Strongly Interacting Electroweak Symmetry Breaking Sector

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Abstract: Using the equivalence theorem we study the scattering of longitudinally polarized gauge bosons in extensions of the Standard Model where anomalous Higgs couplings to gauge sector and higher order $\mathcal{O}(p^4)$ operators are considered. Forcing the amplitudes to be unitary we find new resonances for different values of the coupling constants. Hereby we would be able to narrow the range of values of the chiral parameters with new experiments at LHC.

I. INTRODUCTION

Physicists have been investigating for a long time the features and trying to figure out the nature of the Electroweak symmetry-breaking sector (EWSBS) . There are plenty of theoretical models to describe EWSBS, which can be divided very roughly in two categories: weakly or strongly coupled.

In weakly coupled ones, one expects the appearance of light particles below the TeV scale. The main example of these cases is the minimal Standard Model (MSM) with a light Higgs boson [1]. These models have been largely studied because of the availability to perform perturbative computations. In the strongly coupled case, the strength of the interactions makes the perturbative treatment unreliable. The strong models for the EWSBS predict heavy resonances, as in the longitudinal gauge boson scattering treated in this paper.

Nonetheless, the last data reported from ATLAS and CMS [2] at the time doing this paper, is compatible with the discovery of a scalar particle with a mass around 125 GeV and positive parity, i. e., the Higgs particle predicted by the SM. However, this fact does not discard the strongly interacting EWSBS at all because new higher resonances could be detected during the next years.

This paper is devoted to analyse a strongly EWSBS towards the scattering amplitudes of the weak gauge bosons and the Higgs.

To do so, first one has to be able to describe generically the strong interactions of electroweak gauge bosons. Our procedure is based in chiral perturbation theory (ChPT) [3] which works quite well for pion physics. The idea is to write an effective Lagrangian, the form of its terms is only constrained by symmetry considerations which are common to any strong EWSBS. All the models have to obey the same symmetry, as they have to be capable to reproduce the low energy behaviour of EW. The difference among the theories appears through the values of the parameters in the chiral Lagrangian.

The problem of the chiral approach is that unitarity is not guaranteed. At low energies the violations of unitarity are negligible, but they increase with energy. Then, if we want to extend the range of validity of the theory we will have to make it unitary by hand. There are many ways to unitarize amplitudes obtained with the Electroweak Chiral Lagrangian (EWChL) [2]. The method that is used in this work is the one known as inverse amplitude method (IAM), that has been extensively used in QCD .

The aim of this work is to study the scattering of the longitudinal components of gauge bosons (W^+, W^-, Z^0) and light Higgs boson (H). Using the EWChL approach, we will extract the amplitudes in the massless limit at loop level by making use of the Equivalence Theorem. Knowing the amplitudes, we will expand them in partial wave series, and finally we will explore the parameter space looking for dynamical resonances using the IAM method in order to restore unitarity.

The paper is organized as follows. In section II we write down the Electroweak Chiral Lagrangian, In section III we apply the IAM method to the partial wave amplitudes obtained in [4] to compute the resonances encountered in the parameter space, discussing which resonances have a physical meaning. We have separated the results in two parts: decoupled and coupled channels.

II. THEORETICAL FRAMEWORK

A. Electroweak Chiral Lagrangian with a Light Scalar

Effective Lagrangians of Higgs boson and gauge bosons have already been extensively used to study current data at LHC [2,4-7]. They are useful since one can obtain the amplitudes as truncated series in s, the center of mass energy squared.

The Lagrangian that contains the light degrees of freedom in the standard model, including the Higgs field h, inspired by the non-linear sigma model [1], reads

$$\begin{aligned}\mathcal{L}^{\text{eff}} = &-\frac{1}{2}\text{Tr}W_{\mu\nu}W^{\mu\nu} - \frac{1}{2}\text{Tr}B_{\mu\nu}B^{\mu\nu} \\ &+ \left(1+\frac{h}{v}\right)^2 \frac{v^2}{4}\text{Tr}(D_\mu U)^\dagger D^\mu U \\ &+ \frac{1}{2}(\partial_\mu h)^2 - V(h) + \text{NLO}\end{aligned} \tag{1}$$

where U is the representation of the electroweak-theory

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Goldstone bosons

$$U = \exp\left(i\frac{\vec{\omega}\cdot\vec{\tau}}{v}\right) , \tag{2}$$

where $v \approx 246$ GeV is the SM vacuum expectation value, τ_i are the Pauli matrices and ω_i are the Goldstone fields, related to the charged basis: $\omega^{\pm} = \omega_1 \mp i\omega_2/\sqrt{2}$ and $\omega^0 = \omega_3$. The covariant derivative of U is then defined as

$$D_\mu U = \partial_\mu U + \frac{1}{2} ig W_\mu^i \tau^i U - \frac{1}{2} ig' B_\mu U \tau^3 . \tag{3}$$

So, here the ω are the three Goldstone bosons of the global group $SU(2)_L \times SU(2)_R \to SU(2)_V$ and the Higgs field h is a gauge and $SU(2)_L \times SU(2)_R$ singlet. The NLO are the next-to-leading order terms of $\mathcal{O}(p^4)$, which will correspond to a complete set of $SU(2)_L \times U(1)_Y$ (the gauge of electroweak sector), Lorentz invariant operators containing up to four derivatives. The terms that will contribute of this order are the following:

$$\begin{aligned}\mathcal{L}^{(4)} = {} & a_4 \mathrm{Tr}[V_\mu V_\nu]\mathrm{Tr}[V^\mu V^\nu] + a_5 \mathrm{Tr}[V_\mu V^\mu]\mathrm{Tr}[V_\nu V^\nu] \\ & + \frac{\gamma}{f^4}(\partial_\mu h \partial^\mu h)^2 + \frac{\delta}{f^2}(\partial_\mu h \partial^\mu h)\mathrm{Tr}[(D_\nu U)^\dagger D^\mu U] \\ & + \frac{\eta}{f^2}\mathrm{Tr}(\partial_\mu h \partial^\nu h)[(D^\mu U)^\dagger D_\nu U]\end{aligned} \tag{4}$$

where $V_\mu = D_\mu U U^\dagger$. The two first terms are related to the self-couplings of the Goldstone bosons, while the second terms are the Higgs self-couplings and the couplings at high order of Higgs and the Godstone. Notice that it appears a new parameter f, that should be related to some new-physics scale in the EWSBS. This new dynamics is encoded at low energies in the value of the a_4, a_5 known chiral parameters [1], and with the new parameters γ, δ and η. Varying these parameters in the Lagrangian in a proper way could bring about new resonances, and their detection reveal information about the new dynamics and the new anomalous couplings of the gauge bosons observable by the Large Hadron Collider (LHC).

We could not be satisfied enough with the previous parameter variations and consider more modifications of the EWSBS, such as concerning the third term in (1). We could make an expansion like

$$\left(1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2 + ...\right)\frac{v^2}{4}\mathrm{Tr}(D_\mu U)^\dagger D^\mu U , \tag{5}$$

as gauge invariance poses no restrictions on its form. Therefore, the couplings of h with ω will depart from their MSM values $a = b = 1$ and zero for higher order. We will only consider the expansion up to second order of $\frac{h}{v}$, as no higher orders contribute in the $W_L W_L$ scattering.

The present value of these parameters has been under estimation with LHC results [5], and it has given the following bounds for a, a_4 and a_5:

$$a = [0.67, 1.33],\ a_4 = [-0.094, 0.10],\ a_5 = [-0.23, 0.26]$$

with a 90% of confidence level. The range of a seems to be reasonable for the MSM, but a_4 and a_5 are within a large range yet. The parameter b is almost totally undetermined at present. Needless to say that the other three parameters have not been measured yet.

Now we should discuss the limitations of this effective Lagrangian. One arises from the effective theory itself, and states that the effective approach is only valid up to $4\pi v \sim 3$ TeV, so any result beyond this cutoff should not be considered. The other problem is that when a and b depart from their MSM values ($a = b = 1$), the theory becomes unrenormalizable [8]. However, as we work at one-loop level, scattering amplitudes can be rendered finite by a specific redefinition of the coefficients a_4, a_5, γ, δ and η, performed in [4].

B. The equivalence theorem

As one can see from (3), once included the electroweak interactions, the Goldstone bosons "disappear" and become the longitudinal components of the gauge bosons. Somehow we can identify the Goldsone bosons and their behaviour with that of the gauge bosons. This is a simple explained version of the equivalence theorem (ET), and a deeper look can be found in [1]. Expressing mathematically the theorem is very simple

$$T(W_L W_L \to W_L W_L) \simeq T(\omega\omega \to \omega\omega) + \mathcal{O}\left(\frac{M_W}{\sqrt{s}}\right), \tag{6}$$

where M_W is the mass of the gauge boson and $\sqrt{s}$ is the center of mass energy. Looking at the equation, one realizes that ET allows us to identify the longitudinal components of the gauge bosons with the Goldstone bosons at energies $\sqrt{s} \gg M_W$. This approximation will make much easier the calculation of the amplitudes, since now we are treating scalar particles. Therefore the couplings of the Goldstone bosons with the gauge bosons vanish ($g = g' = 0$), and the only degrees of freedom left will be the GB (massless in the Landau gauge [1]), and the Higgs boson. Moreover, according to LHC data $M_H \simeq 125$ GeV. This mass is quite similar to the gauge bosons mass (~ 100 GeV), consequently, it is a good approximation to consider the massless limit $M_H \simeq 0$, so we have done in this paper.

C. The partial waves method

As long as we have an $SU(2)_V$ symmetry, it is possible to define a weak isospin I. We focus on two particle scattering, thus, our isospin channels will be $I = 0, 1$ and

2. Then, we will be able to make a partial wave expansion as follows.

Let T_I be a scattering amplitude of a process with isospin I. One can Fourier-expand this amplitude as a function of Legendre polynomials, and a certain coefficient or partial wave $t_{IJ}(s)$

$$T_I(s,\cos\theta) = \sum_{J=0}^{\infty}(2J+1)t_{IJ}(s)P_J(\cos\theta)\ , \qquad (7)$$

where J is the angular momentum and θ is the scattered angle in the center of mass. Once computed the total isospin amplitudes, it is straightforward to obtain the partial wave choosing the conventional normalization [4]:

$$t_{IJ}(s) = \frac{1}{64\pi}\int_{-1}^{1}\mathrm{d}(\cos\theta)T_I(s,\cos\theta)P_J(\cos\theta)\ , \qquad (8)$$

and we will only focus on the first non-vanishing partial wave for each isospin, i. e. t_{00}(scalar channel), t_{11}(vector channel) and t_{20}(tensor channel). Then we can expand each partial wave in series of s as well:

$$t_{IJ}(s) = t_{IJ}^{(0)}(s) + t_{IJ}^{(1)}(s) + \mathcal{O}(s^3). \qquad (9)$$

D. The Inverse Amplitude Method

The previous expansion of amplitudes generally does not respect unitarity. It only satisfies the optical theorem perturbatively,

$$\begin{aligned}&\mathrm{Im}t_{IJ}(s) \neq \sigma(s)|t_{IJ}(s)|^2 \ \text{ but}\\ &\mathrm{Im}t_{IJ}^{(1)}(s) = \sigma(s)|t_{IJ}^{(0)}(s)|^2 + \mathcal{O}(s^3)\ ,\end{aligned} \qquad (10)$$

where $\sigma(s) = 1$ in the massless limit.

We will use the Inverse Amplitude Method (IAM) approximation, strictly derived in [9]. This approximation leads to a very simple expression:

$$t_{IJ} \approx \frac{t_{IJ}^{(0)}}{1 - t_{IJ}^{(1)}/t_{IJ}^{(0)}}\ . \qquad (11)$$

The resulting amplitude accomplishes the optical theorem and thus, it is unitary. It is also easy to see that at low energies it reduces to (9). This definition is good enough for the 11 and the 20 channels, but the scalar channel has more processes involved apart from $\omega\omega \to \omega\omega$, like $\omega\omega \to hh$ or $hh \to hh$. Thus, we have to extend the inverse amplitude method in a matrix form, obtaining

$$T \approx T^{(0)}\left(T^{(0)} - T^{(1)}\right)^{-1}T^{(0)}\ , \qquad (12)$$

where $T^{(n)}$ is the n-th order partial wave involving the three processes

$$T^{(n)} = \begin{pmatrix} t^{(n)}_{\omega\omega\to\omega\omega} & t^{(n)}_{hh\to\omega\omega} \\ t^{(n)}_{\omega\omega\to hh} & t^{(n)}_{hh\to hh}\end{pmatrix}, \qquad (13)$$

and then the optical theorem reads

$$\mathrm{Im}T = T^{\dagger}T\ . \qquad (14)$$

The IAM has been proven useful before in nuclear physics. It is able to reproduce the poles related to the ρ meson and the $^*K$ meson in $\pi\pi$ and πK scattering respectively.

III. RESULTS

As commented before, we have three channels to explore: the iso-scalar, iso-vector, and iso-tensor channel. In $\omega\omega \to \omega\omega$ scattering, there is only one contribution to the vector and the tensor channels, but in the scalar channel, one has to take into account the processes involving the Higgs itself. When the parameter $b = a^2$, the amplitudes for the last two processes vanish and one has to consider only the amplitude obtained with the $\omega\omega \to \omega\omega$ scattering. That is why we have separated the results between decoupled ($b = a^2$) and coupled ($b \neq a^2$) channel. The first one is much easier to treat than the second. Therefore, we make a deeper analysis on the decoupled channel, where there are less parameters to play with, while in the coupled channel we will only discuss resonances qualitatively. In both channels, the IAM is applied to the partial wave amplitudes given in [4].

A. Decoupled scalar Channel $b = a^2$

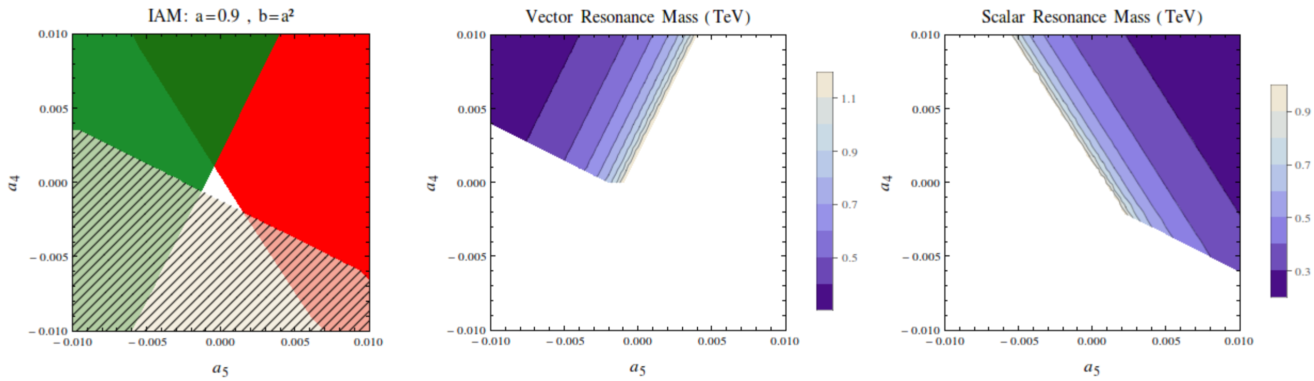
In this case, for the application of IAM, we only need Eq. 11. Then we look for resonances in the parameter space ($a_4 - a_5$), such as

$$t_{IJ}^{-1}(s_{\text{pole}}) = 0\ \ \text{, where}\ \ s_{\text{pole}} = m_{\text{pole}}^2 - im_{\text{pole}}\Gamma_{\text{pole}} \qquad (15)$$

and we only keep the valid ones with positive width ($\Gamma > 0$), accepting only those where $\Gamma < M/4$, which are considered narrow enough to be a resonance.

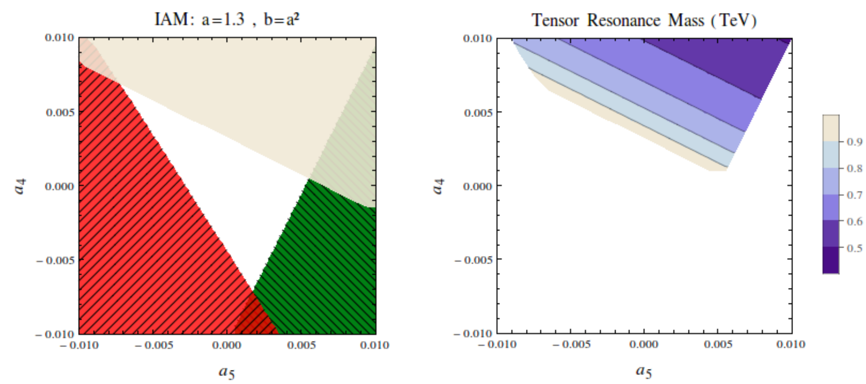
In Figs. 1 and 2 we show the results of the three channel resonances but for two different values of a. For $a = 0.9$ (Fig. 1), where only the vector and the scalar channels have $\Gamma > 0$, we plot the masses obtained for the scalar ($M = [0.280, 1.716]$ TeV, $\Gamma = [0.001, 0.382]$ TeV), and for the vector ($M = [0.309, 1.197]$ TeV, $\Gamma = [0.004, 0.240]$ TeV) channel. For $a = 1.3$ the situation changes radically, and one is only able to find well-behaved resonances in the tensor channel ($M = [0.493, 0.992]$ TeV, $\Gamma = [0.014, 0.121]$ TeV) but not in the other ones.

For $a = 0.9$ we find a very similar picture of the ones found in [5] ($a = 0.9$, $a = 0.95$) and very similar to the one in [7] ($a = 1$), we can say then that the white regions with no resonances around $a_4 = a_5 = 0$ are larger if a differs from 1. So, if no resonances are found at LHC all the way up to 3 TeV, the values of a_4 and a_5 should lie in the central region of Fig 1.



(a) (b) (c)

FIG. 1: For $a = 0.9$ and $b = a^2$, in (a) we show the regions where scalar (red), vector (green), and tensor (gray) resonances appear in the parameter space, where the lines indicate exclusion region ($\Gamma < 0$). In (b) and in (c) we show the resonances for the scalar and vector channels in this case, the ones out of the exclusion region.



(a) (b)

FIG. 2: For $a = 1.3$ and $b = a^2$, in (a) we show the regions where scalar (red), vector (green), and tensor (gray) resonances appear in the parameter space, where the lines indicate exclusion region ($\Gamma < 0$). In (b) we show the resonances for the tensor channel in this case, the ones out of the exclusion region.

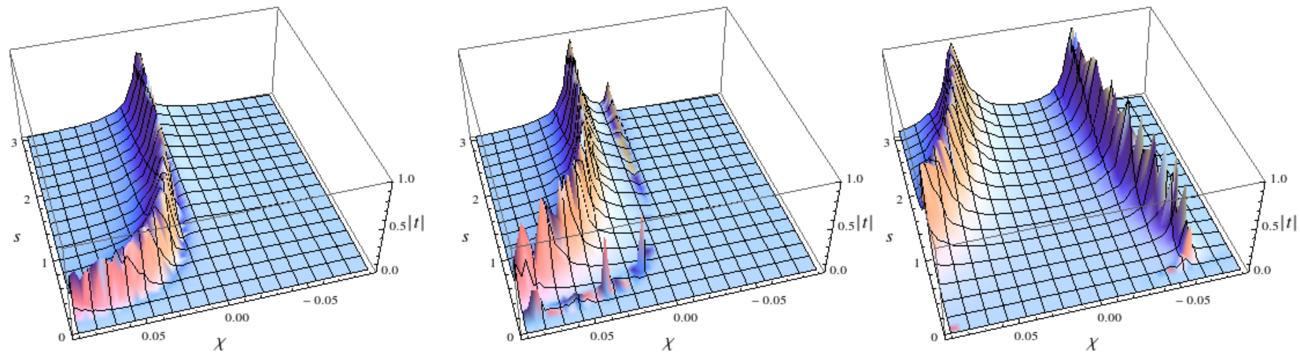
In the case of $a = 1.3$ we found that some poles related to $I = 2$ $J = 0$ channel could exist, but no scalar or vector resonances appear then. However, the region around $a_5 = a_4 = 0$ remains pole-less.

B. Coupled Channel $b \neq a^2$

If we set $b \neq a^2$ in the amplitudes treated before, we obtain a very similar picture, which tells us that b does not play an important role on changing the parameter space behaviour (the variation of b is only noticeable in the widths). So, the iso-tensor and the iso-vector regions remain very close to the regions of $b = a^2$. Nevertheless, in the iso-scalar region this channel is much more complicated in the sense that first we need to use the IAM in its matrix version (12) and thus, there are three more parameters playing a role (γ, δ, η) apart from a_4 and a_5. The new physics scale f is also taking part when the channel is coupled, and we set it to 1 TeV, as it is expected to be larger than the weak scale v [2].

To find the poles in a matrix related to the new states, one must compute the eigenvalues of that matrix and then find the poles for each eigenvalue. So, we set $a = 0.91$ and $b = 0.9^2$ and start the analysis. We do not look values for $a > 1$ as they represented excluded regions on the decoupled channel and because it is complicated enough to focus on one value of a.

We claim that if we set γ, δ, and η to zero, we obtain results similar to those where $b = a^2$, as expected. But if one tries to do a new parameter sweep, more resonances appear. If a_4 and a_5 have the values where there was a pole in the decoupled channel, the same behaviour is observed when we are dealing with the coupled channel. The interesting phenomena occur in the regions of a_4 and a_5 where no resonance was detected. We focus on $a_5 = a_4 = 0$, because that would imply the vanishing of two NLO terms in the Lagrangian (4), although similar results are obtained in the regions where there was no pole on the scalar decoupled channel.



(a) $\gamma = 0$ (b) $\gamma = -0.001$ (c) $\gamma = -0.01$

FIG. 3: For $a = 0.91$ and $b = 0.9^2$, we vary γ in (a),(b) and (c), and plot the modulus of the first eigenvalue of the scalar channel as a function of s (TeV^2) and the new parameter $\chi \equiv \delta + \eta/3$. Notice that for the three figures, when we approach to $\chi = 0$ for a given γ, the pole is heavier, but it becomes significantly wider.

The procedure consists of, for a given γ, plotting the eigenvalue as a function of s and χ, where $\chi \equiv \delta + \eta/3$ is the only combination that appears in the amplitudes of δ and η. In he first eigenvalue, for positive γ there are no poles observed, but when $\gamma \to 0$, we detect physical poles for positive values of χ, as seen in Fig 3 (a). When γ becomes negative, a region of second poles appear, but with negative width (Fig. 3 (b)) making these regions not accessible. As γ becomes more negative, it seems that these poles separate from the first ones remaining with $\Gamma < 0$, until they appear for positive values of χ, but with positive width Fig. 3 (c). In the second eigenvalue, for positive γ there are no resonances, but for negative γ (not very close to 0) there are non-physical poles around $\chi = 0$. The widths have not been calculated but we have been able to estimate their sign by searching whether the phase shift passes trough $\pi/2$ (positive width) or $-\pi/2$ (negative width).

In this channel, one cannot do such immediate conclusions when finding or not resonances. For example, if a new scalar resonance is found at LHC one could not discard the zones where a_4 and a_5 are not allowed in subsection A, because it could be that the new NLO parameters differed from zero. However, the appearance of negative width resonances makes us able to discard certain zones of the new parameter space, excluding some possibilities, specially around $\chi = 0$ where a non-physical pole arises when $\gamma < 0$.

IV. CONCLUSIONS

In this work we have used the partial wave amplitudes of the longitudinal components of gauge boson scattering at one loop level obtained through the equivalence theorem and in the massless limit, and have applied the IAM to unitarize them. We have seen the appearance of new resonances in each isospin channels, with two different values of a , 0.9 and 1.3 implying a new SM physics due to these anomalous couplings. We also have shown that when one departs form $b = a^2$, new processes have to be taken into account and new $O(p^4)$ parameters appear on the amplitudes. If one does so, sees that there is more freedom to estimate scalar resonances than only with the a_4 and a_5 parameter space. We show this fact by finding poles when $a_4 = a_5 = 0$. These results could help in the interpretation of new further observations at LHC, being able to shorten the range of values of the NLO chiral parameters.

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