Effects of damping on the quantum limits to optical phase shifts in Kerr nonlinear media

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It was recently shown [Phys. Rev. A **45**, 1919 (1992)] that the phase shift induced by a control beam on a signal in cross Kerr modulation is limited by the quantum nature of the control. We show that dissipation rapidly restores the classical phase shift.

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I. INTRODUCTION

Considerable interest has been shown in the use of Kerr nonlinear media as all-optical switching devices [1–4]. The switching is produced by the imposition of a π phase shift on the signal field. When this field is mixed with the original field, which was not phase shifted, destructive interference occurs. The phase shift can be produced by mixing two single-mode electromagnetic fields in a Kerr medium. The phase of one of the fields is modified by the intensity of the other and vice versa. This phase shift is known as cross-phase modulation or XPM. A phase shift which is due to the effects of each field's own intensity, also present, will not be used as it cannot be utilized for switching. If the system is treated classically, any desired phase shift can be obtained by choosing appropriate field strengths or interaction times. It was shown by Sanders and Milburn [5] that quantum mechanics imposes a maximum limit on the phase shift which is independent of the interaction time.

Classically the phase shift is given by

$$\phi = \chi I_c, \tag{1.1}$$

where I_c is the intensity of the field which we will call the control field. This intensity is in dimensionless units of photon number and χ can be thought of as the integrated nonlinearity given by

$$\chi = 2\chi^{(3)} \left(\frac{\hbar\omega^2}{2\epsilon_0^2 V}\right) t = 2\chi^{(3)} \left(\frac{\hbar\omega^2}{2\epsilon_0^2 V}\right) \frac{L}{c}, \tag{1.2}$$

where V is the interaction volume, ω is the frequency of the light, c is the speed of light in the medium, and $\chi^{(3)}$ is the third-order nonlinear susceptibility of the medium (one such medium is an optical fiber). The length L in the previous equation is the interaction length. The true quantum-mechanical phase shift was shown to be [5]

$$\phi = I_c \sin(\chi). \tag{1.3}$$

This expression was derived for coherent state inputs. From the above expression we can see that the maximum limit on the phase shift is set by the control field intensity. Taking a longer interaction time does not guarantee a larger phase shift. This limit arises due to quantum-mechanical intensity noise on the control field. This noise translates into phase diffusion in the signal field. If this phase diffusion reaches a sufficient magnitude no net discernible phase will be seen on the signal pulse. This is also true of classical fields with intensity noise. The difference between the quantum and classical cases is the recurrence phenomena which are a consequence of the quantum-mechanical graininess of the field. As the coherent states representing the input fields are superpositions of number states the phase diffusion in the signal field will take on a discrete structure. If the signal field is initially described by a state $|\alpha\rangle$ then the reduced density operator for the signal mode due to the effects of XPM only is [5]

$$\rho = \sum_{n=0}^{\infty} P_\beta(n) |\alpha e^{-i\chi n}\rangle\langle\alpha e^{-i\chi n}|, \tag{1.4}$$

where

$$P_\beta(n) = \frac{|\beta|^2 e^{-|\beta|^2}}{n!}, \tag{1.5}$$

and β is the amplitude of the control field coherent state. That is, the density matrix is a statistical mixture of the density matrices for coherent states with different phases. The weighting factor for these is simply the Poisson photon number distribution for the control field coherent state.

The model described above did not include damping. In this paper we extend the treatment due to Sanders and Milburn by including damping. We show that the effect of damping is to wash out the quantum-mechanical effects and to recover the classical results in the high-damping limit. We use the method of thermofield dynamics [6] to get an exact analytic solution for the full two-mode master equation.

II. THE CLASSICAL MODEL

The classical electrodynamic description of two single-mode fields copropagationg in a Kerr medium leads to the following equation for the classical electric field amplitudes α_s and α_c:

$$\begin{aligned} \frac{d\alpha_s}{dt} &= (-\sqrt{\gamma_s} - i\chi_m|\alpha_c|^2)\alpha_s, \\ \frac{d\alpha_c}{dt} &= (-\sqrt{\gamma_c} - i\chi_m|\alpha_s|^2)\alpha_c, \end{aligned} \tag{2.1}$$

where the subscripts s and c denote the signal and control fields, respectively, and γ is the linear damping rate. The nonlinear coefficient χ_m is the XPM coefficient; we ignored the self-phase-modulation (SPM) terms as they are not useful in switching devices and only complicate the analysis. It should also be noted that dispersive effects were also neglected. This is reasonable as we are only treating the cw case.

We are only interested in the effect of the control field's intensity on the signal phase. We can therefore ignore the damping in the signal field as it does not affect its phase. Equations (2.1) then reduce to the following equations for the signal amplitude and control intensity:

$$\frac{\partial \alpha_s}{\partial t} = -i\chi_m I_c \alpha_s,$$
$$\frac{\partial I_c}{\partial t} = -\gamma_c I_c. \tag{2.2}$$

These equations yield the following form for the signal field amplitude:

$$\alpha_s(t) = \alpha_s(0)\exp\left[-i\frac{\chi_m}{\gamma_c} I_c\left(1 - e^{-\gamma_c t}\right)\right]. \tag{2.3}$$

Thus the classical phase shift is

$$\phi_{\rm cl} = -\frac{\chi}{\tau} I_c\left(1 - e^{-\tau}\right) = -\frac{\chi_m}{\gamma_c} I_c\left(1 - e^{-\gamma_c t}\right), \tag{2.4}$$

where $\chi = \chi_m t$ is the integrated nonlinearity and $\tau = \gamma_c t$ is the total loss. We can write this as

$$\phi_{\rm cl} = -2\mu I_c(1 - e^{-\chi/2\mu}), \tag{2.5}$$

where

$$\mu = \frac{\chi_m}{2\gamma_c}. \tag{2.6}$$

We may regard μ as the effective nonlinearity for the medium.

If we let $\mu \rightarrow \infty$, that is, γ_c is small, we recover the zero damping limit for the phase shift given by

$$\phi_{\rm cl} = \chi I_c. \tag{2.7}$$

III. THE QUANTUM-MECHANICAL MODEL

In the quantum-mechanical treatment we again assume a cw field and ignore other noise sources such as guided-acoustic-wave Brillouin scattering and stimulated Raman scattering. These assumptions are limiting but serve to ellucidate the effects that we are investigating. The Kerr nonlinearity of the medium considered here is modeled, quantum mechanically, by the Hamiltonian for the anharmonic oscillator. This system has been extensively studied in the past due to its usefulness in producing macroscopic superpositions of coherent states [7, 8]. In order to describe the interaction between two light fields in a Kerr medium such as an optical fiber the system is modeled by two coupled anharmonic oscillators. The damping will be included via the master equation. The damped master equation for the two-mode density operator of this system is

$$\begin{aligned}\frac{\partial \rho}{\partial t} = & -i\sum_{i=1}^{2}\omega_i[a_i{}^\dagger a_i, \rho] - i\sum_{i,j=1}^{2}\chi_{ij}[a_i{}^\dagger a_i a_j{}^\dagger a_j, \rho] \\ & +\frac{1}{2}\sum_{i=1}^{2}\gamma_i(\bar{n}_i + 1)(2a_i\rho a_i{}^\dagger - a_i{}^\dagger a_i\rho - \rho a_i{}^\dagger a_i) \\ & +\frac{1}{2}\sum_{i=1}^{2}\gamma_i\bar{n}_i(2a_i{}^\dagger\rho a_i - a_i a_i{}^\dagger\rho - \rho a_i a_i{}^\dagger),\end{aligned} \tag{3.1}$$

where the subscripts i, j denote the different modes, ω_j are the frequencies of the oscillators, and χ_{ij} are the Kerr nonlinear coefficients. When $i = j$ these coefficients describe self-phase modulation while for $i \neq j$ they describe XPM coefficients. The γ_j's are the damping rates for the baths and $\bar{n}_j$ is the mean photon number in the bath which we will take to be zero, i.e., the bath is at zero temperature. We also assume that $\chi_{12} = \chi_{21}$ and will ignore the SPM part of the interaction. The master equation in the interaction picture is then

$$\begin{aligned}\frac{\partial \rho}{\partial t} = & -i2\chi_{12}[a_1{}^\dagger a_1 a_2{}^\dagger a_2, \rho] \\ & +\frac{1}{2}\sum_{i=1}^{2}\gamma_i(2a_i\rho a_i{}^\dagger - a_i{}^\dagger a_i\rho - \rho a_i{}^\dagger a_i).\end{aligned} \tag{3.2}$$

This two-mode master equation can be solved directly to give an analytic solution for the density operator [6]. This method can be generalized to any number of coupled oscillators. The resulting expression is quite complex and of no immediate value in its general form. We will therefore not present it here. The more relevant quantities which can be calculated from the density operator are the mean amplitude and the Q function for the signal mode.

A. The quantum-mechanical phase shift

In order to calculate the phase shift in the signal mode we take mode a_1 to be the signal mode and a_2 to be the control mode. We assume that the signal field is initially in a coherent state $|\alpha_1^0\rangle_1$ and the control field is initially in a coherent state $|\alpha_2^0\rangle_2$. The mean amplitude for the signal mode in the interaction picture is then

$$\langle a_I(t)\rangle = \alpha_1^0 e^{-\gamma_1 t/2} e^{-\eta + i\phi}, \tag{3.3}$$

for

$$\begin{aligned}\eta = I_c\Bigg[& 1 + \frac{1}{1+4\mu^2}\left[e^{-\chi/\mu}\cos(2\chi) - 1\right] \\ & - \frac{2\mu}{1+4\mu^2}e^{-\chi/\mu}\sin(2\chi) - e^{-\chi/2\mu}\cos(\chi)\Bigg], \\ \phi = I_c\Bigg[& \frac{-2\mu}{1+4\mu^2}\left[1 - e^{-\chi/\mu}\cos(2\chi)\right] \\ & - e^{-\chi/2\mu}\sin(\chi) + \frac{1}{1+4\mu^2}e^{\chi/\mu}\sin(2\chi)\Bigg],\end{aligned} \tag{3.4}$$

where $\mu = \chi_m/2\gamma_2$ and $\chi = \chi_m t = 2\chi_{12}t$. The quantity

η characterizes the decay of the signal amplitude caused by the intensity noise in the control field. This corresponds to the classical amplitude decay which accompanies the phase spreading caused by the classical intensity noise considered by Sanders and Milburn [5]. The full quantum phase shift is characterized by the quantity ϕ. This corresponds to the purely quantum-mechanical recurrence phenomena which occur due to the discreteness in the energy states of the field. Both the terms in Eq. (3.4) reduce to the undamped case discussed in Ref. [5].

In the limit of zero damping or very large effective nonlinearity, that is, $\mu \rightarrow \infty$, the classical phase shift in Eq. (2.5) reduces to the standard classical undamped phase shift of χI_c while in the same limit the quantum phase shift becomes $I_c \sin\chi$. In this regime the classical and quantum results are fundamentally different.

In the limit of very large damping or small nonlinearity ($\mu \rightarrow 0$), however, both the quantum and classical treatments yield the same limit of

$$\phi_c = \phi_q = -2\mu I_c = -\frac{\chi_m}{\gamma_c} I_c, \tag{3.5}$$

which is independent of the interaction time. This result shows that the quantum-mechanical effects due to the discrete nature of the field are washed out by the inclusion of damping. We also see that as $\mu \rightarrow 0$ the decay constant η approaches zero, which corresponds to the reduction in the intensity noise in the control field as a result of the decay in its amplitude. This corresponds to the decay of a noisy classical field where the probability distribution for the intensity is Poissonian.

The convergence of the quantum result to the classical result in the presence of dominant damping is clearly seen in Fig. 1. Figure 1(a) shows the low-damping limit where the phase shift is plotted as a function of the effective nonlinearity χ. This difference becomes less pro-

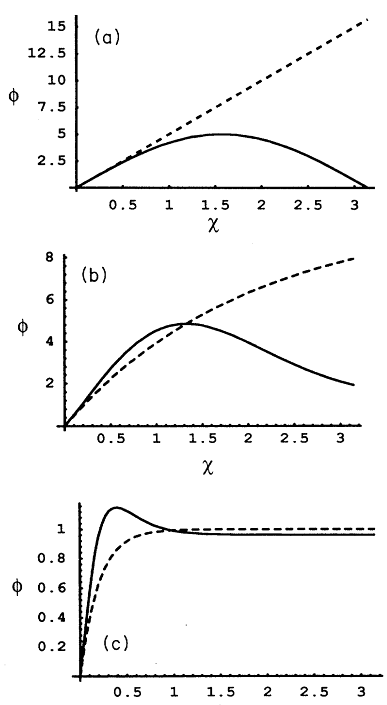

FIG. 1. The classical (dashed line) and quantum (solid line) phase shifts as a function of the effective nonlinearity χ with $I_c = 5.0$, and (a) $\mu = 10.0$, (b) $\mu = 1.0$, and (c) $\mu = 0.1$.

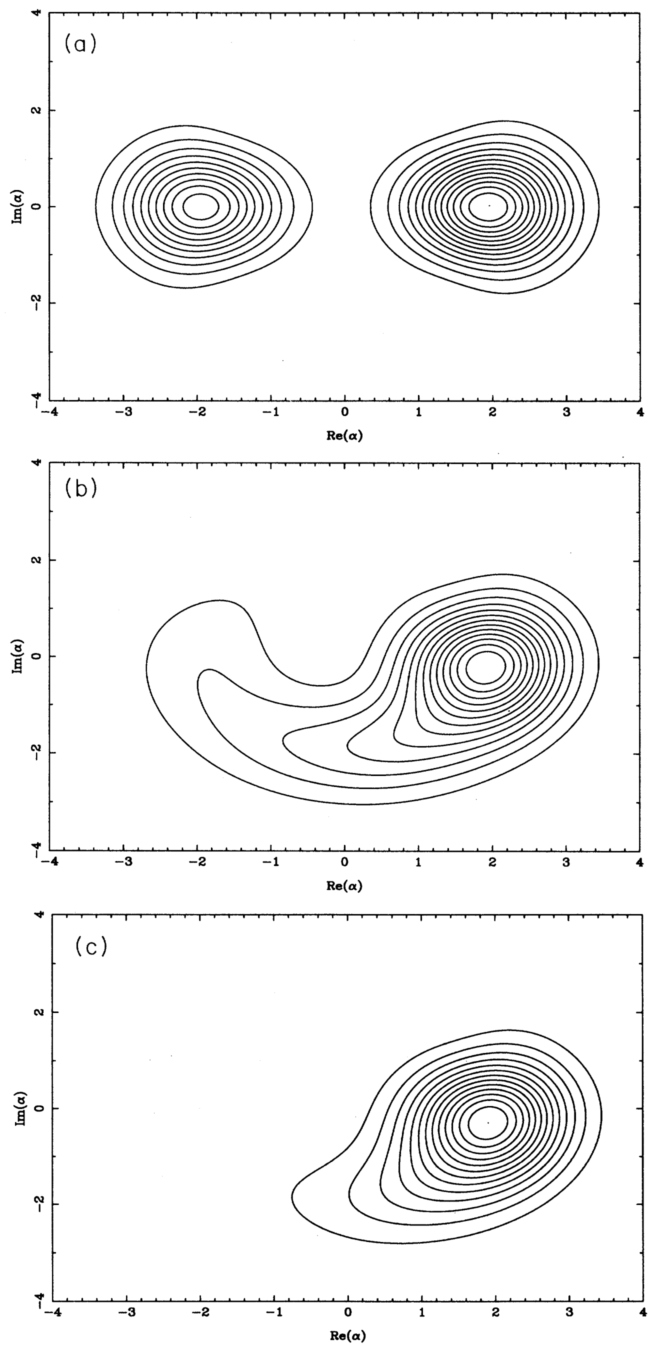

FIG. 2. The Q function for the signal mode with a control intensity of 2, and a nonlinearity $\chi = \pi/2$. The signal field intensity is set at 4 photons. The Q function is shown for different amounts of damping with (a) $\gamma_c = 0$, (b) $\gamma_c = 2$, and (c) $\gamma_c = 5$.

nounced when the relative damping is increased, i.e., μ is decreased. This can be seen in Figs. 1(b) and 1(c). It is interesting that the quantum phase shift is greater than the classical phase shift, for small values of the nonlinearity, in the prescence of damping.

B. Recurrence effects

The quantum-mechanical discreteness of the electromagnetic field is manifested in the zero damping density operator in Eq. (1.4). This reduced density operator for the signal mode shows that the signal field is in a statistical mixture of coherent states with different phases. For certain values of the effective nonlinearity χ these coherent states add up to form a resultant field which is a mixture of two or four coherent states. This occurs for values of the nonlinearity of $\chi = \pi/2$ and $\chi = \pi$, respectively. This behavior is related to the deterioration of the achievable phase shift for large values of the effective nonlinearity χ.

These mixtures can be clearly seen when we calculate the Q function for the signal field. In the case of the master equation with damping the Q function can be computed directly from the density operator obtained by the method of Chaturvedi and Srinivasan [6].

In order to illustrate the behavior of the system under damping we plot the Q function for different values of the damping. We set $\chi I_c = \pi/2$ and $I_c = 2$. This corresponds to a classical phase shift of π. This is the special case when the field is in a mixture of two coherent states. The signal field amplitude for this case is set at $\alpha_1^0 = 2$. This situation is shown in Fig. 2. These two coherent states are centered at $|\alpha_1^0\rangle_1$ and $|-\alpha_1^0\rangle_1$. In the absence of damping the reduced density operator for the signal mode can be simply expressed by [5]

$$\rho_1 = \tfrac{1}{2}\left(1 + e^{-2|\alpha_2^0|^2}\right)|\alpha_1^0\rangle_1\langle\alpha_1^0| + \tfrac{1}{2}\left(1 - e^{-2|\alpha_2^0|^2}\right)|-\alpha_1^0\rangle_1\langle-\alpha_1^0|. \tag{3.6}$$

This corresponds to a phase shift of 0 and no damping as predicted by Eq. (1.3) and can be seen in Fig. 2(a).

When the amount of damping is increased as in Figs. 2(b) and 2(c) we see that the quantum nature of the system is washed out. The phase diffusion in the signal, which is caused by the intensity noise in the control, is diminished. We see that the mixture of the coherent states is degraded. The visibility of the multipeaked structure is reduced as the decay constant γ_c is increased. The decay leads to a reduced amount of intensity noise in the control field as this field has a Poissonian photon number distribution. This reduction in the photon number variance is translated into a diminished phase diffusion of the signal field Q function.

As the decay constant is increased the Q functions also tend to become peaked closer to the initial position of the signal coherent state at $\alpha = 2$. This occurs due to the decrease in the mean intensity of the control which produces a smaller average phase shift. This demonstrates that the system is approaching the classical limit as described in the previous section.

IV. CONCLUSION

In this paper we investigated the effects of damping on the quantum-mechanical limits to phase shifts in a Kerr medium. These limits arise due to the quantum-mechanical noise in the electromagnetic field which result in the spreading in phase of the signal field. This limit corresponds to a classical field with intensity noise. The purely quantum-mechanical limit exists due to the discrete energy spectrum of the field. This leads to quantum-mechanical recurrence phenomena which cause the sinusoidal dependence of the phase shift on the effective nonlinearity χ. The maximum phase shift is given simply by the intensity of the control field I_c. That is, the phase shift is bounded between I_c and $-I_c$.

We showed that when the system was damped the effect of the intensity noise was diminished as the control field amplitude decayed. This was manifested as a reduction in the phase spread of the signal field Q function and a smaller decay in its amplitude. The effect of the discrete nature of the electromagnetic field was also shown to diminish with the increase in damping. This was also due to the reduction in the photon number variance of the control field.

These results are encouraging from an experimental point of view. They show that in the presence of significant damping the magnitude of the phase shift is not as sensitive to propagation distance as in the undamped case.

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