Experimental study of the triple-gluon vertex

DELPHI Collaboration

P. Abreu [a], W. Adam [b], F. Adami [c], T. Adye [d], T. Akesson [e], G.D. Alekseev [f], P. Allen [g], S. Almehed [e], F. Alted [g], S.J. Alvsvaag [h], U. Amaldi [i], E. Anassontzis [j], W.-D. Apel [k], B. Asman [ℓ], P. Astier [m], J.-E. Augustin [n], A. Augustinus [i], P. Baillon [i], P. Bambade [n], F. Barao [a], G. Barbiellini [o], D.Y. Bardin [f], A. Baroncelli [p], O. Barring [e], W. Bartl [b], M.J. Bates [q], M. Baubillier [m], K.-H. Becks [r], C.J. Beeston [q], M. Begalli [s], P. Beilliere [t], I. Belokopytov [u], P. Beltran [v], D. Benedic [w], J.M. Benlloch [g], M. Berggren [ℓ], D. Bertrand [x], S. Biagi [y], F. Bianchi [z], J.H. Bibby [q], M.S. Bilenky [f], P. Billoir [m], J. Bjarne [e], D. Bloch [w], P.N. Bogolubov [f], D. Bollini [aa], T. Bolognese [c], M. Bonapart [ab], M. Bonesini [ac], P.S.L. Booth [y], M. Boratav [m], P. Borgeaud [c], H. Borner [q], C. Bosio [p], O. Botner [ad], B. Bouquet [n], M. Bozzo [s], S. Braibant [i], P. Branchini [p], K.D. Brand [r], R.A. Brenner [ae], C. Bricman [x], R.C.A. Brown [i], N. Brummer [ab], J.-M. Brunet [t], L. Bugge [af], T. Buran [af], H. Burmeister [i], J.A.M.A. Buytaert [x], M. Caccia [ac], M. Calvi [ac], A.J. Camacho Rozas [ag], J.-E. Campagne [i], A. Campion [y], T. Camporesi [i], V. Canale [p], F. Cao [x], L. Carroll [y], C. Caso [s], E. Castelli [o], M.V. Castillo Gimenez [g], A. Cattai [i], F.R. Cavallo [aa], L. Cerrito [p], P. Charpentier [i], P. Checchia [ah], G.A. Chelkov [f], L. Chevalier [c], P. Chliapnikov [u], V. Chorowicz [m], R. Cirio [z], M.P. Clara [z], J.L. Contreras [g], R. Contri [s], G. Cosme [n], F. Couchot [n], H.B. Crawley [ai], D. Crennell [d], G. Crosetti [s], N. Crosland [q], M. Crozon [t], J. Cuevas Maestro [ag], S. Czellar [ae], S. Dagoret [n], E. Dahl-Jensen [aj], B. Dalmagne [n], M. Dam [i], G. Damgaard [aj], G. Darbo [s], E. Daubie [x], P.D. Dauncey [q], M. Davenport [i], P. David [m], A. De Angelis [o], M. De Beer [c], H. De Boeck [x], W. De Boer [k], C. De Clercq [x], M.D.M. De Fez Laso [g], N. De Groot [ab], C. De La Vaissiere [m], B. De Lotto [o], C. Defoix [t], D. Delikaris [i], S. Delorme [i], P. Delpierre [t], N. Demaria [z], L. Di Ciaccio [p], H. Dijkstra [i], F. Djama [w], J. Dolbeau [t], O. Doll [r], M. Donszelmann [ab], K. Doroba [ak], M. Dracos [i], J. Drees [r], M. Dris [aℓ], W. Dulinski [w], R. Dzhelyadin [u], L.-O. Eek [ad], P.A.-M. Eerola [ae], T. Ekelof [ad], G. Ekspong [ℓ], J.-P. Engel [w], V. Falaleev [u], D. Fassouliotis [aℓ], A. Fenyuk [u], M. Fernandez Alonso [ag], A. Ferrer [g], T.A. Filippas [aℓ], A. Firestone [ai], H. Foeth [i], E. Fokitis [aℓ], P. Folegati [o], F. Fontanelli [s], H. Forsbach [r], B. Franek [d], K.E. Fransson [ad], P. Frenkiel [t], D.C. Fries [k], A.G. Frodesen [h], R. Fruhwirth [b], F. Fulda-Quenzer [n], K. Furnival [y], H. Furstenau [k], J. Fuster [i], J.M. Gago [a], G. Galeazzi [ah], D. Gamba [z], J. Garcia [ag], U. Gasparini [ah], P. Gavillet [i], E.N. Gazis [aℓ], J.-P. Gerber [w], P. Giacomelli [aa], K.-W. Glitza [r], R. Gokieli [m], V.M. Golovatyuk [f], J.J. Gomez Y Cadenas [i], A. Goobar [ℓ], G. Gopal [d], M. Gorski [ak], V. Gracco [s], A. Grant [i], F. Grard [x], E. Graziani [p], I.A. Gritsaenko [u], M.-H. Gros [n], G. Grosdidier [n], B. Grossetete [m], S. Gumenyuk [u], J. Guy [d], F. Hahn [r], M. Hahn [k], S. Haider [ab], Z. Hajduk [ab], A. Hakansson [e], A. Hallgren [ad], K. Hamacher [r], G. Hamel De Monchenault [c], F.J. Harris [q], B.W. Heck [i], I. Herbst [r], J.J. Hernandez [g], P. Herquet [x], H. Herr [i], E. Higon [g], H.J. Hilke [i], S.D. Hodgson [q], T. Hofmokl [ak], R. Holmes [ai], S.-O. Holmgren [ℓ], D. Holthuizen [ab], J.E. Hooper [aj], M. Houlden [y], J. Hrubec [b], P.O. Hulth [ℓ], K. Hultqvist [ℓ], D. Husson [w], B.D. Hyams [i], P. Ioannou [j], P.-S. Iversen [h], J.N. Jackson [y], P. Jalocha [am], G. Jarlskog [e], P. Jarry [c], B. Jean-Marie [n], E.K. Johansson [ℓ], D. Johnson [y], M. Jonker [i], L. Jonsson [e], P. Juillot [w], R.B. Kadyrov [f], G. Kalkanis [j], G. Kalmus [d], G. Kantardjian [i], F. Kapusta [m], P. Kapusta [am], S. Katsanevas [j], E.C. Katsoufis [aℓ], R. Keranen [ae], J. Kesteman [x], B.A. Khomenko [f], B. King [y], H. Klein [i],

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W. Klempt [i], A. Klovning [h], P. Kluit [x], J.H. Koehne [k], B. Koene [ab], P. Kokkinias [v], M. Kopf [k], M. Koratzinos [i], K. Korcyl [am], A.V. Korytov [f], B. Korzen [i], M. Kostrikov [u], C. Kourkoumelis [j], T. Kreuzberger [b], J. Krolikowski [ak], U. Kruener-Marquis [r], W. Krupinski [am], W. Kucewicz [ac], K. Kurvinen [ae], M.I. Laakso [ae], C. Lambropoulos [v], J.W. Lamsa [ai], L. Lanceri [o], V. Lapchine [u], V. Lapin [u], J.-P. Laugier [c], R. Lauhakangas [ae], P. Laurikainen [ae], G. Leder [b], F. Ledroit [t], J. Lemonne [x], G. Lenzen [r], V. Lepeltier [n], A. Letessier-Selvon [m], E. Lieb [r], E. Lillethun [h], J. Lindgren [ae], I. Lippi [ah], R. Llosa [g], B. Loerstad [e], M. Lokajicek [f], J.G. Loken [q], M.A. Lopez Aguera [ag], A. Lopez-Fernandez [n], M. Los [ab], D. Loukas [v], A. Lounis [w], J.J. Lozano [g], R. Lucock [d], P. Lutz [t], L. Lyons [q], G. Maehlum [i], N. Magnussen [r], J. Maillard [t], A. Maltezos [v], S. Maltezos [aℓ], F. Mandl [b], J. Marco [ag], M. Margoni [ah], J.-C. Marin [i], A. Markou [v], L. Mathis [t], F. Matorras [ag], C. Matteuzzi [ac], G. Matthiae [p], M. Mazzucato [ah], M. Mc Cubbin [y], R. Mc Kay [ai], R. Mc Nulty [y], E. Menichetti [z], C. Meroni [ac], W.T. Meyer [ai], W.A. Mitaroff [b], G.V. Mitselmakher [f], U. Mjoernmark [e], T. Moa [ℓ], R. Moeller [aj], K. Moenig [r], M.R. Monge [s], P. Morettini [s], H. Mueller [k], H. Muller [i], G. Myatt [q], F. Naraghi [m], U. Nau-Korzen [r], F.L. Navarria [aa], P. Negri [ac], B.S. Nielsen [aj], B. Nijjhar [y], V. Nikolaenko [u], V. Obraztsov [u], R. Orava [ae], A. Ouraou [c], R. Pain [m], H. Palka [ab], T. Papadopoulou [aℓ], L. Pape [i], A. Passeri [p], M. Pegoraro [ah], V. Perevozchikov [u], M. Pernicka [b], A. Perrotta [aa], M. Pimenta [a], O. Pingot [x], A. Pinsent [q], M.E. Pol [a], G. Polok [am], P. Poropat [o], P. Privitera [aa], A. Pullia [ac], J. Pyyhtia [ae], A.A. Rademakers [ab], D. Radojicic [q], S. Ragazzi [ac], W.H. Range [y], P.N. Ratoff [q], A.L. Read [af], N.G. Redaelli [ac], M. Regler [b], D. Reid [y], P.B. Renton [q], L.K. Resvanis [j], F. Richard [n], M. Richardson [y], J. Ridky [f], G. Rinaudo [z], I. Roditi [i], A. Romero [z], P. Ronchese [ah], E.I. Rosenberg [ai], U. Rossi [aa], E. Rosso [i], P. Roudeau [n], T. Rovelli [aa], W. Ruckstuhl [ab], V. Ruhlmann [c], A. Ruiz [ag], H. Saarikko [ae], Y. Sacquin [c], E. Sanchez [g], J. Sanchez [g], M. Sannino [s], M. Schaeffer [w], H. Schneider [k], F. Scuri [o], A.M. Segar [q], R. Sekulin [d], M. Sessa [o], G. Sette [s], R. Seufert [k], R.C. Shellard [a], P. Siegrist [c], S. Simonetti [s], F. Simonetto [ah], A.N. Sissakian [f], T.B. Skaali [af], J. Skeens [ai], G. Skjevling [af], G.R. Smith [d], R. Sosnowski [ak], T.S. Spassoff [f], E. Spiriti [p], S. Squarcia [s], H. Staeck [r], C. Stanescu [p], G. Stavropoulos [v], F. Stichelbaut [x], A. Stocchi [ac], J. Strauss [b], R. Strub [w], C.J. Stubenrauch [i], M. Szczekowski [ak], M. Szeptycka [ak], P. Szymanski [ak], S. Tavernier [x], G. Theodosiou [v], A. Tilquin [t], J. Timmermans [ab], V.G. Timofeev [f], L.G. Tkatchev [f], D.Z. Toet [ab], A.K. Topphol [h], L. Tortora [p], M.T. Trainor [q], D. Treille [i], U. Trevisan [s], W. Trischuk [i], G. Tristram [t], C. Troncon [ac], A. Tsirou [i], E.N. Tsyganov [f], M. Turala [am], R. Turchetta [w], M.-L. Turluer [c], T. Tuuva [ae], I.A. Tyapkin [f], M. Tyndel [d], S. Tzamarias [i], F. Udo [ab], S. Ueberschaer [r], V.A. Uvarov [u], G. Valenti [aa], E. Vallazza [z], J.A. Valls Ferrer [g], G.W. Van Apeldoorn [ab], P. Van Dam [ab], W.K. Van Doninck [x], N. Van Eijndhoven [i], C. Vander Velde [x], J. Varela [a], P. Vaz [a], G. Vegni [ac], J. Velasco [g], L. Ventura [ah], W. Venus [d], F. Verbeure [x], L.S. Vertogradov [f], L. Vibert [m], D. Vilanova [c], E.V. Vlasov [u], A.S. Vodopyanov [f], M. Vollmer [r], G. Voulgaris [j], M. Voutilainen [ae], V. Vrba [p], H. Wahlen [r], C. Walck [ℓ], F. Waldner [o], M. Wayne [ai], A. Wehr [r], P. Weilhammer [i], J. Werner [r], A.M. Wetherell [i], J.H. Wickens [x], J. Wikne [af], G.R. Wilkinson [q], W.S.C. Williams [q], M. Winter [w], D. Wormald [af], G. Wormser [n], K. Woschnagg [ad], N. Yamdagni [ℓ], P. Yepes [ab], A. Zaitsev [u], A. Zalewska [am], P. Zalewski [ak], P.I. Zarubin [f], E. Zevgolatakos [v], G. Zhang [r], N.I. Zimin [f], R. Zitoun [m], R. Zukanovich Funchal [t], G. Zumerle [ah] and J. Zuniga [g]

[a] *LIP, Av. Elias Garcia 14 - 1e, P-1000 Lisbon Codex, Portugal*
[b] *Institut für Hochenergiephysik, Öesterreichische Akademie der Wissenschaften, Nikolsdorfergasse 18, A-1050 Vienna, Austria*
[c] *DPhPE, CEN-Saclay, F-91191 Gif-sur-Yvette Cedex, France*
[d] *Rutherford Appleton Laboratory, Chilton, Didcot OX11 0QX, UK*
[e] *Department of Physics, University of Lund, Sölvegatan 14, S-22363 Lund, Sweden*
[f] *Joint Institute for Nuclear Research, Dubna, Head Post Office, P.O. Box 79, SU-101 000 Moscow, USSR*
[g] *Instituto de Fisica Corpuscular (IFIC), Centro Mixto Universidad de Valencia–CSIC, Avda. Dr. Moliner 50, E-46100 Burjassot (Valencia), Spain*
[h] *Department of Physics, University of Bergen, Allégaten 55, N-5007 Bergen, Norway*
[i] *CERN, CH-1211 Geneva 23, Switzerland*
[j] *Physics Laboratory, University of Athens, Solonos Street 104, GR-10680 Athens, Greece*
[k] *Institut für Experimentelle Kernphysik, Universität Karlsruhe, Postfach 6980, W-7500 Karlsruhe 1, FRG*
[ℓ] *Institute of Physics, University of Stockholm, Vanadisvägen 9, S-113 46 Stockholm, Sweden*
[m] *LPNHE, Universités Paris VI et VII, Tour 33 (RdC), 4 place Jussieu, F-75230 Paris Cedex 05, France*
[n] *Laboratoire de l'Accélérateur Linéaire, Université de Paris-Sud, Bâtiment 200, F-91405 Orsay, France*
[o] *Dipartimento di Fisica, Università di Trieste and INFN, Via A. Valerio 2, I-34127 Trieste, Italy and Istituto di Fisica, Università di Udine, I-33100 Udine, Italy*
[p] *Istituto Superiore di Sanità, Istituto Nazionale di Fisica Nucleare (INFN), Viale Regina Elena 299, I-00161 Rome, Italy and Dipartimento di Fisica, Università di Roma II and INFN, Tor Vergata, I-00173 Rome, Italy*
[q] *Nuclear Physics Laboratory, University of Oxford, Keble Road, Oxford OX1 3RH, UK*
[r] *Fachbereich Physik, University of Wuppertal, Postfach 100 127, W-5600 Wuppertal 1, FRG*
[s] *Dipartimento di Fisica, Università di Genova and INFN, Via Dodecaneso 33, I-16146 Genoa, Italy*
[t] *Laboratoire de Physique Corpusculaire, Collège de France, 11 pl. M. Berthelot, F-75231 Paris Cedex 05, France*
[u] *Institute for High Energy Physics, Serpukow, P.O. Box 35, SU-142 284 Protvino (Moscow Region), USSR*
[v] *Institute of Nuclear Physics, N.R.C. Demokritos, P.O. Box 60628, GR-15310 Athens, Greece*
[w] *Division des Hautes Energies, CRN–Groupe DELPHI and LEPSI, B.P. 20 CRO, F-67037 Strasbourg Cedex, France*
[x] *Physics Department, Universitaire Instelling Antwerpen, Universiteitsplein 1, B-2610 Wilrijk, Belgium and IIHE, ULB–VUB, Pleinlaan 2, B-1050 Brussels, Belgium and Service de Physique des Particules Elémentaires, Faculté des Sciences, Université de l'Etat Mons, Av. Maistriau 19, B-7000 Mons, Belgium*
[y] *Department of Physics, University of Liverpool, P.O. Box 147, Liverpool L69 3BX, UK*
[z] *Dipartimento di Fisica Sperimentale, Università di Torino and INFN, Via P. Giuria 1, I-10125 Turin, Italy*
[aa] *Dipartimento di Fisica, Università di Bologna and INFN, Via Irnerio 46, I-40126 Bologna, Italy*
[ab] *NIKHEF-H, Postbus 41882, NL-1009 DB Amsterdam, The Netherlands*
[ac] *Dipartimento di Fisica, Università di Milano and INFN, Via Celoria 16, I-20133 Milan, Italy*
[ad] *Department of Radiation Sciences, University of Uppsala, P.O. Box 535, S-751 21 Uppsala, Sweden*
[ae] *Department of High Energy Physics, University of Helsinki, Siltavuorenpenger 20 C, SF-00170 Helsinki 17, Finland*
[af] *Physics Department, University of Oslo, Blindern, N-1000 Oslo 3, Norway*
[ag] *Facultad de Ciencias, Universidad de Santander, Av. de los Castros, E-39005 Santander, Spain*
[ah] *Dipartimento di Fisica, Università di Padova and INFN, Via Marzolo 8, I-35131 Padua, Italy*
[ai] *Ames Laboratory and Department of Physics, Iowa State University, Ames IA 50011, USA*
[aj] *Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*
[ak] *Institute for Nuclear Studies, and University of Warsaw, Ul. Hoża 69, PL-00681 Warsaw, Poland*
[aℓ] *Physics Department, National Technical University, Zografou Campus, GR-15773 Athens, Greece*
[am] *High Energy Physics Laboratory, Institute of Nuclear Physics, Ul. Kawiory 26 a, PL-30055 Cracow 30, Poland*

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In four-jet events from $e^+e^- \to Z^0 \to$ multihadrons one can separate the three principal contributions from the triple-gluon vertex, double gluon-bremsstrahlung and the secondary quark–antiquark production, using the shape of the two-dimensional angular distributions in the generalized Nachtmann–Reiter angle θ^*_{NR} and the opening angle of the secondary jets. Thus one can identify directly the contribution from the triple-gluon vertex without comparison with a specific non-QCD model. Applying this new method to events taken with the DELPHI-detector we get for the ratio of the colour factor N_C to the fermionic Casimir operator C_F: $N_C/C_F = 2.55 \pm 0.55(\text{stat.}) \pm 0.4(\text{fragm.} + \text{models}) \pm 0.2(\text{error in bias})$ in agreement with the value 2.25 expected in QCD from $N_C = 3$ and $C_F = \frac{4}{3}$.

1. Introduction

The self-coupling of the gluons is an essential feature of quantum chromodynamics (QCD). It originates from the colour charges of the gluons and is a direct consequence of the non-abelian nature of this gauge theory. The confinement of the quarks is hardly understandable without the existence of the self-coupling of the gluons and a colourless gluon would lead to the reaction $\Upsilon \to 2$ jets [1] which is not observed [2]. The large two-jet rate for medium jet energies at hadron colliders can be considered as direct evidence for gluon–gluon scattering [3], if one accepts the extrapolation of the gluon structure function of the proton from deep-inelastic νN-scattering to collider energies. Notwithstanding these considerations an experiment testing for the presence of the triple-gluon vertex and measuring its strength in the clean conditions of multihadronic events from Z^0-decays at LEP constitutes an important test of QCD. The triple-gluon vertex in e^+e^- annihilation enters in second and higher orders of the strong coupling constant. The principal second order contributions: double-bremsstrahlung, triple-gluon vertex and secondary q$\bar{\text{q}}$-production yield four-parton final states. Thus testing the triple-gluon vertex requires a study of four-jet events. With the four jets ordered according to energy jet 3 and jet 4 correspond preferentially to the secondary partons. The diagrams that yield four-parton final states are shown in fig. 1.

Several observables [4–8] have been proposed to distinguish QCD from a QED-like abelian theory [4] in which the gluon is colourless but the threefold colour of the quarks is retained. In the abelian theory the triple-gluon vertex is absent and secondary quark–antiquark production is eight times higher than in QCD. With the redefinition of the strong coupling constant $\alpha_{\text{Abel}} = \frac{4}{3}\alpha_{\text{QCD}}$ this abelian theory gives in first order the same three-jet rate as QCD and also the second order four-jet rate is approximately the same. Studying the angular distributions of four-jet events the AMY [9], OPAL [10], L3 [11] and VENUS [12] Collaborations have published evidence against this QED-like abelian theory of the strong interaction. The abelian theory is disproved by its much higher secondary q$\bar{\text{q}}$-production since the distributions considered distinguish well between the contribution from secondary quark–antiquark production and that from two gluons due to the different helicity structure. More generally two experiments [10,12] have provided limits on the relative contribution from secondary q$\bar{\text{q}}$-production. However, these results do not give evidence for the existence of the triple-gluon vertex since the one-dimensional angular distribu-

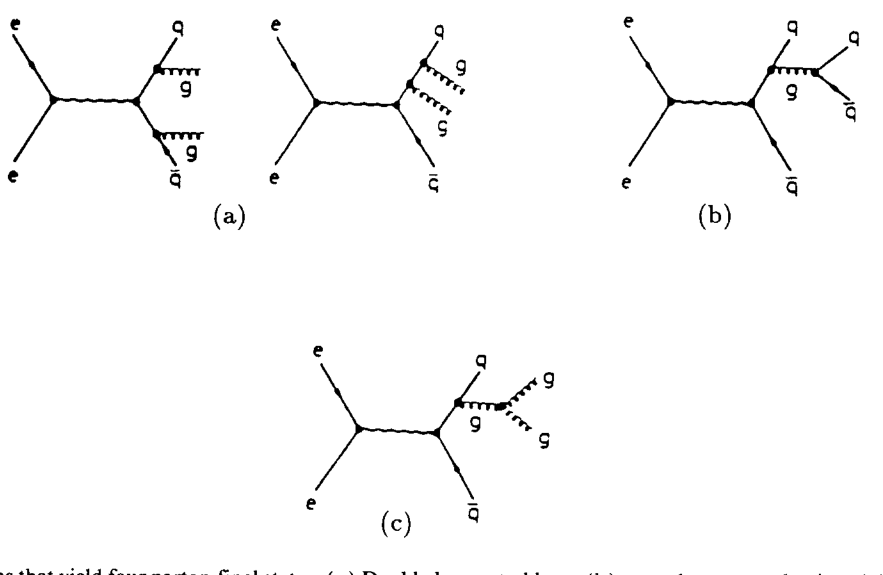

Fig. 1. Diagrams that yield four parton-final states: (a) Double-bremsstrahlung; (b) secondary q$\bar{\text{q}}$-production; (c) triple-gluon vertex.

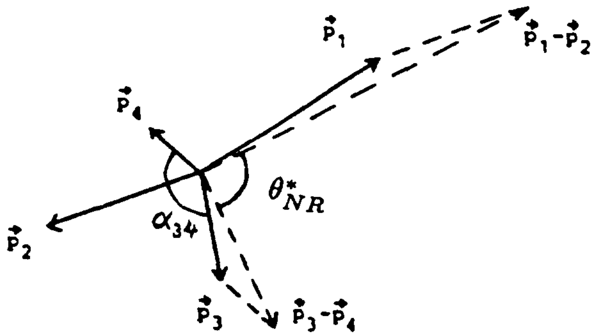

Fig. 2. Definition of the generalised Nachtmann–Reiter angle $\theta^*_{\rm NR}$ in terms of the jet momentum vectors $\boldsymbol{p}_j$ and of the angle α_{34} used in this analysis to distinguish the triple-gluon vertex contribution from that due to double-bremsstrahlung.

tions they studied do not distinguish between the contribution from the triple-gluon vertex and double-bremsstrahlung.

As in the other publications [9–12] we use the generalised Nachtmann–Reiter angle $\theta^*_{\rm NR}$, proposed by Rudolph [8], which is defined as the angle between the two-jet momentum vector differences $(\boldsymbol{p}_1-\boldsymbol{p}_2)$ and $(\boldsymbol{p}_3-\boldsymbol{p}_4)$. Then we use as an additional observable the angle between the secondary jets α_{34} which does distinguish between triple-gluon vertex and double-bremsstrahlung. The two observables of our analysis are illustrated in fig. 2. With the two-dimensional distribution in these two observables one can determine the contribution from the triple-gluon vertex directly.

2. Method

Ellis et al. [13] have calculated the differential cross sections for the production of the four-parton final states in order α_s^2. In figs. 6 and 8 of their paper all topologically distinct graphs for the transition probabilities are shown. For the $q\bar{q}gg$ final state there are 36 contributions which can be grouped into three classes:

(A) planar double-bremsstrahlung graphs with group weight C_F^2;

(B) non-planar double-bremsstrahlung graphs with group weight $C_F(C_F-\frac{1}{2}N_C)$;

(C) graphs involving the triple-gluon vertex with group weight C_FN_C.

Similarly the 36 contributions for $q\bar{q}q\bar{q}$ fall into the classes:

(D) planar graphs with group weight C_FT_R;

(E) non-planar graphs with group weight $C_F(C_F-\frac{1}{2}N_C)$;

(F) graphs with group weight C_F, which give contributions only if the charge of the partons is determined experimentally and are not relevant to this analysis.

For QCD the fermionic Casimir operator is $C_F=\frac{4}{3}$, the number of colours $N_C=3$ and $T_R=\frac{1}{2}n_f$ where n_f stands for the number of quark flavours. For the abelian theory the values are $C_F=1$, $N_C=0$, $T_R=3n_f$.

Since the grouping of the graphs is done in a gauge invariant way one can determine the individual contributions from these classes, and use their relative weights as a test of QCD. We take N_C/C_F and T_R/C_F as free variables which are determined in the analysis by matching the resulting two-dimensional distribution in $|\cos\theta^*_{\rm NR}|$ and $\cos\alpha_{34}$ to our data.

For hard jets, as they are used here for finding the triple-gluon vertex, the use of the second order matrix elements is in principle more thrustworthy than the use of the parton-shower models, see Bengtson [14]. The two approaches are also discussed by Bethke et al. [15].

3. Data-handling

The analysis is based on 21 024 multihadron events from e^+e^--annihilations at CMS energies around the Z^0 resonance. Only tracks from charged particles are used. The same cuts are applied for event selection as in our earlier study [16] of hadronic decays of the Z^0. The most important of these selections are that tracks are kept only if the measured track length is greater than 50 cm and their polar angle is between 25° and 155°. Furthermore, for all events the polar angle θ of the sphericity axis has to be between 40° and 140°. In addition we require that the total momentum imbalance is below 20 GeV/c.

Jets are defined with the algorithm LUCLUS provided with the LUND Monte Carlo program [17] [#1], called JETSET. In this algorithm two jets with momenta p_1, p_2 and opening angle α_{12} are merged together if

$$2\frac{p_1 \times p_2}{p_1 + p_2} \times \sin \tfrac{1}{2}\alpha_{12} \leqslant \text{djoin} .$$

Each time two jets are merged, all particles are reassigned to the nearest jet (in terms of the same distance as above). This procedure is repeated until a stable configuration is reached. The jet resolution parameter djoin is set to 5 GeV E_{vis}/E_{cms} where E_{vis} is the sum of the energies of the accepted charged particles and $E_{cms} = \sqrt{s}$. For the four-jet sample we require $E_{vis} > 0.35\,E_{cms}$. This yields 884 four-jet events.

We have generated 40 000 four-parton events with JETSET 7.2 with y_{cut}, the minimum invariant mass-squared of any two partons scaled by E^2_{cms}, set to 0.010. We passed these events through the full simulation of the DELPHI detector and the DELPHI analysis chain. In JETSET the formulae from Ellis et al. [13] for the graphs of the classes (A), (B), (C), (D), (E) are used to generate the four-parton final states. Each generated event is classified by JETSET as belonging to double-bremsstrahlung [classes (A) and (B)] or triple-gluon coupling [class (C)] or secondary quark–antiquark production [classes (D) and (E)]. The y_{cut} is chosen to be so low that it is below the cut imposed by LUCLUS. From this sample after applying the track and event cuts the LUCLUS algorithm gives 6102 four-jet events.

The simulated four-jet sample has been checked for consistency with the four-jet events extracted from the data. The distributions of the thrust were compared. Also the distributions of the transverse particle momenta in the event plane with respect to the event axis, both defined by the sphericity tensor, and that of the transverse particle momenta out of this plane were controlled. Separately for the jets 1–4, the distribution of the jet momentum, of the charged multiplicity of the jet and of the transverse and longitudinal particle momenta relative to the jet axis were examined. The average values agreed within the typical statistical accuracy of about 5%. The contribution to the four-jet events from fragmentation fluctuations of three- and two-parton events has been determined by passing 2000 events generated with the complete QCD matrix element through full simulation and the analysis chain. This background amounts to 4% of the selected four-jet data.

The simulated events are sorted separately for double-bremsstrahlung, triple-gluon vertex, and secondary $q\bar{q}$ according to the values of our two observables into 10×10 matrices. The same generated events have been passed through a simple simulation of the DELPHI detector, in which the measurement errors and reconstruction inefficiencies are given by parametrisations, and through the analysis. This gives us a 10×10 correction matrix for the relation between this fast and the full simulation. Its elements are around 0.87 with about $\pm 10\%$ maximal systematic variation over the range. Then 20 000 four-parton events have been generated in the abelian theory and passed through the simple simulation and the analysis. The matrices have then been corrected to correspond to the full detector simulation. In principle one has to use specific correction matrices for each subclass. However, as the correction factors are near to unity this effect is negligible within our statistical accuracy.

4. Analysis

The generalised Nachtman–Reiter angle θ^*_{NR} has the advantage that no cuts in opening angles are needed on the four-jet sample. There is some correlation between θ^*_{NR} and the second observable α_{34}. Therefore we perform a two-dimensional analysis. The four-jet events are sorted into a 10×10 matrix according to their values of $|\cos \theta^*_{NR}|$ and $\cos \alpha_{34}$. To these 10×10 bins we fit the theoretical predictions with a maximum likelihood method.

The classes (A), (B) and (D), (E) contribute with different factors in QCD and the abelian theory and this allows one to separate the contribution (A) from (B) and (D) from (E). Class (E) contributes only 0.5% and its shape is therefore not well determined. We have set its shape equal to (D) and checked that varying its contribution by 100% and even imposing totally different shapes do not visibly change the results of our analysis. A direct generation of events of the groups (B) and (E) cannot be done, because for

[#1] We use JETSET version 7.2 (November 1989). LUCLUS is described in ref. [18], pages 240 and 241.

some values of kinematical variables their contributions to the cross-section become negative. The combined contributions of (A), (B) and (D), (E) are of course positive. Class (C) is extracted directly from the QCD-events. For each bin l, m the theoretical prediction is given by:

$$\mathrm{Theo}_{l,m} = \mathrm{Norm} \times \alpha_s^2 [C_F^2 A_{l,m} + C_F(C_F - \tfrac{1}{2}N_C) B_{l,m} + C_F N_C C_{l,m} + C_F T_R D_{l,m} + C_F(C_F - \tfrac{1}{2}N_C) E_{l,m}] ,$$

where Norm is the overall normalisation factor. We perform with MINUIT [19] a maximum likelihood fit to the 10×10 matrix from the data with the three variables $X_1 = \mathrm{Norm} \times \alpha_s^2 C_F^2$, $X_2 = N_C/C_F$, and $X_3 = T_R/C_F$ in the distribution

$$\mathrm{Theo}_{l,m} = X_1 [A_{l,m} + (1 - \tfrac{1}{2}X_2) B_{l,m} + X_2 C_{l,m} + X_3 D_{l,m} + (1 - \tfrac{1}{2}X_2) E_{l,m}] + P3J4_{l,m} .$$

We use Poisson distributions for the likelihood factor in each bin. $P3J4$ represents the four-jet background contribution from three- and two-parton events.

5. Results

Projections of the two-dimensional distributions in $|\cos \theta_{NR}^*|$ and $\cos \alpha_{34}$ are given in figs. 3a, 3b for the groups (A), (B), (C), (D), and QCD after the detector simulation and selections. The relative contributions of the groups (A), (B), (C), (D), (E) are for QCD 34%, −5%, 65%, 6%, −0.1% and for the abelian theory 27%, 32%, 0%, 41%, 0.5%. In θ_{NR}^* secondary $q\bar{q}$ events [class (D)] differ markedly from triple-gluon events [class (C)] and double-bremsstrahlung events [class (A) and (B)], but triple-gluon events and double-bremsstrahlung events look quite similar (the mean values of $|\cos \theta_{NR}^*|$ in the distributions (A), (B), (C), (D) are 0.596 ± 0.006, 0.552 ± 0.015, 0.562 ± 0.005 and 0.447 ± 0.008). In the second observable α_{34} double-bremsstrahlung events and triple-gluon events give different shapes (the mean values of $\cos \alpha_{34}$ in the distributions (A), (B), (C), (D) are -0.269 ± 0.010, -0.263 ± 0.026, -0.187 ± 0.008 and -0.170 ± 0.015). By fitting the two-dimensional distribution of the two observables the three contributions can be determined separately. The 4% background to four jets from two- and three-parton events is treated as part of the theoretical prediction. Varying its contribution by ±50% gives less than 0.1 standard deviation change in the fit results.

Fixing N_C/C_F and T_R/C_F to the QCD values and fitting only the normalisation factor is equivalent to using a directly generated QCD distribution and thus allows a direct comparison of the two-dimensional distribution from the data with that expected from QCD. The influence of the finite Monte Carlo statistics is in this case simply calculated from the number of four-jet events in the QCD Monte Carlo simulation and in the data. Similarly by fixing N_C/C_F and T_R/C_F to the values of the uniquely defined matrix-element version of the abelian theory this model can be tested. Figs. 4a–4c show the two-dimensional distributions in $|\cos \theta_{NR}^*|$ and $\cos \alpha_{34}$ for the data, and as expected for QCD and for the abelian theory.

With the small number of events in each bin a χ^2 test of the goodness of the fit to QCD in the two-dimensional distributions is not meaningful. We check that our data are well described in the projections which are shown in fig. 5. The χ^2 values for the projections in $|\cos \theta_{NR}^*|$ and $\cos \alpha_{34}$ are 9.3 and 7.4 for 10 bins each and show good agreement with the QCD predictions for both observables. For the abelian theory the χ^2 values are 30.7 and 14.9. As in the other experiments [10–12] the matrix-element version of the abelian theory is excluded here by the distribution in the generalised Nachmann–Reiter angle alone with more than 99% confidence level. Other choices for the abelian model were considered by OPAL [10].

For the free fit the available statistics of the Monte Carlo simulations of QCD and abelian theory has to be split up into the classes (A), (B), (C), (D), (E) and this leads to large errors. The propagation of the errors is complicated in this case. The influence of the finite statistics of the Monte Carlo simulation was therefore estimated empirically. With JETSET 7.2 and a simple detector simulation many pairs of simulations with different statistics were generated. The distributions of (A), (B), (C), (D), (E) were deduced from each pair and then another simulated QCD-distribution fitted by these. In the fitting procedure the statistical fluctuations of the distributions (A), (B), (C), (D) and (E) cause the errors in the fits to be underestimated. This influence has been de-

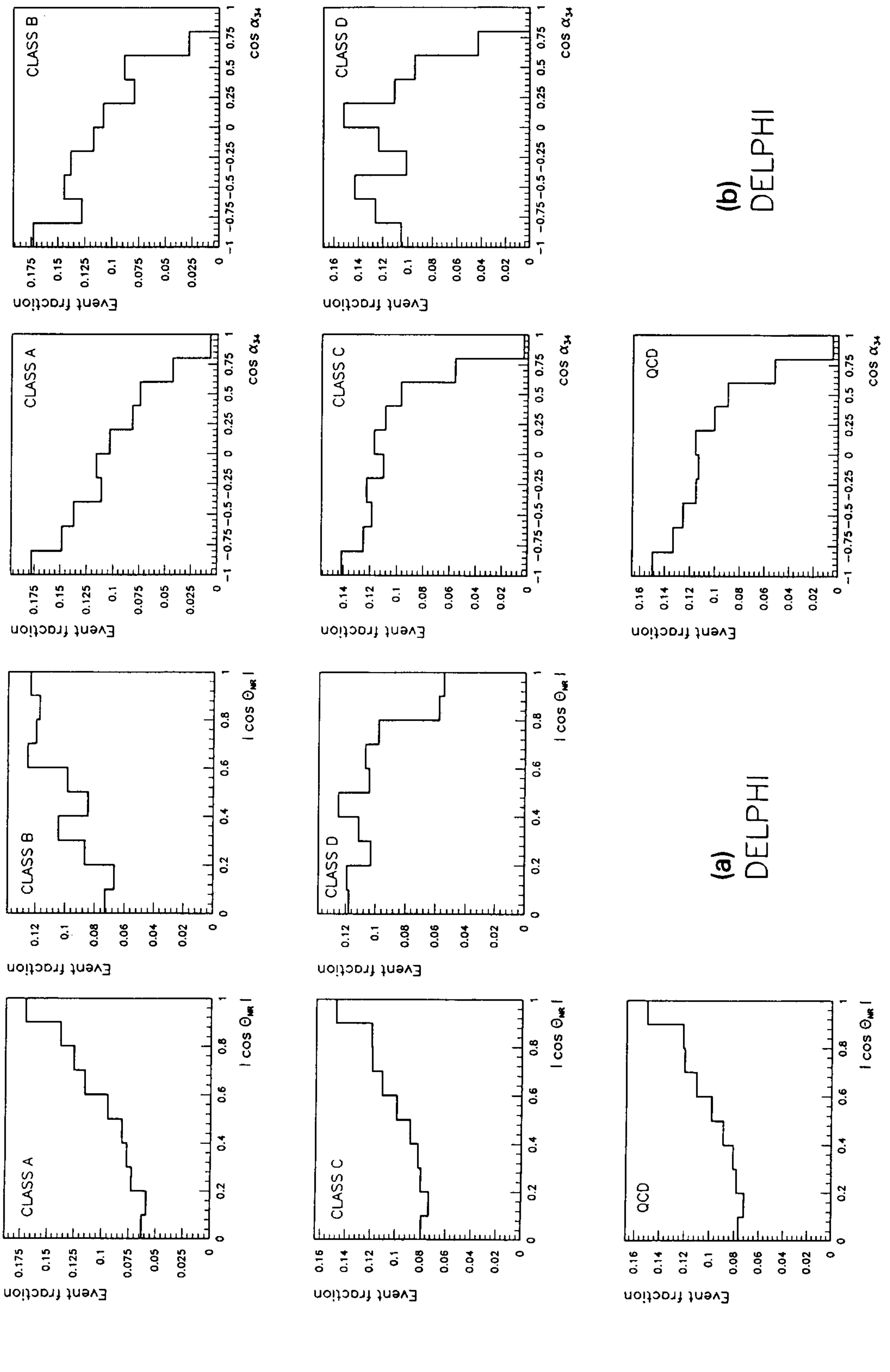

Fig. 3. Distributions of the four-jet events from the classes (A), (B), (C), (D) and from all the four-parton events in QCD (after simulation of the DELPHI-detector): (a) Projection on $|\cos\theta^*_{NR}|$; (b) projection on $\cos\alpha_{34}$.

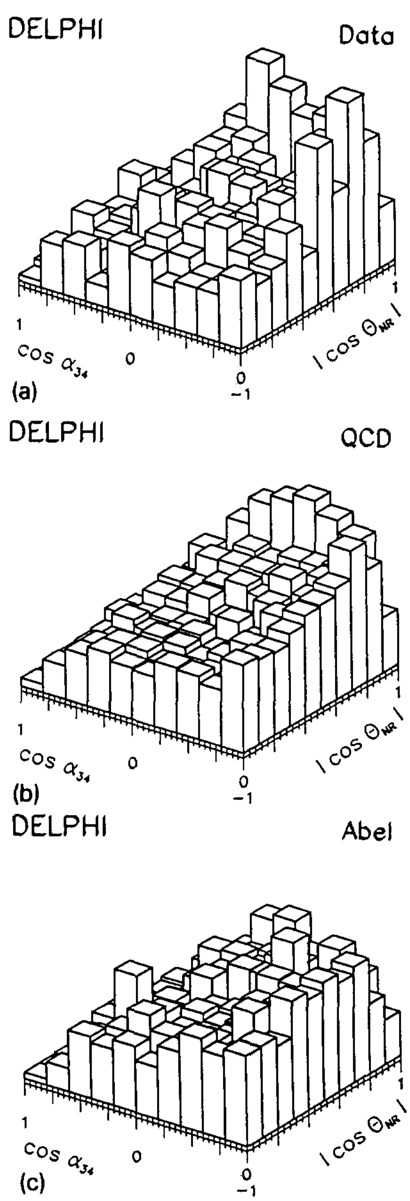

Fig. 4. Two-dimensional distributions in $|\cos\theta^{*}_{\mathrm{NR}}|$ and $\cos\alpha_{34}$: (a) Data; (b) expected distribution from QCD; (c) expected distribution from abelian theory.

termined by comparison with smoothed distributions and is included in the statistical error. Furthermore the fit results for N_C/C_F are systematically shifted toward smaller values and reach the nominal value only asymptotically with increasing Monte Carlo statistics. The estimate of this shift for our simulation statistics is a bias of 0.5 ± 0.2. The value quoted below is corrected for this bias [#2].

The influence of fragmentation and models has been studied with the same simple detector simulation by comparing samples generated with different schemes. We used additionally in JETSET 7.2 in the matrix-element version the fragmentation parameters of JETSET 6.2 [17,18] which are matched to the data from PETRA and PEP in the 30 GeV region, settings for optimised scale [21], and the default parton-shower version with gluon-polarisation and interference. The parton-shower version contains the triple-gluon vertex, but in completely different way. It is based on cascades of $q\to qg$, $g\to gg$, $g\to q\bar{q}$. These simulations were treated as "data" and then N_C/C_F was determined for them by fitting with the classes (A), (B), (C), (D), (E) as they are derived from high statistics QCD and abelian simulations using our standard conditions. The results for N_C/C_F were 2.20 ± 0.22, 2.62 ± 0.23 and 1.69 ± 0.23 for the three cases.

Including the estimates from the studies with the simple detector simulation for the error and bias from the simulation and the fluctuations of the results using the different fragmentations and models we get for the final result of the free fit

$$N_C/C_F = 2.55\pm0.55(\text{stat.})\pm0.4(\text{fragm.}+\text{models})$$
$$\pm0.2(\text{error in bias})\,.$$

The fit gives a significant triple-gluon vertex contribution ($\propto N_C/C_F$) in agreement with $N_C/C_F=2.25$ that is expected for QCD. For T_R/C_F we get 0.1 ± 2.4 which is consistent with the QCD value of 1.875 for five quark flavours and disproves again the abelian theory for which a value of 15 is expected.

6. Conclusions

Within the framework of QCD one can establish directly the triple-gluon vertex contribution with the

[#2] In the contribution to the Singapore conference [20] the preliminary error of $\pm^{0.7}_{0.1}$ given for the influence of the finite statistics of the Monte Carlo simulations contains the increase of the statistical error and the estimated bias, i.e. both effects.

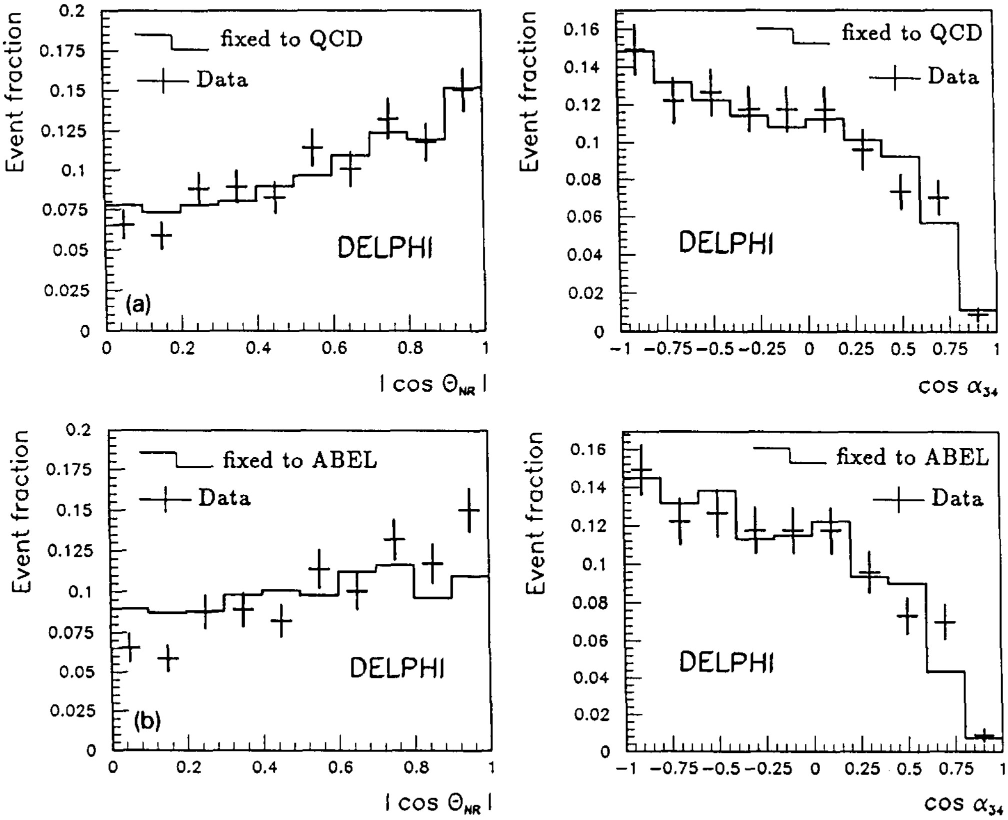

Fig. 5. Projections on $|\cos\theta^*_{NR}|$ and $\cos\alpha_{34}$; (a) comparison of data and QCD; (b) comparison of data and abelian theory.

two-dimensional angular distribution of the generalised Nachtman–Reiter angle and the opening-angle of the secondary jets. This new method has been applied to DELPHI data. It is found that the data require the existence of the triple-gluon vertex contribution in the QCD second order matrix element description. The study of the influence of different fragmentation parameters and the inclusion of the parton shower version which contains higher orders in the leading-log approximation, suggest that the result is not spoiled by fragmentation effects nor by higher order contributions and higher jet-multiplicities.

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