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Are All Booms and Busts Created Equal? A New Methodology for Understanding Bull and Bear Stock Markets

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Abstract: This paper presents a new non-parametric methodology for the description of the evolution of the asset cycle in the stock market. It uses the empirical distribution of the data; in particular the structures of the tails of return distributions to build Boom-Bust Indicators (BBI) that describe whether a given market is a bull or a bear. These indicators, for three different time horizons, perform better than the usual binary sequence of financial crises because they measure both direction and intensity, they have stronger variability than a binary variable, they are strongly associated to the original data and keep some of its underlying characteristics such as serial autocorrelation, and they identify at least the same bull and bear markets as other methodologies. There is no evidence that favors one of the BBI specifications above the others.

JEL Codes: C14, E32, E44, G01, N2.

Keywords: Financial cycle, bull and bear markets, Stock market, Financial crises, Non-parametric models, Stock market history, Economic History.

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Part 1: Introduction

After the bouts of inflation of the early 1970s were controlled, mostly through the use of monetary policy tools, mainstream economists thought that the achievement of low levels of inflation was tantamount to attaining financial stability (Bernanke & Gertler, 1999). This notion gave birth to the modern inflation-targeting policy regime for central banks and monetary authorities all around the world (Borio & White, 2004). However, financial crises are "hardy perennials" which have become more frequent and more severe since 1973 than during the Bretton Woods or the Gold Standard periods (Bordo et. al., 2001, pgs. 72-73). The most diverse types of crises –banking, currency, sovereign debt, stock market and any combination of them—have happened in countries where inflation was closely watched; the most recent crisis just a painful global corollary.

In face of this disconnect between what was theoretically sound and what happened in reality, some economists and researchers, particularly at the Bank for International Settlements (BIS) constructed an alternative hypothesis which decouples financial and price stability. According to Martin (2013), they built on business cycle theory and the forgotten (discarded) works of authors like Bagehot, Minsky or Kindleberger to describe the financial cycle: the joint evolution of asset and credit markets under a scenario of changing risk perceptions. What this view proposes is that economies accumulate financial imbalances (excessive asset price and credit growth) during boom periods and that during busts, when these imbalances unwind, financial crises ensue (Borio, 2014). This new view of the dynamics behind financial instability has led to a burgeoning literature that has intertwined economics and economic history to understand the role institutions like central banks, regulatory authorities and monetary or exchange rate arrangements have played in crises prevention and management[1]. From this fruitful debate even policy recommendations have emerged as tangible evidences of the relevance and reach of the financial cycle proposition. Probably the most relevant ones for overall financial stability are the U.S. Dodd-Frank Act of 2010 and the Basel III accords of 2009.

A pervasive element even in the most recent financial cycle literature is the use of very simple binary variables to discriminate crises and non-crises periods or to distinguish between booms and busts. The results they find using these sequences as dependent variables are intriguing and in most cases draw light on a deep link between monetary, capital flow, and exchange rate

[1] Even though delving into this topic goes well beyond the scope of this paper, the implications of this discussion are extraordinary. If the inflation targeting view is right, then apart from controlling the price level, regulators and authorities can and should do nothing to prevent crises. The only course of action left is to clean the mess once it ensues. This is what Jones (2015) has called the Jackson Hole Consensus or the ex-post clean paradigm. However, if financial and price imbalances follow distinct processes, both institutional arrangements and policies are relevant to prevent crises ex ante and to tend to their negative consequences ex post. This is what Jones (2015) has called the Basel Consensus or the ex-ante lean paradigm. Other important references for the interested reader are Bordo & Jeanne (2002), Bernanke & Kuttner (2005), (Kuttner, 2012), Bordo & Landon-Lane (2013), Bordo (2014), Freixas, Laeven, & Peydró, (2015), and Taylor (2015).

variables and the accumulation of imbalances[2]. However, these binary sequences present researchers with several challenges which, if at least partially addressed may open the door to new research avenues and more robust results.

A first issue that affects the study of the financial cycle in general is an evident "lack of consensus on the mechanics of measurement, such as the choice of indicators and the method used to construct them" (Schüler, Hiebert, & Peltonen, 2015, pg. 2). This can be extended to the diverse methodologies used to obtain binary sequences for crises; there is an assortment of parametric models and non-parametric algorithms that yield diverse dating series. Some methodologies use only information available at the time of calculation (*ex-ante*) while others use information that was not available at the time (*ex-post*). This means that even if it is an accepted fact that there was a stock market crash in a given country at a given time, there might be a divergence of opinions as to the exact dating of its beginning and its end. This is not trivial, in particular if event studies or cycle synchronization analyses are to be performed.

Secondly, these dummy series are usually distilled from financial data –stock market prices, bond prices, credit aggregates, interest rates— which are characterized by their high frequency and strong variability apart from other desirable properties (Pagan & Sossounov, 2003). Trying to summarize their evolution in a yes/no sequence necessarily implies a loss of valuable information and the assumption that the transition from a calm period to a crises period is an instantaneous change of state and not a process that takes time to develop and to resolve. Furthermore, since crises are rare events, researchers are forced to use large panels of data in order to capture sufficient variability as to be able to perform inference

A final issue is that the binary classification does not allow for any kind of distinction between booms or crises in terms of their intensity, their dimension or the way they conformed. In the case of a boom-bust binary sequence, the main assumption is that all booms and all busts are created equal. "This binary classification surely obscures some important information about the variation in the severity of crises. It also means that errors in classification are likely very consequential. As a result, the empirical estimates of the real impact of crisis are likely to be imprecise and potentially inaccurate" (Romer & Romer, 2015, pg. 2). Any nuancing of this assumption requires the use of other variables or of qualitative evidence that cannot be included at once as a dependent variable.

The goal of this paper is to tackle these three issues by developing a new non-parametric methodology that will produce a single variable that measures both the direction and the intensity of the accumulation/unwinding of imbalances. This is relevant both from a methodological perspective and for economists/economic historians who participate in the debate. The higher informational content of the new indicator, when used as a dependent variable may allow for testing of new hypotheses and reaching new stylized facts. Building a continuous variable to explain the evolution of the financial cycle in a given asset class will allow an analysis of the process through which imbalances are accumulated and the way crises ensue.

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[2] For the interested reader, some relevant references that show the breadth of results without being exhaustive are Bordo & Jeanne (2002), Bordo & Wheelock (2009), Schularick & Taylor (2009), Jordà, Schularick, & Taylor (2011), Drehmann, Borio, & Tsatsaronis (2012), Bordo & Landon-Lane (2013), Borio (2014), Borio, James, & Sing (2014), Rey (2015), Schüler, Hiebert, & Peltonen (2015)

This methodology is a necessary first step for a larger research endeavor that wishes to contrast whether the impending monetary arrangement, composed of decisions of exchange rates, capital flows and monetary policy, has an effect on the velocity in which imbalances accumulate and unwind. The mechanisms for this to happen are monetary policy and the credit system, foreign capital flows directed towards portfolio investment and exchange rates and international reserves as proxies for exchange rate commitments. Since dummy variable sequences depend strongly on the methodology used and lose both variability and serial correlation characteristics of the underlying data, we need a new variable, which we call a boom-bust indicator (BBI), that should fulfill at least three goals:

- Serve as a continuous variable that contains more information about the underlying series than the yes/no sequence calculated with the methodologies available today.
- Find at least the same booms and busts as the existing literature while allowing for more nuanced stylized facts,
- Discriminate different types of booms and busts,

We will do this by using a long-run stock market time series for two countries, France and the United Kingdom (UK) in order to date bull and bear markets[3] and describe their different characteristics. Even though we are restricting our database to the stock market, a benefit of the proposed methodology is that it may be applied to all sorts of time series and be used to contrast literature on other types of crises that are characterized objectively through the study of financial or economic time series as we will indicate in the last section of this paper as avenues for future research. As most methodologies focused on historical research, the method proposed here benefits from a large number of observations and from a higher frequency in the data. Thus, using monthly data will result in more granular results than yearly data, and covering a long time period will allow for a more strenuous analysis of the cycle.

As proposed in this paper, working with only two countries evidences an added value of this methodology as it allows for a case by case analysis and not necessarily demand the pooling of information to perform robust inference. The choice of France and the UK as objects of this research responds to several reasons: data availability, development of the financial market during the covered period (1887-2015), availability of historiography and, for the development of a broader research project, their participation in the three different monetary regimes of the twentieth century.

The rest of the paper is structured as follows: Part 2 discusses the usual methodological families found in the literature and the debate surrounding the pertinence of each of them; Part 3 presents the data and some of its time series characteristics; Part 4 builds on the two previous sections to develop a new methodological proposition and studies the time series characteristics of the results; Part 5 compares the results obtained with a selection of methodologies and discusses the main differences; Part 6 presents a discussion of results, concludes and offers future lines of research.

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[3] The origin of the term is uncertain but Wall Street lore indicates that, while bulls attack with an upward movement of their horns, bears usually swipe with their paws downward when they strike. Concurrently, Gonzalez et. al. (2005, pg. 470) state that "bull markets are associated with persistently rising share prices, strong investor interest, and enhanced financial well-being" while bear markets are defined conversely.

Part 2: Usual methodologies for understanding the boom-bust cycles in asset prices

A first debate that takes place in the financial cycle literature has to do with the existence of rational or irrational bubbles in asset markets; this idea implies investors, knowingly or unknowingly, pay more for an asset than is justified (Brunnermeier & Schnabel, 2015). To characterize a bubble a researcher would require a fundamental value or fair price, defined as the discounted future dividend stream from a given stock, in order to contrast it with the market price at any time *t*. However, fundamental pricing models are subject to the same uncertainty as the issuer's future cash flows or the adequate discount rate for investors (Gürkaynak, 2008). Thus, testing for bubbles becomes, in the end, a test for the validity of the underlying pricing model and turns the debate into an ideological one where supporters and detractors from the idea of market efficiency fight over how best to ascertain *the price* of an asset (Borio & Lowe, 2002). The methodologies discussed in this paper are not intended to identify bubbles, nor do they presume that bubbles happen in asset markets. We will steer clear from this issue and analyze the dynamics of stock prices as a cycle where both busts and booms can be observed. In doing so it is not to be understood that a boom is, *per se*, a bubble or that a crash refers to it bursting.

The surveyed methodologies, summarized in Table 1 on the next page, can be characterized according to the way they use information and whether they make assumptions on the statistical characteristics of the underlying series. In the case of the use of information, the literature on early warning indicators, concerned with predicting the unwinding of financial imbalances, focuses mainly on the use of *ex ante* information: data that was available at the time of a given event in order to face the same identification problem as authorities and policymakers when characterizing a financial imbalance (Borio, 2006). On the other hand, a broader body of literature on the financial cycle focused on describing stylized facts and testing diverse mechanisms for the accumulation and unwinding of imbalances, uses all the information available without regard to whether it was accessible at the time of a particular event. The methodology presented in Part 4 partakes in the broader discussion of financial cycle characteristics and aims to offer new insights on the measurement and characterization of the boom-bust cycle for stock market indices. An extension of this methodology to test for its stability with the incorporation of new information goes far beyond the scope of this paper.

Table 1: Surveyed literature

Surveyed literature												
	Author(s)	Year	Information		Parametric				Non - parametric			
						Filtering						
			Ex-ante	*Ex-post*	*Fundamental valuation*	*Band pass*	*Hodrick Prescott*	*Functional form*	*Turning Point Algorithm*	*Empirical distribution*	*Absolute changes*	*Qualitative evidence*
Methodologies	Bry & Boschan	1971		X					X			
	Blanchard & Watson	1982	X		X					X		
	Hodrick & Prescott	1997	X				X		X			
	Stock & Watson	1998		X		X						
	Christiano & Fitzgerald	2003		X		X	X					
Crisis research	Kaminsky & Reinhart	1999	X							X	X	X
	Bordo *et. al.*	2001		X		X		X			X	X
	Bordo and Jeanne	2002	X					X				
	Borio & Lowe	2002	X				X					
	Eichengreen & Bordo	2002	X					X				
	Harding & Pagan	2002		X			X	X	X			
	Mishkin & White	2002		X							X	X
	IMF	2003							X		X	X
	Pagan & Sossounov	2003		X					X		X	
	Borio & Lowe	2004	X				X					
	Borio & White	2004	X				X					
	Bordo & Wheelock	2006		X					X			X
	Harding & Pagan	2006		X					X			
	Barro & Ursua	2009		X							X	X
	Bordo & Wheelock	2009		X					X			X
	IMF	2009	X					X				
	Reinhart & Rogoff	2009	X								X	X
	Schularick & Taylor	2009		X			X					
	Assenmacher-Wesche & Gerlach	2010		X			X					
	Gerdesmeier, Reimers & Roffia	2010	X				X					
	Jordà, Schularick & Taylor	2010		X			X			X		X
	Ng	2011	X		X		X					
	Drehmann, Borio & Tsatsaronis	2012		X		X			X			
	Bordo & Landon-Lane	2013		X					X			X
	Claessens & Kose	2013		X					X	X	X	X
	Dell'Ariccia, Igan, Laeven & Tong	2013		X				X			X	
	Jordà, Schularick & Taylor	2013		X				X	X	X		
	Laeven & Valencia	2013		X				X				
	Shin	2013	X				X					
	Borio	2014	X				X					
	Brunnermeier & Schnabel	2015		X								X
	Greenwald, Lettau & Ludvigson	2015	X		X							
	Passari & Rey	2015	X		X							
	Schüler, Hiebert & Peltonen	2015	X			X			X			

Parametric VS non-parametric models

A relevant methodological decision has to do with the use of parametric or non- parametric models in order to define booms and busts. In an excellent survey of the models for cycle research, Harding & Pagan (2005), identify diverse combinations of time series transformations and methodologies that yield different cycle dates. In Table 1, model families are organized as standing on a continuum that goes from more complex models to less complex ones. The great divide between the two panels resides on whether these models contain any assumption on the underlying statistical characteristics of the studied series (parametric) or they refrain from such assumptions and use only the empirical information available (non-parametric). Reducing the complexity of the model presents the researcher with a tradeoff since less complex models require less inputs but, at the same time, offer numerical results with lower information content. More complex models although more input intensive, offer more information-rich results[4].

[4] Another discussion that goes beyond the scope of this paper has to do with the quality of input variables and the corresponding quality of the final results. Mun (2006) refers to this as the garbage in - garbage out problem with financial modelling. The higher the demand for inputs there is an increasing probability that a given input contains error measurement or some sort of error and thus depletes the quality of the output.

A simple example comes from comparing fundamental value models (first column of the third panel in Table 1) with qualitative models (rightmost column of the fourth panel in Table 1). Fundamental value models demand large amounts of information and perform several assumptions about the future and, in return, they yield a point estimator for the *correct* price of a stock index at any given moment[5]. On the other hand, qualitative models based on historical accounts, secondary sources and the like, are less intensive in calculations and assumptions but may yield, at best, a time series of zeros and ones representing calm and crises periods as in Brunnermeier & Schnabel (2015).

Parametric models for the analysis of cycles can be of diverse types but we will focus only on filtering models since they are the ones that are more readily found in this literature. As a thought experiment the boom bust cycle can be treated as a wave: a succession of alternating peaks and troughs with changing amplitude and frequency. A linear filter is a "linear transformation of the data that leaves intact the components of the data within a specified band of frequencies and eliminates all other components" (Christiano & Fitzgerald, 2003, pg. 436). In the case of business cycle analysis, from where this strand of literature was bred, time series are treated as a linear composition of permanent, cyclical and seasonal[6] components. Thus, to estimate a cycle the permanent component has to be filtered out of the original data and what remains is usually referred to as a gap[7] (Harding & Pagan, 2005). Whenever this gap exceeds an arbitrary threshold set by the researcher the related dummy variable turns on to show a crises or a boom.

In order to filter a series Y_t, some form of averaging, which can have a symmetric window around an observation or use only prior observations, is chosen. A symmetric averaging uses both *ex ante* and *ex post* information and is referred to as a two sided filter, like the band-pass filter described by Stock & Watson (1998) and Christiano & Fitzgerald (2003). If the averaging is done using only *ex ante* information it is a one sided filter like the one described by Hodrick & Prescott (1997) which is widely used in this kind of research.

There are several caveats to the use of filters for the description of cycles. First, Stock & Watson (1998) indicate that this process of separating the trend and the cyclical component in a time series only makes sense when they are independent, but, if there is an underlying factor that affects and links both processes "detrending" Y_t will only result in a loss of information that may otherwise be relevant. "Indeed, the act of removing a stochastic trend actually eliminates one of the major driving forces of the business cycle and is therefore to be avoided" (Harding & Pagan, 2002, pg. 380). This was similarly stated by Bry & Boschan (1971) when they indicated that "the trend will vary with the choice of trend function, the criterion of best fit, and the time period covered. Any of these alternatives (…) influences the computed trend and hence the deviations and the cyclical

[5] The amount of assumptions has made it common place in investment banking and other industries that use these kinds of models to offer not only a point estimator but a confidence interval that reflects the uncertainty from the model, the assumptions and the inputs.
[6] There is no evidence of seasonal components in stock market data so this component will not be considered.
[7] These gaps can be calculated for output, credit, and many more macro and financial time series. Additionally, these gaps have an economic interpretation: "the permanent component is taken to represent supply side influences and the transitory components are demand side, so that an output gap attempts to separate out demand and supply in a simple fashion" (Harding & Pagan, 2005, pg. 152).

measures. It may therefore be preferable to use a different approach" (pg. 13). The assumption of orthogonality of the cycle and trend components in time series is the main reason why we classify filtering models as parametric ones.

A second issue with the applicability of filtering techniques to financial crises lies at its core: the idea of filtering is to smooth a time series as to only show the underlying cycle (Hodrick & Prescott, 1997). However, when studying bulls and bears it does not make sense to smooth out extreme observations which precisely constitute the most relevant information about both booms and busts as they lie in both extremes of the empirical distributions. Thirdly, as Hodrick & Prescott (1997) show, any filtering technique will alter the serial correlation of the underlying data, which, as will be shown in part 3, is a defining characteristic of the cycle in stock prices. Finally, this methodological family leaves many options open for the researcher to determine: the windows of estimation, parameters in the trend model and thresholds above which growth of the variable is excessive. This can potentially affect results and conclusions from studies using the same data (Assenmacher-Wesche & Gerlach, 2010). For the sake of brevity, an in-depth discussion of different filters can be found in Annex 1.

Non-parametric models for the study of the boom-bust cycle of a time series Y_t avoid assumptions on its data generating process. A particular subset of them, termed turning point algorithms, describe local maxima and minima of a time series under a preset group of conditions and instructions and render as a result a sequence of peaks and troughs with intermediate sections classified as contractions (bears) or expansions (bulls) (Harding & Pagan, 2005). One of the best known algorithms in business cycle literature is the one developed by Bry & Boschan (1971) to mimic the recession dates found by National Bureau of Economic Research. All of the papers in Table 1 that use the turning point methodology use the Bry & Boschan algorithm or some transformation of it[8].

In some cases, before using the turning point methodology, the data can be detrended by fitting it with a time series model (Harding & Pagan, 2005). This of course biases the results depending on the filter chosen and has all the caveats of parametric models discussed previously. The pure turning point methodology is not exempt of criticisms. A first issue is that this kind of algorithm leaves a lot of choices for the researcher (Bry & Boschan, 1971) and thus may yield different results for the same inputs. A distilled list of choices taken from Pagan & Sossounov (2003) is the following:

- Keeping or removing extreme values,
- Size of the window of observation from which local maxima and minima are chosen,
- Minimum length of a phase (peak to trough or trough to peak) or a cycle (peak to peak or trough to trough),
- Requirement of alternation between peaks and troughs,
- Minimum phase variation

A more profound criticism of this methodology is that it lacks statistical foundation and thus makes inference and hypothesis testing difficult. (Harding & Pagan, 2002) The usual result is a list of dates for peaks and troughs with no further description of the time series.

[8] A sample of this algorithm under different specifications will be used in part 5 when we discuss the results.

Other non-parametric ways of analyzing time series have to do with an in depth look at the empirical distribution of the data or at its absolute variation over a preset window of time. In this sense, some techniques include the analysis of time-varying volatility with moving windows (Gürkaynak, 2008) or the study of the dispersion of the data vis-à-vis a stable distribution through quantile-quantile (QQ) plots (Jordà, Schularick, & Taylor, 2010). Kaminsky & Reinhart, (1999) study the best and worst returns for a given price series as a means of inspecting the tails of the distribution. Claessens & Kose (2013) follow in this tradition and describe booms and busts in terms of their location (percentile) in the distribution. An even simpler analysis can be performed by looking at the absolute variations (gross returns) within a time series as done by Mishkin & White (2002) and Barro & Ursúa (2009).

The main caveat of this statistical description of data has to do with its lack of standardization. Different researchers will use different tools, different thresholds for their crises events, and the result will be very diverse dummy sequences. Additionally, none of the tools described extracts a cycle from the data since they do not extend their results beyond a description and thus this approach serves mostly as complimentary to others techniques.

From a first analysis of the available tools to study the bull and bear periods in the stock market we find that most of them leave many choices at the discretion of individual researchers while some of them, mostly parametric models, perform unrealistic assumptions about the underlying time series. An additional caveat from all of these methodologies is that the end result, be it a dummy sequence or a series of turning points does not reflect the rich informational content of the original series. To delve deeper into this topic we present the description of the database in the following section.

Part 3: Data and time series characteristics

We will use monthly market-wide value-weighted stock indices expressed in real terms for France[9] and the United Kingdom[10]. Both time series were downloaded from the Global Financial Database. Data for France starts in January 1895 and runs uninterruptedly until March 2015,

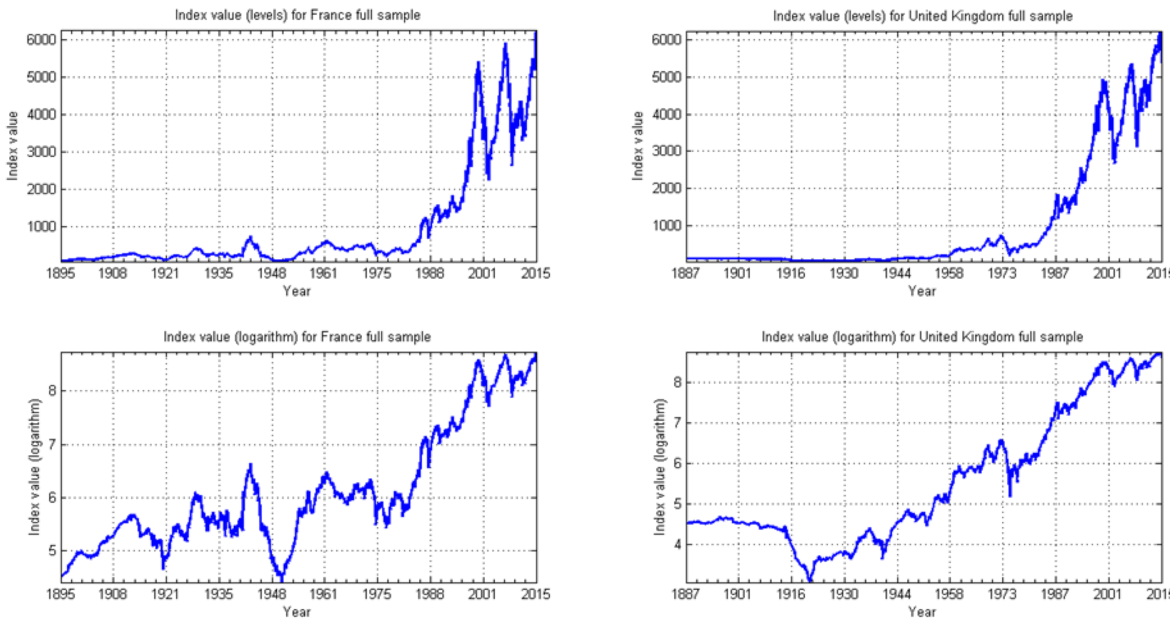
[9] The leading time series for France is France CAC All-Tradable Total Return Index which has a monthly frequency from January 1885 until January 1991 and daily frequency from January 1991 until March 2015. The value of monthly data is recorded as taking the value of the stock market index for the last day of the month. The data was obtained in real terms with CPI index=100 for December 1998.

[10] The leading time series for the UK is the UK FTSE All-Share Return Index which has a monthly frequency from August 1694 until December 1964 and daily frequency from December 1964 until September 2015. The value of monthly data is recorded as taking the value of the stock market index for the last day of the month. The index corresponds to Bank of England Shares exclusively from 1874 to 1922. Thus we will use a secondary source, "UK Banker's Magazine All Securities" which has monthly data from August 1887 until July 1966. We will join the real series in January 1933. The process consists on building back the FTSE series based on the real variations of the Banker's Magazine series. Missing months (17 observations out of 540 between July 1887 an December 1932) in the secondary series were calculated the variations in the FTSE data. The data was obtained in real terms with CPI index=100 for January 1987.

including 1443 observations. Data for the United Kingdom starts in July 1887 and runs uninterruptedly until September 2015, including 1539 observations[11].

Figure 1 presents graphs for the full series for both countries in levels of the index (100=31/01/1950) and their logarithmic transformation.

Figure 1: Time-series evolution in levels and logarithms



Since the top panels are in levels, there is a difficulty when analyzing such a long time series because scale changes in orders of magnitude through time. To correct this, in the lower panels we present a logarithmic transformation of the series that allows for a better viewing of the boom and bust cycles in the stock market. As it is usual with financial data, we calculated monthly growth rates (returns) for both stock indices since they tend to have interesting characteristics: asymmetry, fat tails, long run serial correlation, and clustering (Tsay, 2002). To calculate these returns we followed two alternative specifications. First we calculated the first difference of the logarithmic transformation of the indices which is an approximate growth rate of the form $r_{it} = p_{it} - p_{it-1}$ where $p_{it} = \ln(P_{it})$ and P_{it} corresponds to the value of the index for country i at time t. As a second specification we calculated a linear growth rate (simple return) of the form $R_{it} = (P_{it}/P_{it-1}) - 1$. It is noteworthy that the "real" or effective return and investor would observe in the market is the linear one. Interestingly, $r_{it} \approx R_{it}$ when changes in prices are sufficiently small. If market movements are very brusque, then the different methodologies generate fairly different values as we discuss ahead.

Shape of the distribution: Asymmetry and tail structure

Table 2 presents descriptive statistics for the index time series as well as for the growth rates (monthly returns) calculated in the two fashions described above.

[11] A particular issue with the use of stock market indices for such long periods of time since their composition is not constant. This issue is partially solved by using broad market indices so that the particular weight of any single stock decreases.

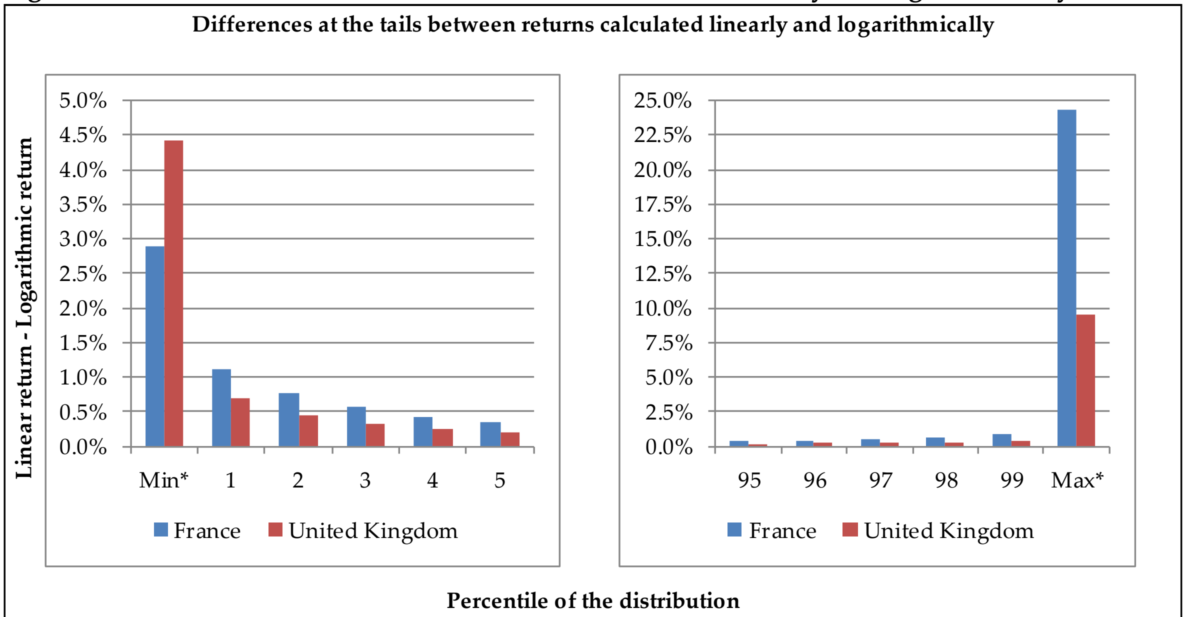
Table 2: Descriptive statistics for indices and logarithmic and linear growth rates

Descriptive statistics for indices and logarithmic and linear growth rates						
	Panel A: Index levels		*Panel B: Log differences*		*Panel C: Linear growth rates*	
	France	United Kingdom	France	United Kingdom	France	United Kingdom
Minimum	82.26	21.60	-25.11%	-31.29%	-22.20%	-26.87%
Maximum	6,221.24	6,252.60	62.47%	40.58%	86.76%	50.05%
Range	6,138.98	6,230.99	87.57%	71.87%	108.96%	76.91%
Mean	944.59	942.61	0.29%	0.27%	0.45%	0.35%
Standard deviation	1,371.71	1,546.27	5.53%	4.12%	5.73%	4.13%
Skewness			0.61	-0.35	2.27	0.57
Excess kurtosis			12.38	11.21	36.24	17.12
J-B statistic (1987)			9,305.47	8,077.93	80,169.12	18,864.92
Observations	1,443	1,539	1,442	1,538	1,442	1,538
Source: Author's calculations and Jarque & Bera (1987)						

The information provided by panel A is rather poor and difficult to analyze in particular because it tries to aggregate the level behavior of an index over a very long period of time. Panel B and C offer a more interesting set of values to analyze. First the minimum and maximum in each series correspond to the worst and best monthly returns and their difference provides an idea of the dispersion in both series. In both panels French data appears to be more disperse than data for the UK. The average monthly return appears to be higher for France than for the UK and, as evidenced in the standard deviation the French series is not only more disperse but also more variable (riskier) than the one for the UK.

When comparing the two methodologies for calculating returns, we find that the minimum and maximum for both series are lower when calculated using the logarithmic approach than the linear one. Additionally, panel B shows consistently lower dispersion and variability than panel C. The use of the logarithmic approximation also appears to change the symmetry in the data, as shown in the skewness measure. The shape of the tails, which contain most of the values of interest when analyzing booms (right tail events) and busts (left tail events) are nuanced as shown by the lower excess kurtosis coefficient in panel B when compared to panel C. To evidence the difference in the structure of the tails Figure 2 presents the differences between the growth rates calculated linearly or logarithmically for the bottom and top five percentiles of each time-series.

Figure 2: Differences at the tails between returns calculated linearly and logarithmically



* Min and Max reffer to the lowest and highest observations in each series

If we calculate the Jarque – Bera (1987) statistic (JB statistic) for both series and both panels we find substantially lower values in panel B than in panel C. This statistic offers a simple way to aggregate non-normal characteristics in the data[12]. The fact that the results from panel B and C are so different is thought provoking since, what has been treated as a monotonous transformation of data, seems to alter in a significant manner the shape of the distribution and, potentially, the results we obtain. Hence, whenever in this paper we refer to growth rates or returns we refer to the linear calculation as performed to obtain the results in panel C.

Naturally, the next step in this exploratory data analysis would consist of determining whether the data conforms to some distribution to motivate the use of parametric models to study the boom and bust cycle. In order to do this we follow Tukey (1977) and perform several transformations both on index levels and linear returns for both countries. These transformations follow a simple form:

$$y = \begin{Bmatrix} x^{\lambda} \text{ if } \lambda \neq 0 \\ \ln x \text{ if } \lambda = 0 \end{Bmatrix} \qquad (1)$$

Additionally we kept the original time series (y=x) and performed an exponential transformation (y=$exp(x)$).

On the transformed variables, different series of y, we performed skewness and kurtosis joint tests of normality (SK tests) following D'Agostino *et. al*. (1990). The null hypothesis is that the transformed variable y is normally distributed. We chose these tests rather than the Jarque-Bera test for which we calculated the statistic in Table 2 or the usual Kolmogorov-Smirnov test since SK test performs better for large samples. The resulting p-values are presented in Table 3 and show that we fail to accept the null hypothesis for all transformation of the 4 time series.

[12] This statistic was first presented in Jarque & Bera (1987) as a means of testing a time series for normality.

Table 3: P-values for SK normality tests on linear return transformations

P-values for skewness and kurtosis joint tests of normality on Tukey's (1977) ladder of transformations						
Transformation	λ	Formula	*Panel A: Index levels*		*Panel B: Linear growth Rates*	
			France	United Kingdom	Levels	Returns
Cubic	3	$y=x^3$	0.00	0.00	0.00	0.00
Square	2	$y=x^2$	0.00	0.00	0.00	0.00
Square root	½	$y=x^{½}$	0.00	0.00	0.00	0.00
Logarithmic	0	$y=\text{Ln}(x)$	0.00	0.00	0.00	0.00
Inverse square root	-½	$y=x^{-½}$	0.00	0.00	0.00	0.00
Inverse	-1	$y=x^{-1}$	0.00	0.00	0.00	0.00
Inverse square	-2	$y=x^{-2}$	0.00	0.00	0.00	0.00
Inverse cubic	-3	$y=x^{-3}$	0.00	0.00	0.00	0.00
Original		$y=x$	0.00	0.00	0.00	0.00
Exponential		$y=exp(x)$	N/A*		0.00	0.00

* Exponential transformations of the index levels yield numbers larger than the computers floating point

Sources: Authors calculations, SK test from D'Agostino *et. al*. (1990), ladder of transformations from Tukey (1977)

These results provide some evidence against the use of parametric boom-bust models that require the assumption of any stable distribution for the underlying time series. In this sense, fundamental value models that presuppose knowledge of the DGP are not adequate candidates.

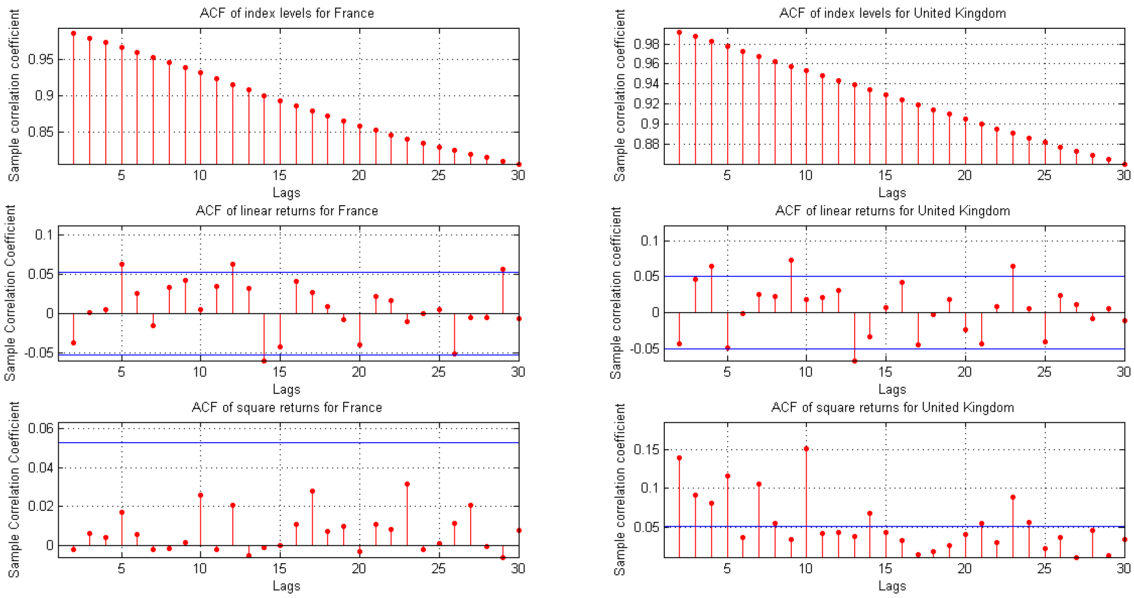
Serial correlation: Indices, returns and squared returns

Financial time series, including stock market indices like the ones treated in this paper, present several "anomalies", including the fact that large positive (negative) returns tend to be followed by large positive (negative) returns (Fama & French, 2008). This particular anomaly, which is called *momentum* can be diagnosed by the use of autocorrelation coefficients. Figure 3 depicts the autocorrelation for the index levels, the monthly returns and the squared monthly returns up to 30 lags. A significant coefficient for lag n means that the return (index level) at month $t-n$ has a significant effect on return (index level) for month t. The blue line that appears in the correlograms refers to confidence bound at the 95% level.

The autocorrelation of the index level should always be positive and decreasing in time since the price process is always path dependent, meaning the value of the index at time $t+1$ depends of its value at time t. This means that prices tend to behave as autoregressive processes with long memory. This analysis can be refined by studying both returns and squared returns. Large and positive values for the autocorrelation of the returns indicate that positive (negative) returns tend to be followed by positive (negative) returns. Positive and significant autocorrelations of the return variable, though incomplete and inconclusive proof, may serve as first evidence not only for the *momentum* anomaly but for the existence of a cycle (Harding & Pagan, 2005). Large and positive values for the autocorrelation of the squared returns indicate that large (small) returns tend to be followed by large (small) returns and thus serve as evidence of volatility clustering[13].

[13] The square return serves as a proxy of the variance of returns if the mean is sufficiently close to 0. From Table 2 we can see that the standard deviation of returns in panel C is 12.78 times the mean for France and 11.79 times the mean for the United Kingdom.

Figure 3: Autocorrelation functions for index levels, returns and squared returns



A first inspection of Figure 3 shows that the top panels yield the expected results, large dependence for the first lags and then decreasing at an almost constant rate as the number of lags is increased. For linear returns it is interesting to see that most of the significant coefficients are positive, and thus provide some initial evidence of momentum in both time series. However, when looking at the bottom panel, we see that while there are very significant coefficients for the UK stock market, the French time series shows no significant correlation coefficients for the first thirty lags. A possible interpretation is that while the UK stock market behaves as an explosive process, the French stock market has some mean reverting characteristics. Testing this hypothesis, however, goes well beyond the scope of this paper.

It is interesting to go beyond the graphical examination and perform a formal statistical test to verify the joint significance of the coefficients for different lags. The Ljung-Box (1978) test verifies if the coefficients from 1 up to l lags are jointly different from 0 and thus confirms if the inference from the correlograms can or cannot be rejected. According to Tsay (2002) the optimal number of lags to perform this test is equivalent to the natural logarithm of the number of observations rounded to the closest integer. Table 4 presents the statistic for each series and number of lags. It was calculated for one, two, three and six months (short term), for seven months which is the optimal number of lags, and then for 1, 1.5, 2 and 2.5 years corresponding to the medium term. The probability of false positives of the test increases with the number of lags so estimations for larger lags are not conducted.

Table 4: Ljung-Box (1978) test statistic for joint significance of autocorrelation coefficients

Ljung - Box (1978) test statistic for joint significance of autocorrelation coefficients												
Lags in months		1	2	3	4	5	6	7°	12	18	24	30
France	Index levels	1,425.58 ***	2,830.88 ***	4,217.91 ***	5,588.57 ***	6,941.39 ***	8,277.17 ***	9,594.43 ***	15,898.14 ***	22,838.18 ***	29,194.77 ***	35,108.84 ***
	Linear returns	18.12 ***	20.17 ***	20.17 ***	20.21 ***	25.84 ***	26.81 ***	27.13 ***	38.8 ***	51.7 ***	55.32 ***	64.13 ***
	Squared returns	0.35	0.36	0.41	0.44	0.86	0.91	0.91	2.5	3.97	5.9	6.87
United Kingdom	Index levels	1,530 ***	3,047.94 ***	4,552.82 ***	6,044.9 ***	7,522.19 ***	8,985.12 ***	10,434.45 ***	17,480.47 ***	25,506.74 ***	33,068.84 ***	40,213.76 ***
	Linear returns	22.68 ***	25.67 ***	28.9 ***	35.43 ***	39.05 ***	39.05 ***	40 ***	51.91 ***	66.61 ***	77.73 ***	81.82 ***
	Squared returns	53.09 ***	83.16 ***	96.18 ***	106.24 ***	127.14 ***	129.23 ***	146.37 ***	193.79 ***	208.7 ***	235.96 ***	244.45 ***

° Reffers to the optimal number of lags according to Tsay (2002). It is the result of L≈Ln(m), where m is the number of observations

Significance: * = 90%; ** = 95%; *** = 99%

As expected, the joint test for any number of lags is significant to the 99% level for all-time series except for the squared returns of the French stock market. This is interesting as it may be related to a lack of evidence for volatility clustering although still consistent with the *momentum* anomaly. We expect all these time-series characteristics to remain present in the BBI measure that we calculate in Part 4.

Clustering: a colorful primer

To summarize the findings from the previous two subsections, returns reflect more information than price levels, even when expressed as logarithms of the original variable. In addition, it is preferable to use simple (linear) returns over logarithmic differences since the first reflect better the changes in wealth of investors. After studying the first four sample moments of the returns and performing several statistical tests there is no evidence that the time series under study conform to any stable distribution which suggests this study should continue using non-parametric measures. Finally, there is evidence of serial correlation for linear returns in both series and for squared returns in the case of the UK's stock market. This follows on the finding by Fama & French (2008) of momentum as an anomaly and conforms to the definition of a bull market as a long succession of positive returns and a bear as a long succession of negative returns. The existence of serial correlation for squared returns is evidence of explosive processes, such as the ones that usually happen during stock market crashes where large negative returns tend to happen in runs.

If present, these bull and bear markets should be visible as waves of positive and then negative returns. However, studying only monthly returns may be counterproductive as short run financial data tends to have more noise than long run returns. Thus we will construct an N-month linear return matrix in which rows will represent time and columns will hold the return from month t-n until t. N will only take successive integer values from 1 to 12 months and then values of 18, 24, 30, 36, 42, 48, 54 and 60. When n=3, for example, the return refers to a quarterly measure, if n=24 it is the return for the two year period. Table 5 depicts graphically the resulting matrix:

Table 5: Graphical representation of an N-monthly return matrix

Graphical representation of an N-monthly return matrix																				
Year	N-monthly return - UK stock market																			
	Short run												Medium run				Long run			
	1	2	3	4	5	6	7	8	9	10	11	12	18	24	30	36	42	48	54	60
1																				
2																				
3																				
4		**A**																		
...						...								...				...		
T-1																			**C**	
T														**B**						

Return A represents the two month simple return between t=2 and t=4. Return B represents the two year return between month T-24 and T. Return C represents the 4.5 year return between month T-55 and T-1. We have separated the returns for the first twelve values of N as short term returns, those between 1.5 and 3 years as medium run returns and the rest as long run returns.

Once the matrix has been constructed we have 20 distinct time series that represent the behavior of each stock market. These vectors are correlated by construction but their correlation decreases as they become more separated (lagged), as can be seen in the heat map in Table 6. Warmer colors refer to higher correlations and cooler colors refer to lower correlations.

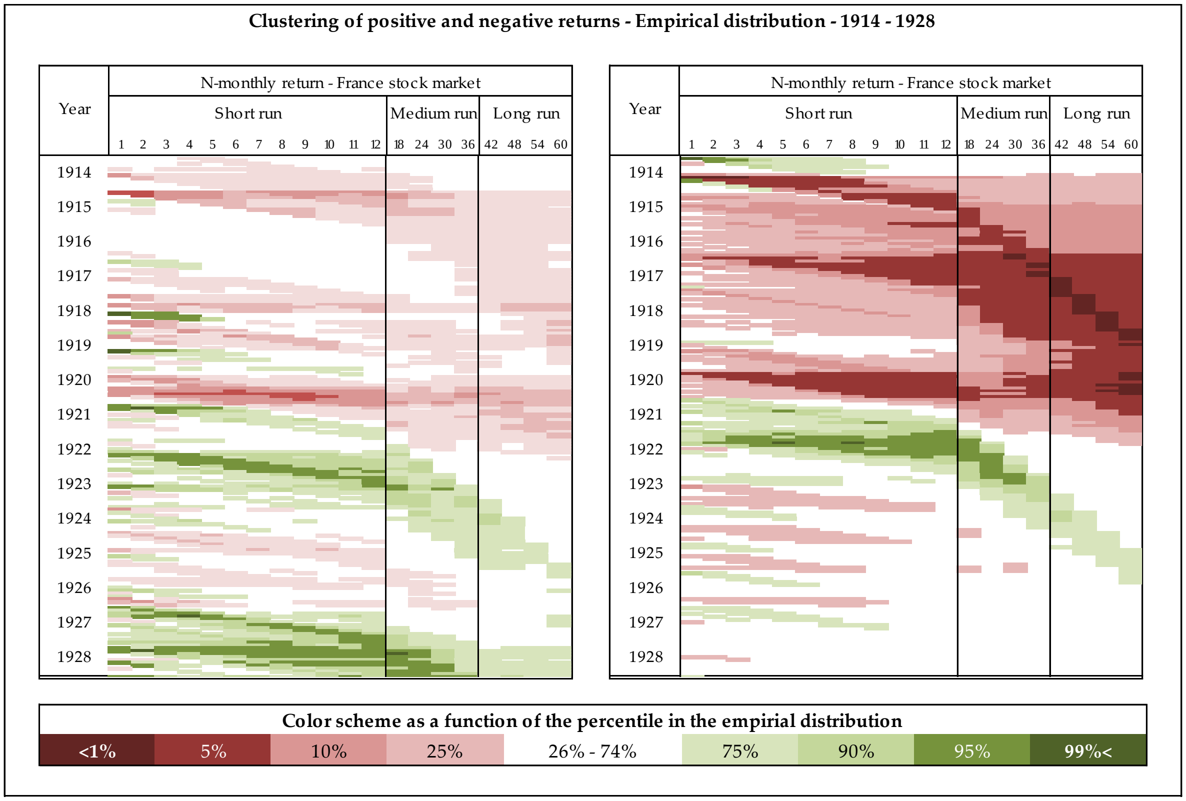
Table 6: Heat map for the correlation matrix of the N-monthly returns for France

Heat map for the correlation matrix of the N-monthly returns for France																				
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*18*	*24*	*30*	*36*	*42*	*48*	*54*	*60*
1	1.00																			
2	0.74	1.00																		
3	0.58	0.84	1.00																	
4	0.50	0.71	0.88	1.00																
5	0.45	0.63	0.78	0.91	1.00															
6	0.42	0.59	0.71	0.82	0.92	1.00														
7	0.39	0.55	0.66	0.76	0.85	0.93	1.00													
8	0.36	0.51	0.63	0.71	0.79	0.87	0.94	1.00												
9	0.35	0.48	0.59	0.68	0.75	0.82	0.88	0.95	1.00											
10	0.34	0.46	0.56	0.64	0.72	0.78	0.84	0.89	0.95	1.00										
11	0.31	0.44	0.54	0.61	0.68	0.75	0.80	0.85	0.90	0.96	1.00									
12	0.31	0.42	0.52	0.59	0.66	0.72	0.77	0.82	0.87	0.91	0.96	1.00								
18	0.26	0.35	0.42	0.49	0.55	0.61	0.65	0.70	0.74	0.77	0.80	0.84	1.00							
24	0.21	0.29	0.36	0.41	0.46	0.51	0.55	0.59	0.63	0.66	0.70	0.73	0.88	1.00						
30	0.19	0.26	0.31	0.36	0.41	0.45	0.49	0.52	0.56	0.59	0.62	0.65	0.81	0.91	1.00					
36	0.17	0.24	0.29	0.33	0.38	0.41	0.45	0.48	0.51	0.54	0.56	0.59	0.73	0.85	0.93	1.00				
42	0.15	0.21	0.27	0.31	0.36	0.39	0.43	0.46	0.48	0.51	0.53	0.56	0.68	0.78	0.87	0.94	1.00			
48	0.12	0.17	0.21	0.25	0.30	0.33	0.37	0.40	0.43	0.46	0.48	0.51	0.63	0.73	0.81	0.89	0.95	1.00		
54	0.11	0.15	0.19	0.23	0.26	0.29	0.32	0.34	0.37	0.39	0.42	0.44	0.58	0.68	0.76	0.83	0.90	0.95	1.00	
60	0.09	0.13	0.16	0.20	0.23	0.27	0.30	0.32	0.34	0.36	0.38	0.40	0.52	0.62	0.71	0.78	0.85	0.91	0.96	1.00

Source: Author's calculations

We will use the N-monthly return matrix **R**, to build a heat map following a simple coloring rule on a column-by-column basis for the full time period. Our focus will be mainly on the tails of the empirical distributions which hold the highest and lowest observed returns. The rule is such that returns farther away in the tails are colored darker, while if closer to the median[14] of the distribution their color is lighter, but any return falling between the 25th and 75th percentiles (both ends excluded) will not be colored. Left tail (first quartile) events will be colored in shades of red and right tail (fourth quartile) events will be colored in shades of green. Figure 4, presents the heat maps of **R** for France and the United Kingdom between January 1914 and December 1928.

Figure 4: Clustering of returns according to the empirical distribution (1914-1928)



We can draw several interesting readings from this Figure. First, focusing on the left panel for France, we can see overall a bear market between 1914 and the early 1920s with short recoveries at the end of 1918 and 1919. These recoveries, however were short lived as they do not affect N-monthly returns beyond n=5 in the first recovery and beyond n=10 in the second one. Another way to put it is that the colors in the graph remain closer to the left edge and do not reach the long run section. A stronger bull starts early in 1921 and, after a correction period that goes until 1922, faces a second wave of very positive returns that lasts until mid-1924. This boom is far longer lived than the previous two recoveries, affecting both medium and long run returns, as it colors the whole row. However, the effect on long run returns was only mild since the shade of green is clear,

[14] Given that the analysis yielded a skewed distribution for both return time series, we prefer the median rather than the mean as the center of the distribution.

representing the 75th percentile. A two year correction of mild characteristics follows and then a new intense bull market that affects long run returns by the end of 1928.

Second, the right panel, for the United Kingdom, shows a different story. There is a long lasting bear market that starts with the beginning of the First World War and that remains until the end of 1920. It has a strong and pervasive effect as it colors in dark red up to the right edge of the panel. Then a short but strong recovery that lasts until mid-1923 and then an indecisive market that alternates between short lived low intensity bulls and bears.

Third, the intensity of the colors in both panels says something about the relevance that the bull and bear markets shown in the figure have on the overall sample. Recall the coloring is performed based on the percentile a given return occupies on the overall sample and this figure only shows a section of it. Since more extreme returns were colored in darker shades, that both bulls and bears are of light color in France indicates that there are more relevant events to be studied elsewhere in the series. Contrarily, the bear market in the United Kingdom was one of the worst in the sample, in particular if we focus on the very dark long run effects that can be seen affecting long run returns (the right side of the panel) well until 1920.

Finally, serial correlation of returns seems to hold not only for monthly returns but to remain present well into long run-five year returns. This is relevant as it allows us to see both the boom-bust cycles in the stock market, which we could infer from the correlogram in Figure 3, and the intensity and pervasiveness of this cycle across time. As stated in the introduction, all the information presented in figure 4, and all the analysis that can be performed from it cannot be reflected in a simple yes/no sequence. Actually when comparing the evolution of the busts in France and the UK during World War I, we have an indication that they do not behave equally. For the sake of brevity a similar analysis of three more sections of the database are presented in Annex 2. Condensing all of this information into one indicator, comparable across time series, is the task that we tackle in the following section.

Part 4: A new methodological proposition

In order to develop the boom-bust indicator (BBI) we will use two different methodologies. The first one will assume a symmetric tail structure while an alternate specification will take into account the skewness in the original data.

Methodology 1: Fixed grading BBIs

Let matrix **R**, where observation $r_{t,n} = [(P_t/P_{t-n}) - 1]$ for $t<n$, refer to the n-monthly return at month t. Each column vector in **R** is a time series of returns $\boldsymbol{r_n}$ with no time subscript. Let $\mathrm{Per}_x(\boldsymbol{r_n})$ refer to the x^{th} percentile in vector $\boldsymbol{r_n}$. Define a grading matrix **G** with the same dimensions as **R** such that:

$$g_{t,n} = \begin{cases} -10 \text{ if } r_{t,n} \leq \text{Per}_1(\boldsymbol{r_n}) \text{ and } r_{t,n} < 0 \\ -5 \text{ if } \text{Per}_1(\boldsymbol{r_n}) < r_{t,n} \leq \text{Per}_5(\boldsymbol{r_n}) \text{ and } r_{t,n} < 0 \\ -2.5 \text{ if } \text{Per}_5(\boldsymbol{r_n}) < r_{t,n} \leq \text{Per}_{10}(\boldsymbol{r_n}) \text{and } r_{t,n} < 0 \\ -1.25 \text{ if } \text{Per}_{10}(\boldsymbol{r_n}) < r_{t,n} \leq \text{Per}_{25}(\boldsymbol{r_n}) \text{ and } r_{t,n} < 0 \\ 0 \text{ if } \text{Per}_{25}(\boldsymbol{r_n}) < r_{t,n} > \text{Per}_{75}(\boldsymbol{r_n}) \\ 1.25 \text{ if } \text{Per}_{75}(\boldsymbol{r_n}) \leq r_{t,n} < \text{Per}_{90}(\boldsymbol{r_n}) \\ 2.5 \text{ if } \text{Per}_{90}(\boldsymbol{r_n}) \leq r_{t,n} < \text{Per}_{95}(\boldsymbol{r_n}) \\ 5 \text{ if } \text{Per}_{95}(\boldsymbol{r_n}) \leq r_{t,n} < \text{Per}_{99}(\boldsymbol{r_n}) \\ 10 \text{ if } \text{Per}_{99}(\boldsymbol{r_n}) \leq r_{t,n} \end{cases} \qquad (2)$$

Note that the grades are assigned arbitrarily with certain conditions: that $r_{t,n}$ exceeds a given boundary; that observations farther away in a tail weigh more than observations closer to the center; that left tail returns must both occupy a given percentile and take on negative values. The rationale behind this last condition is that a bear market is defined as a succession of negative returns. Since the mean of returns is positive it is unnecessary to set a similar condition for positive returns. A first implication is that the tail structure is assumed to be symmetrical since the absolute value of the grade for percentile pairs 1 and 99, 5 and 95, 10 and 90, and 25 and 75 are the same. The only allowed asymmetry comes from the cutoff of positive returns in the first quartile.

Let the boom-bust indicator (BBI) be defined as a column vector of the form:

$$\text{BBI}=\mathbf{G}\boldsymbol{\omega}' \qquad (3a)$$

Or, in algebraic notation,

$$BBI_t = \sum_{n=1}^{N} G_{t,n}\omega_n \qquad (3b)$$

Where $\boldsymbol{\omega}$ is a column vector of weights. For ease of interpretation $\omega_n \geq 0$ and $\sum_{n=1}^{N}\omega_n = 1$. Note that by construction BBI is bounded in the interval [-10, 10]. Let us define three possible $\boldsymbol{\omega}$ to obtain an equal number of BBI's, for the short run, for the long run and an equally weighted one. The weight vectors are presented in Table 7.

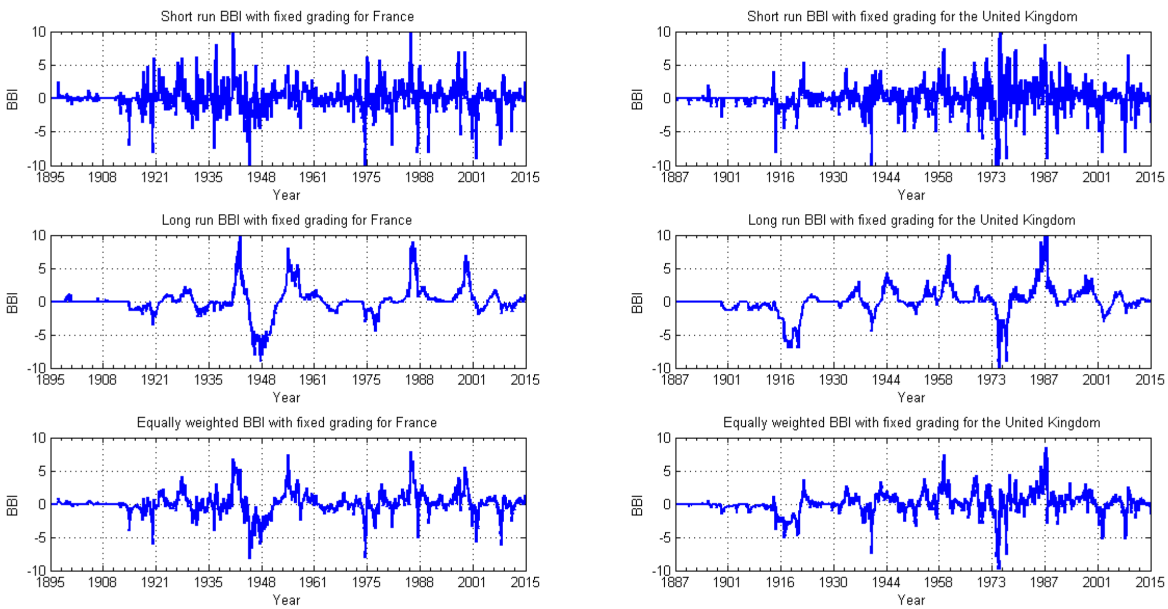
Table 7: Weighing vectors for different specifications of BBIs

Weighing vectors for different specifications of BBIs																				
Months	Short run												Medium run				Long run			
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*18*	*24*	*30*	*36*	*42*	*48*	*54*	*60*
Short run (ω_s')	0.20	0.20	0.20	0.20	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Long run (ω_l')	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.20	0.20	0.20
Equally weighted (ω_e')	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

The motivation for including three distinct weight vectors comes from Figure 4. When comparing the booms and busts for France and the United Kingdom one of the more salient characteristics is that while some booms and busts have very long run effects, coloring the rightmost part of the panels where long run returns are represented, some booms and busts are not as pervasive in time. We expect the short run vector to privilege non pervasive or explosive booms and busts while the long run BBI should show the more economically significant and widespread crashes as it represents more permanent shocks. The equally weighted specification should serve to control for the intensity of booms without discriminating for their pervasiveness.

After calculating the three different BBI series for both countries under this fixed grading methodology we obtained the time series presented in Figure 5.

Figure 5: Boom-Bust Indicator under fixed grading for different weight vectors ω



It is noteworthy that the top two panels (short-run BBIs) hold very noisy time series when compared to the middle panel (long run BBIs). The equally weighted BBI series seem to be a middle road between the other two. An interesting observation, in particular for the long run BBI, is that it does, in effect, resemble a wave. In order to start studying the specific properties of these new time series we present descriptive statistics in Table 8.

Table 8: Descriptive statistics for BBIs under different weights for fixed grading methodology

Descriptive Statistics for BBIs under different weighing specifications for fixed grading methodology						
	Panel A: Short run		*Panel B: Long run*		*Panel C: Equally weighted*	
	France	United Kingdom	France	United Kingdom	France	United Kingdom
Minimum	-10.00	-10.00	-9.00	-10.00	-8.13	-9.75
Maximum	10.00	10.00	10.00	10.00	7.88	8.56
Range	20.00	20.00	19.00	20.00	16.00	18.31
Mean	0.00	0.00	0.00	0.00	0.00	0.00
Standard deviation	1.91	1.91	2.07	2.03	1.66	1.65
Skewness	-0.23	-0.41	0.00	-0.16	-0.07	-0.67
Excess kurtosis	5.86	5.83	5.07	5.79	4.27	6.90
J-B statistic (1987)	2,075.34	2,218.65	1,547.07	2,157.06	1,094.47	3,167.68
Observations	1,442	1,538	1,442	1,538	1,442	1,538

There are various relevant elements worth highlighting. First, the short run BBI is equally disperse for both countries and reaches both the high and the low bound which is consistent with coverage of very explosive processes. In the case of the other two panels, only the UK's long run BBI spans the whole interval. When looking at the equally weighted BBI, it is the least disperse indicator of the three specifications for both countries.

Second, all of the time series except for the long run French BBI are negatively skewed. France presents the highest left skewness in the short run specification while the UK does in the equally weighted one. Skewness in this case would indicate a higher presence of bear markets when compared to bull markets. All BBI time series are leptokurtic and the JB statistic is consistent with these deviations from normality.

Third, when comparing standard deviations between countries results are mixed. The UK data appears more disperse than French data in the short run specification while the inverse happens with the long run BBI. Results seem inconclusive in the third panel. It might be interesting to test whether this is due to qualitatively different booms and busts in France and the UK. If the interpretation of the short run indicator is correct, the UK should have more explosive booms and busts and France should have more pervasive or long run ones.

Finally, although not as leptokurtic as the original linear returns in panel C of Table 2, results seem promising. However, the low values for the skewness indicate that part of the asymmetry is lost in the process of construction of the BBI. This motivates an alternative methodological design focused on the incorporation of said asymmetry.

Methodology 2: Standard deviation grading BBIs

The main difference with the previous methodology has to do with the construction **G**. In this methodology the grades for each vector $\boldsymbol{r_n}$ will be a function of the distance of each datum, expressed in terms of sample standard deviations, to the mean of the time series[15]. This alteration allows the BBIs to reflect the asymmetrical tail structure that occurs in skewed distributions. Let the distance from a given percentile to the mean be measured as,

$$z_x = \frac{Per_x(\boldsymbol{r_n}) - \mu_n}{\sigma_n} \qquad (4)$$

Where μ_n is the mean and σ_n is the sample standard deviation for vector $\boldsymbol{r_n}$. Note that if $|z_1| > |z_{99}|$, the distribution is skewed to the left as it has a longer left tail, while if $|z_1| < |z_{99}|$, the distribution is skewed to the right. For distributions skewed to the left, the grade for $r_{t,n} < Per_1(\boldsymbol{r_n})|Per_1(\boldsymbol{r_n}) < 0$ will be -10. The rest of the grading for the different bins will be built following a recursive process that uses proportions of z. For example, the grade for $Per_1(\boldsymbol{r_n}) < r_{t,n} < Per_5(\boldsymbol{r_n})|Per_5(\boldsymbol{r_n}) < 0$ will be $-10(z_5/z_1)$. The formulae for the different grades in this recursive process are contained in Table 9.

[15] Given that $\boldsymbol{r_n}$may be skewed it might make sense to calculate z_x using the median rather than the mean. When a distribution is skewed to the left, the median is larger than the mean and thus the results for z would shift to the left (decrease). When a distribution is skewed to the right, the mean is larger than the median and thus the results for z would shift to the right (increase). This extension is left for future versions of this paper as it may be useful to better reflect the asymmetry in $\boldsymbol{r_n}$. We expect that using this new measure will increase the variability of the results in the second methodology, so if current results are biased it is towards a more conservative cycle.

Table 9: Recursive process for grades when $\mathbf{r_n}$ is skewed to the left

Recursive process for grades when $\mathbf{r_n}$ is skewed to the left								
Percentile	$r_{n,t}$<Per$_x$ ($\mathbf{r_n}$) | $r_{n,t}$<0				r_{nt}>Per$_x$ ($\mathbf{r_n}$)			
	1	5	10	25	75	90	95	99
Grade g_x	-10	g_5=-10(z_5/z_1)	g_{10}=-10(z_{10}/z_1)	g_{25}=-10(z_{25}/z_1)	g_{75}=-10(z_{75}/z_1)	g_{90}=-10(z_{90}/z_1)	g_{95}=-10(z_{95}/z_1)	g_{99}=-10(z_{99}/z_1)

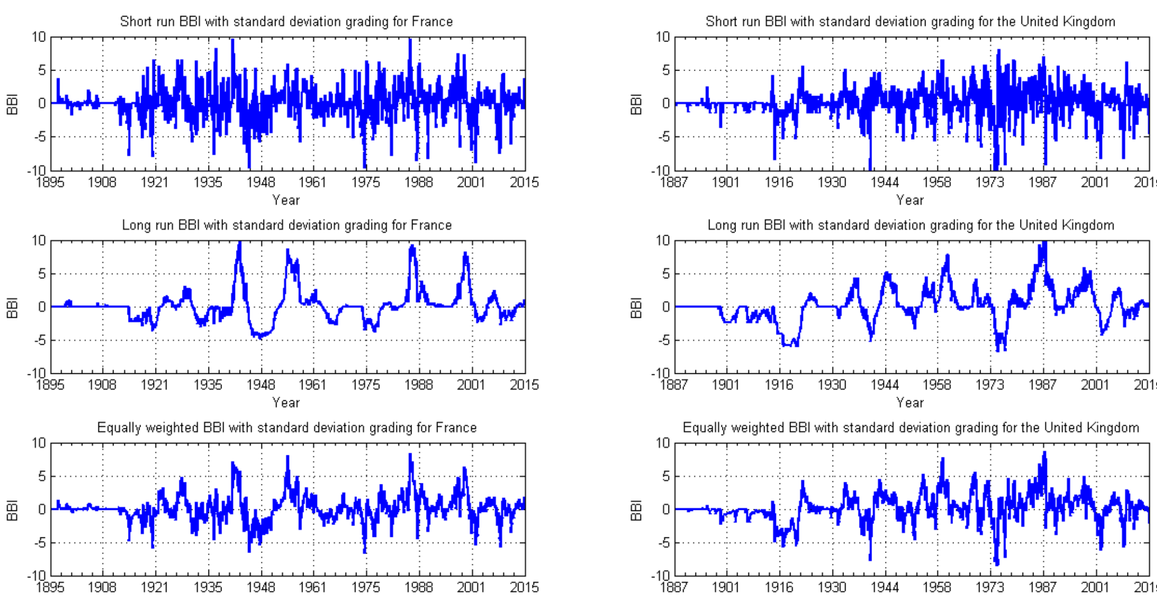
For distributions skewed to the right, the grade for $r_{t,n} > Per_{99}(\boldsymbol{r_n})$ will be 10. The rest of the grading for the different bins will be built following a recursive process that uses proportions of z. For example, the grade for $Per_{95}(\boldsymbol{r_n}) < r_{t,n} < Per_{99}(\boldsymbol{r_n})$ will be $10(z_{95}/z_{99})$. The formulae for the different grades in this recursive process are contained in Table 10.

Table 10: Recursive process for grades when $\mathbf{r_n}$ is skewed to the right

Recursive process for grades when $\mathbf{r_n}$ is skewed to the right								
Percentile	*rnt*<Per*x* (**rn**)				*rnt*>Per*x* (**rn**)			
	1	5	10	25	75	90	95	99
Grade gx	g_1=10(z_1/z_{99})	g_5=10(z_5/z_{99})	g_{10}=10(z_{10}/z_{99})	g_{25}=10(z_{25}/z_{99})	g_{75}=10(z_{75}/z_{99})	g_{90}=10(z_{90}/z_{99})	g_{95}=10(z_{95}/z_{99})	10

Note that this process gives the largest grade (positive or negative) the datum which is farthest away, to the right or to the left, of the center of the empirical distribution. After obtaining the new grading matrix we multiply it by the different vectors of weights **ω** as contained in Table 7 and obtain a new set of BBIs which are also bounded in the [-10 10] interval. We present a graph of these new time series in figure 6.

Figure 6: Boom-Bust Indicator under standard deviation grading for different weight vectors ω



As in Figure 5, the short run indicator seems to be noisier while the long run indicator shows a more wavelike pattern. The equally weighted indicator takes a middle road. In the case of the long run indicator, booms seem to be twice as intense as busts since the minimum value of the indicator is almost half the maximum. To confirm these observations and delve deeper into the series' characteristics, descriptive statistics are presented in Table 11.

Table 11: Descriptive statistics for BBIs under different weights for standard deviation grading methodology

Descriptive Statistics for BBIs under different weighing specifications for standard deviation grading methodology						
	Panel A: Short run		*Panel B: Long run*		*Panel C: Equally weighted*	
	France	United Kingdom	France	United Kingdom	France	United Kingdom
Minimum	-9.61	-10.00	-4.92	-6.79	-6.69	-8.56
Maximum	9.58	8.07	10.00	10.00	8.32	8.61
Range	19.19	18.07	14.92	16.79	15.00	17.17
Mean	-0.03	0.03	0.05	0.10	0.01	0.08
Standard deviation	2.40	2.16	2.34	2.56	1.95	2.08
Skewness	-0.15	-0.53	1.24	0.24	0.38	-0.19
Excess kurtosis	2.09	3.06	2.90	1.44	1.94	2.29
J-B statistic (1987)	268.00	670.47	874.01	147.43	259.99	346.29
Observations	1,442	1,538	1,442	1,538	1,442	1,538
Source: Author's calculations and Jarque & Bera (1987)						

Interestingly, no specification spans the whole [-10, 10] interval but standard deviations are consistently higher when compared country by country with table 8. Still the range for the short run BBI is the highest of the three, which is consistent with it showing more explosive processes and being a noisier indicator. Additionally, means move away from 0 and (mostly) into positive territory while the values of the asymmetry coefficient increase with respect to the fixed grading specification. A first implication of this has to do with the idea that booms are stronger, more intense, than busts at least when comparing absolute real returns. However, excess kurtosis decreases for all of the BBIs from Table 8 to Table 11. This is an indication that the overall BBI distribution flattens in this new specification, which is consistent with the decrease observed in the value of the JB statistic across the board. Annex 3 contains histograms for the different BBI specifications for each country.

When comparing country data, French BBIs are more disperse in panels A and C but not in the long run BBI (panel B). This result contradicts findings from the previous methodology. From this simple descriptive exercise it is impossible to determine if one methodology strictly dominates or outperforms the other. It is necessary to go one step further and asses the real information content of the different BBI specifications. To do so means to investigate how these new measures correlate with the original time series. Table 12 presents correlation matrices for prices, returns, squared returns and BBIs under the different specifications presented in this section.

Table 12: Correlation coefficients between several time series and BBIs

Correlation coefficients between several time series and BBIs

Panel A: Fixed grading BBI's

France	P	r	r^2	BBI_{s1}	BBI_{l1}	BBI_{e1}	United Kingdom	P	r	r^2	BBI_{s1}	BBI_{l1}	BBI_{e1}
P	1						P	1					
r	0.03	1					r	0.03	1				
r^2	-0.01	0.39	1				r^2	0.01	0.17	1			
BBI_{s1}	0.04	0.71	0.12	1			BBI_{s1}	0.03	0.74	-0.01	1		
BBI_{l1}	0.20	0.15	0.00	0.28	1		BBI_{l1}	0.15	0.15	-0.10	0.31	1	
BBI_{e1}	0.13	0.45	0.07	0.75	0.74	1	BBI_{e1}	0.08	0.46	-0.13	0.77	0.73	1

Panel B: Standard deviation grading BBI's

France	P	r	r^2	BBI_{s2}	BBI_{l2}	BBI_{e2}	United Kingdom	P	r	r^2	BBI_{s2}	BBI_{l2}	BBI_{e2}
P	1						P	1					
r	0.03	1					r	0.03	1				
r^2	-0.01	0.39	1				r^2	0.01	0.17	1			
BBI_{s2}	0.06	0.71	0.09	1			BBI_{s2}	0.03	0.73	-0.04	1		
BBI_{l2}	0.19	0.14	0.01	0.28	1		BBI_{l2}	0.16	0.15	-0.08	0.30	1	
BBI_{e2}	0.14	0.46	0.06	0.77	0.73	1	BBI_{e2}	0.10	0.44	-0.11	0.76	0.73	1

Panel C: Fixed grading BBI's VS Standard deviation grading BBI's

France	BBI_{s1}	BBI_{l1}	BBI_{e1}	BBI_{s2}	BBI_{l2}	BBI_{e2}	United Kingdom	BBI_{s1}	BBI_{l1}	BBI_{e1}	BBI_{s2}	BBI_{l2}	BBI_{e2}
BBI_{s1}	1						BBI_{s1}	1					
BBI_{l1}	0.28	1					BBI_{l1}	0.31	1				
BBI_{e1}	0.75	0.74	1				BBI_{e1}	0.77	0.73	1			
BBI_{s2}	**0.98**	0.29	0.75	1			BBI_{s2}	**0.99**	0.33	0.78	1		
BBI_{l2}	0.27	**0.95**	0.71	0.28	1		BBI_{l2}	0.28	**0.96**	0.70	0.30	1	
BBI_{e2}	0.75	0.72	**0.98**	0.77	0.73	1	BBI_{e2}	0.74	0.73	**0.98**	0.76	0.73	1

P: Index level, r: linear return, r^2: squared return, $BBI_{X\#}$: Boom Bust Indicator (s)hort run, (l)ong run, (e)qually weighted, (1) fixed grading, (2) standard deviation grading. Vector of correlations with linear returns shown in dotted columns in panel A and B. Bold data in Panel C shows correlation between different methodologies but same ω.

Both panels A and B in Table 12 show that the BBIs have very weak association to the index levels which is not surprising given that the series was constructed from changes in those levels and not with the levels *per se*. The strongest relation in both panels is between short run BBIs and linear returns followed by equally weighted BBIs. The fact that association between returns and the short run BBI is positive and strong implies that it tracks sufficiently well the linear evolution of returns. Additionally, the association remains strong and for the equally weighted BBIs which seems to indicate that the information content of these two specifications is important.

Panel C shows the association between the different specifications of BBIs under both methodologies. The highest associations are between BBIs with the same weight vector under different grading matrices, as signaled in bold. Most pairwise correlations, especially those involving equally weighted BBIs are positive and significant. Interestingly, the lowest correlations happen between long run and short run BBIs under any methodology which indicates they may actually be showing different phenomena. This idea is furthered by the low association between returns and the long run BBIs presents which may be due to the fact that while short run BBIs are almost contemporary with returns, long run BBIs look from 2.5 to 5 years in the future. Actually when the correlations are calculated lagging the long run BBIs, results not reported in the table, the correlation coefficients increase slightly to 0.21 (France) and 0.18 (UK) with a 12 month lag and to 0.17 (France) and 0.18 (UK) with a 24 month lag. If lags are increased beyond 24 months the correlation coefficients of interest start decreasing below the value of their contemporaneous counterparts. However, the increase in lagged correlations does not reach levels comparable to the correlations between short run or equally weighted BBIs with linear returns. This puzzle, due to its relevance, will be discussed at length at the end of this section.

While Table 12 presents the linear correlation of both variables, it is interesting to see if there are higher order correlations between the new series and the original data. To do so Table 13 presents correlation matrixes between prices, returns and quadratic returns with the squared BBIs under both methodologies and different weightings.

Table 13: Correlation coefficients between several time series and squared BBIs

Correlation coefficients between several time series and squared BBIs													
Panel A: Fixed grading squared BBI's													
France							United Kingdom						
	P	r	r^2	BBI^2_{s1}	BBI^2_{l1}	BBI^2_{e1}		P	r	r^2	BBI^2_{s1}	BBI^2_{l1}	BBI^2_{e1}
P	1						P	1					
r	0.03	1					r	0.03	1				
r^2	-0.01	0.39	1				r^2	0.01	0.17	1			
BBI^2_{s1}	-0.01	0.01	0.39	1			BBI^2_{s1}	-0.01	-0.12	0.51	1		
BBI^2_{l1}	-0.03	-0.02	0.02	0.11	1		BBI^2_{l1}	-0.05	-0.01	0.14	0.33	1	
BBI^2_{e1}	-0.02	-0.01	0.22	0.65	0.57	1	BBI^2_{e1}	-0.04	-0.13	0.19	0.68	0.69	1
Panel B: Standard deviation grading squared BBI's													
France							United Kingdom						
	P	r	r^2	BBI^2_{s2}	BBI^2_{l2}	BBI^2_{e2}		P	r	r^2	BBI^2_{s2}	BBI^2_{l2}	BBI^2_{e2}
P	1						P	1					
r	0.03	1					r	0.03	1				
r^2	-0.01	0.39	1				r^2	0.01	0.17	1			
BBI^2_{s2}	-0.01	-0.03	0.34	1			BBI^2_{s2}	0.00	-0.18	0.45	1		
BBI^2_{l2}	0.05	0.04	0.02	0.14	1		BBI^2_{l2}	-0.02	0.03	0.12	0.27	1	
BBI^2_{e2}	0.01	0.03	0.21	0.64	0.60	1	BBI^2_{e2}	-0.03	-0.10	0.18	0.67	0.63	1
Panel C: Fixed grading squared BBI's VS Standard deviation grading squared BBI's													
France							United Kingdom						
	BBI^2_{s1}	BBI^2_{l1}	BBI^2_{e1}	BBI^2_{s2}	BBI^2_{l2}	BBI^2_{e2}		BBI^2_{s1}	BBI^2_{l1}	BBI^2_{e1}	BBI^2_{s2}	BBI^2_{l2}	BBI^2_{e2}
BBI^2_{s1}	1						BBI^2_{s1}	1					
BBI^2_{l1}	0.11	1					BBI^2_{l1}	0.33	1				
BBI^2_{e1}	0.65	0.57	1				BBI^2_{e1}	0.68	0.69	1			
BBI^2_{s2}	**0.96**	0.14	0.64	1			BBI^2_{s2}	**0.97**	0.32	0.69	1		
BBI^2_{l2}	0.12	**0.82**	0.51	0.14	1		BBI^2_{l2}	0.27	**0.91**	0.58	0.27	1	
BBI^2_{e2}	0.61	0.57	**0.94**	0.64	0.60	1	BBI^2_{e2}	0.64	0.67	**0.95**	0.67	0.63	1

P: Index level, r: linear return, r^2: squared return, $BBI^2_{X\#}$: Squared Boom Bust Indicator (s)hort run, (l)ong run, (e)qually weighted, (1) fixed grading, (2) standard deviation grading. Vector of correlations with squared linear returns shown in dotted columns in panel A and B. Bold data in Panel C shows correlation between different methodologies but same ω.

In the cases of panels A and B the highest associations are found between the squared return variables and the short run BBIs. This result is interesting as it shows that BBIs contain valuable information not only about the first order characteristics of the time series but also for higher orders thus representing correctly the evolution of volatility over time, a benefit that a dummy sequence is unable to offer. As the weight of short run returns in the BBI starts decreasing s does this quadratic correlation as shown by the lower values of the statistic for equally weighted and long run BBIs. It is interesting to see that the decrease is stronger for France (0.02 correlation with squared returns in both panels) than for the United Kingdom (0.14 and 0.12 in panel A and B respectively). Thus, a final step to perform in order to analyze the informational content of BBIs has to do with their serial and quadratic correlations. First It is important to confirm whether they keep the serial autoregressive characteristics of the return data from which they arise and that are summarized in Figure 3 and Table 4. In the case of BBIs, Figures 7 and 8 present the autocorrelation coefficients under the two grading methodologies for France and the United Kingdom respectively.

Figure 7: Autocorrelation functions for different BBI specifications for the France

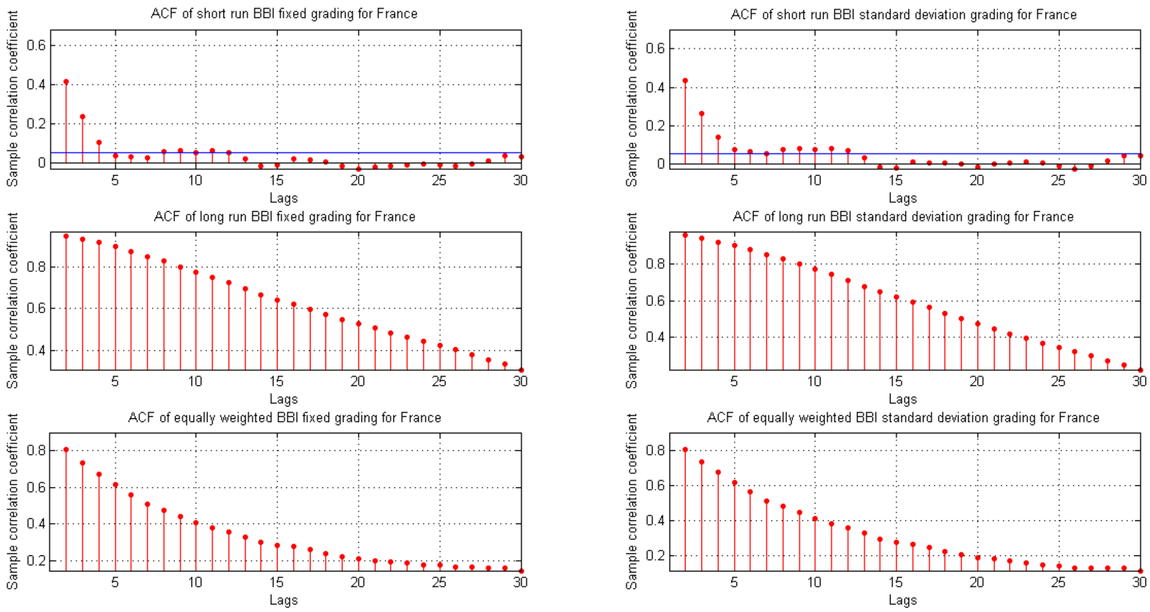
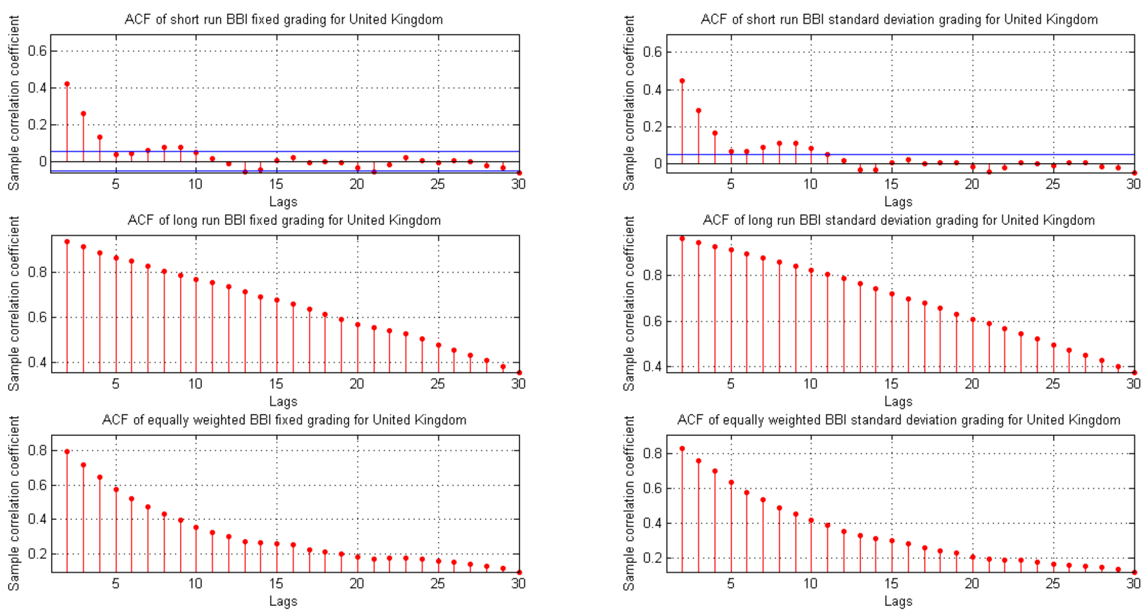


Figure 8: Autocorrelation functions for different BBI specifications for United Kingdom



In both cases we can see that the short run specification has a shorter memory as coefficients decrease rapidly while for the equally weighted and long run specifications they decrease at a slower rate and remain statistical significant for every lag between 1 and 30. The top panel in both Figure 7 and 8 are consistent with the characteristics of short run returns presented in Part 3. The middle panel in both figures gives the strongest indication of path dependence in the formation of the indicator and resembles the serial correlations evidenced in stock index levels. The bottom panels in both figures shows a long memory process with a stronger concavity than the middle panel, where the decrease appears to be almost linear. When performing statistical tests, as the Ljung-Box (1978) test for joint significance of the autocorrelation coefficients presented in Table 14 we find that it does not allow rejection of the null hypothesis of significance under any specification nor for any number of lags tested. These results are consistent with the tests for joint significance of autocorrelations of returns.

Table 14: Ljung-Box (1978) statistic for joint significance of autocorrelation coefficients of BBIs

Ljung - Box (1978) test statistic for joint significance of autocorrelation coefficients of BBIs													
Lags in months			1	2	3	4	5	6	7°	12	18	24	30
France	Fixed grading	*Short run*	672.49 ***	919.64 ***	1,000.99 ***	1,017.48 ***	1,019.56 ***	1,021 ***	1,021.78 ***	1,044.65 ***	1,046.91 ***	1,049.9 ***	1,054.24 ***
		Long run	1,357.64 ***	2,658.08 ***	3,911.83 ***	5,127.3 ***	6,293.61 ***	7,397.79 ***	8,437.87 ***	12,816.2 ***	16,323.42 ***	18,502.35 ***	19,704.52 ***
		Equally weighted	1,162.61 ***	2,093.59 ***	2,865.65 ***	3,515.59 ***	4,060.44 ***	4,512.09 ***	4,885.31 ***	6,128.1 ***	6,835.65 ***	7,186.62 ***	7,418.11 ***
	Standard deviation grading	*Short run*	711.99 ***	988.88 ***	1,089.99 ***	1,117.55 ***	1,125.29 ***	1,131.28 ***	1,134.94 ***	1,175.39 ***	1,178.04 ***	1,178.62 ***	1,185.3 ***
		Long run	1,383.81 ***	2,714.02 ***	3,992.6 ***	5,220.82 ***	6,398.59 ***	7,518.62 ***	8,574.55 ***	12,914.07 ***	16,154.24 ***	17,842.88 ***	18,582.25 ***
		Equally weighted	1,178.26 ***	2,125.15 ***	2,911.98 ***	3,575.62 ***	4,134.6 ***	4,599.51 ***	4,986.22 ***	6,272.05 ***	6,944.64 ***	7,225.56 ***	7,380.37 ***
United Kingdom	Fixed grading	*Short run*	731.69 ***	1,005.69 ***	1,108.7 ***	1,135.04 ***	1,136.91 ***	1,139.96 ***	1,145.73 ***	1,167.52 ***	1,176.68 ***	1,184.79 ***	1,193.56 ***
		Long run	1,443.26 ***	2,805.49 ***	4,093.46 ***	5,312.45 ***	6,472.19 ***	7,587.69 ***	8,645.4 ***	13,251.22 ***	17,388.17 ***	20,201.71 ***	21,855.8 ***
		Equally weighted	1,227.88 ***	2,202.65 ***	2,995.97 ***	3,640.73 ***	4,147.24 ***	4,563.47 ***	4,909.85 ***	5,932.03 ***	6,507.43 ***	6,807.22 ***	6,972.6 ***
	Standard deviation grading	*Short run*	752.64 ***	1,059.45 ***	1,185.69 ***	1,228.98 ***	1,235.55 ***	1,242.59 ***	1,254.94 ***	1,304.8 ***	1,309.13 ***	1,313.51 ***	1,318.94 ***
		Long run	1,471.21 ***	2,891.05 ***	4,260.82 ***	5,589.06 ***	6,871.37 ***	8,110.72 ***	9,298.48 ***	14,542.28 ***	19,252.9 ***	22,385.89 ***	24,199.77 ***
		Equally weighted	1,277.2 ***	2,333.94 ***	3,225.62 ***	3,982.75 ***	4,602.14 ***	5,118.55 ***	5,556.8 ***	6,944.1 ***	7,711.43 ***	8,068.57 ***	8,266.94 ***
° Reffers to the optimal number of lags according to Tsay (2002). It is the result of L≈Ln(m), where m is the number of observations													
Significance: * = 90%; ** = 95%; *** = 99%													

Additional tests, not reported in the paper, were performed on the serial correlations of squared BBIs to identify if the characteristics of squared returns were also contained in the indicator. Results were not qualitatively different for these correlograms with respect of those presented in Figure 7 and 8. The show long memory processes although the value of the coefficient decreases faster for the short run specification than for the equally weighted and long run specifications. All autocorrelation coefficients are statistically significant according to a Ljun-Box (1978) similar to the one presented in Table 14.

From this section we have gathered a basic understanding of the results for both methodologies and both the short run and equally weighted BBIs. However, long run BBIs only show low correlations both with returns and squared returns and their descriptive statistics present contradictory results with the change in the grading methodology. Additionally thy behave as very long memory processes, mor similar to what was seen for prices than for returns. We address this puzzle in the following subsection.

The long-run BBI puzzle: A Principal Component Analysis

The fact that the long-run BBI seems to hold less information than the other two specifications of the indicator under both methodologies is both surprising and counterintuitive since more extreme long run results should be indicative of more pervasive booms and busts which should be also visible in the price and the return series as shown by their strong serial correlation. Thus we recur to Principal Component Analysis as described in Tsay (2002) to achieve a better understanding of this issue.

Let matrix **V**=**R***$\mathbf{R}'$ such that **V** is a square matrix of dimensions txt where t is the number of observations in the vector of returns r_n. Note that if $\mathbb{E}(r_n) \approx 0 \, \forall \, n$, where $\mathbb{E}$ refers to the expected value operator, then $\mathbb{E}({r_n}^2) \approx \sigma_n^2$ and $\mathbb{E}(r_n r_j) \approx \sigma_{nj} \, \forall n \neq j$. Thus $\mathbf{R}'$***R** is an approximation to the variance covariance matrix for **R** and should have dimension nxn. To clarify, σ_n^2 is the variance of vector r_n *across time* and σ_{nj} is the covariance between vector r_n and r_j also *across time*. Changing the order of matrices in the multiplication does not find variances and covariances across time but across n-monthly returns *for a fixed time*. In the case of **V**, observation $v_{5,6}$ is the covariance of all n-monthly returns for month 5 and 6 in the database and observation v_7^2 corresponds to the variance of all n-monthly returns for month 7.

Following Tsay's (2002) rendition on Principal Component Analysis (PCA), let us decompose **V** into eigenvalues and eigenvectors such that,

$$\mathbf{V}=\mathbf{P\Lambda P'} \qquad (5)$$

Matrix **Λ** is a diagonal matrix that contains eigenvalues λ and **P** contains the associated eigenvectors which, by construction, are orthogonal. Ranking within matrix **P** and **Λ** happens in such a way that the higher the corresponding eigenvalue (λ_t) the higher the explanatory power of said eigenvector on the total variability of **V**. Explanatory power for vector P_t (γ_t) is calculated in the following way:

$$\gamma_t = \frac{\lambda_t}{\sum_{t=1}^{T} \lambda_t} \qquad (6)$$

When we correlate the first eigenvector (the one with highest γ_t) of the decomposed matrix **V** for France and the United Kingdom with the long run BBIs under any methodology we find the results contained in Table 15.

Table 15: Correlation coefficients between BBIs and first principal component of V

Correlation coefficient between BBIs and first principal component of V							
	Explanatory power γ_t	Contemporaneous correlation coefficients					
		BBI_{s1}	BBI_{s2}	BBI_{l1}	BBI_{l2}	BBI_{e1}	BBI_{e2}
France	81.38%	0.31	0.33	0.91	0.97	0.75	0.78
United Kingdom	79.05%	0.35	0.37	0.90	0.95	0.75	0.79

This result is surprising and, however, sheds light on a possible interpretation for the long run BBI. Since PCA tries to identify factors that have great explanatory power on the variability of the underlying matrix, we can treat BBI_l under any specification as the principal explanatory factor of the time related variability across returns. Thus, it would explain the variability of short, medium and long term returns at a fixed time. In the panels in Figure 4 as well as the additional heat maps in Annex 2, it would be what explains returns changing at a fixed time from n=1 month to n=60 months. Thus, we can treat this as the factor which explains the pervasiveness of the cycle through time.

A general conclusion for Part 4 is that under both specifications BBIs seem to contain a great deal of the traits of the original data presented in Part 3. A preference for the standard deviation grading methodology seems in place as it shows more skewness than the fixed grading methodology and avoids the use of arbitrary grading scales by researchers. In terms of the weighting vectors, while the short run BBIs track better the overall behavior of returns, equally weighted BBIs seem to be better at measuring the overall intensity of a bull or a bear market. Long run BBIs also hold relevant information on the cross-sectional variability of returns and not only on their time series characteristics. This leads as to an *impasse* since we expected to find a single dependent variable to explain the whole phenomenon but its manifest complexity renders the choice between the three indicators moot. This of course opens a wide inquiry avenue for researchers on the topic.

Before performing comparisons across methodologies in the following section, a set of caveats to this methodology has to be presented. First, the amount and frequency of observations is critical to the value of the indicator if not to its broad-stroke evolution over time. This raises an important issue when dealing with comparability across samples with different starting and ending dates, different frequencies in the data or different numbers of observations. A second caution has to do with the effect that including new observations may have on the behavior of the indicator. It is possible that large changes in the sample require recalculating BBIs for the whole series as they change the empirical distribution of the data, including the shape and structure of the tails.. This implies that this indicator is not appropriate for forecasting and should not be used in its current form in early-warning research. Finally, the indicator is only as representative as the date from which it is derived. Other omitted variables may be relevant in explaining the asset cycle, for example, trading volume or market capitalization measures which are not included in BBIs. Actually, an underlying assumption is that the stock market we deal with is liquid and that price changes are not caused by any market imperfections.

Part 5: Results and contrast between methodologies

Part 1 established the three main conditions a good Boom-Bust Indicator (BBI) should fulfill: be continuous and contain as much information as possible from the original data; find at least the same booms and busts as other methodologies in the literature; and allow for a differential characterization of several types of booms and busts. The previous section assessed the first requisite by studying the information content of the different BBI specifications. This section aims to verify the other two conditions by, first, describing two of the most usual methodologies in the financial cycle literature, the turning point algorithm and the Hodrick & Prescott (1997) filter (HP filter), and then performing a three-wise comparison with the BBI indicator.

Since this literature has a long history and has attracted very diverse researchers, it would be unwise to limit this study to a single parametrization per methodology to perform the contrasts. Thus the turning point algorithm will be run under four different specifications and the HP filter for three different parameter values and several thresholds. When identifying whether the BBI methodology finds at least the same bull and bear markets as the others, both grading matrices and the three different weighing vectors discussed in Part 4 will be employed.

Turning point specification

The four different specifications of the turning point algorithm will depend on the size of the observation window and the amount of restrictions applied to the cycle finding process. The process will be performed on the logarithmic values of the stock market indices since this is a monotonous transformation of the price levels and does not alter the date or the ranking of local maxima and minima. The caveat discussed in part three about the changes this transformation causes to the shape of the distribution does not apply since we will not be using any form of the differenced variable. Whenever returns are reported they will be calculated using the linear specification discussed in Part 3. One set of specifications will be more lax, avoiding the use of alternation or duration censoring rules, while the other set of specifications will implement those rules. Within each type of specification there will be a long run and a short run observation window. A summary of the algorithm is presented in Table 16 and discussed in the following paragraphs[16].

Table 16: Turning point algorithm under different specifications

Turning point algorithm under different specifications				
Instruction	*Panel A: Basic specification*		*Panel B: Censored specification*	
	Short	Long	Short	Long
I. Determination of initial turning points in data				
I.A. Choice of peaks and troughs in symmetric window of X months to both sides of P_t	8 months	12 months	8 months	12 months
I.B. Enforcement of alternation	No	No	Yes	Yes
II. Censoring operations				
II.A. Elimination of turns within 6 months of beginning and end of series	Yes	Yes	Yes	Yes
II.B. Elimination of peaks (troughs) at both ends of series which are lower (higher) than values closer to the end	Yes	Yes	Yes	Yes
II.C. Elimination of cycles with shorter duration than Y months . This is done both for peak-to-peak and trough-to-trough.	No	No	16 months	24 months
II.D. Elimination of phases (peak-to-trough or trough-to-peak) with shorter duration than Z months (unless fall/rise exceeds 20%)	No	No	4 months	6 months
III. Statement of final turning points				

Following Pagan & Sossounov (2003)[17], the idea behind this methodology is to find a set of local maxima and minima that determine when expansion and contraction phases start and end. A cycle can be defined as taking place on a peak-to-peak or trough-to-trough basis while a phase occurs between peaks and troughs (contractions/bears) or troughs and peaks (expansions/bulls). This methodology (I.A.) uses *ex-post* information as it looks at a window around a given observation and checks whether that observation is the maximum or minimum of the surrounding data. Longer windows perform a stronger smoothing than shorter ones. After finding the maxima

[16] For an excellent description of this algorithm and its inner workings see Pagan & Sossounov (2003) and Harding & Pagan (2005).

[17] The methodology was originally set forth by Bry & Boschan (1971). However, they performed calculations of turning points on different smoothed versions of the data, usually eliminating extreme observations. These choices were logical when studying the business cycle and long run economic time series, but these choices are not necessarily applicable to financial data, as discussed in Part 3. In consequence we choose to follow Pagan & Sossounov (2003) who reduce time series smoothing and eliminate data trimming.

and minima, in some cases an alternation rule is enforced (I.B.) so that a peak is followed by a trough and a trough by a peak. In order to fulfill it, the researcher has to choose the highest (lowest) of two consecutive peaks (troughs).

A second stage in this methodology has to do with censoring some of the chosen points. First (II.A) all turning points within 6 months from the beginning and end of the series are eliminated since there is not enough *ex-ante* and *ex-post* information to confirm that those points are truly local maxima or minima. Second (II.B), the same logic indicates that researchers should eliminate peaks (troughs) which are lower (higher) than the last confirmed peak (trough). More stringent specifications also require that peak-to-peak or trough-to-trough cycles have a minimum length (II.C) and that the phases that compose them have either a minimum length or a minimum absolute variation (II.D). After applying this algorithm we obtain four distinct sets of peaks and troughs for each country which are summarized in Table 17.

Table 17: Summary statistics from turning point methodology under different specifications

Summary statistics from turning point methodology under different specifications										
			Panel A: Short uncensored		*Panel B: Long uncensored*		*Panel C: Short censored*		*Panel D: long censored*	
			France	United Kingdom	France	United Kingdom	France	United Kingdom	France	United Kingdom
	Peaks		42	44	25	31	27	29	20	20
	Troughs		41	44	29	29	28	30	20	21
Bears	Number		37	41	24	31	28	30	20	21
Bears	Length (months)	Average	13.76	13.51	21.04	18.52	18.32	17.93	24.20	25.10
Bears	Length (months)	Minimum	1	1	4	2	3	1	4	2
Bears	Length (months)	Maximum	47	63	95	63	53	136	95	136
Bears	Absolute returns	Average	-29.56%	-16.08%	-36.92%	-23.13%	-36.15%	-24.51%	-40.02%	-28.26%
Bears	Absolute returns	Minimum	-59.01%	-72.88%	-89.27%	-75.59%	-65.71%	-75.59%	-89.27%	-75.59%
Bears	Absolute returns	Maximum	-4.29%	-0.50%	-12.15%	-0.50%	-13.41%	-3.19%	-13.73%	-3.19%
Bears	Average CAGR		-31.02%	-19.79%	-29.55%	-21.90%	-30.69%	-26.62%	-29.08%	-24.31%
Bulls	Number		45	45	29	28	27	29	19	20
Bulls	Length (months)	Average	18.67	19.87	28.69	31.32	30.52	31.48	44.89	46.20
Bulls	Length (months)	Minimum	2	4	5	5	5	7	6	7
Bulls	Length (months)	Maximum	61	72	69	83	132	83	132	145
Bulls	Absolute returns	Average	56.33%	37.31%	89.26%	71.10%	100.92%	71.11%	140.20%	102.17%
Bulls	Absolute returns	Minimum	1.96%	0.47%	3.26%	3.35%	10.16%	3.12%	11.92%	3.35%
Bulls	Absolute returns	Maximum	302.21%	305.83%	349.32%	327.18%	349.32%	327.18%	453.97%	362.27%
Bulls	Average CAGR		45.81%	15.97%	33.66%	22.41%	39.21%	20.49%	26.00%	16.98%

Average decrease/increase reffers to absolut growth rates during the phase. CAGR: Compound Annual Growth Rate.

A first element to highlight from Table 17 is the difference in the number of cycles as a function of the different restrictions. This is evidence that the many choices that researchers make do effectively affect results substantially. From the less restrictive, short term algorithm in panel A to the more restrictive long run analysis in panel D the number of cycles almost halves and there are significant changes in the average CAGR[18] for both countries. As expected, the average length of both booms and busts increases from the short to the long run specifications. It is noteworthy that the uncensored specifications tend to have booms and busts with lower average returns and bears that have higher maximums and bulls that have lower minimums. These differences yield substantially different pictures of the financial cycle for the same underlying series.

A very interesting result that can be observed in the previous table is that the characteristics of bull and bear markets change with the different specifications and that average lengths and returns are not necessarily representative of what happens for the more extreme events. This serves to show that not all booms or busts behave in the same fashion. The last subsection in this part, as well as Annex 4 will be dedicated to exploring these differences in a historical context.

While an in-depth analysis of the turning point methodology exceeds the scope of this paper, it is interesting to see whether these different specifications are synchronized with the BBI methodology presented in Part 4. In order to do so we will use the turning point dates to build a time series ($TP_{(b/c)(s/l)}$)[19] that will take values of 1 when in an expansion (bull) and values of -1 when in a contraction (bear). We will build similar sequences from the $BBI_{w,m}$[20] such that they take the value of 1 for months where the BBI is positive, values of -1 when values of the BBI are negative and 0 when the BBI is 0. Then we will establish an average synchronization measure between series x and y (S_{xy}) such that:

$$S_{TP,BBI} = \mathbb{E}\left(\mathrm{TP}_{(b/c)(s/l)}\mathrm{BBI}_{w,m}\right) \qquad (7)$$

Where $\mathbb{E}$ is the expected value operator. When there is coincidence in the sign for two given series at a given month $S_{TP,BBI}$ will take a value of 1, if the two series are opposite, one indicating a bear market while the other indicates a bull market, $S_{TP,BBI}$ will takes a value of -1 for that month and if the BBI series takes a value of 0, then $S_{TP,BBI}$ takes a value of 0 as well for those specific observations. The average synchronization measures between TPs and BBIs are presented in the highlighted sections of Table 18 in the following page. All values above 25% are in bold. It is noteworthy that the different BBIs will take values of 0 in some cases, especially in calm periods when most n-monthly returns are located in the interquartile range. This will impede the indicator S_{xy} to reach values of perfect synchronization (1) or perfect negative synchronization (-1)

A first interesting result is that all values in the highlighted sections are positive, implying that there at least some form of synchronization between the series. In general, the long run BBI has the worst synchronization with the turning point algorithm since, as seen in the last subsection of Part 4, it measures the pervasiveness of a boom or bust across multi-period returns measures and not necessarily its evolution during time. However, we can gather from Table 18 that BBI series do contain some of the information reflected by the turning point methodologies. In particular, short

[18] The use of a Compound Annual Growth Rate (CAGR) is essential to make results across panels and across countries comparable. Using absolute returns assumes that it does not matter if a given gain or loss occurred over a short or a long period of time while the CAGR unifies the returns as yearly measures. This kind of transformation, however, is not common in the literature.

[19] These are 4 resulting series, one for each specification. TP_{bs} for the uncensored algorithm with a short window ; TP_{bl} for the uncensored algorithm with a long window; TP_{cs} for the censored algorithm with a short window; and, TP_{cl} for the censored algorithm with a long window.

[20] Where w corresponds to the weighing vector (s)hort, (l)ong, (e)qually weighted and mreffers to method 1: fixed grading and 2, standard deviation based grading.

run BBIs are consistently more synchronized with the censored short run turning point algorithm. The equally weighted BBI for France has a stronger synchronization with the uncensored long run turning point algorithm while the same measures for the UK emulate the censored long run specification better.

Table 18: Synchronization of TP and BBI methodologies – different specifications

Synchronization of methodologies - Turning Points VS BBIs										
Panel A: France										
	TP_{bs}	TP_{bl}	TP_{cs}	TP_{cl}	BBI_{s1}	BBI_{l1}	BBI_{e1}	BBI_{s2}	BBI_{l2}	BBI_{e2}
TP_{bs}	100%									
TP_{bl}	71.3%	100%								
TP_{cs}	69.7%	79.5%	100%							
TP_{cl}	58.1%	75.2%	88.5%	100%						
BBI_{s1}	**39.0%**	**43.5%**	**45.6%**	**40.2%**	100%					
BBI_{l1}	6.3%	9.7%	8.2%	4.3%	15.3%	100%				
BBI_{e1}	**34.2%**	**43.6%**	**41.0%**	**38.3%**	55.7%	39.5%	100%			
BBI_{s2}	**39.0%**	**43.0%**	**45.6%**	**39.9%**	79.5%	15.2%	55.8%	100%		
BBI_{l2}	6.5%	9.9%	8.4%	4.6%	15.7%	71.9%	39.8%	15.5%	100%	
BBI_{e2}	**35.1%**	**44.1%**	**41.3%**	**38.4%**	56.4%	38.6%	88.4%	56.8%	38.8%	100%
Panel B: United Kingdom										
	TP_{bs}	TP_{bl}	TP_{cs}	TP_{cl}	BBI_{s1}	BBI_{l1}	BBI_{e1}	BBI_{s2}	BBI_{l2}	BBI_{e2}
TP_{bs}	100%									
TP_{bl}	70.7%	100%								
TP_{cs}	54.8%	59.6%	100%							
TP_{cl}	45.3%	52.9%	90.2%	100%						
BBI_{s1}	**30.3%**	**29.2%**	**40.9%**	**38.4%**	100%					
BBI_{l1}	12.2%	6.5%	24.2%	**27.5%**	16.9%	100%				
BBI_{e1}	**31.1%**	**29.8%**	**49.5%**	**49.4%**	51.2%	46.9%	100%			
BBI_{s2}	**31.4%**	**30.4%**	**42.0%**	**39.4%**	74.7%	17.6%	52.1%	100%		
BBI_{l2}	11.5%	5.9%	23.5%	**26.9%**	16.9%	72.4%	46.2%	17.4%	100%	
BBI_{e2}	**29.5%**	**29.2%**	**49.3%**	**50.6%**	48.9%	48.4%	87.9%	49.7%	47.7%	100%

TP(b/c)(s/l): Turning points methodology (b)asic, (c)ensored; (s)hort window, (l)ong window. $BBI_{X\#}$: Boom Bust Indicator (s)hort run, (l)ong run, (e)qually weighted, (1) fixed grading, (2) standard deviation grading. Correlation across methodologies shown in smaller box. Synchronization above 25% between methodologies in bold.

Hodrick & Prescott filter specification

In this section, three different specifications for the HP filter will be performed following what has been done traditionally in that literature. The filter will be run on the stock market indices directly, using rolling windowd of 120 observations (10 years) to predict the gap for observation 121 and then contrast it with what is observed for month 121 by calculating the linear change. If this proportion exceeds a certain upper (lower) threshold, a dummy variable will change from 0 to 1 (-1) indicating a bull (bear) market. Three different filters based on the speed of adaptation to the data are defined: A fast adapting process (λ=14,400), a slow adapting process (λ=400,000) and an intermediate process (λ=100,000). Additionally, 10 different, equally spaced, positive and negative thresholds are tested. Summary statistics for France and the UK are presented in Tables 19 and 20.

Table 19: Summary statistics from HP filter for France under different parameter values

Summary statistics from Hodrick & Prescott filter methodology for France under different parameter values												
Panel A: Fast HP filter (λ=14,400)												
	Threshold		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Bears	Number		41	17	6	0	0	0	0	0	0	0
Bears	Length (months)	Average	5	3	1	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Length (months)	Minimum	1	1	1	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Length (months)	Maximum	16	7	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Average	-16.29%	-18.09%	-14.89%	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Minimum	-54.81%	-36.49%	-22.37%	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Maximum	-2.65%	-5.57%	-5.32%	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Average CAGR		-41.55%	-54.94%	-72.24%	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Number		69	30	7	3	1	1	0	0	0	0
Bulls	Length (months)	Average	5	3	2	2	2	2	N/A	N/A	N/A	N/A
Bulls	Length (months)	Minimum	1	1	1	1	2	2	N/A	N/A	N/A	N/A
Bulls	Length (months)	Maximum	828	13	4	3	2	2	N/A	N/A	N/A	N/A
Bulls	Absolute Return	Average	17.63%	17.47%	20.60%	42.27%	107.97%	107.97%	N/A	N/A	N/A	N/A
Bulls	Absolute Return	Minimum	-27.31%	-5.23%	1.55%	9.87%	107.97%	107.97%	N/A	N/A	N/A	N/A
Bulls	Absolute Return	Maximum	190.55%	187.67%	91.06%	102.94%	107.97%	107.97%	N/A	N/A	N/A	N/A
Bulls	Average CAGR		91.98%	135.49%	197.52%	709.39%	7895.45%	7895.45%	N/A	N/A	N/A	N/A
Panel B: Medium speed HP filter (λ=100,000)												
	Threshold		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Bears	Number		32	21	12	5	1	0	0	0	0	0
Bears	Length (months)	Average	8	5	3	2	2	N/A	N/A	N/A	N/A	N/A
Bears	Length (months)	Minimum	1	1	1	1	2	N/A	N/A	N/A	N/A	N/A
Bears	Length (months)	Maximum	43	19	12	60	2	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Average	-16.29%	-17.58%	-16.48%	-16.53%	-22.37%	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Minimum	-54.81%	-51.29%	-40.14%	-27.86%	-22.37%	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Maximum	-2.65%	-2.77%	-4.31%	-9.31%	-22.37%	N/A	N/A	N/A	N/A	N/A
Bears	Average CAGR		-33.00%	-44.69%	-53.96%	-72.75%	-78.03%	N/A	N/A	N/A	N/A	N/A
Bulls	Number		52	35	14	6	3	4	4	3	3	1
Bulls	Length (months)	Average	8	5	7	10	15	8	6	5	1	1
Bulls	Length (months)	Minimum	1	1	1	1	1	3	2	1	1	1
Bulls	Length (months)	Maximum	624	57	51	43	40	20	17	10	1	1
Bulls	Absolute Return	Average	26.80%	20.96%	28.99%	32.45%	38.55%	31.81%	29.07%	3.81%	4.19%	7.47%
Bulls	Absolute Return	Minimum	-0.43%	2.57%	3.87%	0.79%	2.42%	-14.02%	-11.48%	-1.28%	2.14%	7.47%
Bulls	Absolute Return	Maximum	250.38%	173.64%	186.39%	91.06%	102.94%	102.94%	107.97%	9.95%	7.47%	7.47%
Bulls	Average CAGR		58.89%	89.03%	125.33%	259.03%	591.48%	424.93%	2003.65%	81.36%	69.79%	140.14%
Panel C: Slow HP filter (λ=400,000)												
	Threshold		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Bears	Number		47	23	16	6	2	0	0	0	0	0
Bears	Length (months)	Average	7	7	4	3	2	N/A	N/A	N/A	N/A	N/A
Bears	Length (months)	Minimum	1	1	1	1	2	N/A	N/A	N/A	N/A	N/A
Bears	Length (months)	Maximum	60	44	22	72	2	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Average	-11.97%	-16.12%	-17.71%	-17.70%	-16.55%	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Minimum	-79.13%	-75.76%	-48.54%	-36.92%	-22.37%	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Maximum	-1.32%	-2.57%	-5.57%	-9.31%	-10.72%	N/A	N/A	N/A	N/A	N/A
Bears	Average CAGR		-29.56%	-37.72%	-59.90%	-62.99%	-63.64%	N/A	N/A	N/A	N/A	N/A
Bulls	Number		43	32	21	14	6	7	2	2	1	2
Bulls	Length (months)	Average	10	8	8	8	13	9	24	19	35	16
Bulls	Length (months)	Minimum	1	1	1	1	1	1	3	1	35	1
Bulls	Length (months)	Maximum	516	76	75	70	54	47	45	37	35	31
Bulls	Absolute Return	Average	30.51%	34.25%	35.05%	35.29%	59.41%	36.35%	81.73%	27.87%	41.85%	20.42%
Bulls	Absolute Return	Minimum	3.28%	3.73%	4.77%	3.21%	9.46%	5.57%	60.52%	11.36%	41.85%	6.84%
Bulls	Absolute Return	Maximum	194.30%	250.38%	209.50%	186.39%	103.97%	102.94%	102.94%	44.39%	41.85%	34.01%
Bulls	Average CAGR		51.97%	67.41%	115.53%	132.87%	211.87%	324.78%	785.40%	141.43%	12.73%	64.90%

Average decrease/increase reffers to absolut growth rates during the phase. CAGR: Compound Annual Growth Rate. In red positive return bears or negative return bulls

Table 20: Summary statistics from HP filter for UK under different parameter values

Summary statistics from Hodrick & Prescott filter methodology for United Kingdom under different parameter values												
Panel A: Fast HP filter (λ=14,400)												
	Threshold		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Bears	Number		46	8	2	0	0	0	0	0	0	0
Bears	Length (months)	Average	3	3	4	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Length (months)	Minimum	1	1	1	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Length (months)	Maximum	14	13	7	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Average	-11.67%	-18.92%	-32.80%	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Minimum	-67.81%	-62.84%	-44.60%	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Maximum	-1.42%	-7.00%	-21.00%	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bears	Average CAGR		-47.88%	-68.94%	-78.62%	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Number		35	7	3	2	0	0	0	0	0	0
Bulls	Length (months)	Average	6	4	4	4	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Length (months)	Minimum	1	1	1	2	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Length (months)	Maximum	19	15	8	6	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Absolute Return	Average	17.68%	17.37%	21.23%	24.36%	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Absolute Return	Minimum	-0.57%	5.59%	14.60%	14.60%	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Absolute Return	Maximum	56.78%	39.43%	26.62%	34.11%	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Average CAGR		58.23%	137.13%	490.33%	102.50%	N/A	N/A	N/A	N/A	N/A	N/A
Panel B: Medium speed HP filter (λ=100,000)												
	Threshold		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Bears	Number		45	12	5	2	2	0	0	0	0	0
Bears	Length (months)	Average	4	3	3	4	1	N/A	N/A	N/A	N/A	N/A
Bears	Length (months)	Minimum	1	1	1	1	1	N/A	N/A	N/A	N/A	N/A
Bears	Length (months)	Maximum	24	15	10	7	2	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Average	-10.72%	-14.96%	-20.72%	-32.80%	-14.65%	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Minimum	-56.35%	-51.70%	-58.16%	-44.60%	-16.28%	N/A	N/A	N/A	N/A	N/A
Bears	Absolute Return	Maximum	-1.19%	-6.83%	-5.92%	-21.00%	-13.03%	N/A	N/A	N/A	N/A	N/A
Bears	Average CAGR		-36.60%	-56.25%	-65.02%	-78.62%	-73.58%	N/A	N/A	N/A	N/A	N/A
Bulls	Number		37	25	9	2	0	0	0	0	0	0
Bulls	Length (months)	Average	8	4	3	3	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Length (months)	Minimum	1	1	1	1	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Length (months)	Maximum	30	23	15	5	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Absolute Return	Average	19.28%	12.76%	11.16%	14.67%	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Absolute Return	Minimum	-13.91%	-3.00%	2.73%	8.44%	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Absolute Return	Maximum	85.88%	57.98%	42.69%	20.90%	N/A	N/A	N/A	N/A	N/A	N/A
Bulls	Average CAGR		41.43%	58.71%	75.19%	113.04%	N/A	N/A	N/A	N/A	N/A	N/A
Panel C: Slow HP filter (λ=400,000)												
	Threshold		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Bears	Number		32	15	6	2	1	1	0	0	0	0
Bears	Length (months)	Average	7	4	3	5	5	2	N/A	N/A	N/A	N/A
Bears	Length (months)	Minimum	1	1	1	1	5	2	N/A	N/A	N/A	N/A
Bears	Length (months)	Maximum	36	17	11	8	5	2	N/A	N/A	N/A	N/A
Bears	Absolute Return	Average	-11.40%	-14.77%	-13.84%	-35.36%	-34.74%	-16.28%	N/A	N/A	N/A	N/A
Bears	Absolute Return	Minimum	-49.47%	-45.42%	-37.22%	-49.72%	-34.74%	-16.28%	N/A	N/A	N/A	N/A
Bears	Absolute Return	Maximum	-1.03%	-5.26%	-5.92%	-21.00%	-34.74%	-16.28%	N/A	N/A	N/A	N/A
Bears	Average CAGR		-30.30%	-48.94%	-52.58%	-78.93%	-63.88%	-65.46%	N/A	N/A	N/A	N/A
Bulls	Number		46	31	10	2	1	1	1	1	0	0
Bulls	Length (months)	Average	8	5	6	14	15	10	4	1	N/A	N/A
Bulls	Length (months)	Minimum	1	1	1	2	15	10	4	1	N/A	N/A
Bulls	Length (months)	Maximum	48	35	30	26	15	10	4	1	N/A	N/A
Bulls	Absolute Return	Average	17.49%	13.33%	18.10%	34.24%	38.61%	29.18%	16.01%	6.11%	N/A	N/A
Bulls	Absolute Return	Minimum	-11.80%	-0.59%	5.32%	6.11%	38.61%	29.18%	16.01%	6.11%	N/A	N/A
Bulls	Absolute Return	Maximum	79.25%	70.95%	61.95%	62.36%	38.61%	29.18%	16.01%	6.11%	N/A	N/A
Bulls	Average CAGR		45.15%	50.95%	65.85%	33.85%	29.87%	35.99%	57.09%	105.88%	N/A	N/A

Average decrease/increase reffers to absolut growth rates during the phase. CAGR: Compound Annual Growth Rate. In red positive return bears or negative return bulls

A first relevant element is that in both tables we can find bull markets with negative returns highlighted in red. This is contradictory with the definition of a bull market which is by definition a succession of positive returns.

In the case of France (Table 19), all specifications find consistently more bull markets than bear markets except for the slow specification with a 10% threshold. Bulls tend to be more intense than bear markets since they keep appearing for higher values of the threshold. When comparing average CAGR for bulls and bears under the same parameter and thresholds we find they are not symmetric around 0 but they are positively skewed indicating that booms tend to be stronger than busts. This is consistent with the descriptive statistics presented in Part 2 for the linear returns. However, this characterization has to be taken with a grain of salt since due to the limited liability principal the worst loss a stock holder may have is 100% of his or her wealth while the upside is unlimited. In that sense, bear markets are bounded since stock market indices cannot take on negative values while bull markets are unbounded. This would also explain why, although on average bulls and bears have comparable durations, the maximum length of a bull market always exceeds the maximum length of a bear market.

While Table 19 shows a similar story for the different specifications of the filter, Table 20 shows contradictory results for the United Kingdom. Only the slow adapting filter finds consistently more bull markets than bear markets (Panel C). For the fast and the slow specifications (Panels A and C) bulls are more pervasive than bears but the contrary happens for the medium speed specification. While returns, as measured by the CAGR, appear to be skewed to the right in panels A and B, they seem to be skewed to the left in panel C. These contradictory results show one of the weaknesses of using this methodology since depending on the parameter choice the story that can be told changes. As with the turning point methodology, it is clear that the characterization of a bull or a bear market is contingent on arbitrary decisions such as the parameter of the filter and the choice of threshold.

The use of the HP filter technique, as opposed to the turning point algorithm, yields a time series of gaps that is usually employed by researchers to determine whether a boom or bust is happening. The crisis condition, for example, may be set contingent on the gap series breaking a preset threshold. This value can be set arbitrarily or chosen, as in Borio & Lowe (2002) or Norio & White (2004) to minimize the noise to signal ratio in the early warning literature. Thus, following the methodology from Part 4 the association between BBIs and gap series produced through the HP filters can be determined by finding the correlation matrices for France and the United Kingdom. Results are reported in Table 21 in the following page.

The results are quite telling. First, as in previous exercises, the information content of long run BBIs is the lowest for all time specifications. In the case of France, the fast adapting gap series (λ=14,400) has the highest association with all remaining BBIs, particularly with the short run specifications. In the case of the medium adaptation process (λ=100,000), results remain close to 0.50 both for short run and equally weighted BBIs. The case of the United Kingdom is interesting because it presents overall higher associations between series when compared to France on a case by case basis. Additionally, the slow adapting process (λ=400,000) is significantly associated with the equally weighted BBIs with a correlation coefficient of over 0.70. it is important to recall that the information content should not be exactly the same across methodologies since the added value of the BBI over the HP filter gap is that the first one does not expose the data to any kind of detrending or smoothing. Since the HP filter assumes that trend and cycle are uncorrelated, which is difficult to believe for stock market data, it is methodologically more reasonable to use BBIs as they are shown to share an important association with the HP gap, without the additional assumption or loss of trend information.

Table 21: Correlation coefficient between HP filter gaps and BBIs

Correlation coefficient between HP filter gaps and BBIs									
Panel A: France									
	λ_{fast}	λ_{medium}	λ_{slow}	BBI_{s1}	BBI_{l1}	BBI_{e1}	BBI_{s2}	BBI_{l2}	BBI_{e2}
λ_{fast}	1								
λ_{medium}	0.74	1							
λ_{slow}	0.43	0.77	1						
BBI_{s1}	**0.69**	0.49	0.33	1					
BBI_{l1}	(0.09)	(0.01)	0.12	0.28	1				
BBI_{e1}	**0.54**	0.49	0.40	0.75	0.74	1			
BBI_{s2}	**0.70**	**0.50**	0.35	0.98	0.29	0.75	1		
BBI_{l2}	(0.05)	0.07	0.15	0.27	0.95	0.71	0.28	1	
BBI_{e2}	**0.58**	**0.52**	0.42	0.75	0.72	0.98	0.77	0.73	1
Panel B: United Kingdom									
	λ_{fast}	λ_{medium}	λ_{slow}	BBI_{s1}	BBI_{l1}	BBI_{e1}	BBI_{s2}	BBI_{l2}	BBI_{e2}
λ_{fast}	1								
λ_{medium}	0.82	1							
λ_{slow}	0.58	0.91	1						
BBI_{s1}	**0.73**	**0.62**	**0.53**	1					
BBI_{l1}	(0.01)	0.18	0.37	0.31	1				
BBI_{e1}	**0.62**	**0.69**	**0.71**	0.77	0.73	1			
BBI_{s2}	**0.74**	**0.64**	**0.55**	0.99	0.33	0.78	1		
BBI_{l2}	(0.03)	0.18	0.38	0.28	0.96	0.70	0.30	1	
BBI_{e2}	**0.61**	**0.71**	**0.73**	0.74	0.73	0.98	0.76	0.73	1
HP Filter: λ_{fast}=14,400; λ_{medium}=100,000; λ_{slow}=400,000. $BBI_{X\#}$: Boom Bust Indicator (s)hort run, (l)ong run, (e)qually weighted, (1) fixed grading, (2) standard deviation grading. Correlation across methodologies shown in smaller box. Correlation above 0.5 between methodologies in bold.									

As in the previous subsection, it may be interesting to identify if there is synchronization between the different filters and BBIs. To do so we created three different sequences each corresponding to a filter specification. If the gap for λ_i is positive (negative) for a given month then the variable HP_λ takes a value of 1 (-1) for that month. We define the synchronization coefficient as:

$$S_{HP,BBI} = \mathbb{E}\left(\mathrm{HP}_{\lambda}\mathrm{BBI}_{w,m}\right) \qquad (8)$$

The interpretation is similar to the coefficient described in equation (7) and the results are presented in Table 22 in the following page. Overall synchronization between the results from the

HP filter and BBIs is higher than for the turning points methodology. In this case, short run BBIs present the highest synchronization with the fast HP filter while equally weighted and long-run BBIs have their highest synchronization with the slow HP filter. However, equally weighted BBIs reflect the slow HP filter better than the long run BBIs. These findings are consistent with those obtained from the analysis of the association between BBIs and HP filter gaps.

Table 22: Synchronization of HP filter and BBI methodologies – different specifications

Synchronization of methodologies - HP filter VS BBIs

Panel A: France

	λ_{fast}	λ_{medium}	λ_{slow}	BBI_{s1}	BBI_{l1}	BBI_{e1}	BBI_{s2}	BBI_{l2}	BBI_{e2}
λ_{fast}	100%								
λ_{medium}	69.9%	100%							
λ_{slow}	44.4%	71.1%	100%						
BBI_{s1}	**46.1%**	**40.5%**	**37.5%**	100%					
BBI_{l1}	-8.6%	9.1%	**29.2%**	16.2%	100%				
BBI_{e1}	**40.4%**	**50.4%**	**61.6%**	56.3%	42.1%	100%			
BBI_{s2}	**46.6%**	**40.6%**	**38.1%**	81.6%	16.1%	56.5%	100%		
BBI_{l2}	-8.5%	9.2%	**29.3%**	16.5%	75.2%	42.2%	16.4%	100%	
BBI_{e2}	**41.5%**	**51.4%**	**62.8%**	56.9%	41.2%	90.6%	57.5%	41.4%	100%

Panel B: United Kingdom

	λ_{fast}	λ_{medium}	λ_{slow}	BBI_{s1}	BBI_{l1}	BBI_{e1}	BBI_{s2}	BBI_{l2}	BBI_{e2}
λ_{fast}	100%								
λ_{medium}	63.0%	100%							
λ_{slow}	41.0%	71.0%	100%						
BBI_{s1}	**49.5%**	**41.6%**	**36.0%**	100%					
BBI_{l1}	1.1%	18.7%	**36.9%**	17.7%	100%				
BBI_{e1}	**44.0%**	**56.3%**	**57.5%**	52.7%	49.8%	100%			
BBI_{s2}	**50.1%**	**42.4%**	**36.6%**	77.7%	18.2%	53.3%	100%		
BBI_{l2}	0.4%	18.0%	**36.1%**	17.5%	76.3%	49.0%	17.8%	100%	
BBI_{e2}	**42.3%**	**55.8%**	**58.1%**	50.1%	51.5%	91.5%	50.7%	50.8%	100%

HP Filter: λ_{fast}=14,400; λ_{medium}=100,000; λ_{slow}=400,000. $BBI_{X\#}$: Boom Bust Indicator (s)hort run, (l)ong run, (e)qually weighted, (1) fixed grading, (2) standard deviation grading. Synchronization across methodologies shown in smaller box. Synchronization above 25% between methodologies in bold.

Up to this point we have focused our attention on the generalities of both methods without delving into particularities. The following subsection and Annex 4 present a rich graphical analysis of four time periods for each country. We use the same time periods as the ones for Figure 4 and those in the heat maps in Annex 2 to show the similarities and differences among the turning point, filtering and BBI methodologies. The results for the First World War and the 1920s are in text while the other time periods can be found in Annex 4.

Graphical contrasting of three methodologies: Strengths and weaknesses of BBIs

This subsection presents Figures for France and the United Kingdom for the First World War and the 1920s (1914-1928). Each figure contains four different panels. From the bottom-up they show: the four specifications of the turning point methodology, the three parametrizations of the HP filter methodology, the BBI methodology with a fixed grading matrix, and the BBI methodology with a standard deviation grading. Each BBI methodology shows the results for the three different weighing vectors, short-run, long-run and medium term. In all cases the bull markets are colored green and bear markets are colored red. Finally, in the HP filter and BBI panels, the grading at the left corresponds to thresholds. When the gap (in the case of the HP filter) or the BBI indicator exceed a certain threshold for a certain month, the cells up to that threshold for that month are colored in green or red depending on whether the value of the gap / BBI is positive or negative.

France (1914-1928)

A first issue occurs with the turning point (TP) methodology which identifies a bull market between 1915 and 1916 which no other methodology identifies. Additionally the methodology is indecisive in 1918 and between 1920 and early 1924. Depending on the specification it can be seen as a boom or a bust. This confusion is resolved by both BBI short run specifications which identify a short lived boom amidst two bear markets between 1917 and 1919. The HP filters identify short and not very relevant bear markets. The bear markets that the HP filters identify in 1920 are seen as more intense and pervasive in the BBI specifications.

Additionally, the short run BBIs identify an explosive (fast and intense) boom in 1921 which the HP filter identifies only by 1922. It is noteworthy that while the HP filter sees the 1922-1924 bull market as a single phenomenon, the short BBIs identify it as a three-part explosive boom (1921, late 1922, mid 1923). The fact that the equally weighted and long run BBIs barely register the boom by mid-1922 speaks about its pervasiveness. The long run BBIs however, show a long run bust, consistent with the effects of the First World War that affect long run returns until well into 1921.

The short lived bear market between 1925-1926 shown by the HP filter appears as a period of volatility in the short-run BBI that uses the standard deviation grading. Interestingly, all methodologies agree in the boom that starts in 1927-1928.

Figure 9: Methodological comparison France 1914-1928

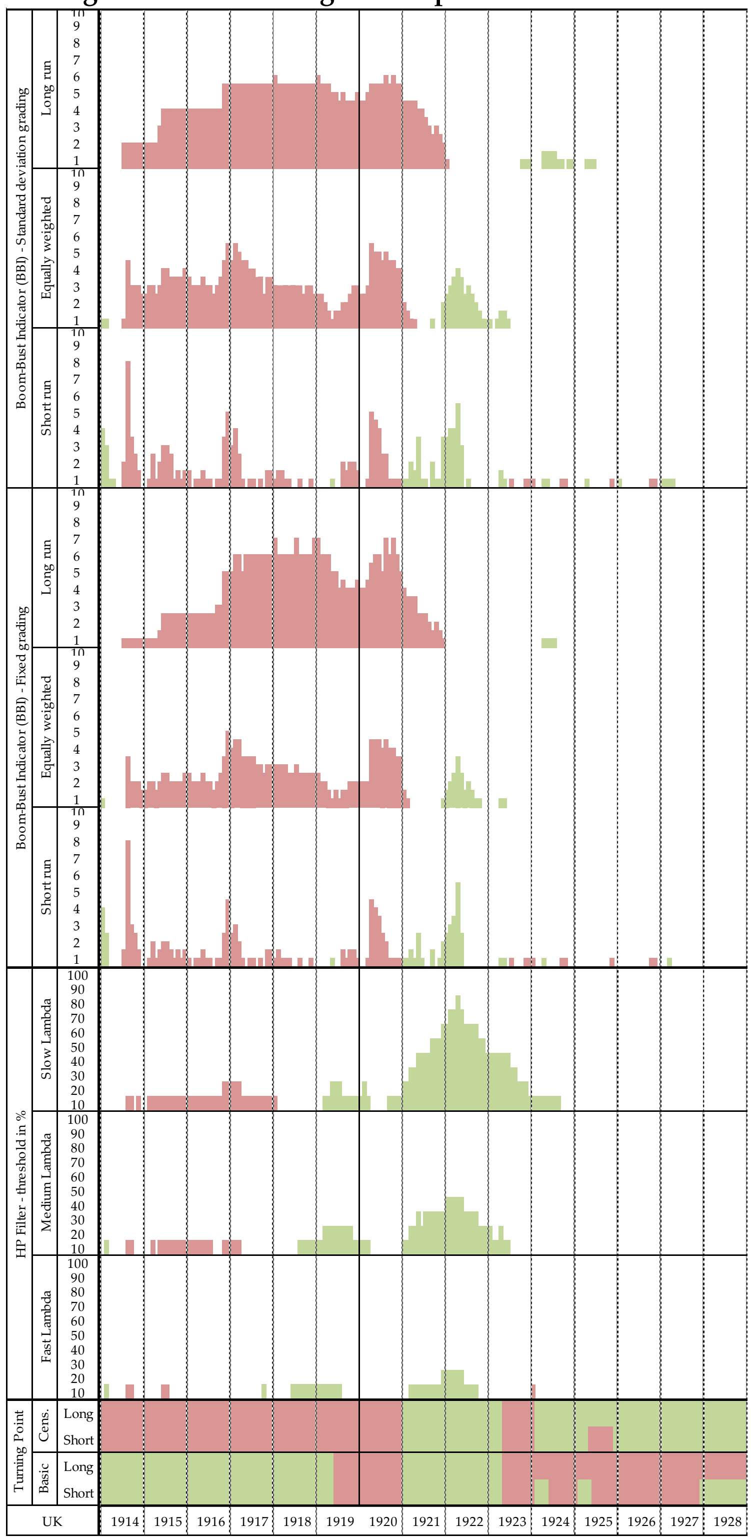
United Kingdom (1914-1928)

During all of the First World War the TP shows inconsistent results. While the uncensored specification shows an expansion, the censored specification reflects a contraction. Additionally, during those same years the HP filter shows only mild bull markets that barely cross the 20 or 30% thresholds depending on the specification. This contrasts with the large values of the BBI indicators and, in the short run specifications, with their volatility. These variables tell the story of a very strong bear market, as evidenced in the heat map in the right panel of figure 4.

In 1919-1920, when all TP specifications coincide in the presence of a bear, the HP filter shows a bull. The BBIs all show a bear market and they actually show it deepening at the beginning of 1920. Both TP and HP coincide in a boom taking place between 1921 and 1922 although the former anticipates it by almost a year. BBIs show the same boom although not as pervasive as the HP filter would imply. Actually its effect on long run returns (pervasiveness) is only mild as evidence in the long-run BBIs.

The bear period shown in the TP for 1923 is omitted by the HP filters and evidenced only mildly by the short run BBIs. The rest of the period remains calm for the HP and BBI methodology while the TP specifications lack synchronization.

Figure 10: Methodological comparison UK 1914-1928



Part 6: Discussion, conclusions and future lines of research

Since the early works of Bagehot in the mid-1800s financial crises have been a relevant subject of study both for economists and economic historians. The seminal works of Fischer, Kindleberger and Minsky paved the way for many others to take steps forward in our understanding of the dynamics underlying both the accumulation and the unwinding of financial imbalances. Literature during the last three decades has been fruitful, the debate spirited and the evolution of policy both for crises prevention and management has been palpable. However, the measures for crises available to modern researchers are too blunt, lacking variability as they usually consist only of a binary sequence that takes values of one for crises or for busts and values of 0 for calm periods or booms, depending on the study. These dummy sequences are plagued by difficulties: they presume all booms and all busts are equal; they treat the transition from a calm period to a crisis period as an instantaneous change of state; there is no consensus on a methodology for their construction; they oversimplify the underlying information up to the point where they barely reflect it; and they can present conflicting results for the same underlying database as shown in the parts 4 and 5 of this paper.

It follows that the criticism to the measures of financial imbalances constitutes a criticism of our current understanding of financial crises and of the value of our inference since its scope is limited by the quality of the inputs we use. It is not that what we know is not useful, but that with better measures we could know more or at least with less uncertainty. This makes the search for a new methodology, as the one proposed here, of paramount importance. By constructing a new variable that at least mitigates some of the current difficulties, new research avenues appear and a rereading of what we currently now becomes possible.

To tend to this issue we have developed a set of Boom-Bust Indicators (BBIs), bounded in a preset interval, calculated through a non-parametric method that exploits the asymmetry and specific tail structure of the underlying price data and their associated linear returns calculated for several time periods. The relevance of a given observed return on the indicator can be based on a preset fixed grading or as a function of its distance to a given measure of central tendency expressed in standard deviations. The fixed grading methodology leaves the option open for the researcher to choose the different grades which may lead to conflicting results across investigations. The standard deviation based grading better reflects the skewness of the underlying data and eliminates this degree of freedom and thus is preferred.

In addressing the criticisms to the binary sequences used in current research, the BBIs under any specification (short run, long run or equally weighted) constitute a continuous variable and thus solve the instantaneous change-of-state issue. In order to understand the financial cycle the researcher does not need to determine whether this variable breaks a threshold, as is common practice with the HP filter gaps, but analyze the evolution of the variable from positive to negative values and for values closer to the lower or upper bound to the center of the interval. Part 4 provides evidence of this in the different panels of Figure 5 and Figure 6, particularly in the ones corresponding to long run and equally weighted BBIs where the wave-like shape of the cycle becomes evident.

Part four of the paper also shows how this resulting methodology contains relevant characteristics of the original database. Even though in the onset of the paper we expected to find a single measure, as we evaluated the different specifications (short run, long run and equally weighted) we found each one contained important and distinct characteristics from the underlying time series. First, we found consistent, strong and positive linear and quadratic correlations between short run and equally weighted BBIs and returns. Additionally, long run BBIs behave similarly to the first principal component of the correlation matrix not across time but across returns for fixed time periods. This can be understood as a proxy for the pervasiveness of bull and bear markets in time. Finally, the wealth of information contained in BBIs is also reflected in their serial correlations which mimic those of the underlying returns and prices. Short run BBIs constitute, like returns, short memory processes while long run and equally weighted BBIs conform to long memory processes like index levels.

Part five contrasts this methodology to the more frequent ones in the literature: the non-parametric turning point algorithm and the parametric Hodrick Prescott filter under several parameters. A first description of each of these methodologies shows that characteristics such as length and average returns for bulls and bears are contingent on the specification. This shows that not all booms or busts are created equal and that the binary sequence is essentially myopic to the specific characteristics of each historic event. Interestingly, we also find contradicting results when contrasting the two traditional methodologies. It may happen, as shown for the UK, that while one methodology shows a bull market, the other shows a bear market. The plasticity of the short run BBIs allows them to change rapidly from a boom to a bust and thus avoid these contradictions or omissions.

Furthermore, in Part 5 there is evidence of an important level of synchronization between the different BBI specifications and the turning point and Hodrick Prescott filter methodologies. This is relevant since BBIs are expected to nuance stylized facts that have already been found in the literature. Their use in future research should allow delving deeper into phenomena that are currently known to be true through economic analysis and historiography. Of course they should also be useful to solve debates on disputed booms and busts. However, the fact that booms and busts identified through BBIs are positively synchronized with other methodologies is taken as a positive first result.

However, these new methodology is not exempt of problems and criticisms. First, the size and frequency of data affect the overall value of the indicator if not its behavior over time. Since changing sample size alters both the percentile position of a given observation and the first and second moments of the full sample, it is possible that the value of the indicator changes. This raises issues about comparability between samples with different starting and ending dates, different frequencies in the data or different numbers of observations. A second caveat has to do with the effect that including new observations may have on the behavior of the indicator. While increasing an originally large sample in one or two observations may not affect the value of the indicator, it must be contemplated that large changes in the sample require recalculating BBIs for the whole series. This implies that this indicator is not appropriate for forecasting and should not be used in its current form in early-warning research. A third caveat has to do with omitted variables that may be relevant in explaining the asset cycle, for example, trading volume or market capitalization measures which are not included in BBIs. Actually, an underlying assumption is that the stock market we deal with is liquid and that price changes are not caused by any market imperfections.

Future avenues of research with this new methodology are wide. In terms of the financial cycle literature, the immediate future requires applying this measure to credit aggregates to identify the credit cycle in several countries since the joint evolution of assets and credit composes the financial cycle. The interpretation of the different BBI weightings may differ, and their information content may also be altered when compared to that for stock market indices. Finding monthly time series for credit aggregates is very difficult and so the methodology will necessarily undergo adaptations.

The resulting measures will then be used to contrast if the different monetary arrangements and institutions (gold exchange standard, Bretton Woods, free floating regime, or euro institutions) have an effect in the way imbalances accumulate and unwind. The proposal consists in testing monetary policy, credit, foreign capital and financial flows, exchange rates and reserves as possible pass-through mechanisms for institutions to impinge on the elasticity with which an economy accumulates imbalances and suffers crises.

Further methodological inquiries may try to assess the diverse informational content of the short run, long run and equally weighted BBIs since they seem to reflect different characteristics of the asset cycle. These differences may be important when choosing one vector over another to explain a phenomenon or when trying to test causality.

Finally, researchers interested in this new methodology can try to evaluate how the BBIs evolve with the incorporation of new information (increased sample sizes) and if overall results are robust when tested for subsamples. Additional robustness checks can be performed by altering the weighing vectors for the construction of BBIs as well as the maximum value of N in the N-monthly return matrix **R**. An issue that has not been resolved in this paper and that can be of some interest has to do with performing statistical tests with BBIs both for univariate and multivariate hypothesis testing as when performing comparisons across time or between countries. This issue, however, is common to most non-parametric methodologies.

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Annex 1: Filters

This annex presents a summary of the specific characteristics of three types of filtering techniques: the band-pass filter, the Hodrick & Prescott (1997) filter and other functional forms.

Band-pass filter

The band-pass filter is a two sided filter designed to minimize the adjustment error of a cycle between a preset bandwidth. The main assumption underlying the band-pass filter, as presented by Stock & Watson (1998) and Christiano & Fitzgerald (2003), is that of a minimum and maximum cycle length. Any cycles or information of shorter (longer) frequency than the lower (upper) bound of the bandwidth will be smoothed out of the time series. A useful characteristic of this method is that it can be used to decompose a time-series in as many cyclical components as the researcher might need by the use of different bandwidths as done by Drehmann *et .al.* (2012). This benefit is what Christiano & Fitzgerald (2003) find is the main difference with the HP filter.

Hodrick & Prescott (1997) filter

The Hodrick & Prescott (HP) filter is a one sided filter designed to decompose a time series Y_t in a growth component g_t assumed to vary smoothly over time so as to have a stable second difference (acceleration), and an independent cyclical component c_t by solving the following programming problem:

$$\min_{\{g_t\}_{t=-1}^{T}} \left\{ \sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\} \qquad (9)$$

Where $c_t = Y_t - g_t$ (Hodrick & Prescott, 1997, p. 3). This is a parametric model since the choice of λ affects the smoothness of the series. As λ tends to infinity the model becomes a linear trend model because it penalizes the growth component over the cycle component. This means that lower values of the parameter result in models that adapt faster to changes in Y_t. The authors find reasonable value of the parameter when dealing with quarterly data is 1,600 although Borio & Lowe (2004) use values of 400,000 as to reduce the short run plasticity of the model. The parameter λ when g_t and c_t are independent and identically distributed, which they are not, can be interpreted as the ratio of the variances of the cyclical component and the second differences of the growth component (Hodrick & Prescott, 1997). Higher λs imply the two variances are very different while lower λs imply they are closer. According to Stock and Watson (1998) this model does not amplify high frequency noise.

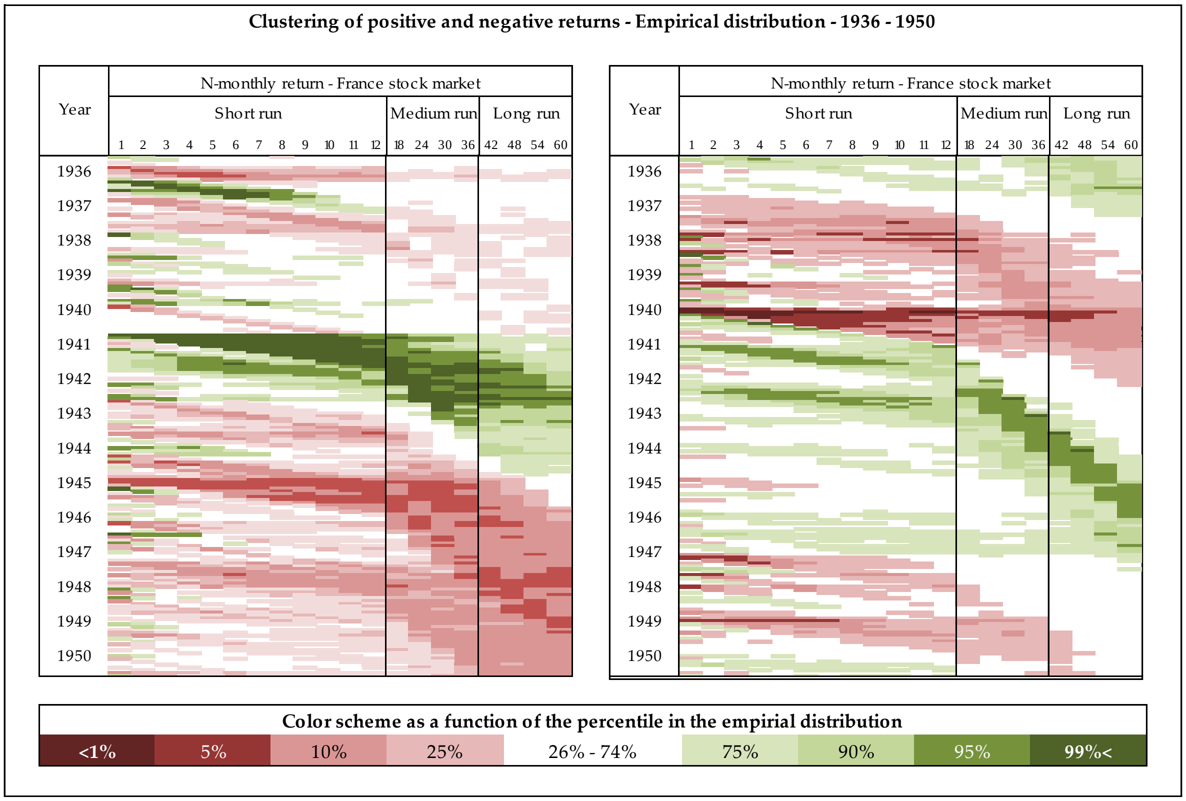
Other functional forms

Filters can be constructed with a variety of functional forms. For example Bordo & Jeanne (2002) use a three period moving average while Eichengreen & Bordo (2002) extend it to five years and Laeven and Valencia (2013) use an of intermediate four years. This is equivalent to fitting the data to an MA process with different parameters. Since the work of Spencer (1904), weighted averages that allow negative weights in the extremes have been used as filters. More complicated functions such as quadratic equations and other polynomials have been used in the literature yielding similar results. A linear time trend can also be removed by regressing the time series against time and then using the residuals, assumed to be stationary, as the cyclical component.

Annex 2: Additional heat maps for France and the United Kingdom

The following three figures contain heat maps for the evolution of returns for the French (left panels) and United Kingdom's (right panels) stock markets. Figure 11 shows the results from the late 1930s until 1950, a time period that contains all of the Second World War and the first few years of the Bretton Woods period. Figure 12 shows result from 1961 until 1975 spanning the first years of the European Community, the last decade of Bretton Woods and the first oil shock of 1973-1974. Figure 13 shows results from 1999 until 2013, showing the beginning of the European Monetary Union and the recent financial and debt crises.

Figure 11: Clustering of returns according to the empirical distribution (1936-1950)

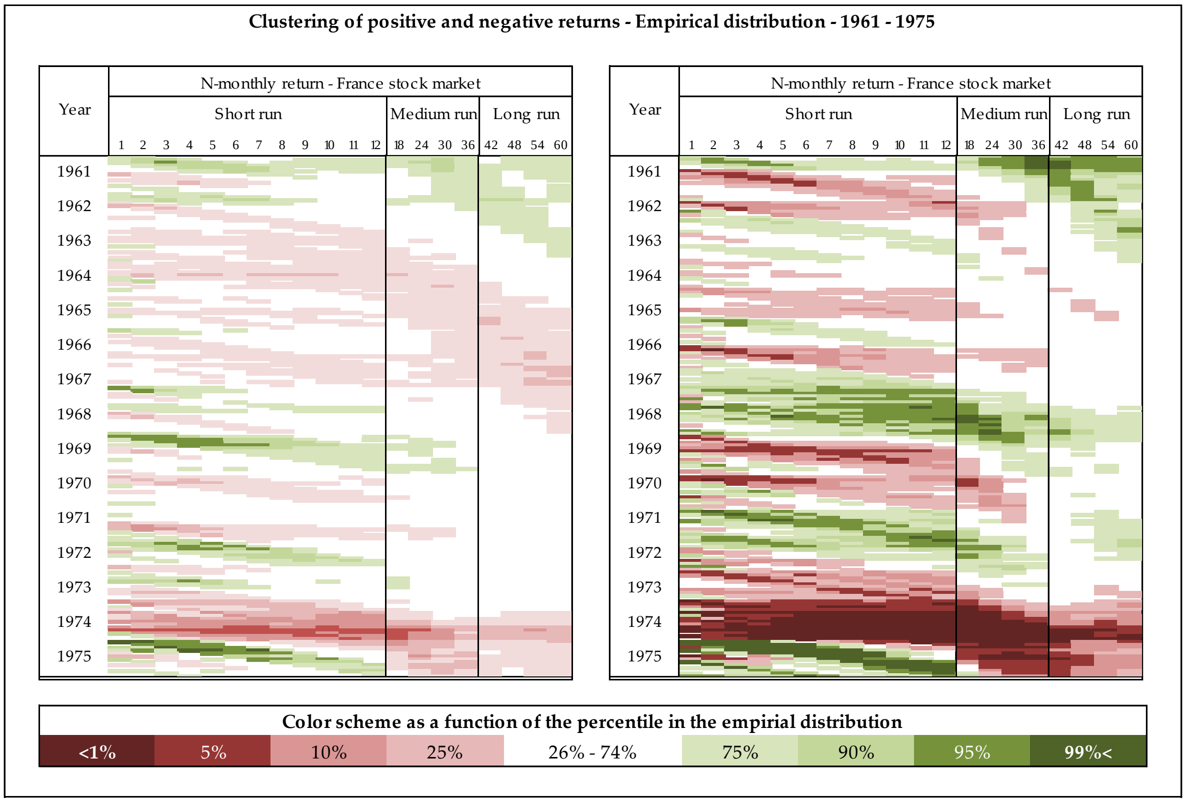


Looking at the right panel (France), we first evidence a strong but short lived boom in 1936 and then a stable market with returns in the interquartile range until we find a very strong and pervasive boom that starts in early 1941 and lasts until mid-1942, affecting positively long run returns well into 1945. Then there is a very pervasive bear market that starts in early 1945 and affects long run returns for the rest of the period.

The story for the United Kingdom is different. The period starts with negative returns during the first few years of the Second World War that consolidate in a pervasive bust that starts late in the 1940 affecting long run returns. Interestingly, returns were consistently negative since 1937, and this extended as a wave toward the largest values of N by 1939. This shows that while pervasive busts may be explosive they can also be caused by consistently negative returns that remain above the 10th percentile. The same happens with a boom period that starts in 1941 and that with consistent short-run returns between the 75th and 90th percentile affects medium and long run returns by the end of 1944. Then we evidence short-lived busts in late 1947 and mid-1949.

The 1941 French boom and the bust in 1940 in the UK are some of the most representative extremes in the full sample as is evidence by the darkness of the corresponding shades of green and red particularly in the short and medium term. Long run effects were darker in the UK in figure 4. It is interesting to see several stability periods, without color in the UK between 1943 and 1946. These kinds of phenomena are cause by N-monthly returns that remain for long periods in the interquartile range.

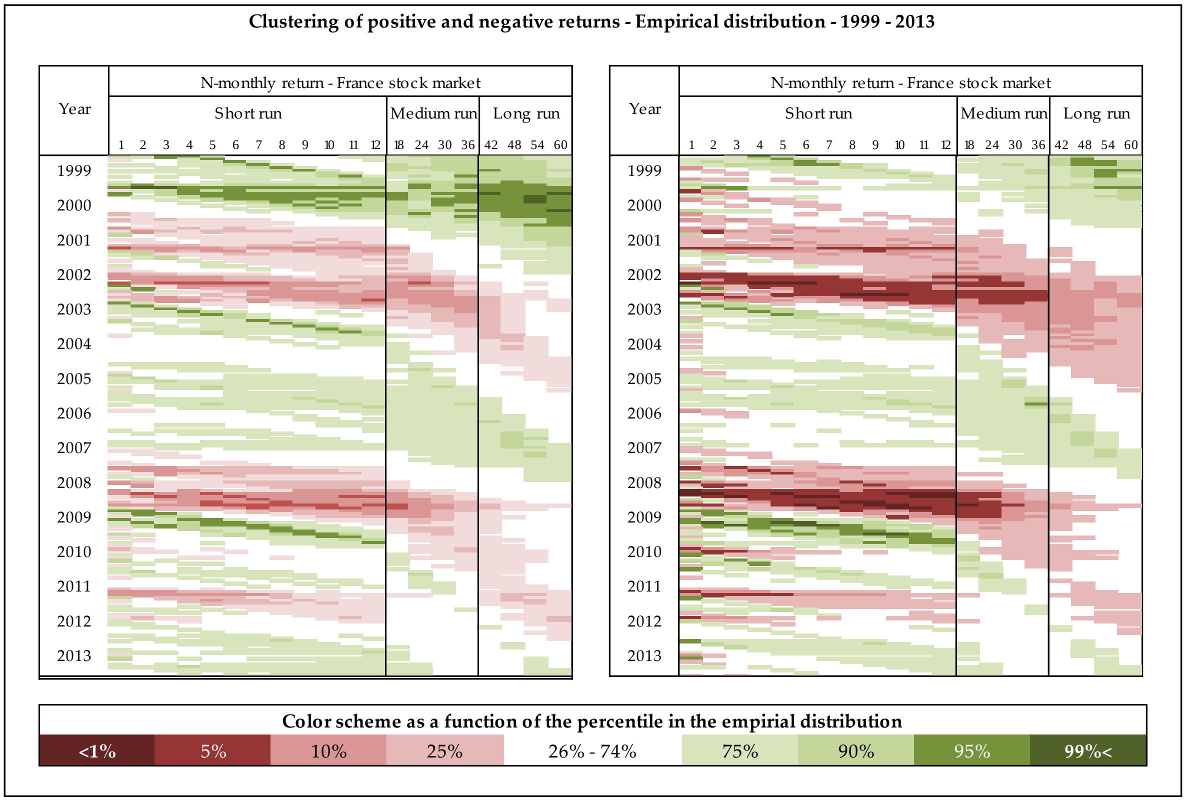
Figure 12: Clustering of returns according to the empirical distribution (1961-1975)



The left panel shows quite a calm period for France, especially when compared with the rest of the sample. There is a mild boom that covers the 1961-1962 period followed by mild negative returns until 1966. A short lived boom happens in late 1968 and early 1969 and after two mild corrections in the early 1970s we see an incipient boom until 1972. There is a stronger bear market in 1973 and early 1974, consistent with the timing of the oil shocks followed by a recovery in early 1975 that does not affect long run returns at least in this sample.

The right panel shows a more volatile story for the United Kingdom with a pervasive bust in 1961-1962 a mild recovery in 1963-1964 and then a bust in two waves in 1964-1967. There is a strong recovery that affects medium term returns in 1967 and does so quite rapidly. Then there is a fall in asset prices in 1969, a short lived but pervasive recovery until early 1972 and then the beginning of a bear market that turns to a full-fledged crash in late 1973 and 1974. We see the beginnings of a quite strong and healthy recovery by mid-1975 but still long run returns are in the lowest percentiles of the whole sample.

Figure 13: Clustering of returns according to the empirical distribution (1999-2013)



The French data show a boom in the late 1990s which is very relevant on the overall sample, probably a recovery after the Asian and emergent market crises that had happened earlier during that decade. Then a bear market ensues, probably associated with the burst of the dot.com bubble in the US. A slight recovery takes place until 2007 when the global financial crisis hit, affecting returns across the whole spectrum. An important recovery starts in early 2009 until a mild crash in late 2011, probably associated with the European debt crises to which French banks were exposed.

The case for the UK is that of two very strong busts (2002-2003 and 2007-2008) amid mild, slow and pervasive recoveries. The most relevant recovery in this subsample takes place in 2009 followed by a short and not very relevant bear market in 2011. The busts we find here are, in the short term, very relevant for the full UK sample.

Annex 3: Histograms by country of different BBI specifications.

The first figure contains the histograms for French BBIs while the second one contains the histograms for the BBIs of United Kingdom. Both X and Y axes cover the same range as to make graphs comparable across methodologies and countries. What we observe is a general flattening of the distributions from the fixed grading to the standard deviation grading methodologies.

Figure 14: Histograms for France

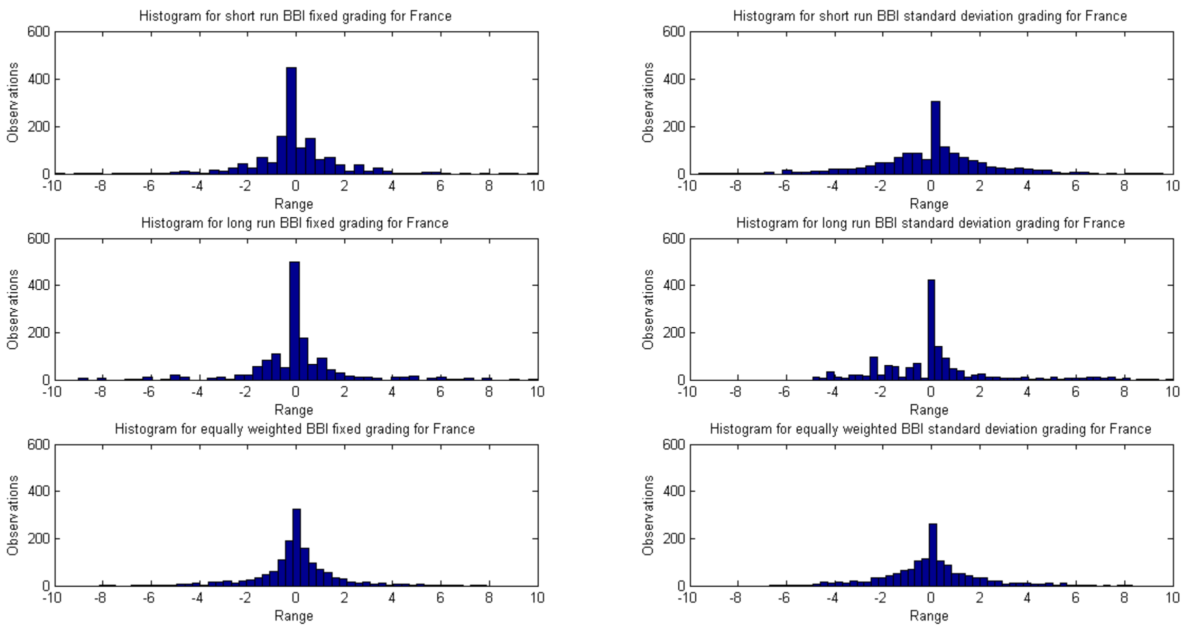
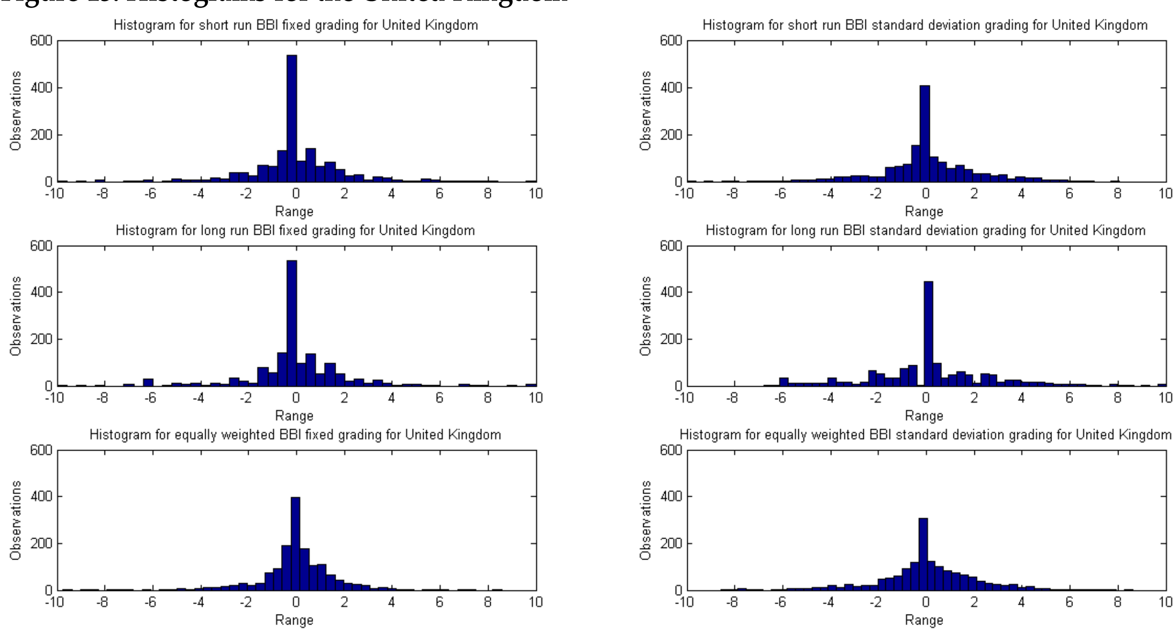


Figure 15: Histograms for the United Kingdom



Results seem to be more spread out for the equally weighted specifications while they present more extreme values in the short run and long run specifications. There is a very strong concentration of observations around zero which is consistent with the fact that both booms and busts are rare events.

Annex 4: Additional graphical contrasting for France and the United Kingdom

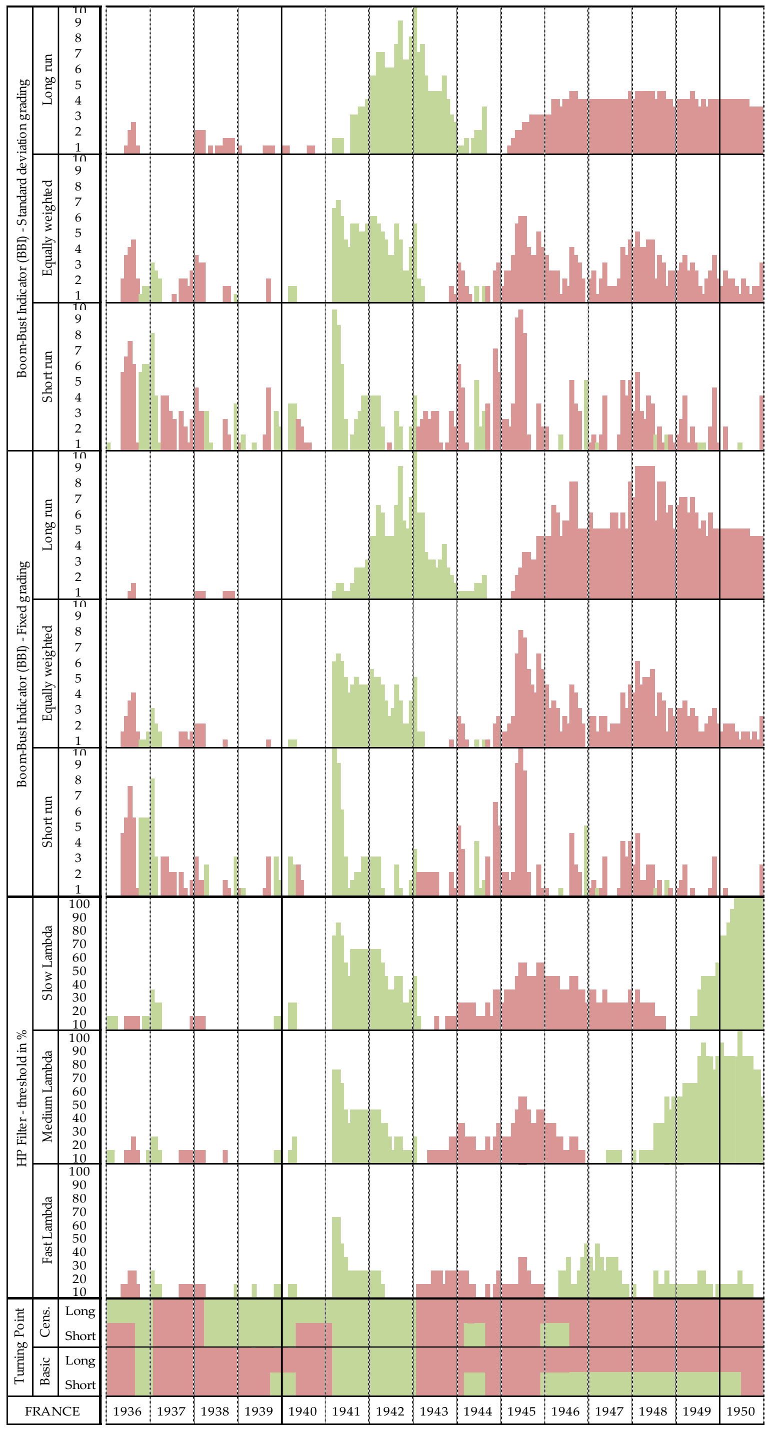
This section presents different figures for Franc e and the United Kingdom comparing three different time periods: the Second World War and the late 1940s (1936-1950); the late Bretton Woods and the first oil shock (1961-1975); and the beginning of the Euro and the Global Financial Crisis (1999-2013). In each page we present one figure by country by period and an analysis of the three methodologies in the same fashion as in the last subsection of Part 5.

France (1936-1950)

The turning point methodology (TP) shows disagreements between the different specifications, in particular between 1938-1940, a short period of 1944 and from 1946 until mid-1950. When all TP specifications coincide generally all other methodologies coincide, except for 1943 where the long-run BBIs are unable to identify a bear market but do show a decrease in intensity of the undergoing bull. This is due, probably to the low pervasiveness of negative returns across time.

The bear market of 1937 is only shown very lightly and in the final months of the year by the HP filters. Short run and equally weighted BBIs do register it as soon as the turning point methodology. The 1938-1939 period where there is no agreement for TPs, is a period of quickly alternating bears and bulls as seen in the short run BBIs. This volatility is not captured by the HP filters. The HP filter shows a boom starting in 1946 that breaks all thresholds in the medium and slow parametrizations. When checking Figure 11 for confirmation there is very small evidence of positive returns, some of which are shown in short run and equally weighted BBIs for 1946. However, the boom that appears in the HP filter did not really happen in the data. One possibility is that the trend was so negative that any positive or less negative return would appear as a boom. This kind of confusions are avoided with the BBI methodology.

Figure 16: Methodological comparison France 1936-1950



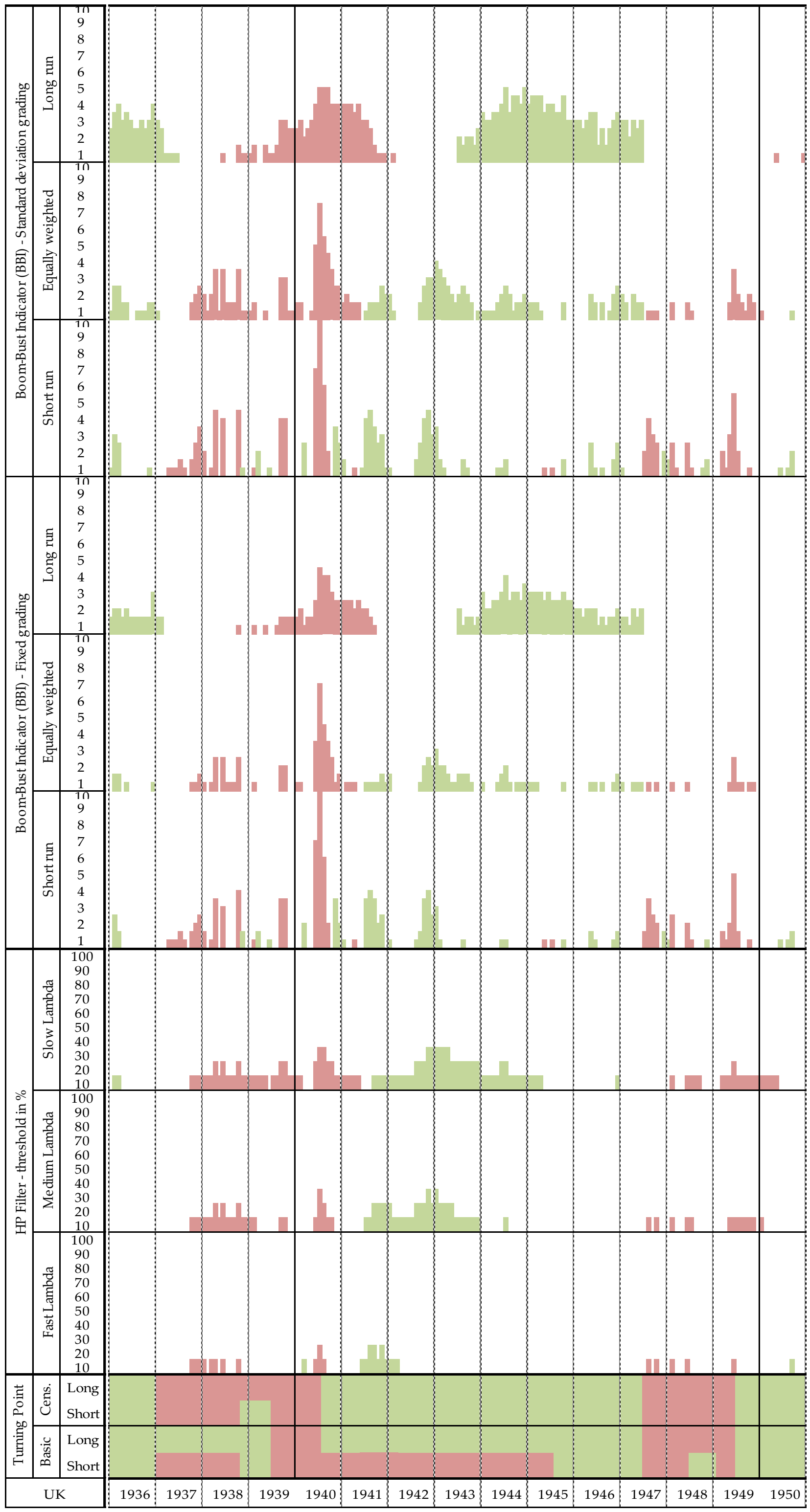
United Kingdom (1936-1950)

Disagreements under TP are present in 1937-1938, from mid-1940 until mid-1945, and a short period at the end of 1948. Only in 1938 and 1948 is there evidence of a fast change of sign in the short run BBI to explain this contradiction. The HP filter identifies a mild bull market amid two mild bear markets. Depending on the parameter the bull appears to last from less than a year (fast adapting process) to almost 3 years and a half (slow adapting lambda).

The BBI methodology coincides with the HP filter in the sense of identifying both bears and bull markets. However, the first bear in the early 1940s appears as a very strong event that affects the long run indicator. The bull market appears to have two different waves, according to the short run indicators, one by the end of 1941 and the other by mid-1942. They affect long run returns though from 1943 and until 1947 the short run BBIs show a stable period.

The bear market of 1947-1948 also happens in two waves although they do not affect long run returns and thus the bear market does not show in the long run BBIs.

Figure 17: Methodological comparison UK 1936-1950



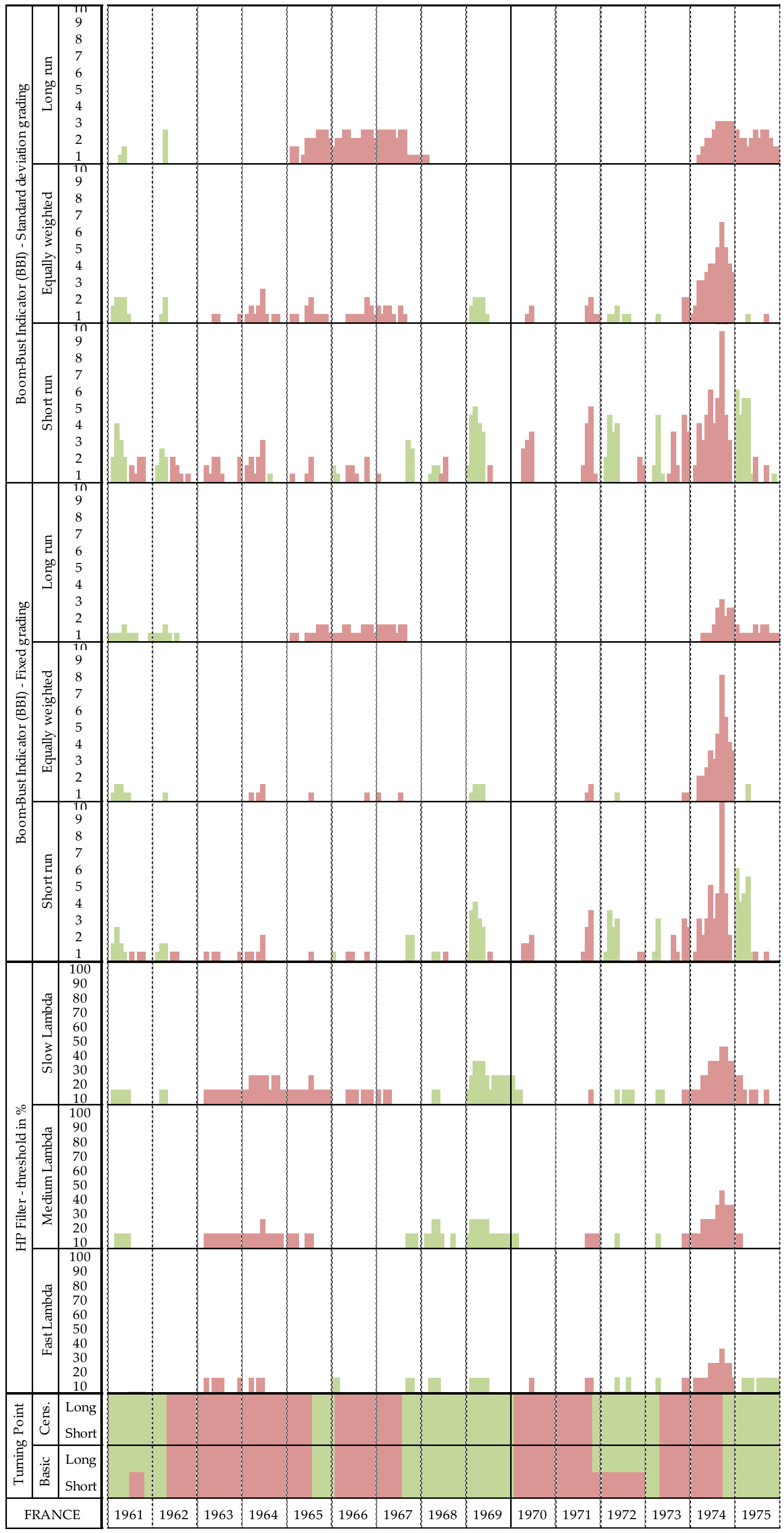
France (1961-1975)

In this case, disagreements in the TP only happen during 1961 and 1972. The HP filters coincide in one bull market lasting between 1 and 2 years amid two bear markets. The first bear market lasts somewhere between one year, under the fast lambda specification, and four years, according to the slow lambda filter. The second bust occurs at the end of 1973 and during 1974. The dates coincide with those of the first oil shock. This bust coincides across all methodologies and all specifications.

BBIs tell a more nuanced story. In particular, the standard deviation grading methodology shows a very telegraphic short run indicator during the 1960s. This means that there are several months with positive or negative values of the indicator but they do not conform either to a solid bull or bear market. It is only by 1974 that the short run indicator starts showing a consistent bear market.

Equally weighted and long run BBIs do show more consistent bear markets between 1964 and 1967 and from 1974 to 1975. The bull shown in the HP filter in 1969 only appears in the short run and equally weighted BBI indicators.

Figure 18: Methodological comparison France 1961-1975



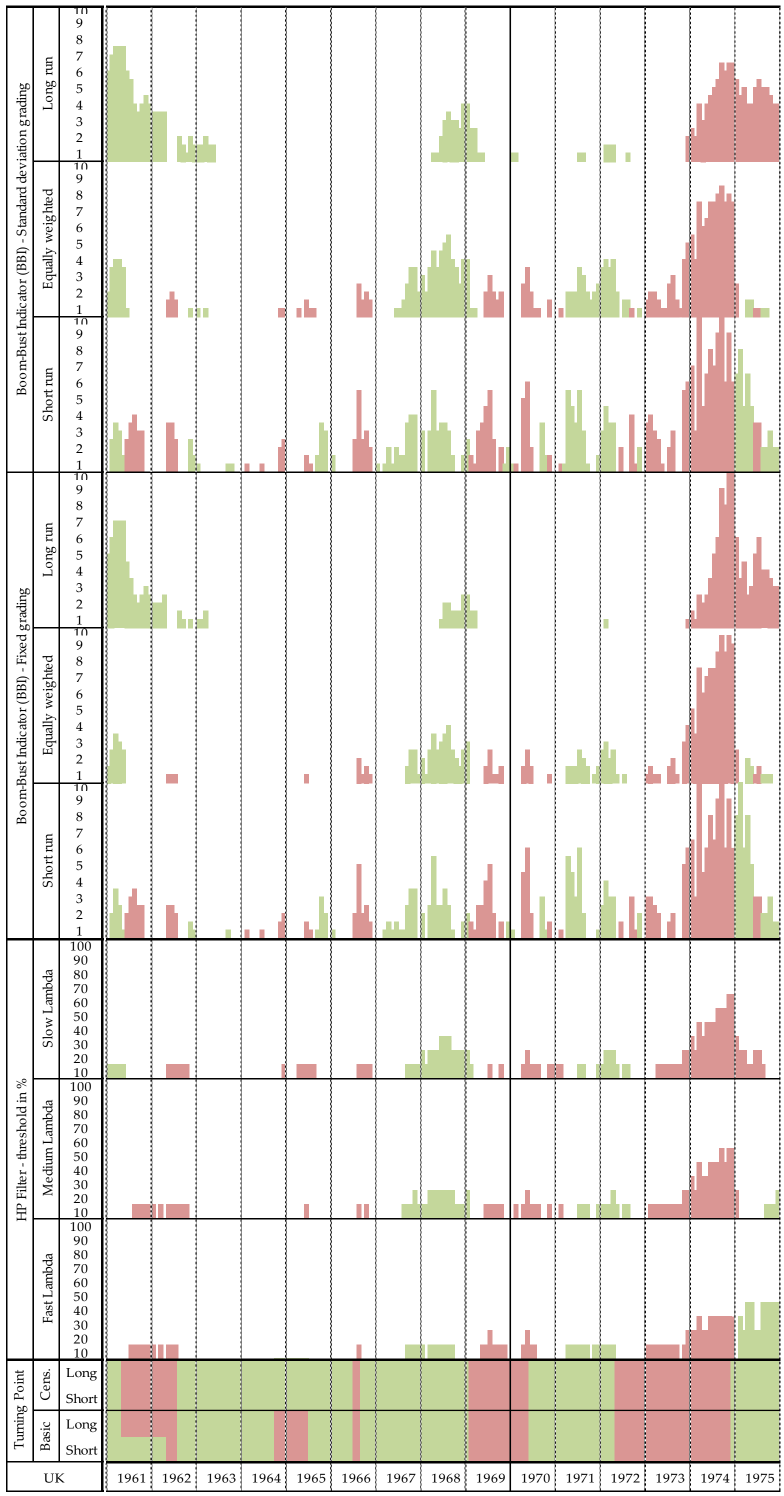
United Kingdom (1961-1975)

TP specifications show disagreements in 1961-1962 and between 1964 and 1965. In the first case all three specifications of de HP filter and the short run and equally weighted specifications of BBIs show the end of a bull market followed by a bear market. Only the long run BBIs show a bull market of decreasing intensity but no bear market to follow. Short run BBIs also show a mild bear that turns into a mild bull between 1964 and 1965, while the HP filters fail to show the mild bull and only show the contraction under the slow adjustment specification. In 1966 there is a shock that occurs over a very short period of time. It is registered by all HP filter specifications and by both short run and equally weighted BBIs. Long run BBIs do not register this phenomenon.

There is a strong consistency in the findings for the period that starts in 1967 and ends in 1975. The HP filters and the short run and equally weighted BBIs all identify a mild bull market (1967-1968) followed by a mild bear market (1969-1970) followed by a milder bull market (1970-1972) and finished by a very pervasive bear market (1973-1975).

Long run BBIs identify a boom that came from a previous period and ended in 1962, the boom in 1968 and then the bust starting in 1974. These seem to be the most pervasive phenomena in this subsample of the series.

Figure 19: Methodological comparison UK 1961-1975



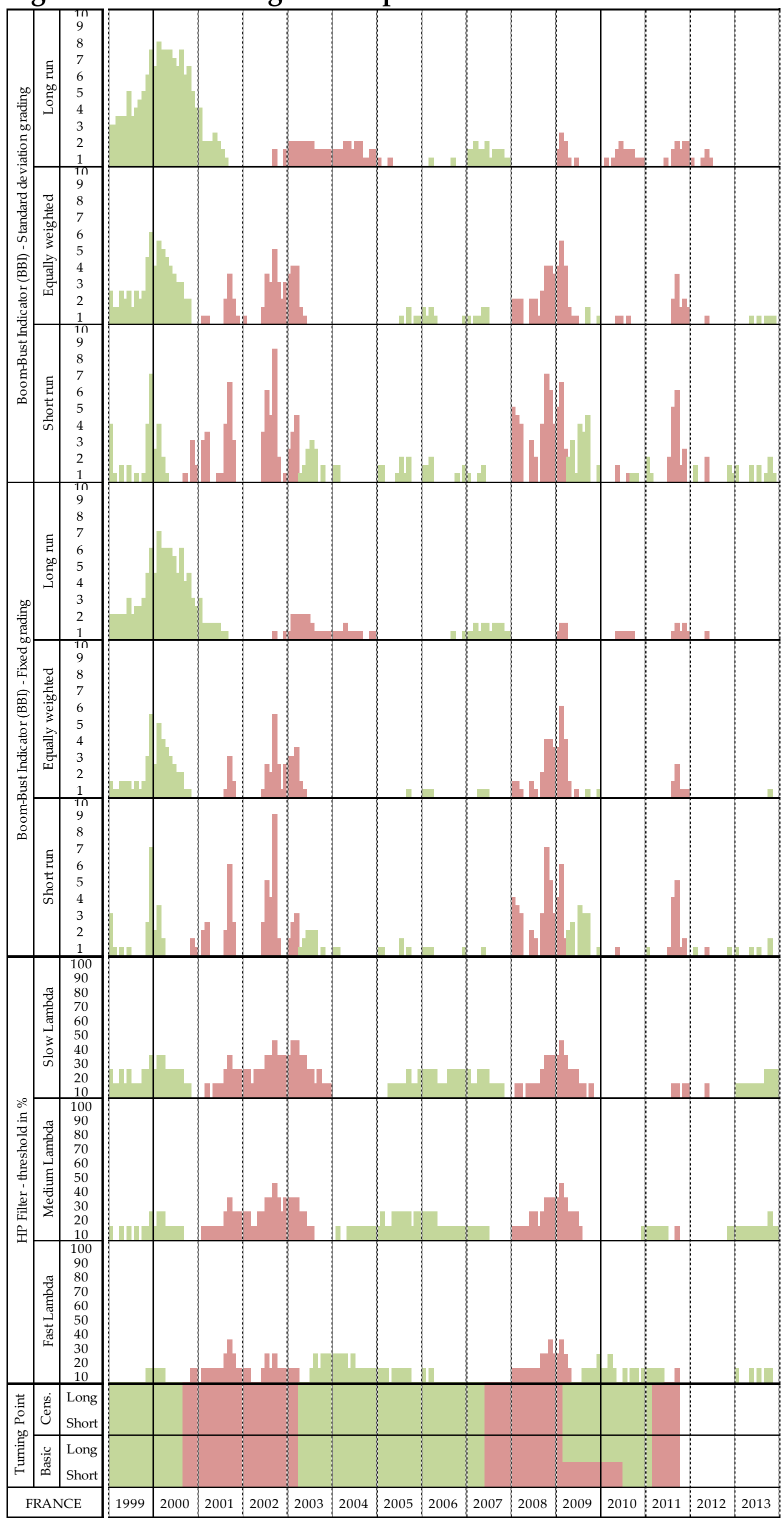
France (1999-2013)

The only lack of agreement in TP occurs between 2009 and 2010. This methodology ends in 2011 because of the censoring restrictions applied. In general, there is agreement between all methodologies in a bull market that covers 1999 and 2000. However, while during 2001 and 2002 the HP filters show a bust as a continuous process, the short and Equally weighted BBIs show two distinct waves for the same bear market: one during the last half of 2001 and another on during the later part of 2002.

The following bull, that registers mildly in the HP filter methodology, registers poorly in the fixed grading methodology and only somewhat better in the standard deviation graded BBIs, in particular in the short run specification. Then, all methodologies show the crises of 2008 and short run BBIs show a slight recovery at the beginning of 2009. Then, all methodologies agree on a short lived bear market at the end of 2011.

Interestingly, the bull market of 2003-2006 registers strongly in the HP filter methodologies but BBIs identify it to be a very mild event that happens to have long run effects only two years after it started. This is a pervasive boom that does not start like an explosive process.

Figure 20: Methodological comparison France 1999-2013



United Kingdom (1999-2013)

There is only a disagreement in TPs during 2009-2010. HP filters show a bear between 2001 and 2002, then a bear between 2003 and 2007 depending on the specification and a bear during 2008. All of these characterizations roughly coincide with the results from the turning point methodology.

BBIs offer a more granular perspective. The first boom between 2001 and 2002, appears to happen in two distinct waves as shown by short run and equally weighted BBIs. The boom that registers between 2002 and 2007 is really a very mild phenomenon that takes more than two years to affect long run BBIs.

Finally the bear market of 2008 is a very intense and explosive process, followed also by a very intense and explosive recovery during 2009.

The story told by the long run BBIs is very similar to the one told by the HP filter. However, short run and equally weighted BBIs seem to offer a more granular and detailed version of the story for the United Kingdom in this time period.

Figure 21: Methodological comparison UK 1999-2013