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Spontaneous emission from two atoms interacting with a broadband squeezed vacuum

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Spontaneous emission from two-level atoms interacting with a squeezed vacuum field is examined, taking account of possible different interatomic separations. It is shown that the steady-state atomic population depends on whether the interatomic separations are comparable to or much smaller than the resonant wavelength. For the extended system the squeezed vacuum introduces a dependence of the atomic population on the interatomic separations. For large interatomic separations the atomic population is identical to that for the thermal field, which is the same for two independent atoms. For small interatomic separations the atomic population is higher than that for the thermal field. This is in contrast to the small-sample model in which the interatomic separation is ignored. In this case the final atomic population in the squeezed vacuum is the same as that for the independent atoms in the squeezed vacuum. Moreover, this population differs from that for the thermal field for which the atomic population is lower than that for the independent atoms. This difference is due to the interatomic correlations whose presence depends on whether the interatomic separation is or is not included. For the extended system the interatomic correlations are induced by the squeezed field and vanish for the thermal field. For the small-sample model, however, the interatomic correlations are induced by the thermal field and the squeezed vacuum changes these correlations in such a way that for the minimum-uncertainty squeezed states these correlations vanish. We also discuss the effect of the interatomic separation on the two-photon transitions and the normalized intensity correlation function in the two-atom system interacting with the squeezed vacuum field.

I. INTRODUCTION

The generation of squeezed states of the radiation field is an objective in quantum optics and provides a test for the quantum theory of light. The attention on the generation of squeezed states stems mostly from a possiblity of reducing quantum fluctuations, or quantum noise, below the vacuum level. This presents obvious advantages for practical applications, for example, in optical communication and in gravitational wave detection, where signals even below the vacuum limit are expected.

Various theoretical methods for the generation of squeezed states have recently been proposed[1] and the first successful experiments have been reported.[2] With the possibility of obtaining squeezed fields, it is quite natural to examine the interaction of such fields with matter. New theories of interaction of atomic systems with the radiation field have been recently developed based on the assumption that the atoms interact with a multimode broadband squeezed vacuum rather than with the normal (unsqueezed) vacuum. Some very interesting deviations from the ordinary decay and ordinary emission and absorption spectra in a normal vacuum environment are obtained under such conditions. In particular, the decay rate of the atomic dipole is seen to depend on its phase relative to the squeezed field. The two quadratures of the atomic polarization are damped at different rates, one exhibiting an enhanced decay rate and the other a reduced decay rate compared with the normal radiative decay of the atom. This opens the possibility of obtaining subnatural linewidths in resonance fluoresence[3] and in the weak-field atomic absorption spectrum.[4] Some corrections to the Lamb shift in a broadband squeezed vacuum also have been reported.[5-7]

Quite recently, cooperative effects arising from the interaction of many atoms with the squeezed vacuum field have been studied. The multiatom resonance fluorescence spectrum in the squeezed vacuum demonstrates asymmetry in the off-resonance excitation by an external coherent laser field.[8,9] However, for exact resonance of the external field with the atomic frequency, the fluorescence spectrum is the same as for the one-atom case.[10] Palma and Knight[11] have studied spontaneous emission from the two-atom Dicke model and have shown that the total atomic population decays at a rate that depends on the extent of squeezing, a feature completely absent in the single two-level atom decay.[3] Moreover, the final equilibrium atomic state is far from being a state of thermal equilibrium. In the Dicke model it is assumed that the interatomic separations are much smaller than the resonant wavelength, as all effects connected with the spatial distribution of the atoms are ignored. In this model the square of the "total spin" of the atomic system is conserved.[12] This assumption, however, may prove difficult in experimental realizations of such a model in free space. Hence, it seems natural to investigate, as an alternative problem, what happens when the atoms are separated at the distances comparable to the resonant wavelength. This is well justified because the experiments in atomic spectroscopy use atomic-beam methods[13] or atoms fixed in organic layers and placed near a plane dielectric plate.[14] In such experiments the atomic separations are on the order of a resonant wavelength and spatial effects cannot be ignored. In this case the square of the total spin of the atomic system is not conserved;[12] i.e., the extended system breaks the S^2 invariance.

Some strikingly different physical behaviors of the resonance fluorescence between S^2-conserving and S^2-breaking systems have been reported. Walls *et al.*[15] have shown that the S^2-conserving system predicts a second-order phase transitions, while the S^2-breaking system predicts a first-order phase transition. It has also been shown[16,17] that the so-called "scaling factor,"[18] or the enhancement of the integrated fluorescence by N atoms, appears only in the S^2-conserving systems. Otherwise, for the S^2-breaking system, the integrated fluorescence by N atoms in N times that for a single-atom case.

In this paper we consider the effect of interatomic separation on the final equilibrium atomic state in the spontaneous emission from two two-level atoms interacting with a broadband squeezed vacuum field and compare this with the case when atoms interact with a thermal (blackbody) field at a temperature T. As we shall show below, this model shows results substantially different from those for the two-atom Dicke model, where the interatomic separation is ignored. We start from the Hamiltonian for the two-atom system, interacting with the squeezed vacuum field, and derive a master equation for the reduced density operator ρ of the two-atom system. We then apply the equation to the problem of spontaneous emission, and we study in detail the steady-state solutions for the total atomic population, the interatomic correlations, and two-photon correlations.

II. TWO ATOMS IN A SQUEEZED VACUUM

We consider a pair of two-level atoms, separated by a distance r_{12} and interacting with the quantized multimode radiation field. The Hamiltonian of the system in the electric dipole approximation is given in the following form:

$$H=\hbar\omega_0\sum_{i=1}^{2}S_i^z+\hbar\sum_k\omega_k a_k^\dagger a_k + \sum_{i=1}^{2}\sum_k[g_{ik}a_k(S_i^+ + S_i^-)+\text{H.c.}] , \tag{1}$$

where ω_0 is the atomic resonance frequency, $S_i^\dagger$, S_i^-, and S_i^z are pseudospin operators for the ith atom satisfying the well-known commutation relations

$$[S_i^+,S_j^-]=2S_i^z\delta_{ij},\quad [S_i^z,S_j^\pm]=\pm S_i^\pm\delta_{ij} . \tag{2}$$

The coupling coefficient is given by

$$g_{ik}=-i\left[\frac{2\pi\hbar\omega_k}{V}\right]^{1/2}(\boldsymbol{\mu}\cdot\mathbf{e}_k)e^{i\mathbf{k}\cdot\mathbf{r}_i} , \tag{3}$$

where V is the normalization volume, $\mathbf{e}_k$ the unit polarization vector, and $\boldsymbol{\mu}$ is the transition electric dipole moment vector.

A master equation for the reduced density operator ρ of the two-atom system is derived from the Hamiltonian (1). The normal treatment of the interaction between atoms and the quantized radiation field assumes that the field is in the usual vacuum state. Here we assume that the quantized radiation field is in a broadband squeezed vacuum state with the carrier frequency in resonance with the frequency ω_0 of the atomic transition. The bandwidth of the squeezing is assumed to be sufficiently broad that the squeezed vacuum appears as δ-correlated squeezed white noise to the atoms. The correlation functions for the field operators a_k and $a_k^\dagger$ can then be written as[19]

$$\begin{aligned}
&\text{Tr}[\rho_R(0)a_k^\dagger a_{k'}]=N\delta_{k,k'} ,\\
&\text{Tr}[\rho_R(0)a_k a_{k'}^\dagger]=(N+1)\delta_{k,k'} ,\\
&\text{Tr}[\rho_R(0)a_k a_{k'}]=Me^{-2i\omega_0 t}\delta_{k,2k_0-k'} ,\\
&\text{Tr}[\rho_R(0)a_k^\dagger a_{k'}^\dagger]=M^*e^{2i\omega_0 t}\delta_{k,2k_0-k'} ,
\end{aligned} \tag{4}$$

where $k_0=\omega_0/c$ and ρ_R is the reduced density operator corresponding to the radiation field; N and $M=|M|\exp(i\phi_v)$ are parameters characterizing the squeezing such that $|M|^2\le N(N+1)$, where the equality holds for a minimum-uncertainty squeezed states, and ϕ_v is the phase of the squeezed vacuum. For $|M|=0$, Eqs. (4) describe the thermal field[20,21] (black-body field) at a temperature T, with N the mean occupation number of the mode k.

Employing Eqs. (1) and (4) we obtain the master equation for the reduced density operator ρ of the atoms interacting with the squeezed vacuum in the following form (see the Appendix):

$$\begin{aligned}
\frac{\partial\rho}{\partial t}=&-M\sum_{i,j}(\gamma_{ij}+i\Omega_{ij}^{(+)})([S_i^+,\rho S_j^+]+[S_j^+\rho,S_i^+])-M^*\sum_{i,j}(\gamma_{ij}-i\Omega_{ij}^{(+)})([S_i^-,\rho S_j^-]+[S_j^-\rho,S_i^-])\\
&-N\sum_{i,j}\gamma_{ij}(\rho S_j^-S_i^+ + S_j^-S_i^+\rho-2S_i^+\rho S_j^-)-(N+1)\sum_{i,j}\gamma_{ij}(\rho S_i^+S_j^- + S_i^+S_j^-\rho-2S_j^-\rho S_i^+)\\
&-i(1+2N)\sum_i\Omega_{ii}[S_i^+S_i^-,\rho]-i\sum_{i\neq j}\Omega_{ij}[S_i^+S_j^-,\rho] ,
\end{aligned} \tag{5}$$

where $\gamma_{ii}=\gamma_{jj}=2\gamma=(4\mu^2k^3/3\hbar)$ is the Einstein A coefficient for spontaneous emission, Ω_{ii} and $\Omega_{ij}^{(+)}$ are connected with the Lamb shift of the atomic levels. In further calculations we shall ignore the parameters Ω_{ii} and $\Omega_{ij}^{(+)}$, which can be shown[5-7] to be negligibly small when the atoms are in resonance with the carrier frequency of the squeezed vacuum. For $i\neq j$ the parameters Ω_{ij} and γ_{ij}, which appear in Eq. (5), describe collective properties of the two-atom system, depending on the interatomic distance r_{12}, and are defined as[20,22]

$$\Omega_{12}=\Omega_{21}=\tfrac{3}{2}\gamma\,\mathrm{Re}[f(k_0r_{12})] \tag{6}$$

and

$$\gamma_{12}=\gamma_{21}=-\tfrac{3}{2}\gamma\,\mathrm{Im}[f(k_0r_{12})]\ , \tag{7}$$

with

$$f(k_0r_{12})=-\left[[1-(\hat{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}}_{12})^2]\frac{1}{k_0r_{12}}+[1-3(\hat{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}}_{12})^2]\left[\frac{i}{(k_0r_{12})^2}-\frac{1}{(k_0r_{12})^3}\right]\right]\exp(ik_0r_{12})\ , \tag{8}$$

where $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{r}}_{12}$ are unit vectors along the transition electric dipole moment and along the vector $\mathbf{r}_{12}=\mathbf{r}_2-\mathbf{r}_1$, respectively. Moreover, $k_0=\omega_0/c=2\pi/\lambda_0$ for λ_0 the resonant wavelength.

In Eqs. (6) and (7) we have assumed that the atomic dipole moments $\boldsymbol{\mu}$ are parallel to each other but may have different spatial orientation with respect to the line connecting both atoms. In Fig. 1, Ω_{12}/γ and γ_{12}/γ, defined in (6) and (7), are plotted for the cases when $\hat{\boldsymbol{\mu}}$ is parallel and perpendicular to $\hat{\mathbf{r}}_{12}$. These two collective parameters strongly depend on the interatomic separation r_{12} and on the spatial orientation of the dipole moments. For large interatomic separations k_0r_{12} goes to infinity, and then γ_{12} and Ω_{12} go to zero, i.e., there is no coupling between the atoms. Otherwise, for very small interatomic separations, k_0r_{12} goes to zero, and thus γ_{12} reduces to γ, and Ω_{12} reduces to the static dipole-dipole interaction potential which, for $k_0r_{12}\to 0$, tends to infinity. For the case when $\gamma_{12}=\gamma$ the two-atom system is referred to as the small-sample model or Dicke model.[8-12] However, this model is valid only when k_0r_{12} is close to zero. This limit presents difficulties for the experimental realization of such a model. The model with $\gamma_{12}\neq\gamma$ is more accessible for observation than the small-sample model. Experiments with atoms placed near a conducting metallic surface have been reported,[14] and a two-atom system described by Eqs. (5)–(7) is equivalent to the case of one atom in the presence of such a surface.[23,24] This model can be also related to the experiments using an atomic-beam method.[13] However, in a gas sample the atomic dipole orientation with respect to the line connecting both atoms is a random variable[25] and we should average Eqs. (6) and (7) over all possible orientations of the vector distance $\mathbf{r}_{12}$. Angular averaging of Eqs. (6) and (7) leads to

$$\Omega_{12}=-\gamma\frac{\cos(k_0r_{12})}{k_0r_{12}} \tag{9}$$

and

$$\gamma_{12}=\gamma\frac{\sin(k_0r_{12})}{k_0r_{12}}\ , \tag{10}$$

which are different from zero and for very small interatomic separations the parameter γ_{12} reduces to γ and Ω_{12} tends to infinity similar to the parameters (6) and (7). Thus Eq. (5), which is fundamental for the theory of radiation of two atoms interacting with a squeezed vacuum, can be applied to derive the radiative properties of a two-atom gas sample as well as for fixed atoms near a metallic surface. For the normal vacuum ($N=|M|=0$) it is the same as those obtained by Agarwal.[20] Aside from the parameters γ_{12} and Ω_{12}, which describe the collective properties of our two-atom system, the radiation depends on the parameters N and M describing the squeezed vacuum. In the following section we apply Eq. (5) to examine the steady-state radiative properties of the two-atom system.

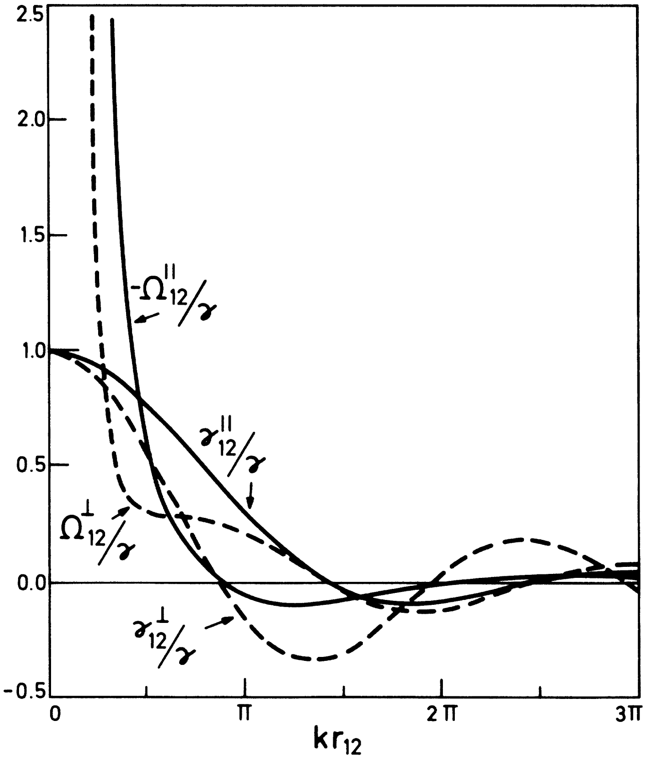

FIG. 1. The damping constants γ_{12} and frequency shifts Ω_{12} as a function of the interatomic separations $\mathbf{r}_{12}$ for dipole matrix elements $\hat{\boldsymbol{\mu}}$ parallel ($\parallel$) and perpendicular ($\perp$) to $\hat{\mathbf{r}}_{12}$.

III. STEADY-STATE ATOMIC POPULATION AND CORRELATION FUNCTIONS

The total atomic population is described by the atomic operators S_1^z and S_2^z. Using the commutation relations (2) we can express the total atomic population by the correlation functions $\langle S_i^+ S_i^- \rangle$ as

$$\langle S^z \rangle = \langle S_1^z + S_2^z \rangle = (\langle S_1^+ S_1^- + S_2^+ S_2^- \rangle - 1) \ . \tag{11}$$

To obtain information about the steady-state atomic population we use master equation (5), which for two atoms leads to a closed set of four equations of motion for the vacuum expectation values of the atomic operators. We can write this set of equations in matrix form as

$$\frac{d}{d\tau}\mathbf{X} = \underline{A}\mathbf{X} + \boldsymbol{\alpha} \ , \tag{12}$$

where $\underline{A}$ is the real 4×4 matrix

$$A = \begin{bmatrix} -n & -a & 0 & 0 \\ -a(2n-1) & -n & -2|M| & 4an \\ -4|M|a & -2|M| & -n & 8|M|a \\ \frac{1}{2}(n-1) & \frac{1}{2}a(n-1) & |M|a & -2n \end{bmatrix} . \tag{13}$$

The column vector $\mathbf{X}$ has the following real components:

$$\begin{aligned} X_1 &= \langle S_1^+ S_1^- + S_2^+ S_2^- \rangle \ , \\ X_2 &= \langle S_1^+ S_2^- + S_2^+ S_1^- \rangle \ , \\ X_3 &= (\langle S_1^+ S_2^+ + S_1^- S_2^- \rangle \cos\phi_v \\ &\quad - i\langle S_1^+ S_2^+ - S_1^- S_2^- \rangle \sin\phi_v) \ , \\ X_4 &= \langle S_1^+ S_2^+ S_1^- S_2^- \rangle \ , \end{aligned} \tag{14}$$

while the vector $\boldsymbol{\alpha}$ has the components

$$\begin{aligned} &\alpha_1 = (n-1), \quad \alpha_2 = a(n-1) \ , \\ &\alpha_3 = 2|M|a, \quad \alpha_4 = 0 \ . \end{aligned} \tag{15}$$

For simplicity, in Eqs. (12)–(15) we have introduced the notation

$$\tau = 2\gamma t, \quad n = 1 + 2N, \quad a = \frac{\gamma_{12}}{\gamma} \ . \tag{16}$$

By setting the left-hand side of Eq. (12) equal to zero we obtain the steady-state solutions of this equation. However, there are two different steady-state solutions of $\mathbf{X}$ depending on whether $a \neq 1$ or $a = 1$. This fact is connected with S^2 conserving for $a = 1$ and the reduction by one of the number of independent variables. In this case the determinant of the $\underline{A}$ matrix vanishes because of the linear dependence of the variables. To explain it we express the square of the total spin of the two-atom system in terms of the $\mathbf{X}$ vector components as

$$S^2 = 2 - X_1 + X_2 + 2X_4 \ , \tag{17}$$

and, in accordance with Eqs. (12) – (14), we have that

$$\begin{aligned} \frac{d}{d\tau}S^2 = -(1-a)[(n-1) - (2n-1)X_1 \\ + nX_2 + 2|M|X_3 + 4nX_4] \ . \end{aligned} \tag{18}$$

This means that S^2 is conserved in the system for $a = 1$ only; otherwise it decays on a time scale $\sim[2(1-a)\gamma]^{-1}$. If a is very close to 1, however, this decay time is much longer than $(2\gamma)^{-1}$ and we can ignore this decay for times of order $(2\gamma)^{-1}$, assuming that S^2 is conserved. This is consistent with the small-sample assumption because γ_{12} tends to γ as the interatomic separation r_{12} becomes small compared to the light wavelength. However, for interatomic separations r_{12} comparable to the resonant wavelength, γ_{12} differs considerably from γ (see Fig. 1) and we can no longer ignore the S^2 decay. This is the S^2 conservation breaking case. The physical interpretation of this difference in the steady-state solutions of Eq. (12) is straightforward: The Hamiltonian (1) can be diagonalized giving collective states[11,12] $|0\rangle = |1\rangle_1 |1\rangle_2$, $|\pm\rangle = (1/\sqrt{2})(|2\rangle_1|1\rangle_2 \pm |1\rangle_1|2\rangle_2)$, $|2\rangle = |2\rangle_1|2\rangle_2$, with energies $E_0 = 0$, $E_\pm = \hbar(\omega_0 \pm \Omega_{12})$, and $E_2 = 2\hbar\omega_0$, where ω_0 is the transition frequency between the ground state $|1\rangle_i$ and the excited state $|2\rangle_i$ of the ith atom $(i = 1,2)$. In this representation, the two-atom system is equivalent to a single four-level system with one upper state $|2\rangle$, one ground state $|0\rangle$, and two intermediate states $|\pm\rangle$. The decay constants[26] to and from the superradiant state $|+\rangle$ are $\gamma + \gamma_{12}$, and to and from the subradiant state $|-\rangle$ are $\gamma - \gamma_{12}$. For the small sample model $(\gamma_{12} = \gamma)$ the subradiant state is not populated[11,20,26] and the two-atom system is equivalent to the three-level cascade system. For the extended system, however, γ_{12} differs from γ and the subradiant state $|-\rangle$ is populated and radiation from this state cannot be ignored.

The steady-state solution of Eq. (12), irrespective of the initial atomic population, for the S^2 breaking system, i.e., for $a \neq 1$, leads to the following expression for the total atomic population:

$$\langle S^z \rangle = -\frac{1}{n}\left[1 - \frac{4a^2|M|^2}{n^4 + 4|M|^2(a^2 - n^2)}\right] . \tag{19}$$

For an S^2-conserving system, i.e., for $a = 1$, when the subradiant state is not populated, the steady-state solution for the total atomic population is

$$\langle S^z \rangle = -4\frac{(n^2 - 4|M|^2)}{n(3n^2 + 1 - 12|M|^2)} \ . \tag{20}$$

The above steady-state solutions, apart from the parameter N, also include the absolute value of the parameter M, which means that the squeezed vacuum changes the steady-state atomic population. For the normal vacuum $(N = |M| = 0)$ in both cases the steady-state atomic population is equal to -1. In this case both atoms are in their ground states. When the atoms interact with the thermal field at the temperature T, $N \neq 0$, $|M| = 0$, and the steady-state atomic population (19) for the S^2-breaking system is

$$\langle S^z \rangle = -\frac{1}{n} \ , \tag{21}$$

whereas for the S^2-conserving system (see also Ref. 11)

$$\langle S^z \rangle = -\frac{4n}{(3n^2 + 1)} \ . \tag{22}$$

It is now possible to see in detail the strong difference between the steady-state atomic population for S^2-conserving and S^2-breaking systems. In both cases the steady-state atomic population is different from that for the ground states. This is due to the presence of the nonzero intensity thermal field.[20,21] For the S^2-breaking system [Eq. (21)] the steady-state population is independent of the atomic interactions and identical with that for the independent atoms.[3] For the S^2-conserving system [Eq. (22)], however, the steady-state atomic population differs significantly from that for the S^2-breaking system and is lower than that for the independent atoms.

If the atoms interact with the squeezed vacuum, $N\neq 0$, $|M|\neq 0$, and the steady-state atomic population is given by Eqs. (19) and (20). In this case the steady-state atomic population differs from that for the thermal field. For the S^2-breaking system [Eq. (19)] the squeezed vacuum introduces a dependence of the atomic population on the interatomic separation, a feature completely absent in the normal vacuum as well as for the thermal field. For the independent atoms $a=0$, and the steady-state population is the same as for the thermal field. Inversely, for small interatomic separations $a\approx 1$, and the atomic population is higher than that for the thermal field. The situation is quite different for the S^2-conserving system. From Eq. (20) and for the minimum-uncertainty squeezed states, i.e., for $|M|^2=\frac{1}{4}(n^2-1)$ we have that the atomic population is the same as that for two independent atoms. Despite this, the atomic population differs from that for the thermal field [see Eq. (22)]. This means that for the S^2-conserving system the squeezed vacuum destroys internal atomic correlations which are induced by the thermal field and lead to an atomic population different from that for the independent atoms. It is the inverse of the S^2-breaking system where the squeezed vacuum induces internal atomic correlations which are absent for the thermal field. In order to show this, we find from Eq. (12) the steady-state solution of the X_2 component of the vector **X** which describes internal atomic correlations. For the S^2-breaking system, the steady-state solution for the X_2 component has the form

$$X_2=\langle S_1^+S_2^- + S_2^+S_1^-\rangle=\frac{-4a|M|^2}{n^4+4|M|^2(a^2-n^2)} . \tag{23}$$

It is evident from the above equation that the interatomic correlations are different from zero *only* when the atoms interact with the squeezed vacuum. Thus the squeezed vacuum induces interatomic correlations which lead to changes in the steady-state atomic population.

For the S^2-conserving system, however, the steady-state solution for the X_2 component is quite different from that for the S^2-breaking system and has the form

$$X_2=\langle S_1^+S_2^- + S_2^+S_1^-\rangle=\frac{n^2-1-4|M|^2}{3n^2+1-12|M|^2} . \tag{24}$$

In this case non-negligible interatomic correlations are induced by the thermal field. The squeezed vacuum, however, changes these correlations in such a way that for the minimum uncertainty squeezed states, i.e., for $|M|^2=\frac{1}{4}(n^2-1)$, these correlations vanish. Thus, for the S^2-conserving system the squeezed vacuum destroys the interatomic correlations, which leads to the steady-state atomic population identical with those for two independent atoms.

From Eq. (12) it is also possible to calculate the changes of the atomic population by two-photon absorption or emission. These changes are described by the X_3 component of the vector **X**. It is not difficult to verify from Eq. (12) that for the S^2-breaking system the steady-state solution for the X_3 component is

$$X_3=\frac{2na|M|}{n^4+4|M|^2(a^2-n^2)} , \tag{25}$$

whereas for the S^2-conserving system

$$X_3=\frac{8|M|}{n[3n^2+1-12|M|^2]} . \tag{26}$$

It is evident from the above equations that the correlation functions describing changes of the atomic population by two-photon absorption or emission are different from zero only when atoms interact with the squeezed vacuum. Moreover, these correlation functions are different from zero independent of whether the system is or is not S^2-conserving. This is in contrast to the one-photon atomic correlations functions (23) and (24) where these correlations functions strongly depend on whether the system is or is not S^2-conserving.

Equations (12) for atomic correlation functions gives us a good starting point to also calculate the second-order correlations of the emitted photons. These correlations are described by the normalized intensity correlation function,[27,28] which in the notation of the components of the vector **X** has the form

$$g^{(2)}=\frac{4X_4}{(X_1+X_2)^2} . \tag{27}$$

A straightforward but lengthy algebraic manipulation of Eqs. (12) and (27) for the S^2-breaking system leads to the steady-state intensity correlation function

$$g^{(2)}=\frac{\{n^2(n-1)^2-4|M|^2[(n-1)^2-a^2]\}[n^4+4|M|^2(a^2-n^2)]}{\{n^3(n-1)-4|M|^2[n(n-1)+a(1-a)]\}^2} , \tag{28}$$

whereas for the S^2-conserving system

$$g^{(2)}=\frac{n(3n^2+1-12|M|^2)[n(n-1)^2-4|M|^2(n-2)]}{4(n-1)^2(n^2-4|M|^2)^2} . \tag{29}$$

For the thermal field ($|M|=0$) Eq. (28) gives the value 1 irrespective of N and interatomic interactions. This implies that for the S^2-breaking system the emitted photons are uncorrelated. For the S^2-conserving system, however, the normalized intensity correlation function can be different from 1. According to Eq. (29) the S^2-conserving system in the thermal field has the intensity correlation function

$$g^{(2)}=\frac{3n^2+1}{4n^2} \ . \tag{30}$$

Since $n \geq 1$, this function is always smaller than 1. This implies that for this system the emitted photons are anticorrelated.

The results are quite different when the atoms interact with the squeezed vacuum field. It is not difficult to verify from Eq. (28) that for the S^2-breaking system and for highly squeezed ($N \gg 1$) minimum-uncertainty states we have $g^{(2)} \approx 1$, whereas for weakly squeezed ($N \ll 1$) minimum-uncertainty states.

$$g^{(2)} \approx \frac{a^2}{N(1-2a+2a^2)^2} \ . \tag{31}$$

It is readily seen that now the intensity correlation function is greater than 1. This reflects the high correlation of the emitted photons for the S^2-breaking system. However, this applies only in the limit of a weak squeezing. Otherwise, for the strong-squeezing case the photons are weakly correlated.

For the S^2-conserving system and for the minimum-uncertainty squeezed states from (29) we get

$$g^{(2)}=\frac{2n}{n-1} \ , \tag{32}$$

which is always greater than 1. Thus we obtain for this system that the emitted photons are highly correlated for all values of N. These high correlations are due to the fact that the squeezed field contains strong internal correlations,[19] which are transferred to the atomic system, and they generate strong correlations between the emitted photons.

IV. CONCLUSION

We have considered here the problem of spontaneous emission from two two-level atoms separated by the arbitrary distances r_{12}, and interacting with the squeezed vacuum field. We have been especially concerned with the steady-state solution for the total atomic population when the interatomic separations r_{12} are comparable to the resonant wavelength (the S^2 conservation breaking case) as well as when are much smaller than the resonant wavelength (the S^2-conserving system).

We have found that the steady-state atomic population is different depending on whether the system is or is not S^2 conserving. For the S^2-breaking system the squeezed vacuum induces dependence of the atomic population on the interatomic separation, a feature completely absent in the normal vacuum as well as in the thermal field. The final atomic population differs from that for the thermal field for which the atomic population is the same as that for noninteracting atoms. This is in contrast to the S^2-conserving system[11] for which the final atomic population in the squeezed vacuum is the same as that for the noninteracting atoms, but differs from that for the thermal field for which the atomic population is lower than that for the noninteracting atoms. This difference between these two systems is induced by the atomic correlations. For the S^2-breaking system the interatomic correlations are different from zero only when atoms interact with the squeezed vacuum, and vanish in the thermal field. For the S^2-conserving system, however, the interatomic correlations are different from zero when the atoms interact with the thermal field and vanish for the minimum uncertainty squeezed field.

We have also shown that for the S^2-breaking system interacting with the thermal field the emitted photons are uncorrelated, whereas for the S^2-conserving system the emitted photons are anticorrelated. The photon correlations are quite different when the atoms interact with the squeezed field. For the minimum-uncertainty squeezed states the S^2-conserving system emits highly correlated photons. The S^2-breaking system, however, emits highly correlated photons only in the limit of weak squeezing. For strong squeezing the emitted photons are weakly correlated.

The present analysis of spontaneous emission from two atoms show that this effect is sensitive to the interatomic separations. However, the role of the interatomic separations cannot be unambiguously declared as "destructive" or "constructive." We hope our paper may contribute towards the clarification of this situation and may prove useful in designing future experiments.

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APPENDIX: DERIVATION OF EQ. (5)

We derive the master equation (5) applying the method used by Agarwal[29] to obtain the master equations for the reduced density operator ρ with the quantized radiation field in the vacuum state. Here we assume that the quantized radiation field is in a broadband squeezed vacuum state with the correlations functions for the field operators a_k and $a_k^\dagger$ given by Eq. (4). The Laplace transform of Agarwal's[29] equation (6.28) with Eq. (4) takes the following form:

$$\begin{aligned} z\rho-\rho(0)&+M\sum_{i,j}(\eta_{ij}^{(+)}+i\Omega_{ij}^{(+)})([S_i^+,\rho S_j^+]+[S_j^+\rho,S_i^+])+M^*\sum_{i,j}(\eta_{ij}^{(+)}-i\Omega_{ij}^{(+)})([S_i^-,\rho S_j^-]+[S_j^-\rho,S_i^-]) \\ &+\sum_{i,j}[(N+1)\gamma_{ij}^{(-)}+N\gamma_{ij}^{(+)}](\rho S_j^-S_i^+ +S_j^-S_i^+\rho-2S_i^+\rho S_j^-)+\sum_{i,j}[(N+1)\gamma_{ij}^{(+)}+N\gamma_{ij}^{(-)}](\rho S_j^+S_i^- +S_j^+S_i^-\rho-2S_i^-\rho S_j^+) \\ &+i(2N+1)\sum_{i}\Omega_{ii}[S_i^+S_i^-,\rho]+i\sum_{\substack{i,j\\ i\neq j}}\Omega_{ij}[S_i^+S_j^-,\rho]=0 \ , \end{aligned} \tag{A1}$$

where

$$\eta_{ij}^{(+)}=\sum_k g_{ik}g_{j,2k_0-k}\frac{z}{z^2+(\omega_0-\omega_k)^2} ,$$

$$\gamma_{ij}^{(\pm)}=\sum_k g_{ik}g_{jk}^{*}\frac{z}{z^2+(\omega_0\mp\omega_k)^2} ,$$

$$\Omega_{ij}^{(+)}=\sum_k g_{ik}g_{j,2k_0-k}\frac{\omega_k-\omega_0}{z^2+(\omega_0-\omega_k)^2} , \qquad \text{(A2)}$$

$$\Omega_{ii}=-\sum_k g_{ik}g_{ik}^{*}\left[\frac{\omega_k-\omega_0}{z^2+(\omega_0-\omega_k)^2}-(\omega_0\rightarrow-\omega_0)\right] ,$$

$$\Omega_{ij}=-\sum_k g_{ik}g_{jk}^{*}\left[\frac{\omega_k-\omega_0}{z^2+(\omega_0-\omega_k)^2}+(\omega_0\rightarrow-\omega_0)\right] ,$$

and z is the complex Laplace transform parameter.

In deriving Eq. (A1), we have made the rotating-wave approximation;[30] i.e., we neglected rapidly oscillating terms with frequency $2\omega_0$ (the so-called counterrotating terms). In Eq. (A1) we have also assumed that the carrier frequency of the squeezed vacuum is in resonance with the frequency ω_0 of the atomic transition.

We will now consider all modes of the radiation field available for spontaneous emission and go over to the free-space continuum limit $V\rightarrow\infty$, so that

$$\sum_k \cdots \rightarrow \frac{V}{(2\pi)^3}\int dk \sum_{s=1}^{2} \cdots , \qquad \text{(A3)}$$

where s represents the polarization of the mode k. Furthermore, we employ the Markov approximation, i.e., we ignore the retardation effects[31] by assuming that the time Δt required for the light signal to traverse the system is small comparable to the atomic lifetime, i.e.,

$$\max(r_{ij}/c)\ll c\Delta t . \qquad \text{(A4)}$$

In this case we can replace $\gamma_{i,j}^{(\pm)}$, $\eta_{i,j}^{(+)}$, $\Omega_{ij}^{(+)}$, and Ω_{ij} by their limiting values as $z\rightarrow0^{+}$. With these approximations and for a sufficiently long time, i.e.,

$$t\gg(1/\omega_0), \quad t\gg\max(r_{ij}/c) , \qquad \text{(A5)}$$

the inverse Laplace transform of Eq. (A1) leads to Eq. (5) with the coefficients γ_{ij} and Ω_{ij} given in Eqs. (6) and (7).

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