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The spin-dependent structure function $g_1(x)$ of the proton from polarized deep-inelastic muon scattering

Spin Muon Collaboration (SMC)

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Abstract

We present a new measurement of the virtual photon proton asymmetry A_1^p from deep inelastic scattering of polarized muons on polarized protons in the kinematic range $0.0008 < x < 0.7$ and $0.2 < Q^2 < 100$ GeV^2. With this, the statistical uncertainty of our measurement has improved by a factor of 2 compared to our previous measurements. The spin-dependent structure function g_1^p is determined for the data with $Q^2 > 1$ GeV^2. A perturbative QCD evolution in next-to-leading order is used to determine $g_1^p(x)$ at a constant Q^2. At $Q^2 = 10$ GeV^2 we find, in the measured range, $\int_{0.003}^{0.7} g_1^p(x)dx = 0.139 \pm 0.006\,(\text{stat}) \pm 0.008\,(\text{syst}) \pm 0.006\,(\text{evol})$. The value of the first moment $\Gamma_1^p = \int_0^1 g_1^p(x)dx$ of g_1^p depends on the approach used to describe the behaviour of g_1^p at low x. We find that the Ellis-Jaffe sum rule is violated. With our published result for Γ_1^d we confirm the Bjorken sum rule with an accuracy of $\approx 15\%$ at the one standard deviation level.

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Polarized deep inelastic lepton-nucleon scattering is an important tool to study the internal spin structure of the nucleon. Measurements on proton, deuteron and neutron targets allow verification of the Bjorken sum rule [1] which is a fundamental relation of QCD. The improved accuracy of data collected by experiments at CERN and SLAC in the past few years has motivated and allowed perturbative QCD analyses of the nucleon spin-dependent structure function $g_1(x,Q^2)$ at next-to-leading-order (NLO) [2–4].

In this paper, we report on a new measurement of the virtual photon proton asymmetry A_1^p by the Spin Muon Collaboration (SMC), obtained by scattering longitudinally polarized muons of approximately 190 GeV energy on longitudinally polarized protons in the kinematic range $0.0008 < x < 0.7$ and $0.2\,\mathrm{GeV}^2 < Q^2 < 100\,\mathrm{GeV}^2$. The data were collected in 1996 with the high-energy muon beam M2 of the CERN SPS using solid ammonia as the polarized target material. They complement earlier data taken in 1993 at the same beam energy using butanol as the target material [5,6]. The statistical precision of the combined A_1^p data sets is a factor of approximately two improved compared to our 1993 data. Using the data with $Q^2 > 1$ GeV^2 and $x > 0.003$ we determine the spin structure function g_1 of the proton. In this paper we present the new data and give a brief description of the analysis. In Ref. [6] we have given a detailed description of the method of the measurement and data analysis for the determination of the spin structure function of the proton.

The cross section asymmetry for parallel and antiparallel configurations of longitudinal beam and target polarizations is given by

$$A_\parallel^p = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}}. \tag{1}$$

The evaluation of the asymmetry $A_\parallel^p$ requires knowledge of the incident muon and target proton polarizations, and of the dilution factor which accounts for the fact that only a fraction of the target nucleons is polarized. The beam polarization was determined by measuring the cross section asymmetry for the scattering of polarized muons on polarized atomic electrons [6,7]. For the average muon energy of 188 GeV, the polarization is $P_\mu = -0.77 \pm 0.03$. The energy dependence of the polarization is taken into account event by event.

The choice of ammonia as the target material rather than butanol which was used in our 1993 measurement [5,6], increased the dilution factor by ≈ 30%. The average longitudinal proton polarization over the entire data taking period was $P_p = \pm 0.89$, known with an overall accuracy $\Delta P_p / P_p = 2.7\%$.

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The polarization, P_N, of the ^{14}N nuclei was determined [8] with an accuracy $\Delta P_N/P_N$ of better than 10% in dedicated measurements. Its value was found to relate to the proton polarization as predicted by the equilibrium spin temperature relation [9]. In the analysis the nitrogen polarization was calculated from the measured proton polarization using that relation. The typical nitrogen polarization was $P_N = \pm 0.14$.

The asymmetry $A_\parallel^p$ and the spin-dependent structure function g_1^p are related to the virtual photon proton asymmetries A_1^p and A_2^p [10,11] by

$$A_\parallel^p = D(A_1^p + \eta A_2^p),$$

$$g_1^p = \frac{F_2^p}{2x(1+R)}(A_1^p + \gamma A_2^p), \qquad (2)$$

in which the factors η and γ depend only on kinematic variables; the depolarization factor D depends, in addition, on the ratio of total photoabsorption cross sections for longitudinally and transversely polarized virtual photons $R = \sigma_L/\sigma_T$. The virtual photon proton asymmetries are defined as

$$A_1^p = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}}, \quad A_2^p = \frac{2\sigma^{TL}}{\sigma_{1/2} + \sigma_{3/2}}, \qquad (3)$$

where $\sigma_{1/2}$ ($\sigma_{3/2}$) is the total photoabsorption cross section of a transverse virtual photon by a proton, with total spin projection $1/2(3/2)$ in the photon direction, and σ^{TL} is a term arising from the interference between transverse and longitudinal amplitudes.

In the kinematic region of our measurement η and γ are small and A_2^p was measured and found to be consistent with zero. We therefore neglect the terms proportional to A_2^p in Eq. (2). The systematic uncertainty due to a possible residual contribution from A_2^p is estimated using the SMC [12] and the SLAC E143 [13] measurements. The E143 results have better statistical accuracy but do not extend to low x. Assuming that $A_2^p\sqrt{Q^2}$ is Q^2 independent, the E143 measurements are evaluated at the Q^2 of SMC data in each x bin. The combined A_2^p data are then parametrized and the parametrization is used in the estimation of the systematic uncertainty.

When calculating $A_\parallel^p$ we correct for the contribution of polarized nitrogen to the longitudinal asymmetry. In the shell model [14], ^{14}N is described as a spinless ^{12}C core with the valence proton and neutron being responsible for the nitrogen spin. The correction is expressed [8,9] in terms of a parametrization of the measured deuteron asymmetry A_1^d from Refs. [15,16]. This correction is found to be less than 3% of $A_\parallel^p$ and introduces a small systematic uncertainty.

Our analysis is limited to the kinematic region with $x \geq 0.0008$ and $Q^2 \geq 0.2$ GeV^2. Cuts are applied to restrict the inelasticity to $y \leq 0.9$, the scattering angle to $\Theta \geq 2$ mrad, the energy of the scattered muon to $E'_\mu \geq 19$ GeV, and the energy transfer to the target to $\nu \geq 15$ GeV. After these cuts 12.5×10^6 events from the 1996 measurement remain for the final analysis.

The new results are in agreement with the 1993 data within the statistical errors so we combine them in the subsequent analysis. The combined results for A_1^p are given as a function of x and Q^2 in Table 1 and shown in Fig. 1. In this figure, we also compare our results and those of EMC [10] to the E143 [17] measurements which are at lower Q^2. No evidence for a Q^2 dependence of A_1 is visible within the accuracy of the present data. Fig. 2 shows A_1^p as a function of x averaged over Q^2 within each x bin. The new results are compared to our 1993 results in Fig. 2(a) and the combined results are shown in Fig. 2(b) along with EMC and E143 data. Our dominant systematic errors at low x are due to radiative corrections, time-dependence of the acceptance ratio r for events from the upstream and the downstream target cells and uncertainties in A_2^p. At high x, the dominant sources of systematic errors are uncertainties in the ratio R and in the beam and target polarizations. Individual systematic errors are added in quadrature to obtain the total systematic error.

We compute g_1^p for data with $Q^2 \geq 1$ GeV^2 using Eq. (2) and parametrizations for F_2^p and R. In our previous publications we used for F_2^p the parametrization provided by the NMC collaboration [18]. Recently, new F_2^p data at lower x became available from NMC [19], E665 [20], H1 [21] and ZEUS [22]. We performed a new fit of F_2^p which includes these new data, the data from SLAC [23], NMC [18] and BCDMS [24], and covers the kinematic range $3.5 \times 10^{-5} < x < 0.85$ and $0.2 < Q^2 <$

Table 1
The virtual photon proton asymmetries A_1^p for different x and Q^2 values, with their statistical errors

$\langle x \rangle$	$\langle Q^2 \rangle$ (GeV^2)	A_1^p
0.0009	0.25	0.001 ± 0.069
0.0011	0.30	0.016 ± 0.085
0.0011	0.34	0.196 ± 0.111
0.0014	0.38	0.139 ± 0.044
0.0017	0.46	0.076 ± 0.053
0.0019	0.55	0.037 ± 0.057
0.0023	0.58	0.020 ± 0.040
0.0025	0.70	0.025 ± 0.044
0.0028	0.82	0.027 ± 0.048
0.0035	0.88	0.038 ± 0.029
0.0043	1.14	−0.011 ± 0.025
0.0051	1.43	0.060 ± 0.030
0.0056	1.71	0.008 ± 0.051
0.0069	1.43	−0.003 ± 0.043
0.0072	1.76	0.016 ± 0.033
0.0077	2.04	0.063 ± 0.032
0.0084	2.34	0.105 ± 0.037
0.0090	2.63	0.099 ± 0.048
0.0095	2.94	−0.041 ± 0.072
0.0114	1.75	−0.075 ± 0.110
0.0120	2.07	0.065 ± 0.072
0.0124	2.36	0.032 ± 0.054
0.0125	2.66	0.017 ± 0.045
0.0127	2.96	−0.014 ± 0.039
0.0133	3.30	0.008 ± 0.033
0.0147	3.74	0.046 ± 0.032
0.0165	4.43	0.112 ± 0.029
0.0184	5.44	−0.029 ± 0.047
0.0231	2.78	0.142 ± 0.111
0.0236	3.31	0.227 ± 0.107
0.0235	3.77	−0.030 ± 0.077
0.0237	4.54	0.083 ± 0.041
0.0241	5.75	0.068 ± 0.030
0.0263	7.42	0.024 ± 0.034
0.0339	4.23	0.058 ± 0.073
0.0342	5.80	0.134 ± 0.050
0.0344	7.77	0.032 ± 0.035
0.0359	10.14	0.082 ± 0.041
0.0472	4.29	0.054 ± 0.108
0.0473	5.86	0.084 ± 0.068
0.0479	7.83	0.103 ± 0.040
0.0485	10.95	0.120 ± 0.029
0.0527	14.72	0.133 ± 0.042

Table 1 (continued)

$\langle x \rangle$	$\langle Q^2 \rangle$ (GeV^2)	A_1^p
0.0736	5.47	0.145 ± 0.101
0.0744	7.88	0.153 ± 0.059
0.0750	11.08	0.196 ± 0.037
0.0762	16.30	0.170 ± 0.029
0.0855	23.05	0.189 ± 0.045
0.1189	7.41	0.368 ± 0.104
0.1196	11.14	0.335 ± 0.068
0.1200	16.48	0.245 ± 0.048
0.1206	24.82	0.248 ± 0.043
0.1293	34.32	0.264 ± 0.060
0.1711	10.19	0.203 ± 0.102
0.1715	16.51	0.293 ± 0.080
0.1717	24.89	0.214 ± 0.068
0.1718	34.94	0.459 ± 0.073
0.1771	45.48	0.361 ± 0.081
0.2368	10.54	0.363 ± 0.132
0.2389	16.54	0.146 ± 0.096
0.2394	24.95	0.424 ± 0.081
0.2398	34.94	0.426 ± 0.084
0.2462	52.74	0.471 ± 0.057
0.3388	15.26	0.514 ± 0.158
0.3404	25.01	0.535 ± 0.149
0.3407	34.96	0.715 ± 0.153
0.3436	61.81	0.555 ± 0.078
0.4688	21.86	0.972 ± 0.179
0.4751	34.98	0.433 ± 0.230
0.4842	72.07	0.616 ± 0.096

5000 GeV^2. We use the same functional form as used by NMC [18]

$$F_2(x,Q^2) = A(x) \cdot \left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]^{B(x)} \cdot \left[1 + \frac{C(x)}{Q^2}\right], \tag{4}$$

where

$$A(x) = x^{a_1}(1-x)^{a_2}\left[a_3 + a_4(1-x) + a_5(1-x)^2 + a_6(1-x)^3 + a_7(1-x)^4\right],$$

$$B(x) = b_1 + b_2 x + \frac{b_3}{(x + b_4)},$$

$$C(x) = c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4,$$

with $\Lambda = 250$ MeV and $Q_0^2 = 20\ \mathrm{GeV}^2$. In the minimization procedure, the data points were weighted by their statistical errors. The normalization uncertainties and the systematic errors were accounted for by additional parameters in the fit. The resulting fit parameters of F_2 are presented in Table 2. This parametrization of F_2 has to be used with consistent values of R such that the measured cross sections are reproduced. For $x < 0.12$ we use a parametrization of R measured by the NMC [19]. In the high x region we use the SLAC parametrization for R [25] as in our previous publications.

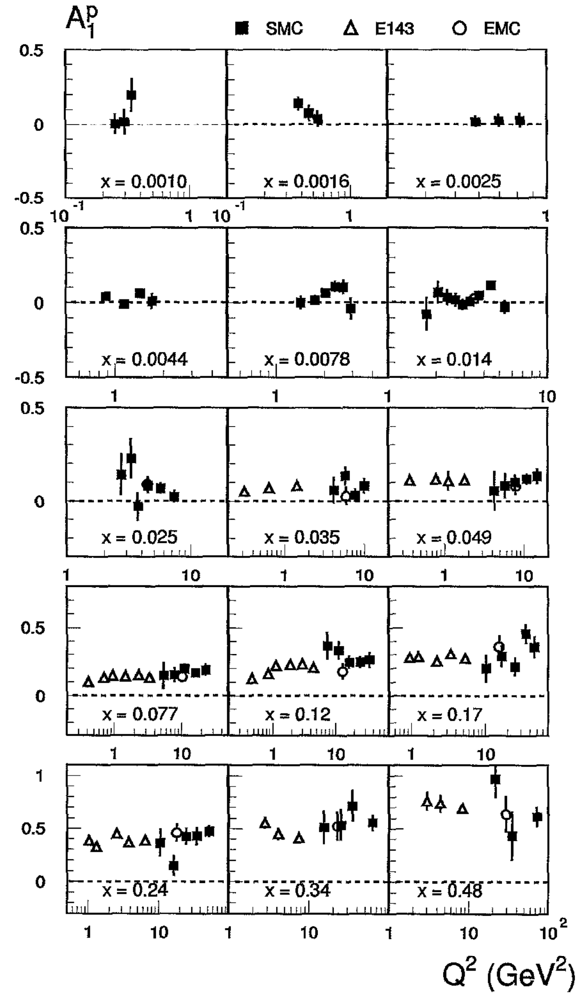

Fig. 1. A_1^p vs Q^2 for different bins of x for the combined, 1993 and 1996, SMC data (squares) where the value of x corresponds to the average in each bin. At higher x values E143 (open triangles) and EMC (open circles) measurements are shown for comparison. Error bars represent statistical uncertainties.

Fig. 3(a) shows g_1^p calculated from our 1996 data using the two sets of F_2 and R parametrizations. The resulting differences in the values of g_1^p are small. In the subsequent analysis, we use the new set of parametrizations. The results for $g_1^p(x)$ at the average Q^2 of each bin in x for 1996 data are compared to our 1993 data in Fig. 3(b). The old and the new results are statistically compatible; however, the lowest x point in the new data has a lower value. The combined results are shown in the same figure and are listed in Table 3. The data do not suggest a rise of $g_1^p(x)$ at low x.

We use our data in the kinematic region $Q^2 \geq 1$ GeV^2, $x \geq 0.003$ to evaluate $\Gamma_1^p = \int_0^1 g_1^p(x) dx$ at a fixed Q^2. The precision of the data and the available Q^2 range do not allow a direct determination of the Q^2 dependence of A_1^p ($\sim g_1/F_1$). Different Q^2 behaviours of g_1 and F_1 are expected from perturbative QCD [2]. The Q^2 dependence of g_1 is then estimated from a perturbative QCD analysis in NLO in the Adler-Bardeen scheme [2] as performed in our previous publications [6,15]. We have updated our analysis to include new published neutron data [26–28] and our 1996 proton data in addition to the data [6,10,15–17,29] used in our previous publications. This results in a small change in the QCD fit. The result of the fit for g_1^p is shown in Fig. 4.

Starting from $g_1(x,Q^2)$ at the measured x and Q^2 of our experiment we obtain g_1 at a fixed Q_0^2 as follows:

$$g_1(x,Q_0^2) = g_1(x,Q^2) + \left[g_1^{\mathrm{fit}}(x,Q_0^2) - g_1^{\mathrm{fit}}(x,Q^2) \right], \qquad (5)$$

where $g_1^{\mathrm{fit}}(x,Q_0^2)$ and $g_1^{\mathrm{fit}}(x,Q^2)$ are the values of g_1 evaluated at Q_0^2 and at the Q^2 of the experiment, using the fit parameters. We choose $Q_0^2 = 10\,\mathrm{GeV}^2$ which is close to the average Q^2 of our data. The resulting g_1 is given in Table 3.

In the measured range, $0.003 < x < 0.7$, the con-

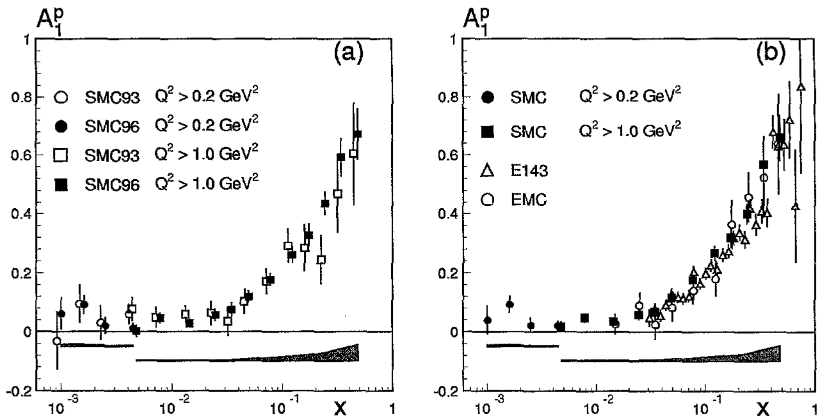

Fig. 2. (a) Comparison of A_1^p vs x for 1993 and 1996 SMC data at the measured Q^2. (b) A_1^p vs x from 1993 and 1996 SMC data combined is shown along with the measurements from EMC and E143 experiments. Statistical errors are shown as error bars while the shaded band below indicates the SMC systematic uncertainty.

tribution to the first moment of the proton structure function is

$$\int_{0.003}^{0.7} g_1^p(x,Q_0^2)\,dx = 0.139 \pm 0.006 \pm 0.008 \pm 0.006 \quad (Q_0^2 = 10\,\mathrm{GeV}^2), \tag{6}$$

where the first uncertainty is statistical, the second is systematic and the third is due to the uncertainty in the Q^2 evolution. The uncertainties on the integral of g_1^p in the measured range are separated by source in Table 4. In addition to several sources of uncertainty on A_1^p and uncertainties from F_2^p and R, contributions due to kinematic smearing and residual biases of the extraction and combination of the asymmetries are also listed. These contributions were studied with Monte Carlo techniques simulating realistic data taking conditions and found to be small. Fig. 5 shows xg_1^p as a function of x. In this figure the area under the data points represents the integral given in Eq. (6). Evaluating the integral in the measured x-region from the QCD fit gives 0.136 which is consistent with Eq. (6).

Table 2
Values of the fitted parameters for the F_2 function given in Eq. (4). In the second column we give the parameters for the central value of the fit, parametrizations for an upper and a lower limit for F_2 are given in the last two columns

Parameter	Central value	Upper limit	Lower limit
a_1	−0.24997	-0.24810	−0.25196
a_2	2.39635	2.36324	2.42968
a_3	0.22896	0.23643	0.21913
a_4	0.08498	-0.03241	0.21630
a_5	3.86079	4.22681	3.46446
a_6	−7.41428	−7.81197	−6.98874
a_7	3.43422	3.58225	3.27710
b_1	0.11411	0.09734	0.13074
b_2	−2.23556	−2.22540	−2.24648
b_3	0.03115	0.03239	0.02995
b_4	0.02135	0.02233	0.02039
c_1	−1.45174	−1.43613	−1.47152
c_2	8.47455	8.10840	8.91079
c_3	−34.3791	−33.3057	−35.7143
c_4	45.8881	44.7175	47.3385

To estimate the contribution to the first moment from the unmeasured high x region $0.7 < x < 1.0$, we assume $A_1^p = 0.7 \pm 0.3$ which is consistent with the data and covers the upper bound $A_1 \leq 1$. We obtain

$$\int_{0.7}^{1} g_1^p(x,Q_0^2)\,dx = 0.0015 \pm 0.0006. \tag{7}$$

To estimate the contribution from the unmeasured low x region we consider two approaches:

1. Consistent with a Regge behaviour $g_1^p \propto x^{-\alpha}\,(-0.5 \leq \alpha \leq 0.0)$ [30], we assume $g_1^p =$ constant at 10 GeV^2. This constant, 0.69 ± 0.14,

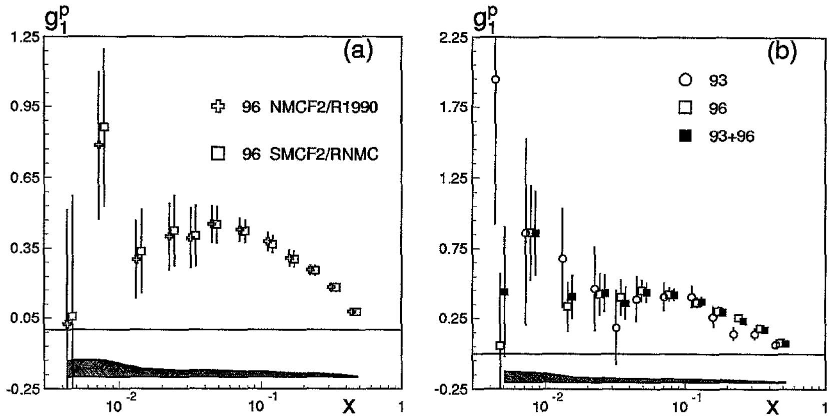

Fig. 3. (a) A comparison of g_1^p at measured Q^2 from 1996 SMC data using the old and the new set of F_2^p and R parameterizations (see text). (b) SMC g_1^p values at measured Q^2 from 1993, 1996 and 1993 + 1996 combined data sets. In both figures error bars show the statistical uncertainty and the shaded band indicates the systematic uncertainty.

obtained from the three lowest x data points evolved to 10 GeV^2, leads to

$$\int_{0.0}^{0.003} g_1^p(x,Q_0^2)dx = 0.002 \pm 0.002$$
$$(\text{Regge assumption}), \qquad (8)$$

where we assign a 100% error to this extrapolation, as was done in our previous publications [5,6]. The area under the dot-dashed curve in Fig. 5 and its inset corresponds to this low x contribution.

2. Alternatively, we calculate the low x integral from the QCD fit. Integrating this fit in the low x region gives

$$\int_{0.0}^{0.003} g_1^p(x,Q_0^2)dx = -0.011 \pm 0.011$$
$$(\text{QCD analysis}). \qquad (9)$$

The area under the QCD fit for $x < 0.003$ in Fig. 5 and its inset corresponds to this low x contribution. The uncertainty in the low x integral is obtained using the same procedure as for the

Table 3
The virtual photon proton asymmetry A_1^p, the spin-dependent structure function $g_1^p(x)$ at the measured Q^2 and $g_1^p(x)$ evolved to $Q_0^2 = 10$ GeV^2. The first error is statistical and the second is systematic. In the last column, the third error indicates the uncertainty in the QCD evolution

x Range	$\langle x \rangle$	$\langle Q^2 \rangle$	A_1^p	g_1^p	$g_1(Q_0^2 = 10.\text{GeV}^2)$
0.003–0.006	0.005	1.3	0.017 ± 0.018 ± 0.003	0.44 ± 0.46 ± 0.08	0.73 ± 0.46 ± 0.08 ± 0.71
0.006–0.010	0.008	2.1	0.047 ± 0.016 ± 0.004	0.86 ± 0.30 ± 0.07	1.12 ± 0.30 ± 0.07 ± 0.26
0.010–0.020	0.014	3.6	0.035 ± 0.014 ± 0.003	0.41 ± 0.16 ± 0.04	0.56 ± 0.16 ± 0.04 ± 0.09
0.020–0.030	0.025	5.7	0.058 ± 0.018 ± 0.005	0.43 ± 0.14 ± 0.03	0.50 ± 0.14 ± 0.03 ± 0.02
0.030–0.040	0.035	7.8	0.067 ± 0.022 ± 0.005	0.36 ± 0.12 ± 0.02	0.38 ± 0.12 ± 0.02 ± 0.01
0.040–0.060	0.049	10.4	0.115 ± 0.019 ± 0.008	0.44 ± 0.07 ± 0.03	0.44 ± 0.07 ± 0.03 ± 0.00
0.060–0.100	0.077	14.9	0.176 ± 0.019 ± 0.013	0.42 ± 0.05 ± 0.02	0.40 ± 0.05 ± 0.02 ± 0.00
0.100–0.150	0.122	21.3	0.267 ± 0.025 ± 0.018	0.37 ± 0.04 ± 0.02	0.35 ± 0.04 ± 0.02 ± 0.00
0.150–0.200	0.173	27.8	0.318 ± 0.035 ± 0.021	0.30 ± 0.03 ± 0.02	0.28 ± 0.03 ± 0.02 ± 0.01
0.200–0.300	0.242	35.6	0.400 ± 0.036 ± 0.028	0.23 ± 0.02 ± 0.01	0.23 ± 0.02 ± 0.01 ± 0.01
0.300–0.400	0.342	45.9	0.568 ± 0.058 ± 0.042	0.17 ± 0.02 ± 0.01	0.19 ± 0.02 ± 0.01 ± 0.01
0.400–0.700	0.480	58.0	0.658 ± 0.079 ± 0.055	0.08 ± 0.01 ± 0.01	0.09 ± 0.01 ± 0.01 ± 0.01

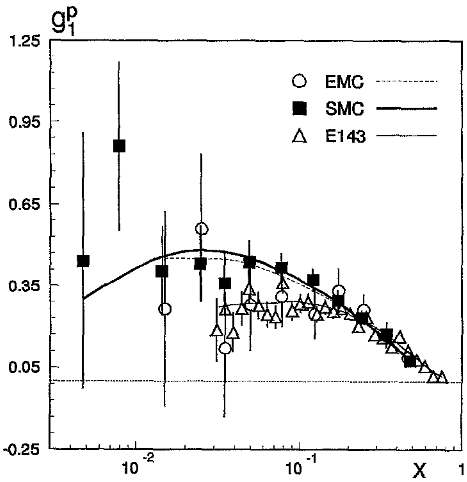

Fig. 4. Published data sets on g_1^p are shown. The curves represent the QCD fit at the measured Q^2 for each data set. Error bars represent the total error.

estimation of the uncertainty in the QCD evolution described in [6]. For the low x region, it is dominated by the uncertainties in factorization and renormalization scales.

We note that the two approaches described above lead to different contributions. The inset in Fig. 5 illustrates this difference. The corresponding values for the first moment $\Gamma_1^p(Q^2) = \int_0^1 g_1^p(x,Q^2)dx$ of g_1^p over the entire range in x are

$$\Gamma_1^p(Q_0^2 = 10\,\mathrm{GeV}^2) = 0.142 \pm 0.006 \pm 0.008 \pm 0.006 \quad (\text{Regge}), \qquad (10)$$

$$\Gamma_1^p(Q_0^2 = 10\,\mathrm{GeV}^2) = 0.130 \pm 0.006 \pm 0.008 \pm 0.014 \quad (\text{QCD}), \qquad (11)$$

where the first uncertainty is statistical and the second is systematic. The third uncertainty is due to the low x extrapolation and the Q^2 evolution, both of which have theoretical origins, and due to the high x extrapolation. The data do not allow us to exclude either approach so we keep the two numbers using the larger value for the third uncertainty

$$\Gamma_1^p(Q_0^2 = 10\,\mathrm{GeV}^2) = \left.\begin{matrix} 0.142\,(\text{Regge}) \\ 0.130\,(\text{QCD}) \end{matrix}\right\} \pm 0.006 \pm 0.008 \pm 0.014. \qquad (12)$$

Assuming $SU(3)_f$ symmetry within the baryon octet we determine the flavor singlet axial charge $a_0(Q^2) = a_u + a_d + a_s$ of the nucleon using the experimentally determined first moment of the proton and the relation

$$\Gamma_1^p(Q^2) = \frac{C_1^{NS}(Q^2)}{12}\left[a_3 + \frac{a_8}{3}\right] + \frac{C_1^S(Q^2)}{9} a_0(Q^2). \qquad (13)$$

For $a_3 = g_A/g_V = F + D$ and $a_8 = 3F - D$ we take values calculated from the experimental measurements, $g_A/g_V = 1.2601 \pm 0.0025$ [31] and $F/D = 0.575 \pm 0.016$ [32]. For the singlet and non-singlet coefficient functions C_1^S and C_1^{NS} we use values calculated to 3rd order in α_s [33]. Using the relations $a_8 = a_u + a_d - 2a_s$ and $a_3 = a_u - a_d$ we can calculate the individual quark flavor matrix elements. Results based on our proton data are given in Table 5. Assuming $SU(3)_f$ and $a_s = 0$, Ellis and Jaffe predicted a sum rule which gives for the above given couplings a theoretical value of $\Gamma_1^p = 0.170 \pm 0.004$ [34]. Irrespective of whether we take the Regge or the QCD approach in the low x region our result for the first moment Γ_1^p is smaller than the Ellis-Jaffe

Table 4
The sources of uncertainties in the integral of g_1^p in the measured range $0.003 < x < 0.7$

Source of the error	$\Delta\Gamma_1(0.003 \to 0.7)$
Beam polarization	0.0052
Target polarization	0.0039
Dilution factor	0.0029
Uncertainty on F_2	0.0025
Acceptance variation	0.0016
Radiative corrections	0.0007
Asymmetry extraction	0.0006
Polarized background	0.0005
Neglect of A_2	0.0005
Kinematic resolution	0.0003
Momentum measurement	0.0003
Uncertainty on R	0.0001
Total systematic error	0.0078
Evolution	0.0060
Statistics	0.0056

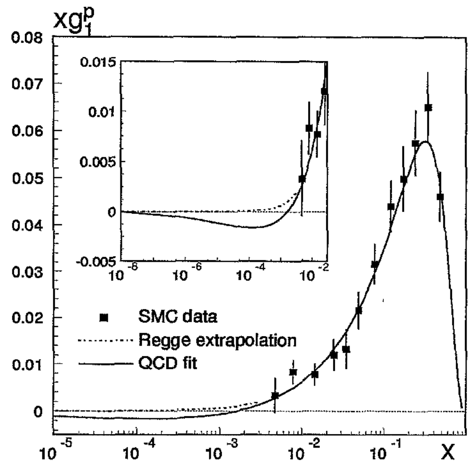

Fig. 5. xg_1^p as a function of x; SMC data points (squares) with the total error are shown together with the result of the QCD fit (continuous line), both at $Q^2 = 10$ GeV^2. For $x < 0.003$ the extrapolation assuming Regge behaviour is indicated by the dot-dashed line. The inset is a close-up extending to lower x.

prediction and our value of a_s is negative. The more conservative estimate of the uncertainty in the low x extrapolation results in the increase of uncertainties shown in Table 5 compared to our previous publications.

In the naive QPM the axial coupling $a_0(Q^2)$ is identified with $\Delta\Sigma$, the quark spin contribution to the nucleon spin. In the QCD improved QPM because of the U(1) anomaly there is a contribution of the gluon spin to $a_0(Q^2)$ which makes $\Delta\Sigma$ strongly scheme dependent. In the Adler-Bardeen scheme used in our QCD analysis, $a_0(Q^2)$ is decomposed into quark and gluon contributions in the following way:

$$a_0(Q^2) = \Delta\Sigma - n_f \frac{\alpha_s(Q^2)}{2\pi} \Delta g(Q^2), \tag{14}$$

where Δg is the gluon spin contribution to the nucleon spin. In this decomposition $\Delta\Sigma$ is Q^2 independent which enables it to be interpreted as the intrinsic quark-spin content of the nucleon. When we make the assumption $\Delta\Sigma = a_8$ corresponding to an unpolarized strange sea, our measurement of a_0 corresponds to $2 < \Delta g < 3$ at $Q^2 = 10$ GeV^2.

The QCD analysis done with all the published data along with the data presented in this paper results in $\Delta g = 0.9 \pm 0.3(\text{exp}) \pm 1.0(\text{theory})$ at $Q^2 = 1$ GeV^2 and the corresponding value of Δg at $Q^2 = 10$ GeV^2 is 1.7.

In Ref. [15] we have presented Γ_1^d using the Regge extrapolation approach for the unmeasured low x region. This is similar to the approach leading to Eq. (10). Combining the result from Eq. (10) with $\Gamma_1^d = 0.041 \pm 0.008$ at $Q_0^2 = 10\,\text{GeV}^2$ [15] we obtain for the Bjorken sum

$$\Gamma_1^p - \Gamma_1^n = 0.195 \pm 0.029 \quad (Q_0^2 = 10\,\text{GeV}^2). \tag{15}$$

which agrees with the theoretical prediction at $Q_0^2 = 10\,\text{GeV}^2$

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6}\left|\frac{g_A}{g_V}\right| C_1^{NS} = 0.186 \pm 0.003 \quad (Q_0^2 = 10\,\text{GeV}^2). \tag{16}$$

This conclusion is obviously unchanged if we use the result from the Regge extrapolation with an enlarged error from Eq. (12).

An alternative test of the Bjorken sum rule using QCD has been performed [35], which uses a QCD fit leaving g_A/g_V free, whereas g_A/g_V is held fixed in our fit.

In summary, we present a new measurement of the spin-dependent structure function of the proton, $g_1^p(x,Q^2)$, from polarized deep inelastic muon proton scattering. The new results are in agreement with our previous data and the statistical errors are reduced by a factor of 2. They do not confirm an earlier indication of a possible rise of $g_1^p(x)$ at low x. The reduction of the statistical error is not reflected in the final error on the first moment Γ_1^p

Table 5
Results for the first moment and the axial charges at $Q^2 = 10$ GeV^2 from our proton data

Quantity	SMC Results	
	Regge approach	QCD approach
Γ_1^p	0.142 ± 0.017	0.130 ± 0.017
a_0	0.34 ± 0.17	0.22 ± 0.17
a_u	0.84 ± 0.06	0.80 ± 0.06
a_d	-0.42 ± 0.06	-0.46 ± 0.06
a_s	-0.08 ± 0.06	-0.12 ± 0.06

because the uncertainty in the low x extrapolation has been enlarged in view of recent theoretical developments. This uncertainty which is now the dominant source of error in Γ_1^p can only be reduced significantly by future measurements [36] of the structure function in the very low x region. Such uncertainties however do not prevent us from confirming the violation of the Ellis-Jaffe sum rule. Combining the new value for Γ_1^p with our published Γ_1^d confirms the Bjorken sum rule with an accuracy of 15% at the one standard deviation level. Large uncertainties in the estimation of Δg from the QCD analysis exist at present due to the theoretical uncertainties. This points to the need of direct measurements [37] of Δg through processes in which the gluon polarization contributes at leading order.

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