GeV/c.

For topologies 1–n, where $n > 2$, no further selection was required after the polar angle and isolation cuts had been made as described earlier. From Monte Carlo studies the contamination in these topologies from hadronic events was estimated to be about 6%. The losses of $\tau^+\tau^-$ events due to γ conversion and secondary interactions producing topologies with $n > 5$ were estimated to be also about 6%. As a result no additional corrections were made for these effects, but a systematic uncertainty of 6% in the selection procedure has been included for this topology.

Events of topology 1–2 can arise from pattern recognition failure causing the loss of a track in 1–3 topology events or from the inclusion of a spurious track to events of the 1–1 topology. The identification procedure adopted was as that for the 1–1 topology.

These selection procedures gave a total of 158 events, of which 84 were in the 1–1 topology, 28 in the 1–2 and 46 in the 1–n ($n > 2$). For all topologies the $\gamma\gamma$ background was calculated to be negligible.

The efficiency of the selection procedures has been determined to be $64.5 \pm 3.6\%$ in the selected polar angular region using DELSIM/KORALZ. The division of the simulated events between the various topologies agreed, within the statistics, with the data. The systematic error has been determined by varying the different cuts and studying the stability of the results.

For each energy, the uncorrected numbers of events within the selected angular region and the corresponding cross sections are given in table 3. Taking into account the systematic errors discussed above, the cross section ratio corresponding to the full angular range was determined to be

$R_\tau = (4.72 \pm 0.38 \pm 0.29)\%$.

A fit, shown in fig. 1c, to the $\tau^+\tau^-$ cross section, using the method of Bardin et al. and the program ZCUTCOS [10] with values for M_Z and Γ_Z given above, gave the result

$(\Gamma_e \Gamma_\tau)^{1/2} = 82.6 \pm 3.3 \pm 3.2$ MeV .

The systematic error includes the uncertainties in the values of M_Z and Γ_Z given above.

Table 3
The number of selected events and cross sections for the $\tau^+\tau^-$ final state. The cross sections are for the angular interval $50° < \theta < 130°$ and the errors are statistical only. The corresponding integrated luminosity is 457 nb^{-1}. The overall systematic error on these points is ±6.5%.

$\sqrt{s}$ [GeV]	Number of $\tau^+\tau^-$ events	$\sigma(\tau^+\tau^-)$ [nb]
88.28	7	0.27±0.10
89.28	5	0.36±0.16
90.28	12	0.41±0.12
91.04	34	0.85±0.15
91.28	34	0.62±0.11
91.54	47	0.93±0.14
92.29	12	0.47±0.14
93.28	3	0.12±0.07
94.28	2	0.25±0.18
95.04	2	0.18±0.13

6. Discussion and summary

The results for the cross section ratios R_e, R_μ and R_τ are in agreement with each other. The partial widths for the three leptonic channels are also in good agreement. Thus these results are in agreement with the hypothesis of lepton universality and test it at values of $s \sim 8300$ GeV^2.

The results of the cross section ratios for the three leptonic channels have been combined by taking a weighted average to give a value

$$R_\ell = (4.96 \pm 0.20 \pm 0.12)\% .$$

From this value the ratio of the partial widths Γ_ℓ/Γ_h has been obtained by subtracting the estimated s-channel photon contributions. The result is

$$\Gamma_\ell/\Gamma_h = (4.89 \pm 0.20 \pm 0.12) \times 10^{-2} .$$

The leptonic widths from the three channels have also been combined, giving

$$\Gamma_\ell = (83.6 \pm 1.8 \pm 2.2) \text{ MeV} .$$

These results are also compatible with the values expected from the standard model. Taking the QCD coupling constant to be $\alpha_s = 0.12$ and the mass of the Higgs scalar to be 100 GeV/c^2, the predicted standard model values are $\Gamma_\ell = 83.4$ MeV for a top quark mass of $m_t = 90$ GeV/c^2 and $\Gamma_\ell = 84.6$ for $m_t = 220$ GeV/c^2. The predicted value of $\Gamma_\ell/\Gamma_h = 4.81\%$ with a variation of less than 0.01% for top quark masses in the range $90 \leqslant m_t \leqslant 220$ GeV/c^2.

The results of the above measurements can also be used to estimate the number of light neutrino species. Assuming lepton universality the total width of the Z^0 can be written as $\Gamma_Z = \Gamma_h + 3\Gamma_\ell + \Gamma_{inv}$, where Γ_h and Γ_{inv} are the hadronic and invisible widths respectively. This can be rearranged as

$$\frac{\Gamma_{inv}}{\Gamma_\ell} = \frac{1}{B_\ell} - \frac{\Gamma_h}{\Gamma_\ell} - 3 ,$$

where $B_\ell = \Gamma_\ell/\Gamma_Z$ is the leptonic branching ratio. It can be seen from the expression for the cross section that the leptonic cross section at the Z^0 pole is proportional to B_ℓ^2. The μμ final state is used to estimate Γ_{inv}/Γ_ℓ as this channel is free from uncertainties of t-channel subtraction and has smaller errors than the ττ channel. From the μμ cross section data and the measured value of R_μ this gives

$$\frac{\Gamma_{inv}}{\Gamma_\ell} = 6.22 \pm 0.47 \pm 0.50 .$$

The small statistical error is a consequence of the strong (anti)correlation between R_ℓ and B_ℓ as stressed by Feldman [11].

Assuming only the standard model for the ratio $\Gamma_\nu/\Gamma_\ell = 1.993 \pm 0.001$ one obtains for the number of ν families

$$N_\nu = 3.12 \pm 0.24 \pm 0.25 .$$

The main source of systematic error is the uncertainty on the luminosity.

In conclusion, these results are in agreement with the predictions of the standard model. The results are also in agreement with the results of other experiments at LEP and at SLC [12].

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