

# Correction factor for ablation algorithms used in corneal refractive surgery with gaussian-profile beams

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**Abstract:** We provide a correction factor to be added in ablation algorithms when a Gaussian beam is used in photorefractive laser surgery. This factor, which quantifies the effect of pulse overlapping, depends on beam radius and spot size. We also deduce the expected post-surgical corneal radius and asphericity when considering this factor. Data on 141 eyes operated on LASIK (laser in situ keratomileusis) with a Gaussian profile show that the discrepancy between experimental and expected data on corneal power is significantly lower when using the correction factor. For an effective improvement of post-surgical visual quality, this factor should be applied in ablation algorithms that do not consider the effects of pulse overlapping with a Gaussian beam.

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OCIS codes: (170.1020) Ablation of tissue; (170.4470) Ophthalmology

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## References and links

1. S. M. MacRae, R. R. Krueger and R. A. Applegate, "What is customisation?," in *Customized corneal ablation: the quest for supervision*, S. M. MacRae, R. R. Krueger and R. A. Applegate, eds. (SLACK Incorporated, Thorofare, NJ, 2001).
2. J. R. Jiménez, R. G. Anera, J. A. Díaz and F. Pérez-Ocón, "Corneal asphericity after refractive surgery when the Munnerlyn formula is applied," *J. Opt. Soc. Am. A* **21**, 98-103 (2004).
3. M. Mrochen and T. Seiler, "Influence of corneal curvature on calculation of ablation patterns used in photorefractive laser surgery," *J. Refract. Surg.* **17**, S584-S587 (2001).
4. J. R. Jiménez, R. G. Anera, L. Jiménez del Barco and E. Hita, "Effect on laser-ablation algorithms of reflection losses and nonnormal incidence on the anterior cornea," *Appl. Phys. Lett.* **81**, 1521-1523 (2002).
5. R. G. Anera, J. R. Jiménez, L. Jiménez del Barco and E. Hita, "Changes in corneal asphericity after laser refractive surgery, including reflection losses and nonnormal incidence upon the anterior cornea," *Opt. Lett.* **28**, 417-419 (2003).
6. J. R. Jiménez, R. G. Anera, L. Jiménez del Barco and E. Hita, "Influence of laser polarization on ocular refractive parameters after refractive surgery," *Opt. Lett.* **29**, 962-965 (2004).
7. D. Huang and M. Arif, "Spot size and quality of scanning laser correction of higher-order wavefront aberrations," *J. Cataract Refract. Surg.* **28**, 407-416 (2002).
8. A. Guirao, D. Williams and S.M. MacRae, "Effect of beam size on the expected benefit of customized laser refractive surgery," *J. Refract. Surg.* **19**, 15-23 (2003).
9. F. Manns, J. Shen, P. Soderberg, T. Matsui and J. Parel, "Development of an algorithm for corneal reshaping with a scanning laser beam," *Appl. Opt.* **21**, 4600-4608 (1995).
10. M. Mrochen and T. Seiler, "The Erbium Laser," in *Photorefractive Keratectomy*, R. Steiner, H. Wu, S. Slade and V. Thompson, eds. (Thiem Verlag, New York, 1988).
11. D. Huang, M. Tang and R. Shekhar, "Mathematical model of corneal surface smoothing after laser refractive surgery," *Am. J. Ophthalmol* **135**, 267-278 (2003).
12. D. A. Atchison and G. Smith, *Optics of the Human Eye* (Butterworth Heinemann, Oxford, 2000), Chap. 18.
13. H. S. Ginis, V. J. Kasanevaki and I. G. Pallikaris, "Influence of ablation parameters on refractive changes after phototherapeutic keratectomy," *J. Refract. Surg.* **19**, 443-448 (2003).

## 1. Introduction

A great number of factors (decentration, physical aspects of ablation, biomechanics of cornea, flap, wound healing, type of laser, among others) influence laser-ablation in refractive surgery [1], thereby prompting a marked discrepancy [2] between corneal experimental data and data expected by the ablation equations. Therefore, it is essential for refractive surgery to quantify the influence of these parameters in order to propose corrections in the ablation algorithms. This demand is even more important for customized refractive surgery (a current trend in which eye aberrations are corrected to improve visual performance), in which high accuracy is needed during corneal ablation. In this sense, it has been shown that any ablation algorithm should be modified by adjustment factors that take into account physical factors such as reflection losses on the cornea [3-5], non-normal incidence on the cornea when the laser beam is moved parallel to the optical axis of the cornea [3-5], and laser polarization [6].

Concerning the influence on corneal ablation of the type of laser and some variables of laser beam, only a few studies have been conducted [7-11]. It is known that the finite size of the laser beam limits the accuracy of the ablation and causes a pulse overlapping that also affects the corneal ablation. In relation to the beam profile, top-hat (uniform beam intensity) or Gaussian beams are used, although most lasers used today in refractive surgery are scanning-spot excimer lasers with a Gaussian profile [7-8]. Despite that numerical simulations [7-9] have been performed to evaluate the effects of overlapping in a Gaussian beam during refractive surgery, no works available in the literature analytically quantify this effect. This would improve the ablation algorithms used in refractive surgery that do not consider this effect. Manns *et al.* [10] studied the effect of using a Gaussian beam for a single pulse, though no analytical analysis of pulse overlapping is done.

In the present work, we propose a simple model that provides a correction constant that quantifies the overlapping effect with a Gaussian beam. This correction constant can provide a more accurate ablation depth per pulse and can be used in any photorefractive ablation algorithm which uses a Gaussian beam. Experimentally, we evaluate whether the use of this correction constant reduces the discrepancy between the theoretic and experimental data when using a scanning spot excimer laser with a Gaussian profile. For this, we quantify the effects of this factor in two important ocular parameters: the corneal radius and the  $p$ -factor (a parameter that quantifies corneal asphericity). The results confirm that, at least for the excimer laser used in our study, for a large set of experimental data from subjects having undergone refractive surgery, the prediction of corneal power is significantly higher when the correction constant is used than when it is not.

## 2. Method

### 2.1. Background

Photo-ablation of corneal tissue at one point of the cornea during refractive surgery is approximated by Lambert-Beer's law [3-4]:

$$d_p = m \cdot \ln\left(\frac{F_0}{F_{th}}\right) \quad \text{if } F_0 > F_{th} \quad (1)$$
$$= 0 \quad \text{if } F_0 \leq F_{th}$$

where  $d_p$  is the ablation depth per pulse,  $m$  is the slope efficiency of the ablation,  $F_0$  is the incident exposure of the laser pulse (energy per illuminated area) and  $F_{th}$  is the threshold exposure for the ablation.  $F_0$  is a constant that depends on the type of laser and usually ranges from 120 mJ/cm<sup>2</sup> to 400 mJ/cm<sup>2</sup>, with  $F_{th}=50$  mJ/cm<sup>2</sup>. If the effect of pulse overlapping with Gaussian beams is taken into account, a correction factor in Eq. (1) is needed.

For a Gaussian beam (the type most widely used in refractive surgery) the incident exposure (called fluence in other works [3]) is:

$$F(x, y) = F_0 \exp \left[ -2(x^2 + y^2) / w^2 \right] \quad (2)$$

where  $x$  and  $y$  are spatial coordinates perpendicular to the optical axis,  $w$  is the beam radius and  $F_0$  is the peak exposure. This exposure is limited to a spot size of  $b$  millimetres.

### 2.2. Deduction of a correction factor for the depth ablation

For a finite-size beam, there is an overlap and therefore a point of the cornea is affected by the ablation at surrounding points (see Figure 1). Thus, any point on the cornea receives the exposure due to ablation at surrounding points with a value that depends on the expression given by Eq. (2) in case of a Gaussian-profile beam.

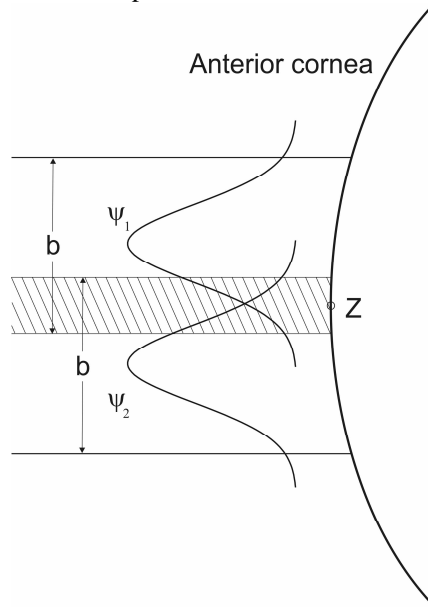


Fig. 1. Effect of overlapping due to the finite-size of the incident beam. A point on the cornea ( $Z$ ) is affected by the ablation of surrounding pulses (e.g.  $\Psi_1$  and  $\Psi_2$  in the drawing).

We can define a correction factor for the depth ablation per pulse,  $c$ , that takes into account the overlapping effect, by:

$$c = \frac{\int_0^{r'} m \cdot \ln \left( \frac{F_0 \exp \left[ -2(x^2 + y^2) / w^2 \right]}{F_{th}} \right) dx dy}{\int_0^{r'} m \cdot \ln \left( \frac{F_0}{F_{th}} \right) dx dy} \quad (3)$$

The numerator in Eq. (3) calculates the contribution to the photo-ablation due to exposure at surrounding points. Any point on the cornea receives exposure from surrounding pulses, and the value of this exposure depends on the distance according to the Gaussian function given by Eq. (2). Thus, the total irradiance is calculated by adding (integral) the contribution of surrounding points within a limit,  $r'$ . The value of the limit,  $r'$ , to which the integral extends, depends on the spot size,  $b$ , and the value  $r = (x^2 + y^2)^{1/2}$  from which there is ablation, given that values lower than a threshold value do not produce an ablation. That is, according to (1),  $r$  is obtained from the following condition:

$$\left( \frac{F_0 \exp(-2r^2 / w^2)}{F_{th}} \right) > 1 \quad (4)$$

After computations, we find that  $r = \frac{\sqrt{2}}{2} w \left( \ln \frac{F_0}{F_{th}} \right)^{1/2}$ . Therefore, integrals shown in Eq.

(3) are thus extended to  $r' = \min \{r, b/2\}$ . The numerator is normalized by the value (denominator) calculated when considering the homogeneous beam (top-hat). Therefore, the correction factor  $c$  quantifies the deviation of the depth ablation per pulse (with respect to a uniform exposure) due to pulse overlapping with a Gaussian beam. The denominator should guarantee that  $c=1$  for an uniform beam, and this occurs, since  $c=1$  for  $w=\infty$  (uniform beam). Computing Eq. (3), we get:

$$c = \frac{m \int_0^{r'} \left( \ln \left( \frac{F_0}{F_{th}} \right) - 2 \frac{x^2 + y^2}{w^2} \right) dx dy}{m \cdot \ln \left( \frac{F_0}{F_{th}} \right) \int_0^{r'} dx dy} \quad (5)$$

taking into account that  $\int_0^{r'} dx dy = \pi r'^2$  and  $\int_0^{r'} (x^2 + y^2) dx dy = \pi r'^4 / 2$  we get:

$$c = 1 - \frac{r'^2}{w^2} \left( \ln \frac{F_0}{F_{th}} \right)^{-1} \quad (6)$$

Factor  $c$  depends on the characteristics of the Gaussian beam: the parameter  $w$ , the spot size  $b$  and exposure. If we consider a Gaussian beam, any ablation algorithm that does not consider overlapping effects should be multiplied by the constant  $c$  deduced here.

The next step would be to check experimentally whether the parameter deduced here improves the prediction on refractive surgery data. First of all, we consider the parameter  $c$  jointly with an adjustment factor, which takes into account reflection losses and non-normal incidence mentioned above [4-5]. This adjustment factor ( $\rho$ ) is given by [4]:

$$\rho(a, y, R, p) \cong (1 - 0.0435a) - \frac{ay^2}{2R^2} + \frac{a(0.232 - 0.5p)y^4}{R^4} \quad (7)$$

where  $R$  is the pre-surgical corneal radius,  $p$  is the pre-surgical  $p$ -factor of the cornea,  $y$  is the incidence height and  $a = 1 / \ln(F_0 / F_{th})$ . If we consider the effect of the correction factor,  $c$ , for Gaussian beams jointly with the adjustment factor ( $\rho$ ), we must multiply both factors to correct the deviation for ablation depth per pulse (see Ref. [4] and [5] for a more extensive procedure). Therefore, any ablation algorithm should be modified multiplying it by  $c'$ :

$$c' = c \cdot \left[ (1 - 0.0435a) - \frac{ay^2}{2R^2} + \frac{a(0.232 - 0.5p)y^4}{R^4} \right] \quad (8)$$

To evaluate the practical influence of  $c'$  in refractive surgery, we calculated the predicted corneal power and  $p$ -factor after surgery and we compared them with experimental results [Corneal power ( $\varphi$ ) is calculated as:  $\varphi = \Delta n / R$  with  $R$  being the radius and  $\Delta n = 0.375$  (refraction

index difference between air and cornea)]. For this, we applied Eq. (8) for a particular algorithm. Although, as mentioned above, most algorithms are proprietary, practical non-customized surgery is based on Munnerlyn's paraxial formula [2,4]. This formula, denoted by  $z(y)$ , is given by:

$$z(y) = \frac{4Dy^2}{3} - \frac{Dd^2}{3} \quad (9)$$

where  $D$  is the number of diopters to correct and  $d$  is the ablation diameter. With the application of this equation, the expected post-surgical radius,  $R'$ , that emmetropizes the eye is given by [2,9]:

$$D = \frac{\Delta n}{R'} - \frac{\Delta n}{R} \quad (10)$$

In our procedure, we assume [4,5,12], that the anterior cornea obeys the equation of a conicoid:

$$x^2 + y^2 + pz^2 - 2Rz = 0 \quad (11)$$

where  $R$  is the radius of curvature and  $p$  represents the  $p$ -factor asphericity parameter, with the  $z$  axis being the optical axis and  $x, y$  being the spatial coordinates.

To determine the expected post-surgical corneal radius and asphericity when the ablation performed is given by  $c'z(y)$ , we used a mathematical analysis based on a minimum-square procedure. [This mathematical procedure has been extensively explained in Refs. [2, 4, 6]]. With the same procedure, after mathematical computations, the post-surgical corneal radius  $R'$  and corneal asphericity  $p'$ -factor as a function of pre-surgical data and laser characteristics are given by:

$$\frac{1}{R'} = \frac{1}{R} + \frac{8c'D}{3} - 0.116ac'D + \frac{ac'd^2D}{3R^2} \quad (12)$$

$$p' = \frac{R^3}{R^3} \left[ p + \frac{ac'D}{R} (d^2 [-0.62 + 1.333p] - 1.333R^2) \right]$$

The values of  $R'$  and  $p'$  are calculated from pre-surgery ( $R, p, c, d$  and  $D$ ) data. We did not have access to data for  $a$  and  $w$  for the Esiris laser. As is usual in many cases [5,6,7], companies and patents do not provide quantitative parameters of the laser, except (in our case) the spot diameter:  $b=1$  mm. We used a value of  $w=1.29$  mm, a value used in theoretical simulations by other authors [7]. In our computations, we tested three different values for  $a$ :  $a=0.48, 0.62$  and  $1.14$ , corresponding to an incident exposure of  $400, 250$  and  $120$  mJ/cm<sup>2</sup> and a correction factor, obtained from (6), of  $c=0.93, 0.90$  and  $0.83$ , respectively. Testing these three values of incident exposure, we checked two extreme and one intermediate exposure value within the range  $400-120$  mJ/cm<sup>2</sup>. As mentioned above, according to Eq. (6),  $c=1$  for the correction factor means that there is no overlapping effect for a Gaussian beam and only the effects of non-normal incidence and reflection losses are included [2,4,5]. An analysis of the differences between experimental cornea data and desired (or expected) data considering only the Munnerlyn formula, Eqs. (9)-(10), showed [5] that corneal discrepancies were very large, decreasing when the effect of these two physical factors were included.

### 2.3. Observers

We measured the pre- and post-surgical radius and corneal asphericity for 141 myopic eyes (78 subjects). All observers, before participating in the study, gave their informed consent in accordance with the Helsinki Declaration. They were operated on with LASIK (laser in situ keratomileusis) in a clinic specializing in refractive surgery using the Esiris scanning spot excimer laser (SWCHIND) with a non-customized procedure. With this procedure, no aberration correction is intended and the ablation algorithm is based on the Munnerlyn formula (Eq. (9)). The age of the patients ranged from 21 to 48 years. Their mean pre-operative spherical refractive error was  $-4.80 \pm 2.4 D$  (standard deviation), ranging from  $-1.25 D$  to  $-8.5 D$ . We studied subjects for whom the operation could be considered satisfactory in order to avoid variables or surgical complications that might affect cornea shape and thereby mask the aim of our study [5]. Thus, the subjects were required to fulfil the following conditions: after three months (we checked whether corneal parameters remained stable from this time period on [5]) they were satisfied with the outcome of the surgery and no longer used any form of optical correction. No patient had pre-existing abnormal conditions that might affect visual acuity (glaucoma, corneal or neuro-ophthalmic diseases, cataracts, etc.).

Data on corneal topography (curvature radius and  $p$ -factor) were taken by a previously calibrated EyeSys-2000 topographer. The  $p$ -factor data provided by the topographer were taken for a pupil diameter of 4.5 mm. This pupil size ensures that corneal data do not include transition zones having radii and asphericity that differ from those of the optical zone [5].

### 3. Results and discussion

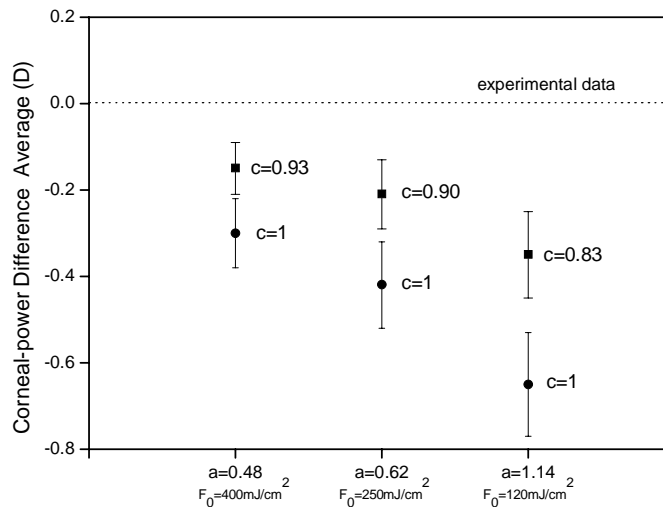


Fig. 2. Average of corneal-power difference between experimental data and theoretical data for 3 values of  $a$ . For each value of  $a$ , theoretical data are computed with or without ( $c=1$ ) the use of the correction factor that takes into account the effect of pulse overlapping during corneal ablation with a Gaussian beam. Data include standard errors.

We compared the theoretical results using Eq. (12) with the experimental results. The results (Fig. 2 and Table 1) reflect the average difference between the experimental and theoretical results, Eq. (12), for the 141 eyes tested. The theoretical results were checked under two conditions for each of the 3 values of  $a$  tested—i.e., with and without ( $c=1$ ) the correction factor. Therefore, six points are plotted in Fig. 2, with their standard errors, corresponding to the difference between experimental corneal-power data and the six theoretical conditions

tested. The experimental results for corneal power, the most important refractive parameter, showed a higher prediction when using the correction factor. Discrepancies with experimental results were significantly lower ( $p < 0.05$ , see Table I) when using this correction factor. For the  $p$ -factor, however, no significant differences ( $p > 0.05$ , see Table I) were found when using the correction factor with respect to not using it. The lack of significant influence found in the  $p$ -factor may be because this parameter takes into account the eye geometry off the optical axis where biomechanical factors (not quantified) are more important.

Table 1. Average difference between experimental and theoretical data for corneal-power ( $\Delta\phi$ ) and asphericity ( $\Delta p$ ). Theoretical data are computed with and without the use of the correction factor that takes into account the effect of pulse overlapping.

	$\Delta\phi^c$	$\Delta\phi$	$\Delta p^c$	$\Delta p$
$a=0.48$	-0.15 D*	-0.30 D*	0.28**	0.28**
$a=0.62$	-0.21 D*	-0.42 D*	0.25**	0.24**
$a=1.14$	-0.35 D*	-0.65 D*	0.15**	0.14**

<sup>c</sup> Computations include the correction factor

\*Test for means difference show significant differences ( $p < 0.05$ ) for corneal-power when using the correction factor or not.

\*\*Test for means show non-significant differences ( $p < 0.05$ ) for  $p$ -factor when using the correction factor or not.

Results concerning corneal power showed the importance of using the correction factor in ablation algorithms. Corneal-power differences average between experimental data and theoretical data, when using the  $c$ -parameter or not, ranged from 0.15 D to 0.35 D for  $a=0.48$  and  $a=1.14$ , respectively. Such differences could significantly influence the visual performance of the observer. It is well known that low defocusing values deteriorate visual quality. For example, a difference of 0.25 D diminishes effective visual acuity, due to a loss of spatial sensitivity, to roughly 22 cycles per degree [12]. In addition, the differences found clearly exceed the one-quarter-wavelength tolerance criterion [5], indicating that the possible improvements due to the correction factor are significant for the correction of aberrations [5].

Although for the three  $a$  values tested, the differences found for corneal power proved significant, we find that corneal-differences increase as the exposure intensity diminishes as might be expected by Eq. (6). On this point, it is important to point out that if the Gaussian parameters and spot size of a laser beam satisfied the condition  $r < b/2$ , we would have  $r' = r$  and we would get  $c = 0.5$  from Eq. (6). In this case, the correction factor is constant and it does not depend on laser parameters.

Other authors [13] have shown that a hyperopic shift appears after-surgery, resulting in a real curvature radius larger than expected. Note in Fig. 2 and Table 1 that the average difference between experimental and theoretical data (with and without the correcting factor) is negative, indicating that the experimental post-surgery radius is greater than expected as in Ginis *et al.* [13]. In any case, the use of the correcting factor provides a more accurate ablation depth per pulse, significantly reducing this hyperopic shift detected experimentally.

Our analysis shows that the prediction of corneal data is higher when considering the correct factor because real corneal-power is closer to the corneal power given by Eq. (12). We are not saying that, for removing D diopters, the desired post-surgical is given by Eq. (12); the desired radius to emmetropize the eye continues to be the one given by the Munnerlyn formula, Eq. (10). The point is that laser designers should use the ablation depth per pulse corrected in order to compute the number of pulses necessary for removing an initial emmetropia. When correcting the ablation depth per pulse, the post-surgical radius will be closer to the desired one.

Despite that discrepancies with respect to experimental results are lower when using the correction factor, there are nevertheless differences between the theoretic data and the

experimental ones. This is because the factors mentioned above (decentration, physical aspects of ablation, flap, biomechanics of the cornea, wound healing...) influence laser ablation, although their influence has not been quantified and therefore has not been corrected in ablation algorithms. Our results show that it is also necessary to quantify and model the influence of these parameters in refractive surgery in order to attain greater accuracy during ablation.

Because the algorithms are proprietary, we cannot totally know whether the effect of overlapping with a Gaussian beam is considered or not by companies. In light of our information, no patents or descriptions by companies report it. Examining different articles [7,9,11] on algorithms we find that laser companies modify corneal ablations based on clinical experience [11], so that they modify ablation algorithms from experimental data. These adjustments can partially minimize the effects of the different variables (including overlapping) that influence corneal ablation, although the discrepancies arise between the desired and real corneal shape and are significant for visual performance. According to the results of the present study, it would be expected for the Esiris laser to consider overlapping in some way, given that when we include our proposed adjustment factor in the analysis of the experimental data the discrepancies with real corneal shape diminish.

Even if companies consider overlapping, it may be restricted to their own devices and not provide a correction valid for all laser devices, whereas in the present paper we propose a correction factor as a function of laser-beam parameters that can be used in a general way.

#### **4. Conclusions**

To consider a correction factor that quantifies the effect of pulse overlap during corneal ablation with a Gaussian beam could provide significant improvements in the prediction of corneal data (corneal power). The correction factor proposed here can be taken into consideration in all types of ablation algorithms but especially in customized ablation algorithms that require high accuracy. This correction factor can help minimize the hyperopic shift found after surgery, that could interfere with an effective improvement of post-surgery visual quality and correction of eye aberrations.

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