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Measuring Messy Mathematics: Assessing learning in a mathematical inquiry context

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Abstract

The pedagogy of inquiry to teach mathematics presents a seemingly messy yet busy classroom, where children are engaged in using purposeful mathematics to collaboratively generate effective and creative solutions to open-ended, ambiguous questions. Problems arise when describing student learning in inquiry settings, when assessment practices are chosen that do not align with or are not designed to capture learning in this context. This study will present an inquiry into mathematics inquiry classrooms, to analyse and interpret the relationships between three classroom elements of assessment, teaching and learning. Distinctive to this study, the researcher was also the classroom teacher which offers the reader close insight into school practices in these classrooms. The researcher aimed to understand if an alignment between these classroom elements could support teaching and learning in inquiry settings. Using design research as a methodology (Cobb et al., 2003), two primary, inquiry classrooms (Years three and six) are presented as three iterative phases of study, each using particular theoretical lenses and analytical tools. In the first phase of study, theoretical analysis of formative assessment practices was based on Dewey's (1891) conception of thinking as a process involving abstraction, comparison and synthesis. Vygotsky's (1978) zone of proximal development was the analytical tool in the second phase of study and was used to analyse how the classroom teacher adjusted her teaching based on feedback she received through formative assessment. The third and final phase looked closer at the mathematical learning revealed through formative assessment using the DNR framework (Harel & Koichu, 2010): a Piagetian-influenced framework. Duckworth's (2006) belief framework was also used to consider an interrelated synthesis of findings from this study, to assist in the development and refinement of assessment practices that align with using the inquiry pedagogy to teaching mathematics.

Findings from three phases in this study revealed the inadequacy of summative assessment practices in capturing and describing student learning and thinking, fostered at higher levels through inquiry. In the first phase of study, analysis of assessment completed by students as part of their everyday, classroom curriculum reflected how such assessment only requires students to perform lower-level, reproductive thinking. In contrast, formative assessment opportunities encouraged students through inquiry to conceptualise their mathematical thinking in connected and abstract ways. The second phase of study focused on teaching in one inquiry classroom and characterised the difficulties classroom teachers face as they implement inquiry into their mathematics curriculum. Analysis of inquiry teaching and learning in this phase characterised how the teacher needed to be an engineer: able to interweave student ideas as potentialities, into the scaffolding of particular learning goals.

Interweaving by the teacher, of students' connections to the mathematical topic being explored, highlighted the complexity and messiness of the inquiry classroom where frequent interactions generated feedback about students' thinking. Analysis of student learning in the third phase of study reflected a complex journey for students which considered interactions with peers in an inquiry context. Student thinking was provoked in these interactions shifting some responsibility for learning to the student as they tried to make sense of conflicting ideas.

In all phases of this study, the inquiry pedagogy supported deep and connected mathematical learning, engineered by the classroom teacher towards particular learning and assessment goals. The learning process for students, as an ongoing journey of testing and refining mathematical processes and skills, was neglected when assessment did not value these characteristics. In inquiry, when assessment of learning values the messy and personal learning journey students face, there is potential for students to continue learning beyond the constraints of narrow curriculum objectives. Further research into 'what else' is learned through mathematics inquiry is required, to refine the pedagogy and to make its intentions clear. This study presents potential innovations to consider for future research.

Declaration by author

This thesis is composed of my original work, and contains no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

I have clearly stated the contribution of others to my thesis as a whole, including statistical assistance, survey design, data analysis, significant technical procedures, professional editorial advice, and any other original research work used or reported in my thesis. The content of my thesis is the result of work I have carried out since the commencement of my research higher degree candidature and does not include a substantial part of work that has been submitted to qualify for the award of any other degree or diploma in any university or other tertiary institution. I have clearly stated which parts of my thesis, if any, have been submitted to qualify for another award.

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Publications during candidature

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Fry, K. (2013). Students' holding' the moment: learning mathematics in an inquiry mathematics classroom. In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics Education: Yesterday, Today and Tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia, pp. 306-313). Melbourne, VIC: MERGA.

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Keywords

Inquiry-based learning, mathematics education, formative assessment, Dewey, Piaget, Vygotsky

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Chapter 1 Introduction

In a year three classroom students are seated in rows. Some have their hands up in eagerness to answer the teacher's question. Others are looking at the floor, or are pretending to be busy aligning their erasers on their desktop. The children are learning about how data can be collected, organised and interpreted as part of their mathematics lesson. A graph on the board shows the names of five students who don't really exist, and the amounts of fruit they had in their lunchboxes over one week. Students record these totals in their book before neatly entering the data onto their own graph. Being a pictograph, the children are then required to count the number of fruit items to interpret who has the most fruit in their lunchbox, who has the least amount of fruit in their lunchbox and which two students have the same amount of fruit as each other. They will use this information to determine the healthiest child. The students write these answers in their books to be handed in to the teacher before their own lunch break. Later, the teacher gathers the students' work and ticks the correct answers. Errors are graded with a cross next to the answer and the total number of correct answers is recorded in the students' books and the teacher's grade book. They will return to the lesson next week. What learning has occurred here? What understandings have resulted from the experience? Will the teacher use this information to inform her teaching? How will the students interpret their results and use this information to inform their learning?

In another year three mathematics classroom which seems somewhat noisy and chaotic, a small group of three students can be heard arguing whether or not a sandwich made of brown bread with chocolate spread is healthy or not. They are looking at an untidy table of data in a scrapbook, obviously constructed by the students themselves, and are trying to find other students' names and what they had in their lunchboxes. Previously this group of students had collected data about the number of items contained in their classmates' lunchboxes and are revising their plan to see if they have missed anything. Upon the teachers' request the students end their conversations and bring their work to the floor. A discussion takes place around the decisions students are making, whether their class is healthy or not. The children and the teacher can't decide whether a healthy student must have a lunchbox where all the items are healthy, or if it is alright to have biscuits, muesli bars and other 'half-healthy' items. The students need to make their own decisions around this and re-form groups to discuss it. One group is very strict and allows no unhealthy items to be present in a lunchbox whereas others seem to be more flexible, being more interested in comparing the total amount of healthy items to unhealthy items. They record these ideas in their scrapbooks to use the following day. What learning has occurred here? What understandings have resulted from the *experience? Will the teacher use this information to inform her teaching? How will the students interpret their results and use this information to inform their learning?*

In both classrooms, the mathematical concepts of collecting, organising and interpreting data are being explored. The first classroom presents a teacher-centred scenario where the data to be analysed has been provided by the teacher. Students are working independently to complete the set task correctly. Valuable information is being gathered by the teacher regarding how the students construct pictographs and if they can read them to interpret who is the healthiest student. The second classroom looks and sounds much different. This is an inquiry mathematics classroom where students are encouraged to investigate meaningful problems which are often ill-structured; to "engage in the epistemological processes of coming-to-know used by experts" (Makar, 2007, p. 48). Here, in this inquiry classroom (Allmond, Wells & Makar, 2010; Makar, 2008; 2012), the students can determine what data they are to collect by mathematically reasoning with their peers about what healthy means. The data are collected using a suitable method decided by the group and how this happens may change once they see how their peers are collecting data (Makar, 2008). As students work through the process of inquiry, students are expected to defend the mathematical ideas they propose while they in turn respond thoughtfully to the mathematical arguments of their peers (Goos, 2004). The teacher may decide to present the students with a variety of different graph styles to provide the students with further ideas and the whole class could then determine effective data gathering techniques and the best ways to communicate this information mathematically. In both classrooms the same mathematical content is covered yet how students learn in each classroom is different.

Even though the mathematical content in each classroom is the same, differences exist in what is learnt. However, this may not be evident in assessment that is designed to capture and describe learning in a more traditional, non-inquiry classroom. In classrooms, decisions around assessment, teaching and learning are often informed by views of testing to do with mastery learning, dominant in the field of education (Black & Wiliam, 1998a; Shepard, 2000; Torrance, 2012). These approaches may have supported past models of curriculum and instruction yet continue to be implemented in classrooms regardless of pedagogy. Such decisions may be made by educators above the classroom level, such as principals or regional officers, leaving teachers little choice. Possibly, pedagogical decisions are swayed by popularity, the parent community, high-stakes testing regimes or clearly-positive results that show academic gains (in high-stakes testing) (Dingman, 2010; National Research Council, 2001a; Schoenfeld & Kilpatrick, 2013). Difficulty lies in assessing mathematical learning in such complex domains (Spector, 2006). Although there is substantial research into the pedagogy of inquiry in mathematics classrooms (Fielding-Wells, 2014;

Fielding-Wells & Makar, 2012; Goos, 2004; Makar 2008; 2010; 2012), there is little in regards to how assessment can support or align with learning through inquiry.

It is the vast difference in assessment and pedagogical choices in an inquiry classroom, compared with a traditional mathematics classroom, which requires further scrutiny through educational research. Much research has compared traditional classrooms to socio-constructivist pedagogies (Alsup, 2004; Cobb, Yackel & Wood, 1992; Cobb & Yackel, 1996; Confrey, 1991) and it would seem that both classrooms could be as far away from each other on a continuum as possible, yet similar practices and expectations surrounding assessment prevail. When choices around assessment, teaching and learning do not align then these elements do not support each other and a mismatch can result in unfair and unreasonable assumptions of what students know and can do in mathematics. This mismatch is the foundation for my own inquiry as researcher, into the classroom elements of assessment, teaching and learning in an inquiry mathematics classroom.

A Personal Note

As a classroom teacher being introduced to inquiry pedagogy, I intuitively felt that inquiry presented benefits to teaching and learning that I did not fully understand. The school I taught was not averse to inquiry style pedagogy as a framework for improving student learning of mathematics, yet it seemed to me that the assessment measures expected by the school were not designed to measure student learning in inquiry. Design research (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003) enabled me to conduct an inquiry into what was happening in my own inquiry classroom; to engineer inquiry classroom environments for research purposes that closely reflected or took into consideration my own regular classroom practices. Regular classroom practices were informed by school planning at the year level, including common, standardised assessment tasks as well as any other accommodations for students in the classroom. A clear definition of this methodology can be found in the literature section of this thesis (Chapter 2). The inquiry units of work that I planned and implemented aligned with mathematical content as prescribed by curriculum documents, including Essential Learnings (Queensland Studies Authority, 2007a; b) and the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority ACARA, 2012). The inquiry pedagogy was not part of any pedagogical framework adopted by the school but I was drawn to teach mathematics in this way through being intrigued by the possibility of enhanced student learning outcomes in mathematics.

Teachers face many challenges in the transition to adopting inquiry-based practices (Makar, 2007; 2012). As a participant in a larger inquiry study, I was aware that I was developing my own personal knowledge and capabilities of teaching mathematics using the inquiry pedagogy and became very interested in how assessment aligned with this teaching and learning approach. Due to the complexity of the inquiry pedagogy, planning inquiry teaching and learning experiences proved to be time-consuming and challenging and often lessons appeared disastrous in terms of behaviour management, resources and time requirements. The time and efforts spent on these challenges were not reflected in the academic reports I completed to describe my students' learning. My accountability to the research project and the value I saw in this teaching and learning approach meant that I continued to teach mathematics through inquiry in my classroom alongside other approaches. As a research participant, I was encouraged to reflect on and share successes and challenges with other teachers participating in the project and learnt to collaboratively plan inquiry units of work.

As a classroom teacher, my initial beliefs about learning and assessment aligned more with scientific notions of measurement, as the school culture valued students being able to get accurate answers quickly. Classroom assessment was driven by quantitative data which only narrowly focused on students' conceptual understandings, or the mathematical processes they could do. School mathematics assessment data portrayed students' mathematical ability and understanding as a single number or percentage. As the classroom teacher, I knew that students developed mathematical understandings through inquiry, but this was not portrayed through current classroom assessments. I wanted to find assessment that valued learning as a journey, enhanced further learning, and that valued teacher judgment. My reading of formative assessment processes resonated with the learning I felt I was observing when students engaged in inquiry. I engineered inquiry units of work to make such learning apparent.

I present this explanation of my learning journey, as it contributed to my understanding of what an effective teacher of inquiry mathematics does to deepen their students' mathematical understandings and because it shaped my own implementation of this pedagogical approach. It also describes part of my journey in becoming a teacher-as-researcher; a reflective practitioner who explores "the implications of teaching acts for their pupils' developing understanding(s) of mathematics" (Jaworksi, 1998, p. 4). The inquiry units of work presented in this thesis are only three of the many I designed and trialed as a teacher in the large inquiry study. Yet they were designed for my classroom and not only for the purposes of the study in which I was a participant. I claim this to demonstrate my own ability to plan and implement inquiry mathematics teaching units because it became my own personal challenge to try to and align classroom elements to the field of inquiry mathematics and to explore how to assess learning in inquiry, what assessment information to capture and how to use that assessment information to improve student learning. I do not propose that these units are exemplary inquiry practice and have chosen three particular inquiries out of many that reflect the development of mathematical ideas in the classroom, or learning. All units of work reflect the characteristics of inquiry (Allmond et al., 2010).

My dissatisfaction with school expected and directed assessment practices fueled my inquiry into my development as an inquiry teacher. Whilst I adhered to socio-constructivist pedagogies (Alsup, 2004; Cobb, Yackel & Wood, 1992; Cobb & Yackel, 1996; Confrey, 1991), my assessment pedagogies often were closely associated with traditional mathematics pedagogy. My voyage into inquiry pedagogy jolted my realization that when assessment, teaching and learning do not align, unfair and unreasonable assumptions of what students know and can do in mathematics become starkly apparent. This mismatch is the foundation for my own inquiry as researcher, into the classroom elements of assessment, teaching and learning in an inquiry mathematics classroom.

Study Aims

This thesis illustrates my own inquiry as a researcher, into teaching and learning mathematics through the pedagogy of inquiry, in my own classroom. As a classroom teacher I felt assessment practices offered little detail about the students I taught in terms of learning mathematics in an inquiry setting. I wanted to explore through research what was revealed by descriptions of learning in inquiry settings, captured through assessment, and to determine whether there was potential to consider learning in other ways. The complexity of teaching in an inquiry classroom meant that my students often went on unanticipated learning pathways that pushed them into higher levels of thinking about mathematics. I wanted all my students to experience this learning all the time through inquiry, so I wanted to research how a teacher can make this happen. To do so would require a deep analysis of mathematical learning in my classroom. I became aware of a nexus around the three elements of assessment, teaching and learning as exploration into one informed and was informed by the other two in a cyclical way. A relational understanding of the three classroom elements of assessment, teaching and learning in the context of inquiry formed the aims of this study:

1. Through assessment, identify and describe learning in ways that are well-informed and respectful of students as learners of mathematics in inquiry (Assessment).
2. Consider how information gained through assessment informs my teaching about how to extend children's thinking further through inquiry (Teaching).
3. Elaborate on the process of learning (understandings, skills and procedures) in an inquiry context (Learning).

This thesis represents my own inquiry into understanding learning in an inquiry mathematics classroom and through assessment, qualitatively analyse that learning in ways that reflect the richness and complexity of learning in an inquiry context. This thesis contributes a teacher-as-researcher perspective to the field of work on mathematical inquiry as distinctly, I was also the classroom teacher in each phase of study. The nature of inquiry is that it is qualitative, whereas science measures change in quantitative terms (Dewey, 1938b). I do not intend to disprove theories regarding learning mathematics, or assessment of learning in mathematics yet there is little research which relates the fields of assessment, teaching and learning in an inquiry mathematics context. Similarly, I do not pose research questions where the answer is already known. Just as pathways in inquiry are open-ended, research in the field of inquiry in mathematics education reflects various methodologies and analysis frameworks. Through three iterative phases of study, this qualitative study attempts to ground the classroom elements of assessment, teaching and learning in the pedagogy of mathematical inquiry (Cobb et al., 2003; Flick, 2009).

The Study

My own inquiry into this pedagogical approach to teaching and learning mathematics as reflected in this thesis, is structured using principles of formative assessment. As a researcher, the notion of feedback (the information gained through formative assessment) was seen to relate to the information I gained from my own students, informed my own practice as a teacher of inquiry, to inform the research field of mathematical inquiry. I therefore applied principles of formative assessment to the chapters in this thesis in relation to three key questions recognised for generating effective feedback (Black & Wiliam, 2009; Hattie & Timperley, 2007). These questions are:

1. *Where am I going?* In a classroom sense, this question relates to the learning goals of particular tasks or performances. Feedback associated with this question makes clear to the learner, the learning intentions. In this thesis, the review of literature extended from the intentions currently valued in the fields of mathematics education, to inquiry pedagogy, and views of assessment and learning that align with this pedagogy. This serves to provide a sense of what is expected in mathematics classroom. Gaps and inconsistencies in literature provided direction for my own research to highlight *where my research needed to go*.
2. *How am I going?* This question relates to the intended learning goal/s identified in the literature as effective practice in classrooms. This aspect of feedback provides “information about progress and/or about how to proceed” (Hattie & Timperley, 2007, p. 89). In this thesis, *How am I going?* guides the data and analysis chapters. In answering this question, my journey of progress towards aligning assessment, teaching and learning mathematics in inquiry classrooms

is presented. Data and analysis thus offers an evaluation of the interrelatedness of these elements in an inquiry context to provide guidance on how to proceed the pedagogy for mathematics inquiry classrooms.

3. *Where to next?* This question provides feedback that leads to greater possibilities for learning as it becomes clearer about what is or what is not understood (in terms of the learning goals identified in this instance, in the Chapter 2 of this study). Classroom instruction is often sequential and so examples of this kind of feedback from teachers in a classroom could be the suggestion of more strategies to complete tasks, or the offer of enhanced challenges (Hattie & Timperley, 2007). In this thesis, the Discussion Chapter relates theoretical findings from data and analysis to inform mathematics education research about *where to next* in terms of effective classroom practice, replete with suggestions of how to get there.

The purpose of this study is to gain insight into *where to next* in terms becoming a more expert teacher of inquiry. Rather than present an isolated view, three classroom elements provide the research focus to offer insight into inquiry mathematics classrooms: assessment, teaching and learning. The study does not intend to find only one answer or solution of best practice, nor does it intend to produce a particular assessment tool. My own view is that learning is a journey or process, and principles of formative assessment support the view that learning is ongoing. As applied to the structure of this research, the findings of each phase offered feedback that contributed to new, related understandings about assessment, teaching and learning mathematics and the inquiry pedagogy.

Literature Review: Where am I going?

Just as formative assessment principles first identify where the learner is "going" by clarifying targets or goals (around the teaching and learning of mathematics in this instance), the literature review in this thesis explores four main areas, to highlight the direction of the research in this study. The literature selectively addresses (1) what is currently valued in mathematics classrooms, (2) the pedagogy of inquiry to teach mathematics, (3) how assessment practices influence learning in mathematics classrooms, and (4) how learning in mathematics classrooms is understood through different theoretical lenses. Summarised below, this review highlighted criteria for successful learning intentions in mathematics, and I present this as a goal for the pedagogy of inquiry.

Literature to identify knowledge, understandings and skills that are currently valued in mathematical classrooms.

This section of the review considered the current state of mathematics education by outlining historical factors that have influence on what happens in mathematics classrooms today. Research places different amounts of emphasis on the importance of content, procedure, fluency, reasoning, strategy, proficiencies, competencies and thinking skills. The results are views that conflict with and often disagree on what is most important to include in curricula and effective ways to teach it. It can be difficult for teachers to adopt pedagogies for teaching mathematics in their classrooms when dominant pedagogies are already seen as effective in terms of achieving improved student learning outcomes, and explicated in current research in mathematics education. The review of literature serves to identify what is currently valued in mathematics classrooms to offer a sense of 'effective mathematics education' to contemplate in inquiry settings.

Current research in the field of mathematics education reflecting teaching and learning through the inquiry pedagogy.

The pedagogy of inquiry is not new but specific classroom based research using inquiry pedagogy in mathematics is relatively sparse. Research is broad, and spans a continuum of inquiry definitions from guided inquiry to open exploration of topics. The pedagogy of inquiry also extends across different subject areas and research into teaching and learning mathematics through an inquiry approach is in process. In this thesis, the review of literature explicitly focuses on teaching and learning through inquiry applied in this study. Just as it is important to understand how research has informed what is valued in mathematics, the literature reviewed illustrates values aligned with the pedagogical approach of inquiry mathematics. This is used to provide clear goals implementing the pedagogy of inquiry.

Literature exploring assessment to illustrate the related views of learning valued by different assessment practices.

Research exploring assessment is reviewed to illuminate the values and beliefs of education that are endorsed both generally and in mathematics. Literature describes how assessment practices value and describe learning (or what is learnt) to inform goals for assessment. Key characteristics of assessment that align with teaching and learning principles in inquiry are highlighted to identify assessment practices that are suggested to be effective in an inquiry context Assessment, as part of an inquiry pedagogy, needs to be broad and flexible to capture the construction of mathematical knowledge and understandings that develop in inquiry. Just as assessment may capture understandings of content, skills or processes, the review of literature identifies the content, skills and process that are not measured. The literature exploring assessment and inquiry will be used to inform my study.

Literature exploring learning in mathematics classrooms.

Literature exploring how students learn, in mathematics in particular, are outlined to characterise learning in an inquiry context. Researchers continually strive to understand the learning process in mathematics, to capitalise on learning opportunities and enhance learning. Constructivist research that aligns with descriptions of learning in inquiry are pertinent to this study and are used to clarify current beliefs of how students learn using this approach. For this thesis, knowledge of how students learn assisted me in evaluating how assessment tools and strategies describe learning. This assisted in clarifying when learning takes place in an inquiry context.

The rationale for the Australian Curriculum: Mathematics, which is the policy document currently informing what is taught in Australian classrooms, states the importance of learning mathematics to enrich lives, and developing numeracy capabilities for personal, work and civic life. The curriculum intent is for our students to develop mathematical capabilities that are "increasingly sophisticated and refined" (ACARA, 2012), by linking mathematics to other disciplines; providing opportunities for students to reason mathematically and apply their mathematical understandings creatively and efficiently. Working mathematically was also defined (ACARA, 2012) as developing through inquiry and the application of problem-solving approaches where students select and use technologies, ways to communicate, and reason and reflect in ways that are appropriate. The pedagogy of inquiry seems to offer a promising way to support these broad intentions, although the evidence gained through assessment does not yet capture this intent.

Little is known about how assessment information from inquiry learning experiences can be identified, stored and used to inform future teaching and learning. Although there is research on teaching and of learning mathematics in inquiry, there is a gap in literature exploring the use of assessment in this field in relation to these classroom elements. Research on classroom assessment was used to inform the design of phases of inquiry teaching and learning in this study, to gain insight into how these practices align with and support student learning in three classroom contexts. A range of formative assessment tools are available that provide ongoing feedback to teachers and students. For inquiry, there is a need to identify ways to assess student learning, that values multiple types of understandings rather than focusing and reporting only on particular content. A greater knowledge of how and what students learn in inquiry is required before refinement of assessment can take place.

Theoretical framework for data analysis

Three theoretical contributions to inquiry pedagogy are (1) the importance of experience (Dewey, 1938a), (2) the importance of collaboration (Vygotsky, 1978) and (3) the importance of doubt (Piaget, 1952). These three theoretical foundations underpin inquiry pedagogy, and they are used to analyse data from each of the three phases of this study. Chapter 3 elaborates the theoretical frameworks. In simple terms however a view of experience in mathematical inquiry is presented from a Deweyan perspective, the work of Vygotsky emphasises the role of the teacher in the inquiry learning environment, and Piagetian principles are used to describe learning that values the challenges in solving problems that students face through inquiry. More importantly, the theoretical contributions offered a framework in which to consider the alignment of the classroom elements of assessment, teaching and learning mathematics with inquiry. This framework is considered at the classroom level, and from the teacher's and researcher perspectives in Chapter 3.

Data and Analysis: How am I going?

Based on principles of feedback used to inform learners about their current progress, findings from this study have served to inform my own goal of becoming an expert teacher of inquiry. The question *How am I going?* guided the analysis of data in this study in relation to the goals established in the literature. This thesis presents qualitative analyses of three episodes of inquiry practice, in two different classrooms, to illustrate the richness of this learning environment. Distinct to this study, as the researcher, I am also the classroom teacher. This thesis presents a teacher-as-researcher perspective, illustrating both viewpoints through analyses. Analyses focus on three essential classroom elements: assessment, teaching and learning. Due to the nature of design research being used as the methodology for this study (Cobb et al., 2003), three phases or iterations of study are presented to explore these three elements. Each phase contributed to the design of each subsequent phase. Each phase is presented as a separate chapter in this thesis, with each phase including the related data, analysis and findings; as outlined below.

Phase One: Assessment

There are three research questions to frame envisioned assessment and the means of supporting it in the inquiry setting (Cobb et al., 2003). These questions guided analysis of data in this first phase of study:

1. How is mathematical learning in one inquiry classroom assessed currently and does this align with learning in this inquiry context?

2. Are there characteristics of assessment that support learning in inquiry and can illustrate insight into what students know?
3. What understandings, skills and procedures are developed by students through inquiry as they learn mathematics?

Data were used to characterise assessment practices currently in a Year 6 inquiry classroom. Analysis of a summative assessment task designed to assess learning was undertaken to provide information about learning through inquiry, gained through formative assessment. This serves to illustrate the current climate of assessment in one inquiry classroom to consider how assessment elements support and enhance learning. Analysis of data in this phase attempts to highlight understandings, skills and procedures that are developed in a typical primary, inquiry classroom. Some assessment frameworks (e.g. OECD, 2009) offer definitions of mathematical literacy that seem to align with the inquiry pedagogy, and these are used to offer insight into mathematical learning in this classroom context. Theoretical analysis is informed by Dewey's (1891; 1938a; 1938b) relational view of learning that posits learning based on personal experience. Findings inform subsequent analyses of chapters exploring teaching and learning, contributing to the body of knowledge surrounding this approach and assisting in refining this field of study.

Phase Two: A teaching focus

This phase of the study was guided by the following research question: How does one teacher of inquiry mathematics respond to feedback gained during formative assessment, to guide student learning towards particular learning goals? This research question considered the extent to which teaching in inquiry is responsive to feedback gained through formative assessment. This second phase of the study built on findings from the first study phase on assessment, reflecting the intertwined nature of these two dimensions of my thesis.

Data collected from a Year 3 mathematical inquiry classroom is used to provide insight into teaching practices in this context, for the purpose of exploring how responsive it is to the needs of the students. Analysis of data explored elements of teacher practice that supported and enhanced learning in the inquiry pedagogy. It incorporated findings from Phase One of this study to determine alignment. Data included feedback gained through pre-assessment, teacher planning and reflections, audio recordings and video observations and products of students work. Whilst analysis in the first phase considered the information about student learning gained through formative assessment, the focus for analysis in this phase shifted to consider how teaching was influenced by teacher engagement with feedback generated through formative assessment. Vygotsky's (1978) notion of Zone of Proximal Development (ZPD) was drawn upon to consider how I, as the classroom teacher, guided students' learning towards particular learning goals with the intention also of extending the mathematical thinking of students in the class.

Phase Three: Learning mathematics in an inquiry classroom

Findings from the previous two phases informed the design of classroom teaching and learning in this final phase of investigation. Principles of formative assessment were included to make student learning visible, and findings from the previous phase included an awareness of how feedback informed the teacher about how to progress learning. An element missing from the previous two phases was an understanding of how students learn in inquiry settings. Although implementing a constructivist approach to teaching and learning, this study had not yet considered how findings from the previous phases aligned with models of learning in inquiry settings. Two research questions guided this phase of the study:

1. What learning opportunities arise in an inquiry classroom to develop mathematical understandings?
2. How can a student learn mathematics through inquiry?

The learning opportunities that arise in inquiry are often unexpected. Students can develop: ideas about particular content or procedures, fluency, reasoning, strategy, proficiencies, competencies or thinking skills in any one lesson without the teacher intending on this focus. Opportunities to learn in the Year 3 inquiry mathematics classroom were made visible through the frequent opportunities for formative assessment. In this phase of the study, collected data focused on student learning, and included video observations, products of student work and teacher-reflection notes. Analysis of data identified elements that supported and enhanced learning in the inquiry pedagogy. Analyses of teaching and assessment were considered to show how alignment may take place. In this phase, theoretical analysis was informed using Piagetian frameworks that value learning as a journey or process that includes getting stuck (Harel & Koichu, 2010).

Discussion: Where to next?

Returning to the three key questions used in this thesis to generate feedback about teaching and learning in inquiry settings, the discussion chapter considers the findings from all three phases of this study in an interrelated fashion. *Where to next?* as a formative assessment question, provided me with feedback to consider greater possibilities for my own teaching, research and for mathematical learners in my classroom. Feedback is most powerful when it informs the learner about three different aspects of their learning: the learning goal/s, the learner's current progress and what the learner can do to reach the identified goal/s, and by providing information that leads to deeper understanding. The notion of ‘feed-forward’ (Hattie & Timperley, 2007) corresponds to the question of *Where to next?* which guided this chapter. It aimed to provide more information about what is and what is not understood about inquiry in my classroom. The Discussion Chapter considers how the classroom elements of assessment, teaching and learning align to support mathematical learning in an inquiry classroom and hence, seek ways to reduce the current discrepancy between these elements. In this chapter an interrelated analysis of data collected in each phase is related to the literature to consider the complex learning environment of inquiry mathematics. This analysis also informed using Duckworth’s (2006) belief framework as a way to broadly depict learning in inquiry. Findings contribute to the body of knowledge surrounding this pedagogy and offer insight into ways that assessment, teaching and learning can work together to support mathematical development in a ‘messy’ inquiry context. Of most interest is how the practices of assessment, teaching and learning foster learning and deepen mathematical understanding. Assessment principles that align with inquiry are developed to illustrate the complexity of learning in this context.

The Inquiry Classroom

Assessment in an Inquiry Context

Research into learning in inquiry classrooms has described benefits of the approach for teaching mathematics (Goos, 2004; Fielding-Wells, 2014; Fielding-Wells & Makar, 2012; Fry, 2013; Fry & Makar, 2012; Makar, 2012). In inquiry, planned learning outcomes remain a focus yet what is learnt seems much broader. Rigid styles of summative assessment offer criteria for teachers to use that refer to particular learning intentions, with little concern for any other kind of information. Due to the openness of inquiry, it is not quite known what mathematics (knowledge or content) students may use to solve the problem, or which invented or known procedures they might follow. Difficulty in assessment processes arise when students are creative knowledge builders, exploring and co-constructing their learning (Coles & Banfield, 2012). Although a teacher can carefully plan inquiry lessons around learning intentions and assessment criteria, students are encouraged through inquiry to make connections between mathematical strands, experiment with mathematical solutions, make mistakes and think creatively. Research into teachers’ assessment practices has focused on how to honour complex mathematical problem solving (OECD, 2009; Silva, 2009; Spector, 2006; Suurtamm, Koch & Arden, 2010) and may show promise for assessing learning in inquiry. An inquiry classroom presents a complex mathematical problem-solving environment. Inadequate assessment can result in interpretations of student understandings that are incomplete, narrow and lack respect for the learner.

Through mathematical inquiry, students develop thinking and ideas that are relevant to the context that is invented or explored. Assessment, on the other hand, tends to focus on content which is taught, devoid of context. In other words, contextual elements often do not factor in the evaluation of learning. This point has been considered in relation to assessing the numeracy capabilities of students. To assess numeracy would require observing students and their independent ability to make appropriate choices about: methods, tools, strategies, degree of accuracy, representations of interpretations and reasonableness of solutions; based on context (Perso, 2011). In inquiry, learning is meaningful for students as they consider how certain mathematical ideas suit a particular purpose of the inquiry. For example, in the inquiry “Which bubble gum is best?” (Makar, 2012) the children developed quantitative ways to characterise bubble gum to help them make comparisons between brands. Students experimented with measurement instruments to connect the usefulness of instruments for measuring the size of bubbles, the quality of flavor, how long the flavour lasted and its elasticity. Problems posed in this inquiry encouraged multiple pathways for students to follow. Groups of students may use entirely ‘different’ mathematics to that of their peers. In inquiry, the information that a teacher records about her/his students’ learning may present as messy, and may not match the original teaching intent. It can reveal particular insights into different students’ ways of thinking and understanding, dissimilar to information recorded on black line master-generated rubrics. This means that often what is learnt in one inquiry unit of work does not match what a student *should* know and do according to planned assessment.

Assessment has a great effect on what is valued in terms of learning and how this learning takes place. When assessment does not align with teaching and learning in inquiry, there is a risk that what is learnt is no longer valued by the teacher and the students. The National Research Council (1989) critiqued the use of objective, multiple-choice tests as the norm in U.S. schools. In U.S. classrooms, this style of assessment led to teaching which emphasised lower-order thinking over original thinking and promoted teaching to the test. This kind of thinking was not preparing students to compete in the technological, global future that America faced.

In the U.S. a shift in focus turned to standards-based testing in schools (National Research Council, 2001a). Assessment became linked to classroom instructional efforts as State competency tests in mathematics were used to improve instruction to be more effective. Test designers chose which mathematics was important to learn and tests assessed whether students met specific goals (p. 43). The connection between assessment and school improvement resulted in a deep and intensifying crisis (Stiggins, 2002) as schools used assessment to find the wrong answers, how to make test scores go up in high-stakes testing and how to generate valid and reliable test scores. Stiggins (2002) explained how this flaw was "a direct manifestation of a set of societal beliefs about what role assessment ought to play in American schools" (p. 760). As Wiliam (2010a) questioned, to whom are schools accountable other than parents? Too much testing can result in a narrowing of the curriculum (Silva, 2009) and in an inquiry setting, may result in schools and teachers placing less value on the flexibility and messiness of this approach to exploring mathematics.

It is important to consider the dominance of standardised testing in classrooms and the effects it has on classroom-based assessment. In discussing accountability, Klenowski (2011) highlighted how a classroom emphasis on testing in Queensland could product negative effects and a loss in trust in teacher professionalism. She argued the importance in building and maintaining "teachers' assessment capacity and their assessment literacy" (p. 80). The foreword in Master's (2013b) review of Australian assessment reform, noted how student performance on national and international tests is used to judge whole education systems. This in turn "creates pressure at every level of education systems" (Masters, 2013b, p. iii) including a choice in pedagogy.

High or low student results have been used to influence economical and operational decisions at the school level. Formative evaluations do not seem to contribute or weigh as heavily as the information gained through common or standardised-type tests (Harlen & James, 1997). Often, information gained from summative assessment is used to inform administration and education officials, parents and teachers, about how well teachers, students, year levels and schools may be performing (Black & Wiliam, 1998b; Brown, 2011; Masters, 2013b). The concern is that using information from high stakes, summative assessment, to describe learning in an inquiry context, may not capture all there is to know about learning in this context. For example, how the learner can improve or how the teacher can adjust their teaching to suit the needs of the learner is overlooked. Although formative assessment data can depict a more holistic view of the student as a learner, it becomes very difficult to use such qualitative information on a large scale and often remains with the teacher in the classroom (Perso, 2011). If what is assessed determines what is to be taught and learnt, then it is the responsibility of educators to make sure that assessment in inquiry classrooms aligns with teaching and learning in an inquiry context. In this study, principles of formative assessment will be explored in inquiry classrooms. Analyses will be used to further assessment practices in the field of inquiry, to help define the process of capturing and describing student learning in innovative and creative ways.

Teaching Mathematics in an Inquiry Classroom

The teachers in each of the classrooms illustrated in the beginning of this chapter were both responsible for teaching particular mathematical content. Their beliefs about how students should learn mathematics differed greatly in each classroom. It may be that explicit attention to beliefs is required for fundamental changes to mathematics assessment to take place (Borko, Mayfield, Marion, Flexer & Cumbo, 1997). A goal of mathematics education in traditional classrooms seems to favour a scientific emphasis on 'skill' acquisition (Boaler, 2002; Schoenfeld, 2007; Sfard, 1998a; Shepard, 2000). In the first classroom above, it was important that students knew the right answer or correct procedure for working out mathematical problems, as taught by the teacher. This environment catered for practice of correct procedures, and assessment would be able to easily describe whether students accurately completed the taught procedures or not. Teaching in the second classroom reflected a different view of mathematics. In this classroom, students determined the accuracy of the procedures they chose, in relation to the context in which the mathematics was being used. These students made decisions around which data to collect and how to organise the data in useful ways; the teacher permitted creative solutions to be negotiated with peers. This environment was fostered by the classroom teacher through the expectations for learning she set for her students, and her own view of how students learn mathematics. Shifting thinking about assessment, away from the idea of single correct answers in mathematics, is difficult for teachers (Borko et al., 1997). This study does not aim to identify the differences in teaching that these kinds of extreme scenarios depict. Instead, it is hoped that research in an inquiry classroom may provide information about teaching that aligns with and supports learning and assessment in this context.

The choices a teacher makes about how to teach mathematics influences the development of their students as mathematical beings. Two different contexts existed for students to learn about data in the classrooms above. These were established by the classroom teacher and understood by the students. In the first classroom there did not appear to be a purpose for collecting and using data other than to learn how to follow this procedure or develop this skill or understanding. The teacher chose to use students' lunchboxes as the context for exploring data and may have considered that this was relevant to students or a familiar context to explore. Either to save time, or to ensure all students completed the work, the teacher placed constraints on the parameters to define what being healthy meant. The student who had the most fruit during a week would be the healthiest and the student with the least amount of fruit, the least healthy. In the second classroom the same context was being explored but it appeared much messier as the students determined the parameters for what was healthy or unhealthy. Children collected different kinds of data, in different ways, to suit their own inquiries into the healthiness of children's lunches. The pedagogical choices a teacher makes can influence how the students in each classroom might view mathematics; as isolated and abstract, as an absolute truth, as relative, as purposeful to solve real inquiries or a mixture (Goos, 2004; Sfard, 1998b). This places responsibility on the classroom teacher as to how they establish the learning environment.

In the real-world, notions of being healthy are communicated through advertisements in a variety of ways: going regularly to the gym, choosing low-fat milk or making healthy choices at fast food restaurants. Being healthy can be as extreme as participating in marathons or as safe as walking your dog a couple of times a week. Students might be faced with many different versions of what being healthy means and in this inquiry setting, it would seem relevant and real to explore many different versions of this in the classroom. The context for learning in this classroom reflected mathematics that was real-life and ill-defined, a key characteristic of inquiry (Makar, 2007; 2012). The openness of the ambiguous term 'healthy' meant that multiple pathways could be explored including notions of quantifying healthy. The mathematical learning in this classroom could be described as messy and student-centred because what one student learnt would be difficult to compare with another's learning. The students determined the qualitative criteria surrounding key terms in the question and then chose the mathematics that would be useful in this context.

As much as a teacher can teach, it is up to a student to learn (Nicol & Macfarlane-Dick, 2006). The teacher in the first classroom determined the context and purpose for collecting data. Students in this classroom would learn to recognise the teaching-learning patterns and values in their classroom. They may develop beliefs that classroom mathematics is not relevant in the real world, or that mathematics in the real world does not count in the classroom. Teachers, with care, try to remove obstacles that may hinder a child in their class from learning particular mathematical content yet in the real world, solving problems with mathematics is not such a straightforward task. In education, where learning is fundamental, it is worth considering how students are learning and how different pedagogies may influence the mathematical beliefs of students.

Learning Mathematics in an Inquiry Classroom

Just as teaching mathematics in an inquiry classroom has been explored above, learning mathematics in an inquiry classroom also requires consideration. How mathematics is learnt in an inquiry setting will differ to learning mathematics in other settings. Students are required to make many decisions throughout the inquiry process involving how to proceed to solve the problem being explored (Fielding-Wells, Dole & Makar, 2014; Fielding-Wells & Makar, 2012; Makar, 2008; 2012). The social norms and expectations established in the classroom which make the community of inquiry (Goos, 2004; Makar, 2012) will also influence how students think about mathematics as they: contemplate the ideas of others, approach tasks with a focus on creation and finding-out rather than on taught rules and procedures, respond to set-backs they encounter and analyse their own thinking to clearly communicate their ideas to others. For the mathematics learner in inquiry, classroom peers become the audience as well as problem posers and fellow learners (Fielding-Wells, 2010). Educational research which strives to understand mathematics conceptual learning does so with the hope of informing teaching practice and pedagogy to support student learning (Simon, 1995; Simon et al., 2010; Steffe, 2003). Although various aspects about learning mathematics in inquiry classrooms has been explored, this has not been considered in relation to classroom elements of assessment and teaching in inquiry settings, even though changes in one element could potentially influence all others.

The mathematical ideas that students develop in an inquiry classroom are based on decisions the students make which are important to them and the context they explore. This is different to mathematics classrooms where the teacher makes decisions regarding the mathematical ideas important to consider. For instance, children use mathematics to justify solutions and may argue with peers to reach consensus about generalisations they make; children make decisions about how they will communicate their findings and reflect on how effective the choices their peers make in doing this; children determine what data they want to collect and the procedure/s for doing this; and children can change their minds when the data they collect does not 'go to plan' (Fielding-Wells & Makar, 2012; Makar, 2008; 2012). From a constructivist perspective, inquiry appreciates the mathematics evident in the lives of the students and the processes of finding out that they determine (Confrey, 1991; Jaworski, 1994). Learning mathematics in such a student-centred environment becomes important to students as they choose the process of solving problems they encounter. This may support the notion that learning in inquiry settings may need to be considered beyond the scope of mathematics alone.

When explicit teaching of mathematics is used to show how a particular method can correctly solve a problem, students are offered effective solutions suited to the carefully engineered problems. Of course, to the teacher it is the most efficient way to solve the problem being presented or else such a well-structured, neat example would not have been selected (Makar, 2012). In inquiry, open-ended tasks encourage students to generate and experiment with multiple methods to solve problems. Often, students will encounter 'roadblocks' on their learning journey even when consensus between peers is reached and seemingly good mathematical ideas are applied (Confrey, 1991; Makar, 2012). Facing and overcoming challenges is a key characteristic of learning in inquiry and students can build resilience as they persist with different, often unproductive pathways (Kapur, 2010; Makar, 2012). Describing learning in inquiry mathematics classrooms needs to reflect the value in students following their own pathways to solve a problem, and possibly recognise roadblocks to their learning. These may be important parts of the learning journey to successfully solving mathematical problems.

Traditionally, students completed mathematical assessment independently and in this sense there would not appear to be a need to develop skills to work with peers. Wenger (1998) described such tests as one-on-one combat with students. In this sense, classroom instruction would tend to focus on students working in seats, on independent tasks. In an inquiry classroom there are many opportunities for students to learn collaboratively (Fielding-Wells & Makar, 2012; Goos, 2004; Makar & Rubin, 2009). Generally, students work in small groups and together define a common mathematical goal, closely related to the inquiry question. They choose the mathematics to use to solve the problem and may work on refining the original inquiry question to suit. Learning through collaboration becomes important as decisions students make are dependent on those of their peers (Goos, 2004; Sfard, 1998a). The choices that are made by students in this process, and how they come to make these choices, offer insight into student mathematical reasoning (Makar, Bakker & Ben-Zvi, 2011). These choices are often beyond responses to answers on a summative test where no justification of method is required. Tests which only consider how a student correctly follows a taught procedure to correctly solve a mathematical problem, independent of their peers and the teaching process or learning journey do not seem to align with learning in an inquiry classroom which values how students come to know mathematics through collaboration with peers.

The challenge in inquiry mathematics education is to adapt or design assessment in ways that align with how mathematics is taught and learnt in inquiry classrooms. When the pedagogy of inquiry has been chosen to explore mathematical concepts in a classroom, and a notion of assessment is applied that values knowledge attainment and procedural fluency, then the information gained may present only a narrow perspective of what a student is capable. Findings from research into how and what students learn in an inquiry classroom can be used to inform subsequent ways to assess or capture this learning.

Although there appears to be a vast difference in pedagogy in the two classrooms presented at the beginning of this chapter, both teachers will be required to assess their students for similar reasons. Results may be used by the teacher to help evaluate the effectiveness of their own teaching methods. For example, when students do not do well on classroom assessments the teacher may adjust future lessons, use different classroom resources, or research alternative ways to help their students learn, calibrating their teaching to their students' current understandings and skills

(National Research Council, 2001a). Assessment information could also be used to inform students about how close they are to reaching specific learning goals. Teachers interpret and share assessment data with parents, informing them about their child's progression either individually or in relation to their peers. At a higher level, schools can also use data from assessment to compare results between classes, across year levels and among junior and senior grades. Like schools are then able to make comparisons about teacher and student performances, which contribute to understandings about how effective chosen pedagogies or strategies are within geographical regions. Wiliam (2010a) acknowledges that assessment may be the central process in education. As assessment informs so many aspects of the classroom, decisions surrounding assessment need to be carefully considered by teachers if fair and accurate interpretations of student learning are to be reported.

Thesis Overview

This research will explore the inquiry pedagogy at the classroom level, to make sense of how the classroom element of assessment can support teaching and learning in an inquiry setting. Chapter 2 is a summary of pertinent literature, as outlined in this chapter. Chapter 3 will outline the theoretical influences used as analysis frameworks in each phase of this study. The methodology is presented generally in Chapter 4. The three phases of analysis are presented in Chapters 5, 6 and 7 including more specific explanations of the methodology used in each phase. Finally, Chapter 8 will provide a synthesis of the findings of each phase of study in an interrelated fashion.

Chapter 2 Literature Review

Where Am I Going?

As outlined in the Introduction Chapter to this thesis, my own inquiry is guided by three key questions related to formative assessment: *Where am I going?, How am I going?* and *Where to next?* The literature reviewed in this chapter addresses the first of these three questions to provide a sense of what is expected in mathematics classrooms; in a sense, to set my own learning goals.

Choices surrounding assessment, teaching and learning that are based on traditional principles of knowledge acquisition seem to support models of scientific measurement (Shepard, 2000), convergent-type assessment (Torrance & Pryor, 2001), explicit-style teaching approaches and mastery learning of taught procedures. When these principles are applied to an inquiry setting, a mis-alignment between the means of education that inquiry hopes to foster becomes clear. It is this difference which requires scrutiny through research. Assessment practices that have been refined to align with dominant traditional approaches (Shepard, 2000) are designed to support teaching and enhance learning in this context. When applied to learning in an inquiry classroom, these traditionally-aligned approaches to assessment do not seem to align with the inquiry approach to teaching and learning. Assessment that measures learning in terms of acquiring particular curriculum objectives (Sfard (1998a) uses the acquisition metaphor to describe when learning is perceived as something to be acquired) may not align with the values and beliefs surrounding teaching and learning through an inquiry approach. The elements of assessment, teaching and learning need to align if they are to support and enhance learning in a mathematics inquiry classroom, as is the importance of alignment of these elements in any classroom. In the field of inquiry, little research has articulated how all three elements can interact to support learning in the context. This is important if educators are to acknowledge what their learners know and can do, in ways that are respectful to the learner and useful for teachers, parents and students to understand.

Chapter Outline

Findings from this study will consider theoretical and empirical research already conducted in the areas of mathematics education, the inquiry pedagogy, assessment and learning, with an aim to build on what is already considered effective practice, and to connect these fields of study. Considerations from this literature assisted me in my research, to reflect on what is already considered effective in mathematics education and assessment, and in regards to research already in the field of inquiry and mathematical learning. An awareness of this literature assisted in providing a current benchmark or reference level (Sadler, 1989); an audit of 'where we are' in terms of the three classroom elements of assessment, teaching and learning in relation to the pedagogy of inquiry to teach mathematics. The review of literature is structured to reflect these four main areas of understanding:

1. *Knowledge, understandings and skills valued in mathematics classrooms,*
2. *Inquiry mathematics; an explanation of pedagogy,*
3. *Assessment and the views of learning valued by different assessment practice, and*
4. *Learning in a mathematics classroom.*

A brief outline will firstly present historical perspectives on the kinds of mathematical knowledge, understandings and skills valued in classrooms over time, including moments of reform in mathematics education. This is to make clear the changes to curriculum development at both the international level and in an Australian context. The inquiry pedagogy as a way to teach and learn mathematics, is defined and considered in contrast to school mathematics traditions to identify gaps and alignments in literature. Ambiguity as a key characteristic of the inquiry approach to teaching and learning mathematics is outlined as well as the teaching and learning cycle used in this study. Next, a review of literature on assessment is presented to make clear the principles of formative assessment and feedback relevant to this study. Finally, literature turns to perspectives on learning in mathematics classrooms. A constructivist view of mathematical learning that aligns with the inquiry pedagogy is provided, as well as social perspectives of learning and the importance of getting stuck as part of the process of learning. Duckworth's (2006) theoretical beliefs framework is also considered, to ponder the question: 'what is to be learnt?'

Assessment literature on feedback explains how effective feedback must answer three major questions: *Where am I going?*, *How am I going?*, and *Where to next?* (Hattie & Timperley, 2007). This communicates the learning goal to the learner, which serves to progress learning (for the person receiving the feedback) by outlining what needs to be done to reach this goal. The over-arching question of *Where am I going?* frames this Literature Review where 'I' considers my own perspective as a researcher in mathematics education. Literature focuses on research that describes 'what is expected' in mathematics education to set goals for this study around the classroom elements of assessment, teaching and learning. Explicating goals in mathematics education in these three areas informs the design of teaching and learning experiences in this study. Similarly, literature explored in this chapter also provides insight into data interpretation in educational settings.

Knowledge, understanding and skills valued in mathematics classrooms

> The history of educational theory is marked by opposition between the idea that education is development from within and that it is formation from without; that it is based upon natural endowments and that education is a process of overcoming natural inclination and substituting in its place habits acquired under external pressure. (Dewey, 1938a, p.1)

Already in 1938, John Dewey was questioning the discontent with traditional education. He described a traditional scheme where one imposed adult standards, subject-matter and methods upon the young. Alternatively, he argued how progressive education valued more intimate learning, through personal experience of subject-matter and acquainting (students') past experiences to appreciate the living present. Mathematics classrooms are often depicted as traditional schemes with an imposition of standards from above; lacking constructivist values of experience and constructing personal meaning, (Dewey, 1938a). The kinds of knowledge and understandings that are valued in mathematics classrooms can be influenced by both traditional and progressive-type schemes, if not some sort of balance between the two. A mismatch occurs when beliefs about mathematical knowledge which align more closely with traditional views, are transferred to a more constructivist approach to teaching and learning. Discontent occurs when learning expectations are to replicate taught skills and procedures that the classroom teacher has deemed important to solve in a progressive classroom. Or when personally constructing mathematical meaning through experiences that aim to be relevant and important to the learner to solve are applied to a traditional classroom.

The classrooms presented in the introductory chapter of this thesis reflected two different views of education similar to the contrasting perspectives described by Dewey, although this thesis does not intend to present only a dichotomous view. The National Research Council (2001a) acknowledged that mathematics is bound by history and culture and that participation in mathematical situations will help students learn. The report presents the claim that mathematics is universal and eternal; where students have to absorb clearly presented mathematical ideas. Although, they claim further that by itself, this kind of view of mathematical knowledge is incomplete in its ability to capture the complexity of learning and teaching mathematics, and of mathematics itself (National Research Council, 2001a). In fact, Schoenfeld (1992) critiqued perspectives of mastery, and mathematics as a body of facts and procedures calling such a curriculum impoverished. The assumption that mathematics is a discipline of pure truth makes inquiry in the mathematics classroom a challenge (Makar, 2012), especially when inquiry encourages students to apply mathematics to situations that are messy and often ill-defined. In each phase of study in this thesis, the mathematical content to be taught matches the curriculum expectations of the school yet through inquiry pedagogy, characteristics of the classroom elements of assessment, teaching and learning will be explored.

In mathematics classrooms, tensions arise when theoretical influences that align with particular pedagogies, are applied to all teaching and learning approaches. For example, throughout the 20th century the paradigm of "scientific measurement" dominated much educational research informing effective practices in mathematical classrooms (Shepard, 2000). Shepard considered how maximising learning in mathematics classrooms in the 20th century meant teaching mathematics in steps where building blocks of understanding were specifically developed (associationist learning theory). Behaviorists such as Skinner focused on the reinforcement of accumulating small, successive steps as stimulus-response associations (Shepard, 2000). Mathematics classroom curricula in the 20th century were informed by social efficiency ideas and development included detailed objectives and tracking by ability (Shepard, 2000). Hereditarian theory emphasised the biological nature of intelligence testing and curriculum became highly differentiated. Similarly, these views all assumed mathematics offered both a "foundation of truth and a standard of certainty" (National Research Council, 1989, p. 31). Ideas of measuring ability and achievement, in increments of pre-determined steps and in relation to a body of pre-existing mathematical knowledge, continue to influence mathematics classrooms today (Spector, 2006; Suurtamm et al., 2010; Torrance & Pryor, 2001). When this kind of view is applied to an inquiry classroom it becomes extremely difficult for teachers to track the accomplishment of small, successive steps when directions students take in their learning are broad and varied.

The terms 'knowledge, understandings and skills' that head this subsection are all nouns that reflect knowledge as something to acquire (Shepard, 2000; Sfard, 1998a). When considered in this way, educational research strives to articulate what it is that students should acquire and how this process might unfold. In early studies by Duckworth in the 1970s, her team set out to try and identify the knowledge or concepts that students were learning in one particular study. Reflecting on these self-contained studies, her work described the difficulties her team had in trying to articulate what was learnt (Duckworth, 2006). She noted how when concepts were stated as nouns they presented a limiting way to describe learning. I will return to Duckworth and her beliefs about learning in the learning section of this chapter but mention her work here to highlight how difficult it can be to articulate and track all the knowledge, understandings and skills there is to be learned in mathematics. For teachers of inquiry, assessment that focuses on particular mathematical concepts may also miss other key information about students' learning.

It soon became clear that the mathematical skills being taught were not adequate in solving real-world mathematics problems (National Research Council, 1989; Schoenfeld, 2007; Suurtamm, 2004). The idea of mathematics being a set of skills that one could gain was still apparent in curricula developed in the late 20th century (Shepard, 2000). Educators became interested in knowing which mathematical concepts, skills and procedures students needed to know and an emphasis on “understanding” mathematics was called for.

Reform

Systematic change to mathematics curricula led to a wave of reform, emerging as early as the 1950s yet taking hold in America later in the 20th century (Kilpatrick, 2009). Principles of scientific measurement were challenged and a cognitive revolution began where cognitive theorists considered learning as sense making and active construction of ideas (Anderson, Reder & Simon, 1996; Shepard, 2000). An emphasis was placed on understanding mathematics over knowledge acquisition (Suurtamm, 2004) and reform efforts required teachers to think about instruction (Brophy, 1986). By 1989, the National Research Council acknowledged that in the U.S. there was an urgent national need to revitalise mathematics (and science) education. The benchmark report *Everybody Counts: A report to the nation on the future of mathematics education* (National Research Council, 1989) analysed the problems of American education and outlined a national strategy to revitalise mathematics. The report emphasised “there is no place in a proper curriculum for mindless mimicry” (p. 44). The report on the future of mathematics education in the U.S. described mathematics as being much more diverse than decades previously, forcing a “revolution in the nature and role of mathematics” (p. 4) to ensure students would be prepared for tomorrow’s world.

The approaches to teaching and learning mathematics of the past were considered no longer adequate as research was indicating that learning was more than listening and imitating and involved students personally constructing their own understandings of mathematics in their daily lives. Numeracy was described as coping confidently with the mathematical demands of life and negative public attitudes towards learning mathematics required change (National Research Council, 1989). Mathematics needed to be more visible in culture and it was suggested that the curriculum developed more of the breadth and power of mathematics emphasising an increased importance to society of mathematics. This trend was visible globally in mathematics education also (Clarke, Clarke and Sullivan, 1996; Kilpatrick, 2009; Suurtamm, 2004) and educational research focused on how to generate meaningful learning (Cobb, Wood, Yackel, & McNeal, 1992; Confrey, 1991; Gregg, 1995; Lerman, 1989). Yet the dominance of traditional practice seemed ubiquitous making the development of inquiry in mathematics classrooms a difficult task (Gregg, 1995). Key authors in the field of U.S. mathematics education (Schoenfeld & Kilpatrick, 2013) also referred to schooling traditions making the implementation of inquiry-based learning in mathematics in the U.S., difficult.

Classroom reform was shifting pedagogy in mathematics classrooms away from more traditional approaches of mathematics 'skills' acquisition through rote memorisation (Shepard, 2000; Suurtamm, 2004; Suurtamm et al., 2010). Situated learning and constructivism gained influence on education highlighting a mismatch between school mathematics and mathematics in real world situations. Socio-cultural models of learning proposed cognition in practice, situated in the relationships with other people through culture in everyday life (Cobb, 1994; Lave, 1988). Learning was viewed as constructive activity (von Glasersfeld, 1987) and a variety of pedagogies claimed to support a constructivist view of mathematics, all which seemed to contrast with more traditional ways of teaching mathematics (Confrey, 1991). The National Research Council (2001a) asserted that effective teaching in mathematics could take a variety of forms. Instructional practices recommended by the National Mathematics Advisory Panel (2008) avoided one-sided alignments with either "student-centred" or "teacher-directed" instruction. In this study, the model of inquiry to be used allows opportunities for a balance of student-centred and teacher-directed instruction. There are times when direct teaching may be used in inquiry although attention must be paid to students' interpretations to provide guidance where necessary (Fielding-Wells & Makar, 2008). The difficulty lies in implementing the inquiry pedagogy into school settings where more traditional practices are well-established.

Curriculum development

Decisions around what is taught in mathematics and how it is taught continue to change as research into mathematics education transforms ideas about 'best' methods of practice (Black & Wiliam, 1998a; Hattie, 2009; National Research Council, 2001a). This is turn drives beliefs about what should be learnt. Indicative of changes in a Queensland context, there have been more than four significant developments in curriculum since 1999 setting the trend of, on average, new curriculum policies implemented every four years. Each new policy document has been informed by research articulating 'best' practice. In the development of the current Australian Curriculum (National Curriculum Board, 2009; ACARA, 2012), policy maker decisions were informed by *The Final Report of the National Mathematics Advisory Panel* (National Mathematics Advisory Panel, 2008) from the United States. The panel from the U.S. Department of Education had reviewed more than 16 000 research publications and policy reports that were considered as "the best available scientific evidence" (p. xvi). They presented individual findings about curricula content; learning processes; teachers and teacher education; instructional practices and materials; research policies and mechanisms; and assessment. Recommendations for the learning process that resulted included developing conceptual understandings, computational fluency, and problem-solving skills for all content areas. The ability of students to transfer these learnt skills to solving problems was considered vital to mathematics learning and an inquiry approach to teaching and learning could be an approach that supports this.

At the local context, what students should learn mathematically in primary classrooms is mapped out as Content Descriptions in the Australian Curriculum (National Curriculum Board, 2009; ACARA, 2012). The ways students explore or develop mathematical understandings is described through the proficiency strands of *Understanding, Fluency, and Problem solving* and *Reasoning*. These strands were based on the five intertwined strands of proficiency chosen by the National Research Council (2001a), with the exception of *Productive disposition*. Many curricula provide a focus on the actions or processes students engage in when learning and using mathematical content. The Australian Curriculum (National Curriculum Board, 2009) valued a robust, adaptable and transferable *Understanding* of mathematical concepts and suggested this occurred when students could connect ideas, represent concepts in different ways and describe their thinking mathematically. *Fluency* is a skill that is developed when students calculate answers efficiently by choosing appropriate methods and approximations. The Proficiency strand of *Problem solving* values how students formulate and solve problems through the design of investigations or by planning their approaches. Finally, student *Reasoning* in mathematics occurs when students reach conclusions and explain their thinking to justify their use of strategies and the conclusions they reached. Atweh and Goos (2011) critiqued how in the implementation of these proficiencies, there existed a heavy focus on understanding and fluency in mathematics classrooms, over reasoning and problem solving. It is the intent of the curriculum to foster all four proficiencies in students in mathematics classrooms and the pedagogy of inquiry can be most useful in this sense. Yet little research has been undertaken in inquiry classrooms, in terms of the proficiency strands and how this approach may support values in the Australian Curriculum.

There seems to also be value placed on the development of real-life numeracy skills in students internationally. The Programme for International Student Assessment (PISA) is an internationally standardised assessment program which has been developed by the Organisation for Economic Co-operation and Development (OECD, 2009). The assessment program focuses on students solving real-world problems that are less-structured, do not include clear directions, and where the student has to make decisions about selecting mathematical knowledge which is relevant to solve the problem. Although PISA does not advocate an inquiry pedagogy, the assessment task aims to present mathematical problems that are less structured to move students “beyond the kinds of situations and problems typically encountered in school classrooms” (OECD, 2009, p. 84). Spector (2006) discussed how dynamic and ill-structured problems are often avoided in school-based instructions, even though they occurred frequently outside of school settings. Similarly in inquiry, the problem may initially be posed by the teacher as a way to introduce or engage students in the topic, but the focus is on students using mathematics to solve problems that are made problematic, either to the students themselves or related to real-world topics. There may be potential to explore the use of the PISA Assessment Framework (OECD, 2009) in inquiry classrooms to offer an analytic tool for categorising student learning.

Inquiry mathematics

Inquiry mathematics encourages the connection of classroom learning to the lives of students to promote a learning culture where students learn to speak, think and act mathematically (Fielding-Wells, 2010; Fry & Makar, 2012; Goos, 2004; Makar, 2007; 2012). Solving real-world problems through inquiry has been an acceptable pedagogy in the discipline of science where it is recognised that learning emulates many of the same activities and thinking as real scientists in the real world (Artigue & Blomhøj, 2013; Center for Science, Mathematics, and Engineering Education, 2000; Schoenfeld & Kilpatrick, 2013). Just as in the literature referenced to in this chapter, in this thesis inquiry mathematics may be referred to as an inquiry unit of work, inquiry pedagogy, mathematical inquiry, an inquiry classroom, inquiry learning experiences, guided inquiry or as inquiry generally.

The mathematical inquiry foregrounding chapter one of this thesis illustrated part of a larger inquiry experience exploring “How healthy is our class?” In inquiry, an answer to this question would be sought after using mathematics both as the process of finding out and in the final conclusion/s students make. Whereas in a mathematical investigation the solutions are already known (either as an established mathematical proof without need for reference to local context, or by the teacher contextually), the answer to an inquiry question aims instead to open pathways for investigation and to encourage divergent thinking in response to the question. Mathematical solutions that students present in inquiry, require justification that relies on evidence and are typically not “right” or “wrong” (Makar, 2012).

Learning to speak, think and act mathematically, make and defend conjectures in a community of peers, and search for innovative solutions may seem like worthy practices for students to develop. Educational research has become interested in knowing what kinds of skills and abilities students may need to cope with the effects of globalisation (Boaler, 2008) and living in the 21st century (Makar, 2012). A new workforce reality demands that students will require newly important thinking and reasoning skills as well as the basics (Silva, 2009). A recent article in the New York Times (Friedman, 2014) described the attributes of prospective employees that Google look for when hiring. The senior vice president of Google listed first and foremost general cognitive ability, which he further described as an ability to learn; being able to process or pull together different bits of information (Friedman, 2014). A model for student mathematical argumentation, a key goal of inquiry mathematics, has been developed for the classroom to incorporate students making a claim about the particular problem, providing evidence to support the claim and reasoning to justify the link between the two often also relying on the inclusion of qualifiers (Fielding-Wells, 2014). Learning mathematics through inquiry could be said to emulate the same activities and thinking as required of adults generally in the real world as students develop skills in argumentation to pull together different bits of information.

The process of mathematical inquiry

Although appearing messy, an inquiry unit of work is structured to provide opportunities for teachers to listen to and better understand their students as mathematicians. Collaboration between classroom teachers and researchers exploring the process of inquiry in classrooms (Allmond et al., 2010) framed inquiry using the 4D model. Their model described inquiry pedagogy for teachers to assist them in engaging their students in iterative cycles of investigating and reporting and is the model used in this study to guide the design and implementation of inquiry units of work. This is opposed to a view of inquiry as a “minimally guided approach”; an ineffective an inefficient support for learning (Kirschner, Sweller & Clark, 2006). The 4D model informed the planning of all inquiry learning experiences presented in this thesis. The model consists of 4Ds: *Discover*, *Devise*, *Develop* and *Defend*. In the *Discover* phase, the teacher introduces the inquiry task and students consider the mathematics in context. Collaboratively, students plan how they will answer the question in the *Devise* phase and many adjustments are made as ideas are shared with others. Students implement their plan of action in the *Develop* phase and have opportunities to reflect upon their plans and the efficiency of the mathematics that they have chosen to solve the problem. An important phase in inquiry is the opportunity for students to justify their solutions in the *Defend* phase. Although presented as phases, the 4D Model is not intended to be linear in nature and can be used by teachers as a planning tool to guide the direction of the inquiry. The model also incorporates an opportunity to *Diverge* students’ thinking through reflection on how new learnings could be applied to new topics of exploration.

Careful consideration of lesson sequencing or inquiry phases can foster occasions where students' mathematical understandings are challenged, making thinking explicit and visible. Makar (2012) described these moments as roadblocks; "not as "errors" but as opportunities for learning" (p. 378). The classroom teacher can orchestrate the sharing of incomplete reasoning to encourage negotiation of ideas and the construction of meaning-making (Makar, 2012). Although incomplete ideas can be naïve to begin with, careful scaffolding by the teacher in an inquiry setting can strengthen conjectures and solutions shared by students (Fry, 2013). This study will explore the opportunities that road-blocks present students to consider as important information about learning.

There are many opportunities throughout an inquiry for the class to break into smaller groups to collaborate, as well as to come together as a class to share findings. The teaching and learning sequences offer iterations where the classroom community can share their ways of thinking and understanding about mathematics (Allmond et al., 2010; Makar, 2012). Developing communication skills and problem-solving capacities in students are also goals for school mathematics in Australia (ACARA, 2012; Goos, 2004). In her work on argumentation, Fielding-Wells (2014) described how students developed understandings about equiprobability through iterative experiences of testing their ideas. In an inquiry on developing the best card for playing addition bingo, the students shared their numbers, based on evidence, reasoning and perceived success with the classroom community (Fielding-Wells, 2014). In inquiry, multiple chances to articulate their reasoning, with peers in small groups and the wider classroom community, can assist students in developing connected ideas about mathematical topics.

A contrast for school mathematics education researchers

I would like to contrast the inquiry pedagogy to the tradition of school mathematics in particular as the dominance of school mathematics in many classrooms presents challenge to the implementation of the inquiry pedagogy. This was a challenge I faced in my own classroom as I implemented the pedagogy of inquiry to teach mathematics. For instance, there are differences in mathematical discussions in inquiry and traditional classrooms. Richards (1991) described the awkwardness for both the teacher and students when the rules of the classroom shifted towards exploration and reflective inquiry, especially when neither the teacher nor the students were used to the new process of inquiry. Tension between school and inquiry mathematics traditions has been described as taking place when dominant traditional practices such as teacher-centred instruction in mathematics education were considered as failures at that point in time (Gregg, 1995). From a broader perspective, Schoenfeld and Kilpatrick (2013) highlighted a range of factors in the U.S. that seemed to challenge the implementation of inquiry-based learning projects. This included schooling traditions as well as perceived societal needs and the readiness of teachers being able to implement inquiry-based pedagogies.

Implementing the pedagogy of inquiry into a school mathematics culture can be challenging when mathematics is viewed as "a discipline of pure truth" (Makar, 2012). In her chapter devoted to making clear the pedagogy of inquiry to teach mathematics, Makar (2012) presented one case study depicting an inquiry to find the best bubble gum. It was noted in the chapter how the teacher considered the inquiry as a disaster. The teacher was concerned with how to balance taking control of students' behaviours and giving students opportunities to develop their own ways of collecting data. In a school mathematics classroom culture the teacher deems which is the best method of data collection and makes the decision regarding how to organise this. Rather than teaching mathematics in an isolated fashion, for little purpose other than knowing mathematics, research has explored how inquiry opportunities to teach mathematics are established in authentic contexts particularly in the field of data and statistics (Allmond & Makar, 2010; Gil & Ben-Zvi, 2011; Makar & Fielding-Wells, 2011; Makar, 2010; 2012). This can be challenging for the teacher and students when the culture of a school mathematics tradition is already well-established.

In mathematics classrooms with a mix of pedagogies, teaching styles, topics and learners it can be difficult to classify the classroom dichotomously as either a school-tradition or inquiry mathematics-type culture. Views of mathematics in a more traditional, school mathematics classroom may have to do with certainty and "following the rules laid down by the teacher" (Lampert, 1990, p. 32). Students need to remember and apply the correct rules and student answers are ratified by the teacher (Lampert, 1990). Yackel and Cobb (1996) described a socialization in schools where children learn "to rely on social cues for evaluation and on authority-based rationales" (p. 467). In the sense that all classrooms are communities of practice, Goos (2004) depicted school mathematics classrooms as having norms or patterns of interaction associated with students learning mathematics by memorising and reproducing procedures as they listen to and watch teacher demonstrations. The difficulty with this environment is in the transfer of mathematics to real-life problems. Inquiry embraces the context and purpose of problems, focusing on the process of mathematisation to investigate, communicate and justify solutions (Makar, 2012).

Teaching and learning in inquiry

Yet the student-centeredness of an inquiry classroom does not imply there is less importance placed on the classroom teacher's role. Teacher expertise is required to know how to balance assisting students with scaffolding and allowing them to grapple with challenges (Makar, 2012; Stigler, Fernandez & Yoshida, 1996). Goos (2004) examined the pivotal position of the teacher in a secondary mathematics inquiry classroom in terms "structuring learning activities and social interactions to facilitate students' increasing participation in a culture of mathematical inquiry" (p. 264). In this paper (Goos, 2004), research was conducted over a two-year period acknowledging that the development of a classroom community of inquiry takes time especially noting pressures from high-stakes assessment favoring more traditional approaches to teaching mathematics. The pedagogy of inquiry illustrates a complex classroom environment which is highly focused on mathematics and the processes of student investigation; fostered through teaching and learning interactions.

In an inquiry classroom the teacher's role changes, as do the expectations of students. Yackel and Cobb (1996) explained how the teacher is a representative of a classroom mathematical community where students personally and meaningfully develop their own ways of knowing. The teacher is no longer the unquestioned authority and students are encouraged to propose and defend their own mathematics ideas while responding thoughtfully to the arguments of others (Goos, 2004). Children work collaboratively in the inquiry classroom and are required to justify and defend their thinking with peers (Fielding-Wells, 2010; Goos, 2004). The classroom is noisy (Makar, 2007) and full of movement. It also appears full of enthusiasm as students solve problems in ways that are meaningful to them (Makar, 2007). The curriculum is broadened as real issues or examples from the outside world are considered as part of the problem-solving process requiring skills which are typically not part of the school curriculum (Makar, 2008). An aim of the Australian Curriculum: Mathematics (ACARA, 2012) is to ensure that students "are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens."

Mathematical inquiry presents problems to solve that are important to students (Fry, 2013) and places value on the many different ways students approach solving the problems they encounter. Student willingness to make mistakes is thought of as important (Makar, 2012) and successes and ideas are shared to build on the knowledge of others. The use of resources to model or represent thinking is encouraged (Makar, 2012) and students select what they need rather than use what they are given. Real world resources are often integrated into the classroom such as bubble gum, online data sets, Google maps, or even people-resources within the school community (Allmond et al., 2010; Makar, 2012). Such resources can be found in a more traditional classroom, but are not often used for the purposes of which the student decides.

Opportunities to share ideas in a classroom community of inquiry offer the classroom teacher chances to consider her students' mathematical understandings and abilities for using mathematical procedures. Confrey's work (1991), describing constructivist approaches to teaching mathematics, argued that examination of students' problems and solutions allowed a teacher to reconsider the mathematics involved. Goos' (2004) work on classroom communities of inquiry depicted a sociocultural view of mathematics that included a focus on mathematical communication, in whole class and within small-group settings. In particular, Goos (2004) noted how in discussions the teacher often reinterpreted students' language to introduce more technical, mathematical terms, later insisting that students use the conventional mathematical language in their reasonings. With opportunities for students' mathematical thinking to be made visible through discussions, it is worth exploring if or how teachers in inquiry classrooms benefit from listening to and thinking about their students' contributions to discussions.

Sometimes, students will select unproductive methods for solving a problem and be allowed to consider their strategy to think of a more effective way to solve the problem (Makar, 2012). For instance, in working out how healthy their class is, one group of students could collect data on each brand of food item in their peers' lunchboxes. A resulting bar graph may offer little information when considering the variety of brands, flavours and sizes of lunchbox items yet seeing the ineffectiveness of gathering and organizing this data can be more powerful to the student than the teacher offering a correct process or solution to begin with. A social perspective in inquiry mathematics places emphasis on mathematical discussion among students (Gregg, 1995) and students working collaboratively with their peers to solve problems, building a community within which students propose and defend their mathematical ideas, as well as to respond thoughtfully to conjectures their peers also make (Goos, 2004).

With research focusing on the richness of learning in inquiry, further efforts are required to be able to capture, understand and use this information in meaningful and respectful ways, as part of the regular classroom processes of assessment. Ways to collect evidence of and interpret improved learning are emerging generally (Hattie, 2005; Spector, 2006; Wiliam, 2011b) although aspects of learning and effective assessment practices in an inquiry setting have not been operationalised. Research into classrooms is needed to pinpoint the mechanisms responsible for learning (Artigue & Blomhøj, 2013), to inform how assessment can take place. Inquiry mathematics presents opportunities for teachers to listen to and understand their students, yet such information is difficult to assess in ways that will enhance learning. For an inquiry approach to be widespread and have practical implications research is required into how students develop mathematically through the experience, considering the dynamics of teaching and assessment from the students' perspective also.

Ambiguous and ill-structured questions

Just as there is ambiguity in real life problems involving mathematics, inquiry question are ill-structured to contain ambiguities allowing students to *hook* into the problem and draw on their own views about the topic (Makar, 2007; 2008; 2012). Ill-structured, open-ended inquiry questions mean that students are required to make a number of decisions as they work on the problem (Allmond et al., 2010), often needing to form their own criteria around ambiguous terms such as 'best' or 'biggest' in terms of the context which they are building. The consideration of relevance to the students' worlds subscribes to a constructivist approach which values an organic connection between education and personal experience to foster learning connections (Dewey, 1938a). In contrast to problem solving that is independent of the learner, Confrey (1991) presented a constructivist view which aligned with descriptions of solving important problems using an inquiry approach. She described how constructivists imagined problems through the eyes of the student, and viewed the problem as dependent on the learner. In contrast, problem solving as a way to teach mathematics presents well-organised structures where the teacher is prematurely concerned with proof, ignoring the process of invention (Confrey, 1991). Teachers of inquiry tend to carefully select contexts which are relevant to the students, or they succeed in making the context relevant to the students.

A broad inquiry question is often refined through joint construction with students and the teacher as part of the inquiry process (Fielding-Wells, 2010). When introducing such a broad question, the teacher foregrounds the mathematics as it supports the classroom curriculum. Often dynamic and ill-structured problems are avoided in school-based instructions, even though they occur frequently outside of school settings (Makar, 2012; Spector, 2006). Makar (2007; 2012) explained how nearly all everyday problems in life can be ill-structured and she suggested using ambiguous words to open the pathways of investigation and to 'set up' complications for students. Ambiguity in language choice forces students to define the criteria that they will use. For example, when finding which bubble gum is *best* (Makar, 2012), students had to connect quantitative measures with the qualitative characteristics that they deemed to be important, such as; size of bubble, flavor quality, flavor duration and elasticity. Determining the best bubble gum could ultimately end up being a personal choice, especially if you value blowing lots of big bubbles over and over.

With such ambiguity in inquiry questions, the criteria students develop to measure success also plays an important role in the justification of their answer. In inquiry, students justify solutions mathematically rather than rely only on personal choice. As students progress towards answering an inquiry question, they are reminded as part of the process of inquiry to carefully consider the evidence that they collect and how this evidence will link to the inquiry question and support their own conclusion (Fielding-Wells, 2010). Mathematics is required to justify the direction of a student's inquiry, claims the students make and the results of the student's investigations (Fielding-Wells & Makar, 2012). Making and justifying claims with mathematical evidence, relevant to the inquiry question context, is described by Fielding-Wells and Makar (2012) as argumentation. Argumentation practices, including connecting and applying mathematics beyond procedural levels in the classroom, requires explicit teaching to become part of an inquiry community of practice (Fielding-Wells & Makar, 2012). Ambiguity in inquiry questions presents students with a need to justify the conclusions they make, and it is important that mathematical evidence they generate supports the claims they make.

Assessment

The purpose of assessment in education is to inform: inform the teacher, students, parents and schools. A general sweep of literature about assessment in educational settings reflects how research is interested in how information gained through assessment informs teaching and learning practices (Black & Wiliam, 1998a; 1998b; Harlen, 2007; Suurtamm et al. 2010). In an inquiry classroom, there seems to be a mismatch between the descriptions of learning described through assessment and the learning experiences of students. Assessment is generally used to illustrate what students have learnt in terms of specified learning goals (Black, Harrison, Lee, Marshall & Wiliam, 2004; Shepard, 2000). Careful alignment of learning goals with assessment ensures tests are reliable and valid ensuring that what is assessed is what is taught, and hopefully what is learnt (National Research Council, 2001b). It is possible to integrate assessment with instruction, and desirable, yet what is assessed in a classroom does not necessarily match what is being taught (Wiliam, 2007; 2010a, 2011a). In an inquiry context particularly, it seems that assessment does not match what has been taught or learned.

This section of literature will focus on two main kinds or purposes of assessment: summative and formative assessment. When assessment takes place at the end of a unit of work, and intends to measure or summarise what has been learnt, this is known as summative assessment (Sadler, 1989). At the classroom level, often summative-style assessment consists of common tasks with some expected standardisation of processes on how to conduct the task. Consistency may be required at this level when information gained from summative assessment is used for reporting purposes and comparisons are made across different classes or schools. Formative assessment, in contrast, takes place during learning and often at many times and is used to shape and improve a student's competence (Sadler, 1989). Although various interpretations exist of formative assessment, Black and Wiliam (2009) included in their definition of formative assessment the idea of eliciting evidence about student achievement that is “interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited” (p. 9). It is this view (that formative assessment provides key information for the teacher and the learner to consider in terms of improvement) that was used to inform formative assessment practice in this study.

Summative assessment takes place at the completion of a learning experience and generally reports upon notions of student success in relation to particular learning goals. Summative assessment can also be generated in a number of ways: by the student, by peers, by the students of the teacher, by the teacher of themselves or by the teacher of the students. Traditionally, it was hoped that summative-style assessment would pressure students to get better grades, motivating students to put in greater effort and learn more (Stiggins, 2005). Binary views of success, or being unsuccessful in learning, have dominated classrooms where an *acquisition* metaphor of learning (Sfard, 1998a) presented a picture of learning as equivalent to knowledge accumulation. Much assessment practice continues to hold these beliefs where notions of scientific measurement describe measurements of learning according to set objectives and goals. These principles may match a classroom where teaching and learning align with knowledge *acquisition.* In contrast, Sadler (1989) explained how formative assessment could provide qualitative judgments of students’ work using multiple criteria, beyond notions of correct or incorrect. In inquiry, value is placed on how students are coming to know and their participation in a community (Goos, 2004; Wenger, 1998); their ability to choose certain knowledge structures or strategies, alternate conceptions, prior knowledge and how they modify this to form successful and meaningful ways of working (Cobb, Yackel & Wood, 1992). The function of summative assessment in inquiry mathematics needs to move beyond aspects of measuring acquired knowledge in the summative sense, to consider learning more broadly.

There has been much research on the benefits of formative assessment practices in general education although missing are descriptions of what this might look like in an inquiry classroom. Refining the principles of formative assessment led to a view of assessment as assessment *for* learning; to promote students’ learning through formative design and practice (Black & Wiliam, 1998a; Black et al., 2004). Principles of formative assessment may prove useful in an inquiry context to prevent the measure of mathematical achievement being synonymous with curriculum goals (Goldin, 1992). Different approaches to formative assessment were identified by Torrance and Pryor (2001) as they sought to use a collaborative, action research approach to bringing about changes in classroom assessment. They identified two types of formative assessment which seemed to associate with different views of learning: convergent and divergent assessment. The first kind was concerned with closed approaches to tasks with precise planning, tick lists and can-do statements. These notions implied a behaviourist view of learning where the view of assessment was something accomplished by the teacher (Torrance & Pryor, 2001). Divergent assessment on the other hand, focused on the learner's understanding and offer potential for highlighting what students know and can do.

Elements of formative assessment offer potential ways to support and describe learning in inquiry mathematics. For instance, information about learning gained during formative assessment is qualitative in nature and multidimensional rather than sequential (Sadler, 1989). Elements of formative assessment that make it effective have been identified that may be used to characterise assessment in an inquiry context. Wiliam is a prolific writer in the field of formative assessment in education and his early work (Black & Wiliam, 1998a; 1998b; Leahy, Lyon, Thompson & Wiliam, 2005) led to the refinement of effective formative assessment strategies (Figure 2-1). Their work described how student learning improved when pedagogy included careful and planned consideration of these formative assessment strategies. More locally described as an artful process, Clark (2010) noted how the pedagogical processes of formative assessment were gathering momentum among research and practitioners. With evidence that consistent and frequent use formative assessment supports learning by raising standards in classrooms generally (Black, 1998; Black & Wiliam, 1998b; Black et al., 2004; Sadler, 1998; Stiggins, 2005; 2007), this study will focus on how formative assessment structures support learning in an inquiry mathematics classroom.

1. **Clarifying and sharing learning intentions and criteria for success;**
2. **Engineering effective classroom discussions, questions, and learning tasks that elicit evidence of learning;**
3. **Providing feedback that moves learners forward;**
4. **Activating students as instructional resources for one another; and**
5. **Activating students as the owners of their own learning.**

Figure 2-1 Five key strategies of effective formative assessment (Wiliam, 2011a; b).

Wiliam's (2011a; b) strategies have been used in pilot work to elaborate on formative assessment opportunities in an inquiry mathematics classroom (Fry, 2011; Fry & Makar, 2012), although elaborations on the strategies do not clarify what is worthwhile assessing "in the moment". General to classroom practice, these strategies for effective formative assessment do not specify what to 'look out for' to determine where the learner might be situated in the process of learning. Nor do they inform the teacher if feedback has been used to improve learning. Missing is how to help a teacher to describe what learning is taking place in terms of content. Although not designed for inquiry pedagogy these strategies may prove useful in capturing learning in the inquiry classroom, in ways that might support and promote mathematical learning.

Formative assessment and feedback

A central feature in determining the quality of learning through formative assessment is the quality of feedback that is generated (Black & Wiliam, 2009; Sadler, 1989). Although in the context of management theory, Ramaprasad (1983) described feedback as the "information about the gap between the actual level and the reference level" (p. 4). It is this notion of feedback which became applicable to education. In this sense, feedback supports learning by considering where learners are in their learning (actual level), where they are going (reference level) and how to get them there (information used to reduce the gap between these levels). Information from formative assessment is used to close the gap between these levels (Sadler, 1989). This notion of formative assessment is also commonly described as identifying where the learner is going (what are the goals?), where the learner is right now (progress being made towards the goal) and informing the learner how to get there (activities that could lead to better progress) (Black & Wiliam, 2009; Hattie & Timperley, 2007; Wiliam, 2011a; b). Feedback can be powerful when the information it generates is able to answer all these three aspects of learning to combine aspects of instruction and feedback (Hattie & Timperley, 2007). In an inquiry classroom the referent level may differ between students even though they are working on the same task and so feedback must encompass a wide variety of solutions. Research is required to gain insight into the types of and effectiveness of the feedback process in an inquiry mathematics classroom.

Feedback takes place in the interactions between the student and the teacher, and between students, to provide learners with a wide range of information. An understanding of good feedback practice was developed from self-regulation models by Nicol and Macfarlane-Dick (2006) as "anything that might strengthen the students' capacity to self-regulate their own performance" (p. 205). They developed seven principles of good feedback practice where the students were assumed to occupy a central and active role in all feedback processes. The fourth principle included encouraging peer dialogue as a source of external feedback other than the teacher. Discussion and collaboration in a climate of intellectual challenge, are key values in a classroom community of inquiry (Goos, 2004) offering ample opportunities for feedback to be exchanged. The feedback that is generated through formative assessment practices in an inquiry setting may provide a wide range of information about the learner and learning. How to capture this kind of information may prove difficult when students are exploring mathematics in different ways and for different purposes.

Although research describes the benefits of effective feedback on improving learning for students (Black & Wiliam, 1998a; 1998b; 2009; Black et al., 2004; Hattie & Timperley, 2007; Hattie, 2009; Nicol & Macfarlane-Dick, 2006; Shepard, 2000; Stiggins, 2007), descriptions of quality are vague, offering little to improve the quality of mathematical learning in classrooms (Callingham, 2008). In Hattie's (2009) synthesis of over 800 meta-analyses relating to achievement, he described how diagnostic formative assessment that provided high-level, frequent and specific teacher feedback was a feature of mastery learning; a strategy emphasising success criteria. He ranked formative evaluation third out of 138 possible influences on increasing achievement. As feedback is now given much credit and empirical evidence shows that providing formative evaluation enhances learning (Hattie, 2009), what counts as quality feedback in a mathematics inquiry classroom needs to also be defined. Providing quality feedback for the learner to improve performance is very difficult (Wiliam, 2011a). Feedback needs to be sensitive to the open-ended nature of inquiry as well supporting students to improve performance.

PISA assessment framework

This study will also consider assessment designs that already consider mathematical learning in ways that are similar to the inquiry approach to teaching and learning mathematics. The assessment framework developed by PISA (OECD, 2009) defined mathematisation as the process of transforming complex real-life problems into mathematical problems. Eight characteristic cognitive competencies were identified (OECD, 2009) to describe how fifteen-year old students were able to mathematise: (1) thinking and reasoning, (2) argumentation, (3) communication, (4) modelling, (5) problem posing and solving, (6) representation, (7) using symbolic, formal and technical language and operations, and (8) use of aids and tools. These competencies were not intended to be individually assessed yet provided language to describe how students engaged successfully in mathematisation. Inquiry mathematics presents ambiguous and broad problems for students to solve and it is the process of mathematisation that assists in foregrounding the mathematical content to be taught and learned. A simplification of the process of mathematisation in inquiry settings was presented by Allmond and her colleagues (2010) as: *Find the maths*, *Do the maths* and *Share the maths*. The competencies offered by PISA (OECD, 2009) offered a more elaborate way to consider the process of mathematisation in inquiry settings, as learning.

The PISA assessment (OECD, 2009) takes place every three years to gain an academic profile of fifteen-year old students in participating OECD countries. Countries can use the data gained through PISA assessments to monitor, for example, trends in the knowledge and skills of their students, in consideration of country and subgroup demographics (OECD, 2009). In response to

Australia's results in PISA in 2012, a call for research into micro-changes was thought of as a way to address concerns about low levels of achievement (Masters, 2013a). Masters determined 'micro' to mean the interactions between a teacher and a student that monitor learning progress and celebrate progress learners make (among other actions). More recently, he described how micro-strategies have been implemented into classrooms to sustain improvement of literacy and numeracy (Masters, 2015a). Analyses of high-quality, micro-interactions in the inquiry classroom will help to qualify learning, gained through the process of formative assessment in a complex classroom.

Standardised assessment

Demands for improving student outcomes, and for quality data to inform educational decision-making, have led to standardised assessment that can 'benchmark' student achievement levels against national and international standards (Masters, 2013a; 2013b). In the current climate of schools in Queensland, Australia, schools are relying more on information from National and International standardised tests to understand how their students are 'going.' This is also being felt in nearby countries such as New Zealand, where research agrees that "Schools are awash with data" (Hattie, 2005, p. 11). In Australia, data from the National Assessment Program – Literacy and Numeracy (NAPLAN) have been available to schools since 2008 to illustrate how their Years 3, 5, 7 and 9 students have performed in English and mathematics. The Trends in International Mathematics and Science Study (TIMSS) is an international study directed by the International Association for the Evaluation of Educational Achievement (IEA), managed in Australia by the Australian Council for Educational Research (ACER). A goal of TIMSS described by Thomson et al. (2012) was to provide "comparative information about educational achievement across countries to improve teaching and learning" (p. v) in mathematics at Years 4 and 8. PISA is yet another internationally standardised test that enables Australia to compare the mathematical performance of their fifteen year old students with students from of over sixty countries in the Organisation for Economic Co-operation and Development (OECD, 2009). The inquiry pedagogy does not aim to standardise the learning of mathematics and subsequent assessment processes, and qualitative data gained about students does not always translate to easily recognised benchmarks. Before assessment information about students gained through inquiry can be valued by educational-decisions makers, a rethink about what is important to assess is needed.

Assessment in the classroom seems to be growing more global allowing Government officials to make comparisons between 'like' schools in different states and even different countries. Yet quantitative data from National and International standardised tests describes learners in ways that narrowly consider what students can and can't do, know or don't know. It is unknown how these descriptions correlate to understandings of learners in an inquiry classroom, especially when the culture of the assessment does not match the culture in the classroom.

Assessment in inquiry: A mismatch

Assessment information is often recorded in ways that are efficient, objective and consistent for classroom teachers. Where it is an intention in education that students continue to learn, assessment can measure this progression (Masters, 2013b). In inquiry, teachers may track learning anecdotally and ponder moments when student seems to struggle; or consider why students take certain directions in solving the problem; or wonder what assisted a student in finally overcoming challenges to their mathematical solution. Qualitative judgments about learning in complex learning settings, such as inquiry classrooms, “cannot be conceptualised as neatly packaged units of skills or knowledge” (Sadler, 1989, p. 123). The type of assessment information gathered in an inquiry classroom may be considered by educational stakeholders as subjective and messy and less reliable than standardised testing. Spector (2006) noted the problem of assessing improved understandings in complex domains when assessment was focused on single approved solutions. Complex domains are problem-centred learning environments where the problem is dynamic and ill-structured, similar to problems posed in an inquiry classroom. A summarising note to his paper, Spector (2006) recommended asking the learner to represent their own thinking about the problem. Although difficult to record on a rubric as a grade, or to quantify as a number in a test, valuable messy learning in inquiry can offer valid and reliable assessment information. It is just unknown which valuable messy moments these are in an inquiry classroom and how the classroom teacher can capture this.

To better understand how students developed mathematical beliefs through inquiry in a second-grade classroom, Yackel and Cobb (1996) found that they had to broaden their focus from purely a cognitive perspective to one which included sociological perspectives. With development of social norms of justification and argumentation for example, the teachers they observed began to capitalise on the learning opportunities that arose by listening to their students’ explanations. Situated in a secondary mathematics classroom, Goos (2004) also explored learning in a classroom community of inquiry. Her research explained how difficult it was for teachers to establish a culture of inquiry when pressures from high-stakes assessment seem to favour more traditional teaching approaches. Many assessment methods do not match teaching and learning experiences in an inquiry classroom and research is required which considers all aspects of this classroom environment to help the development of more refined formative assessment practices in inquiry settings which enhance learning.

Beliefs about assessment that view learning as ‘hitting’ achievement targets or objectives, and only in terms of correct and incorrect understandings, do not cater for student misconceptions or how new understandings are currently being accommodated for in the mind of the learner (Confrey, 1991; Masters, 2013b; Sfard, 1998a; Torrance, 2012). When a view of assessment, which focuses only on the attainment of particular goals, is used to describe learning in an inquiry setting, which does place importance on students coming to know, a mismatch occurs. A classroom culture of inquiry to learn mathematics places value on students; making accommodation for new ideas (Piaget, 1952); solving problems in reasonable and productive ways that are personal (Confrey, 1991); participating in a community of learning (Sfard, 1998a); and ‘knowing’ as an ongoing process or continua instead of a point in time (Harel & Koichu, 2010). Research undertaken by Black and his colleagues (Black et al., 2004) on how to improve formative assessment practices, highlighted the importance (to teachers particularly) of understanding models of how students learn. While there seems to be much support for using formative assessment techniques to improve learning, Leahy et al. (2005) considered how “figuring out how to make them work in your own classroom is something else” (p. 23). A greater alignment of assessment practices with the pedagogy of inquiry to teach mathematics will need to consider how students learn mathematics in this setting. Learning theories that align more with an inquiry approach to teaching mathematics may better inform assessment practices in inquiry pedagogy. Literature that has informed effective assessment practices reviewed in this chapter, will next be related to theories on learning relevant to the inquiry pedagogy.

Learning in a mathematics classroom

If educators assumed that what students learned was the result of the instructional practices of teachers, then it would be unnecessary to assess learning (Wiliam, 2010a); teaching does not cause learning (Simon et al., 2010). In order to assess student learning through inquiry, I will firstly clarify views of learning that align with this pedagogy. This will contribute to the development of assessment that understands and aligns with learning in inquiry, in turn, presenting a relational link between the two classroom elements. A theoretical view of learning will be presented in the theoretical framework (Chapter 3). A classroom is a complex environment where educators need to keep in mind all aspects of the classroom including access to resources, time constraints, behaviour management and curriculum content. Assessment cannot be considered in isolation to relationships with teaching and learning and in fact should strive to be closely aligned with instruction and the goals of curriculum (National Research Council, 2001b; Torrance & Pryor, 1998). The literature presented below illustrates the complexity behind what is valued as learning in an inquiry mathematics classroom. Assessment practices need to consider characteristics of learning mathematics in inquiry settings if it is to capture and report on learning in respectful and meaningful ways.

A constructivist view of mathematical learning

Constructivism as a grand theory (Confrey & Kazak, 2006) presented a way to consider mathematics from the perspective of the student. In the classroom, constructivism focused on actively involving the strengths and resources that students brought to tasks. Confrey and Kazak (2006) described how traditions rooted in constructivism recognised "that the difficulty or ease of learning could not be explained simply by looking at the complexity of the material, but rather that other factors were needed to account for the path learning traversed and levels of success or failure" (p. 307). Constructivism as a theory of learning seems to closely align with learning in an inquiry classroom and offered a lens in this study for analysing student learning in inquiry settings.

Perspectives of constructivism in mathematics classrooms, as a theory of learning, aligns with learning in inquiry settings. Shepard's (2000) description of an emergent, constructivist paradigm in mathematics classrooms at the time, drew on a blend of cognitive, constructivist and social-constructivist learning theories which closely matches views about knowing and learning in an inquiry classroom. Illustrations of a classroom reflecting an emergent, constructivist paradigm (Shepard, 2000) described a focus on the learners' construction of knowledge and the development of intellectual abilities, and a teacher's close assessment of students' understandings and feedback from peers. In comparing and contrasting the constructivist epistemology to alternative traditions of discovery learning, problem solving and misconceptions, Confrey (1991) described the development of students' conceptions within a constructivist framework. She described value in diversity of problem-solving approaches, considering problems as crucial in the construction of knowledge and an interactive process where an appropriate answer to the problem will evolve for the student. Constructivists view mathematics as a human creation, where reflection and communication and negotiation of meaning occur socially and culturally (Alsup, 2004; Cobb, Yackel & Wood, 1992; Confrey, 1991; Confrey & Kazak, 2006; Lerman, 1989; Nuthall, 2002). It is this view of learning that seems to also characterise learning in a mathematics inquiry classroom.

Constructivist educators encourage their students to express their beliefs and accept alternative perspectives to their own. Constructivist teachers listen to their students' perspectives and try to understand them (Confrey, 1991). Although it is known that inquiry mathematics present cycles of investigation, reflection and communication where students construct understandings about mathematics (Allmond et al., 2010; Makar, 2012), further research is required to identify how students construct knowledge and the interactions that influence this construction, specific to an inquiry context. Assessment which focuses on mathematical content and correct conceptions, does not generally consider what students do know (as alternate conceptions to the teacher) and their processes of coming to know. This study explored the processes of construction of mathematical knowledge in inquiry settings, to inform assessment practices in mathematical inquiry.

A social view of learning

Social constructivist Lev Vygotsky (1896 – 1934) considered the relations between human beings and their environment in intellectual development. The large body of work he contributed to understanding learning and development socially, pervades educational research today. A key idea of his work relevant to this study focuses on learning in social interactions, particularly through mediated activity where the teacher expertly guides student learning towards their potential (Vygotsky, 1978). He included the notion of the zone of proximal development (ZPD) in the process of learning to describe the gap between what learners have already accomplished to do by themselves, and the problems that they are yet incapable of handling without expert assistance. Vygotsky (1978) referred to independent problem solving as the *actual developmental level* of a learner. When learners require adult guidance or collaboration with more capable peers to solve problems this determines their *level of potential development*. Scaffolding became the term to describe this process (Bruner, 1986, p. 74). In educational research, much attention has been paid to the gap between unassisted and assisted performance levels (ZPD), and what can be done to close this gap (Anghileri, 2006; Goos, 2004; Lerman, 2000), even at a whole class level (Smit, van Eerde & Bakker, 2013). It is worth exploring the social interactions in an inquiry classroom to see how mathematical learning is supported using this approach.

In inquiry, students often work in small groups to solve problems and are required to test their conceptions in interactions with their peers. Shepard (2000) described the "new" emergent paradigm where learning and development were primarily considered social processes, including a willingness to persist in solving difficult problems. The inquiry pedagogy can be seen to fit within this "new" paradigm as students develop personal understandings through a culture of participation. This describes a situative view of learning where participation in social practices of sense-making include, "abilities to use a rich variety of social and material resources for learning and (contribution) to socially organised learning activities" (Greeno, Collins & Resnick, 1996, p. 28). Situated perspectives shift attention from the behaviours and practices of individual students in mathematical situations, towards broader communities and the relationships formed through interactions within (Boaler, 1999). Social collaboration is valued as part of the learning process in inquiry and as such, assessment in inquiry will need to consider sociocultural views of learning.

A difficulty with trying to measure learning in inquiry may stem from the common idea that mathematical knowledge is something to be acquired. Sfard (1998a) used the acquisition metaphor to compare learning concepts or units of knowledge to accumulating material goods. Alternatively, she provided the metaphor of participation to describe learning as a constant flux of *doing* rather than *having*. Participation, she noted, is to do with taking part; part of a greater whole in a community of practitioners. In inquiry, the complex interactions that reflect students participating involve a wide range of characteristics. Simon and his colleagues (Simon et al., 2010) stated that learning occurred in the context of mathematical communication with peers, such as negotiating meanings, or sharing and comparing solutions. In these situations, students are required to justify their thinking when challenged by others, establishing shared mathematical ideas that provide the basis for subsequent work. In an inquiry classroom, Goos (2004; 2014) described this social context as a classroom community of inquiry. A classroom community of inquiry includes a sociocultural perspective of learning as students participate in a community of practice that is normal and accepted. Assessment that values the independent acquisition of mathematical concepts and ideas conflict with norms that value becoming a participant in a classroom community of inquiry.

The importance of getting stuck

Inquiry presents opportunities for students to 'get stuck' in their learning, or to grapple with ideas, and to refine conceptions through challenge, practice and discussion with peers. For constructivists, a problem is a roadblock or problematic, in relation to the solver (Confrey, 1991; Fielding-Wells & Makar, 2008; Koellner, Jacobs & Borko, 2011; Stigler et al., 1996). These roadblocks or perturbations are useful in activating assimilatory schemes that the students have available, becoming an important part of a teaching process (Simon et al., 2010). Often inquiry situations can be engineered by the teacher to evoke misconceptions and to capitalise on any unanticipated problems that arise (Makar, 2008; 2012). Moments when students seem to be stuck in an inquiry context present research opportunities to explore student learning. Reflecting a constructivist perspective, Harel and Koichu (2010) provided an operational definition of learning as a continuum of disequilibrium-equilibrium phases which also supports this notion of 'getting stuck' in learning. Although complex and multi-dimensional itself, the definition of learning they present (Harel & Koichu, 2010) was later used to describe learning in a chapter devoted to the pedagogy of mathematical inquiry (Makar, 2012), and served to inform the theoretical analysis of learning in an inquiry classroom, in this study.

Harel and Koichu's (2010) premise was based on the work of Piaget (1952) that considered how knowing "proceeds through a continual tension between assimilation and accommodation, directed toward a (temporary) equilibrium" (Harel & Koichu, 2010, p. 116). Their conceptual framework for learning and teaching mathematics was developed in earlier work by Harel (2008a; 2008b; 2008c) as *DNR-based instruction in mathematics*. This is the conceptual framework that describes learning taking place when the dual relationship between perturbations and knowledge construction (duality -D) arise from the intellectual and psychological necessity (N) to develop knowledge or understanding. Repeated-reasoning (R) of the new ways of thinking and understanding developed assist the learner to internalise, organise and retain the new knowledge, described by Harel and Koichu as essential to the process of learning. Harel and Koichu (2010) described how when one carries out a mental act (similar to the PISA competencies used in Chapter 5), the cognitive product is called a *way of understanding* (*WoU)* and is associated with the mental act. Cognitive characteristics of the newly formed *WoU* are referred to as a *way of thinking* (*WoTh),* associated with the original mental act. Harel (2008c) offered helpful examples of different *WoU*s related to the concept of fractions such as a part-whole *WoU* (¾ is 3 out of 4 objects) and a unit fraction *WoU* (¼ + ¼ + ¼). How a student thinks about or comes to these understandings is to do with their *WoThs*. For example, a related *WoTh* could be that "a problem can have multiple solutions" or "a concept can have multiple interpretations" (Harel, 2008c, p. 903). In inquiry, students think about the mathematics they are using, in an inquiry context where mathematical problems become important to students to solve. It will be interesting to consider the reciprocity between *WoUs* and *WoThs* about mathematics, developed in an inquiry classroom. This is especially the case when students' carefully planned decisions in inquiry don't go to plan, and students become stuck or challenged by 'road blocks' they meet. These perturbations and how students overcome challenge in inquiry may present opportunities for teachers to glimpse into how their students are thinking, to provide valuable assessment information.

A critical response to the DNR-oriented definition of learning presented by Harel and Koichu (2010), was the idea that learning did not always involve problem solving (Simon, 2013). A complementary instructional design approach was constructed by Simon (2013) which offered a contrasting approach to the DNR research program in terms of goals and methodology, to "promote conceptual learning for intractable concepts and students struggling to master particular concepts" (p. 282). Based on a previous research program (Simon et al., 2010), a contrast between DNR-based instruction and Simon's (2013) work on learning mathematics was that Simon's research program focused on the process of learning to consider the transitions students pass through from one conceptual state to another. However, both frameworks by Harel and Koichu (2010) and Simon

(Simon et al., 2010; Simon, 2013) focus on the changes in students' thinking about mathematics. A focus of this study was how and when changes to thinking occurred for students in inquiry settings.

Theories that view learning or knowing as an ongoing process or continua, instead of a point in time, also inform assessment practices in an inquiry pedagogy as cycles of investigation inform future learning experiences (Cobb, 1994; Harel & Koichu, 2010; Makar, 2012). Skemp noted as early as 1976 that thinking is never complete as the process of building relational ideas is self-continuing and consequently, self-rewarding. In vignettes offered by Makar (2012) to illustrate inquiry experiences in the classroom, she noted how after just one inquiry, aspects of learning were not apparent to the teacher. It wasn't until the students encountered the next inquiry that the classroom teacher noticed the value in previous learning experiences, including student struggle. Summative assessment practices that focus on an end-point in the learning process may need to reconsider that learning progresses over time when applied to an inquiry context.

What is to be learnt?

When learning is considered as a *thing* to be learnt, or a concept to acquire, research strives to articulate what each of these concepts must be. Learning trajectories have been generated to describe fractional schemes for example, in an effort to understand how students learn (Steffe, 2003). Duckworth (2006) considered that learning does not always need to be described as concepts or curriculum content. She suggested the importance instead of broader educational goals. It could be said that an aim of the inquiry approach to teaching and learning mathematics is for students to become aware of themselves as numerate citizens as they draw on their own conceptions of mathematics to solve real-life problems. For example, a classroom, mathematical inquiry to find out how healthy a class of students is may not have a mathematical aim articulated in mathematics curricula, yet is an example of an important social issue that can be explored mathematically. A mathematical inquiry to find out how many steps children take each day can also be a real-life mathematical problem; of interest to designers of shoes for children for example. What is to be learnt by students in either of these inquiry examples is the reliance on mathematics in the solution process although little is known about what else is valuable to students to learn.

Approaches to assessment that draw on views of learning conducive to an inquiry learning environment need to be identified, for teachers to see the teaching pedagogy as a valuable one. In this study, constructivist learning principles were used to frame how learning is viewed in an inquiry classroom, which in turn informs assessment practices in this context; assessment which is broad and flexible, and that considers learning in an inquiry context.

Where This Research Needs to Go

I return to the key question at the beginning of this chapter, *Where am I going?* This question relates to establishing some kind of benchmark for the findings of my own study to consider. The literature reviewed in this chapter highlighted gaps and inconsistencies when applied to an inquiry classroom context. For example, the kinds of knowledge, understanding and skills valued in mathematics classrooms are mis-aligned with the kinds of knowledge, understanding and skills valued in an inquiry setting. Inquiry pedagogy presents a contrast to the tradition of school mathematics, which then conflicts with assessment practices that are carefully designed to suit the school mathematics tradition. It is clear that there is a mismatch between the elements this study explores in relation to each other.

There appears to be consensus among researchers that the inquiry mathematics approach provides rich opportunities for learning (Fielding-Wells, 2010; 2014; Fry, 2013; Fry & Makar, 2012; Goos, 2004; Makar, 2007; 2008; 2012). The breadth of findings is widening as different researchers apply their own lenses to teaching and learning in inquiry settings. A variety of frameworks have been offered to describe learning in inquiry classrooms, each with their own lens. Goos (2004) established her work on Vygotskian frameworks to offer a generalised template for teachers to facilitate student learning. Allmond and her colleagues (2010) co-wrote three illustrative resource books for teachers condensing research and teaching experience. These books highlighted mathematical knowledge to be learnt and numerous valued skills to be fostered in students such as: working collaboratively, using tools to solve inquiry problems, communicating thinking and sharing findings, constructing representations and self-evaluating effectiveness of chosen approaches. Research by this author (Fry, 2011) suggested using the PISA assessment framework (OECD, 2009) to identify and describe learning that valued multiple types of understandings.

Questions still linger regarding the validity of this approach (Hattie, 2009; Kirschner et al., 2006). Kirschner and his colleagues (2006) described inquiry learning as a minimally guided approach as opposed to direct instructional guidance. They claimed there was no body of evidence supporting instructional techniques with minimal guidance and uniformly supported “direct, strong instructional guidance” (Kirschner et al., 2006, p. 83) as most effective. The teacher resources designed by Allmond and her colleagues (2010) emphasise a highly structured inquiry cycle of teaching and learning experiences: the 4D model. Ewing (2011) provided a very clear description of direct instruction, offering research supporting the approach in regards to purpose e.g., memorisation, mastery of facts, etc. The critique she offered highlighted a didactical, authoritative approach where transmission of mathematical knowledge takes place with no discussion and a competitive process defines and labels winners and losers. This is not an aim of inquiry pedagogy. Hattie (2009) ranked inquiry-based teaching 86th out of 138 influences on teaching and learning in terms of effect size. In terms of achievement outcomes, Hattie noted that inquiry or problem-solving methods seemed to promote the learning of surface concepts, whereas deeper concepts needed more specific and direct teaching. I would argue in response to their work (Hattie, 2009; Kirschner et al., 2006) that a goal of inquiry *is* the interweaving of mathematical, formal knowledge with the familiar, where mathematical thinking is generated and tested, and through specialising, generalising, conjecturing and convincing, mathematical thinking becomes a process of sense-making (Goos, 2004). When such criticism of the inquiry approach to teaching mathematics is widespread, it makes it difficult for teachers and schools to see value in the pedagogy and to adopt inquiry practices in classrooms. Findings from this study hope to contribute to the field of work already exploring the richness of inquiry in mathematics classrooms, in terms of teacher guidance and the richness of learning experiences.

Ways to collect evidence of improved learning are emerging. In this study, aspects of an inquiry mathematics classroom considers assessment practices as integral to all other 'runnings' in the classroom. The National Research Council (2001b) reported on the importance of curriculum, pedagogy and assessment being closely aligned in order not to disrupt or imbalance the teaching and learning process. They described how difficult it was to achieve this as approaches to curriculum, pedagogy and assessment change, and decisions made about these areas are taken at different levels of the educational system. Formative assessment practices are one method that may shed light on student learning as it takes place in inquiry. This research hopes to extend research in the field of formative assessment by looking more closely at the information it reveals about learning in an inquiry context, and understanding how the classroom teacher uses this information to give feedback to students to improve their learning.

This thesis aims to understand assessment, teaching and learning in the inquiry classroom. The Literature Review for this thesis was therefore structured to explicate current learning goals in mathematics classrooms, by identifying:

- *Knowledge, understandings and skills valued in mathematics classrooms,*
- *Inquiry mathematics; an explanation of pedagogy,*
- *Assessment and the views of learning valued by different assessment practices, and*
- *Learning in a mathematics classroom.*

There is potential for research to consider formative assessment as a way to identify learning in an inquiry classroom that considers learning mathematics as participation in a classroom community of inquiry. The difficulty will be in capturing the information about learning that generates from the frequent complex interactions as part of this process, and describing it in ways that are useful and that thoughtfully reflect what students decide to share.

Chapter 3 Theoretical Framework

This chapter will theoretically ground the pedagogy of inquiry, to make clear the purpose of this approach to teaching and learning. Research into classroom assessment has been critiqued for under-problematising theoretical issues (Torrance & Pryor, 1998). Multiple perspectives of the pedagogy of inquiry will be outlined throughout this chapter and their theoretical influences made clear. Firstly my own philosophical stance will be presented to frame the study within the constructivist paradigm. Duckworth's theoretical framework (2006) will be outlined next as this framework will ultimately assist in synthesising analyses of findings in this study, to draw together the theoretical contributions in succinct ways. A brief overview of the key theoretical ideas will be presented next which act as the frameworks for each phase of this study. To elaborate, contributions from three grand theorists in education, Dewey, Vygotsky and Piaget, will be applied to teaching and learning mathematics through the pedagogy of inquiry. Within each theorist's contribution, the notion of inquiry will be discussed through three perspectives: the student, classroom teacher, and the researcher. These three perspectives will be informed by the theoretical contributions to highlight the importance of experience, the importance of collaboration and the importance of doubt and equilibration in the pedagogy of inquiry. A brief summary of these contributions will be included at the end of this chapter (Table 3-1).

Using design research (Cobb et al., 2003) as the chosen methodology in this study, three phases of study will be engineered to explore the classroom elements of assessment, teaching and learning in inquiry mathematics classrooms. This thesis aims to understand learning in an inquiry mathematics classroom and through assessment, describe it in ways that become useful for a teacher in the day-to-day running of the classroom. Just as inquiry might be used to explore mathematics in a classroom, this thesis will present my own inquiry as a researcher, into the classroom elements of assessment, teaching and learning in an inquiry context.

To orient this, my own philosophical stance will be provided to situate my own qualitative inquiry into mathematical learning in an inquiry classroom. Corbin and Strauss (2008) elaborate on the ways that a researcher's theoretical orientation determines a committed approach to the research (p. 42). They describe how theoretical frameworks can be useful in determining methodological choices although they should not structure the research. My own philosophical stance will be presented as a guiding approach to the research in this thesis.

Philosophical Stance

Contributions to the field of knowledge production in mathematics education reflect various perspectives on conducting research in classrooms (Cobb, 2007). Decisions based on ontology consider and interpret order and reality in varied ways. Positivists want to capture and understand reality whereas feminists consider how differences in race, gender and class influence the apprehended world (Hatch, 2002). The view a researcher aligns with in turn influences the epistemological and methodological choices she or he makes, and knowledge produced will take different forms. Analysis may be inductive or deductive, or a combination of the two. It is within the constructivist tradition that I describe my own theoretical stance.

The overarching lens positions this thesis through a constructivist paradigm which considers the construction of multiple realities. Constructivist instruction in mathematics presents opportunities for students to construct their own mathematical understandings (Alsup, 2004; Confrey, 1991; Greeno et al., 1996). As a researcher and teacher, I aim to consider learning in the inquiry mathematics classroom from the perspective of the student, to consider their thinking. Constructivist research in the classroom places value on listening to students to see learning from their point of view (Confrey & Kazak, 2006; Steffe & Tzur, 1994). Cobb and Steffe (2011) presented their work on building mathematical models from a constructivist perspective. They explained how the models they formulated would be distinguished from what went on in children's heads. Similarly in this study, questions around learning will not be a priori based on themes in literature. Instead, concepts of learning will reflect interpretations of children's thinking from my own teacher-as-researcher perspective, through a constructivist lens.

I chose this stance due to the frustration I felt as a classroom teacher when information about student learning did not appear to match student learning observed through inquiry. It was essential through this research, for me to view student learning as a natural event in the classroom, without bias from myself as teacher and researcher, and without bias from assessment instruments designed to measure learning in different contexts. A constructivist perspective would allow me to consider what the students could know, learn and do through inquiry, unrelated to what I (as their classroom teacher) or the curriculum wanted the students to know. Qualitative in nature, this research is thus situated in a natural classroom setting myself as researcher as the data gathering instrument, and myself as teacher as participant in the study. Choosing to study this context and how learning takes place forced me to look more closely at the theories informing my beliefs about assessment, teaching and learning mathematics through inquiry pedagogy. It is through explicating these epistemological and paradigmatic stances that I am able to create a lens for interpreting the data I collect.

Beliefs, as things we might say to ourselves while learning

A challenge of this inquiry will be in describing what it is that students learn beyond what is currently measured through assessment. The title of this thesis highlights that there is difficulty in measuring learning that results from messy explorations of mathematical concepts. Literature reviewed in this thesis highlights the mismatch between assessment and the mathematics that students learn through inquiry. Constructivist researcher Eleanor Duckworth (1935-) was also concerned with how educational practice could recognise and value the many different ways that learners come to understand, ways that are all perfectly adequate. She challenged the belief that there is ever only one best way to do things, while accepting the difficulty in sensing the representations of others. A key feature of the inquiry pedagogy is the belief that students construct understandings in personal and productive ways (Goos, 2004) as they follow multiple pathways to solve mathematical problems (Fielding-Wells & Makar, 2012; Makar, 2012). A key goal in this study is to represent student understandings in ways that are respectful and that will contribute to the body of research about the inquiry pedagogy.

A particular challenge for Duckworth (2006) in analysing her research was the difficulty in the language used to describe learning. Her concern was the neglect of goals that sparked human interest or that developed self-confidence for example. She described how the aims or goals of learning could be stated as sentences rather than as concepts, the kinds of sentences that learners might say to themselves. She suggested four different kinds of beliefs that could categorise all types of statements about learning: '*the-way-things-are*', '*it's-fun*', '*I-can*' and '*people-can-help*' beliefs. The notion of describing all things to be learnt as beliefs will be used as an analytic tool in this thesis as a way to characterise student learning without limiting it to concepts, curriculum, or teaching objectives.

The first kind of belief (Duckworth, 2006) to categorise statements about student learning is termed '*the-way-things-are*' beliefs. These kinds of statements have to do with knowledge of the world; bodies of information and skills often described as curriculum content. Duckworth was concerned that classroom assessment was mostly addressing these kinds of beliefs. She recommended teachers be aware of the emphasis such assessment can make if the importance of learning becomes focused on accumulating such bodies of information or knowledge. She felt that teachers should place as much importance on teaching all four kinds of beliefs. This idea resonated with teaching mathematics through inquiry, and in this setting, assessing student learning.

Three other kinds of beliefs are described by Duckworth (2006). '*It's-fun*' beliefs are to do with self-motivation; sometimes we learn because we are interested to know. For example, it could be fun to collect data from peers in a classroom to find out how to plan a pizza party. Mathematics will be required to work this out but the topic may be meaningful to students who would love to have a pizza party. '*I-can*' beliefs are to do with self-confidence, such as teaching children to persist with solving mathematical problems when they are stuck. '*People-can-help*' beliefs incorporate a social aspect to learning where learners draw on others as resources, or consider learning as shared understandings. Each of these beliefs show promise for analysing what students may learn through inquiry mathematics, beyond '*the-way things*-are,' that is not be currently articulated.

Theory Informing Inquiry: An Overview

A collective influence of Dewey, Vygotsky and Piaget are used to outline the theoretical influences informing the pedagogy of inquiry to teach mathematics. Three key contributions have been selected spanning the works of these grand theorists and will be used to explain teaching and learning in an inquiry classroom. They are: the importance of experience, the importance of collaboration, and the importance of doubt and equilibration. These contributions will be applied in parallel to the notion of inquiry as a student who is learning mathematics, to a teacher's inquiry into her own students' understandings, and to a researcher's inquiry into how the classroom elements of assessment, teaching and learning align to support learning in this context. A summary can be seen in the conclusion of this chapter (Table 3-1).

One main tenet of the work of Dewey (1938a) in the early 20th century was the emphasis he placed on experience in education. He described education then as "pattern(s) of organisation" (p. xxx) where classroom schedules, rules of order, and bodies of information and skills from the past were transmitted to a new generation. He seemed to challenge the existence of '*the-way-things-are'* beliefs (Duckworth, 2006) as being the only knowledge to consider in the classroom. In Dewey's (1938b) book on Logic: The Theory of Inquiry, he explained that the necessary laws of thought were logic, and inquiry was the methodology that led to testing these laws of thought. Inquiry in the classroom allows students to test their own thoughts or logic leading to "expression and cultivation of individuality" and "learning through experience" (1938a, p. xxx). He described the relationship between actual experience and education as intimate and necessary. I have chosen this relational view between experience and education claimed by Dewey (1938a) to reflect the importance of everyday activity in mathematical inquiry, as an intimate and necessary experience. Inquiry is the attempt to link mathematical theory and everyday activity in ways that are meaningful to students, to foster '*it's-fun*' and '*I-can*' beliefs that urge learners to continue for enjoyment and satisfaction (Duckworth, 2006). In regards to the classroom teacher, everyday activity consists of a need to understand what their students have learned (Wiliam, 2010a), judged through the use of assessment. In this study, the researcher is undertaking an inquiry into the everyday experiences in an inquiry mathematics classroom for insight into ways that assessment, teaching and learning align to support learning in this context.

Social collaboration in inquiry is a notion already well-established in the field of contemporary mathematics education research (Goos, 2004; Sfard, 1998a; Steffe & Tzur, 1994). Vygotsky (1978) placed value on the social interactions during learning. When frustration occurred at being unable to solve a problem, Vygotsky explained how children enlisted the assistance of others through verbal appeal. Duckworth (2006) saw importance in teaching '*people-can-help*' beliefs explicitly to learners. In an inquiry classroom students work collaboratively to solve problems and are encouraged to share successes and challenges in problem solving with their peers (Fielding-Wells, 2010; Fielding-Wells & Makar, 2012). The teacher has a responsibility in the inquiry classroom to foster opportunities for productive collaboration (Goos, 2004; Makar et al., 2011; Makar, 2012) and to scaffold students through the inquiry and process (Fielding-Wells & Makar, 2012). The importance of social collaboration will be highlighted in the student interactions in inquiry classrooms explored in this study, and the role of the teacher in establishing the classroom environment; to guide students to assist them in achieving success with learning. This assistance can also involve fostering in students the development of '*I-can*' beliefs (Duckworth, 2006). Similarly, social collaboration from a researcher perspective is opportunity to collaborate with a community of researchers, to collectively analyse mathematical learning and share research findings.

Finally, to consider learning in the inquiry mathematics classroom I will refer to the importance of doubt and the process of equilibration. Dewey's (1891) early work on thinking described three aspects of the process of thinking as: conception, judgment and reasoning. Although judgment, he explained, was a typical act in this process, one's doubt challenged one's own judgments. Dewey noted how through doubt, the mind "learns to assume a state of suspended judgment" (p. 219). Later, Dewey (1938b) described how inquiry was related to doubt. His theory of inquiry explained how inquiry terminates when doubt is settled, yet the resulting knowledge often leads to further inquiry. Piaget (1952) similarly applied this notion of settling doubt as part of the learning process. He described learning new ideas by reaching equilibrium through processes of assimilation and accommodation. Piaget (1964) called upon four factors to explain cognitive development in children, with the final being equilibration: a process of equilibrating maturation, the total coordination of actions (from logical-mathematical experiences and physical experiences), and social transmission (the previous three factors leading to cognitive development in children). He described the process of equilibration as an active process of self-regulation by the learner, taking "the form of a succession of levels of equilibrium" (p. 181). This notion of equilibration can be seen to align with a view of learning in the inquiry classroom in the moments when students overcome challenges to their learning and achieve success. For students, learning in inquiry can be considered as a continuum, as a rocky pathway of successes and failures especially when the teacher presents an ambiguous problem to solve (Makar, 2012). For students, persisting on these pathways relates to learning '*I-can*' beliefs to do with self-confidence, and having the belief that one can do something to find out (Duckworth, 2006). The classroom teacher in inquiry places value on these 'ups and downs' as they learn to listen to their students (Confrey, 1991; Steffe & Tzur, 1994) to consider misconceptions or ideas that are different to their own. From the perspective of researcher, doubt has driven the need for my own inquiry into a mathematics inquiry classroom, to gain understandings of how the classroom elements of assessment, teaching and learning interrelate to support learning in inquiry. Dissatisfaction with how current assessment practices describe what students learn in the inquiry classroom has driven my own need to look more closely at inquiry in a classroom context.

Alignment of notions shared by these theorists reflects a strong link to constructivist orientations. Although constructivists describe a cognitive theory of learning, inquiry as a teaching approach values constructivist views of learning and supports a classroom environment that fosters these approaches to learning. This chapter does not intend to compare the ideals of Dewey, Vygotsky and Piaget, although Mayer (2008) does point out one assumption shared by all three: "each questioned how children might be taught to think in new ways and so move beyond lockstep re-enactment of the known" (p. 8). The theoretical perspectives presented in this study will support the pedagogical choices important to teaching and learning mathematics in the inquiry classroom, including the importance of: everyday activity, collaboration and establishing the classroom environment, and doubt and equilibration.

Dewey: The importance of experience

In the inquiry classroom

In inquiry, actively exploring mathematics in personally meaningful ways can assist in making mathematical problem solving more purposeful and relevant to the child's world. Already in 1891, John Dewey was presupposing that every fact could not be considered in isolation. In his chapter on thinking he referred to the element of relation which he explained, is thinking that is always embodied in relation and not separate or distinct from other thinking. By 1938(a), Dewey described progressive education as a product of the discontent with traditional education; insisting on the first to be more child-focused. He depicted traditional mathematics classrooms as lacking constructivist values of experience and instead, imposing subject-matter upon students that is foreign. Alternatively, Dewey described progressive education as valuing more intimate learning, through the personal experience of subject-matter and accessing or acquainting (students') past experiences to appreciate the living present. The relevance of students' worlds in the classroom, or connecting learning to students' past experiences, is a sound educational principle that subscribes to a constructivist approach. Dewey (1938a) placed emphasis on valuing an organic connection between education and personal experience to foster learning connections. It is this belief that learning comes about through experience that resonates with inquiry: the belief that students learn through involvement with rich contextual experiences of solving challenging, mathematical problems.

Through inquiry, in-depth and open-ended exploration of mathematical topics that are real-life or everyday, or that are important for students to solve, attempt to assist students in building deep understandings of the mathematics they are using. The US reform in teaching mathematics in the 1980s and 1990s led to reviews of education such as *Adding it up: Helping children learn mathematics*, and *How students learn: Mathematics in the classroom* (National Research Council, 2001a; 2005) which both supported the argument that understanding was vital to learning mathematics, as opposed to acquisition of procedural knowledge or solving problems that are disconnected from meaning making. Comparisons that contrasted teaching mathematics for understanding as opposed to teaching skills can be found in the work by Boaler (2002) amongst others (Brophy, 1986; Cobb, Yackel & Wood, 1992; Kilpatrick, 2009; Makar, 2012; Sullivan, 2011). Now recognised for her work in reform approaches to teaching, Jo Boaler has been a key figure in the implementation of the Common Core Mathematics Standards (National Governors Association & The Council of Chief State School Officers, 2014) in the US. Her input is reflected in the descriptions of the importance of mathematical understanding, being equally important to procedural skill. Recognition of the need to transform mathematics education (Gravemeijer, 2014) has encouraged a shift in schools towards inquiry-like practices that are realistic and which allow students to construct their own knowledge. In inquiry, realistic or everyday topics are used to link mathematical learning to personal learning experiences.

Context plays an important role in situating and providing meaning for answers to an inquiry question. Everyday contexts produce mathematics that is often messy yet meaningful. *How healthy is our class?* is an inquiry question that could launch students into collecting data about lunchboxes, to compare with exercise habits of their friends, or measurements collected to compare ratios of body parts. Artificial data can de-contextualise a problem for students although it may be carefully selected to demonstrate a particular mathematics purpose. Makar et al. (2011) showed how statistical inquiries can help students understand our world and saw value in students participating in the entire process of investigation. In their paper they connected statistical inferences emerging from inquiry data investigations to puzzling questions about the world. In an inquiry classroom, children are encouraged to solve problems purposefully; through essential connections to personal experience every day.

Teacher perspective

Repeating Dewey's educational principle of linking learning to personal experience as everyday activity in the classroom, we begin to consider what the everyday activity or experience of a classroom teacher is like in an inquiry classroom. If we broadly consider a goal of education as learning, then teaching must strive to know what or when learning is taking place (Black & Wiliam, 1998a; Hattie & Timperley, 2007; Sadler, 1989). I will apply this view to describe the 'everyday' for teachers in the classroom as assessment. Whether inquiry is the pedagogical choice or not, teachers constantly evaluate or judge the amount of learning that takes place in their class through assessment.

In inquiry, the challenge lies in employing assessment practices designed for more traditional classroom settings. In a similar sense to the traditional school described by Dewey (1938a), traditional notions of scientific measurement also influenced assessment approaches. These influences are referred to as the dominant 20th century paradigm by Shepard (2000) and are elaborated in Chapter 2 in the section describing knowledge, understandings and skills in influences are referred to as the dominant 20th century paradigm by Shepard (2000) and are elaborated on in Chapter 2 by the section describing knowledge, understandings and skills in mathematics classrooms. Sfard (1998a) used a metaphor of acquisition to describe when learning is considered as the activity of accumulating material goods. This is the view which seems to be reflected in traditionally-aligned assessment that values notions of students 'getting it' right. In inquiry this raised questions of *what* to consider as ‘right’; is it the acquisition of taught knowledge (considered as right by the teacher); or the knowledge applied to solving a contrived problem? Schooling traditions such as assessment have presented challenge to US schools implementing inquiry-based learning approaches (Schoenfeld & Kilpatrick, 2013). High-stakes tests led to teachers increasingly “teaching to the test.” This resulted in less chance to conduct curriculum projects that weren’t related to the goal of test performance, such as implementing an inquiry based learning-inspired curriculum. Assessing mathematical learning, as part of the everyday activity of a classroom teacher, needs to be broad and flexible if it is to value learning in an inquiry context.

Often in the inquiry classroom, decisions around which context to explore are made by the teacher. Although ideally inquiry ideas that students generate will be of importance to those students, initially not all students will have wonderful ideas to begin with. Duckworth (2006) explained how “making new connections depends on knowing enough about something in the first place to provide a basis for thinking of other things to do” (p. 14). A teacher’s responsibility in inquiry is to engage students in the issue and assist them in connecting experiences (Makar, 2012). Although inquiry as a constructivist notion is a student-centred pedagogy, we start to see the teacher’s role in valuing, guiding and selecting learning experiences as part of their everyday activity and regarding assessment, valuing the related everyday learning experiences that unfold.

Researcher perspective

This study presents my own inquiry into mathematics classrooms as part of my everyday experience as a researcher. Teachers and educators can use inquiry as a tool for critically engaging with issues in mathematical teaching practice (Jaworski, 2006). This section will elaborate on my experience as researcher, of engaging with inquiry as a theoretical principle and present my own inquiry into mathematics teaching and learning. Research into mathematical education *is* inquiry into what happens in mathematical classrooms. Studies vary in methodologies in an attempt to glimpse into what is happening in these contexts from different vantage points. A landmark longitudinal study into classroom practices by Boaler (2002), discussed the question of which approach to use to teach mathematics. She presented this through stories which gave detailed insights into the ways that mathematics *teaching* affected mathematics *learning.* This robust and well-documented body of findings explored what students could do in the contexts of the classroom communities in which the learning took place, noting that “the activities of different practices are central to what is learned” (p. 132). Similarly, it is my own inquiry into the activities in inquiry classrooms that will be central to understanding teaching and learning in an inquiry context.

The experience of my own research inquiry is distinct in that I play the dual role of teacher and researcher. This meant that I had the privilege of being close to the issues that would drive my research, with an aim to provide solutions to the dilemmas facing mathematical learning through inquiry. When the researcher acts as teacher they become an interactive participant in communications with the child (Steffe, 1991). The benefits of researchers acting as teachers is that there is no substitute for the intimate interactions involved in teaching children when exploring children's constructions of mathematical concepts and operations (Cobb & Steffe, 2011). Acting as teachers when conducting research allows researchers to continually test and revise research understandings, consider their own influence in the students' construction of knowledge, and place importance on the context where construction of knowledge takes place. Although my interactions with the students was intertwined with my research focus, how my students learned mathematics through inquiry reflected the day-to-day 'runnings' in the classroom. Exploring the interactions in my own classroom through research would seem less intrusive to students, yet could be more insightful than an outsider's perspective.

Summary: The importance of everyday activity

Situating learning in the experience/s in which the learning took place is a key principle of progressive education described by Dewey (1938a). This section has applied the notion of experience to reflect three different perspectives, of the student in the classroom, the teacher and the researcher. In the mathematics classroom, inquiry experiences can assist in making mathematical problem solving more purposeful and relevant to the child's world. For example, finding ways to consider area and perimeter, or using large numbers, can be purposeful and meaningful when the situation in which to explore this is also relevant to students. Inquiry questions present students with problems to be solved that are open-ended to encourage in-depth exploration. An inquiry question can hook students into solving a mathematical problem where meanings become purposeful and intertwined with the context. The teaching experience in an inquiry classroom still includes classroom elements of making teacher judgements of performance and academic achievement. When these processes are teacher-directed and narrowly defined by curriculum, they become inflexible in supporting wonderful ideas and learning in an inquiry context. Finally, my perspective as teacher and researcher reflects my own experience of inquiry into the richness in learning that presents in inquiry.

Vygotsky: The importance of collaboration

In the inquiry classroom

Student learning in an inquiry classroom is guided carefully by the teacher as they balance teaching and facilitating. Vygotsky (1978) questioned how children could be expected to learn without the influence of an expert, entertaining “the notion that what children can do with the assistance of others might be in some sense even more indicative of their mental development than what they can do alone” (p. 85). He saw value in teachers understanding their students’ zone of proximal development (ZPD) to assist a child to be able to ‘do’ by herself, tomorrow. Feedback gained through formative assessment can inform the teacher about where students are in their learning, making students’ ZPDs visible. In the inquiry classroom, the teacher fosters opportunities for children to work in their own ZPD as they interact and cooperate with peers; and supports students through the openness of the inquiry question which allows students to access mathematical problems at many levels. Although the ZPD describes the interactions between the classroom teacher and students specifically, this notion has been applied to collaboration more broadly to include collaboration between students and the teacher and their students (Goos, 2004). Working collaboratively in an inquiry context, with teacher guidance, allows peers to see other successful and creative ways of solving problems.

Cultural activity plays a key role in the inquiry classroom as mathematical problems are explored in relation to the context they are presented in. Students work collaboratively in inquiry, in ways that may be similar to the real world (Goos, 2004; Makar, 2012). Conversations students conduct, with their teacher and with their peers, offer a network of relations in which a student can become entangled. In understanding intelligence from a biological perspective, Piaget (1952) described how intelligence did not happen independently of the relationships one had with the environment. Processes of externalization and internalization are supported in inquiry when students are encouraged to pause to reflect and share successes and challenges in solving the problem. Vygotsky (1978) referred to Piaget as he discussed the importance of communication as a source of development, noting “that internal speech and reflective thought arise from the interactions between the child and persons in her environment” (p. 90). In the inquiry classroom, conversations that students have with each other and their teacher, whether structured by the teacher or not, allow for students to externalize and internalize their own mathematical thinking.

The collaborative nature of solving inquiry problems presents a view of learning that is different to learning in a conventional mathematics classroom. In a conventional classroom, students often work independently to complete decontextualised algorithms to illustrate correct acquisition of knowledge, and the teacher is the only provider of feedback. In contrast, the social environment in inquiry views learners as interested in becoming a member of the community, contributing to the shared knowledge being developed in context. Sfard (1998a) uses the metaphor of participation to describe learning in this way, "a 'constant flux of doing' rather than an end point which signifies a halting signal" (p. 6). More recently, Goos (2004) associated this school of thought with Vygostkian frameworks to analyse learning in the social environment in inquiry. More specifically, she termed the learning environment as *communities of mathematical inquiry,* placing emphasis on the interactions between students and the knowledge they interweave as the mathematical culture of the inquiry classroom. This culture takes time to establish and is fostered by the classroom teacher.

Teacher perspective

Establishing the classroom environment in inquiry places strong emphasis on the teacher's role in guiding and scaffolding learning, through participation in a community of inquiry. Glassman (2001) compared perspectives on educational practice from Dewey and Vygotsky and provided strong reasons from both to show that the teacher facilitates learning, differing only in what they suggest should be facilitated. He highlighted Vygotsky's views on experience emerging through direct communication between social interlocutors and neophytes. In inquiry, the teacher models and supports the social interactions that support learning, scaffolding to draw the child closer to the socially defined goals of solving a specific mathematical problem. Glassman (2001) presented Dewey's interpretation of facilitator as more distant, where the teacher discovers doubt along with the child. The teacher in an inquiry classroom is able to expertly guide students towards the use of successful mathematical strategies and content knowledge by drawing collaborative knowledge together in ways that make sense to students. The teacher respects challenges students encounter and attempts to further student thinking about their own mathematical solutions. This in turn establishes the classroom environment as supportive, more child-centred and one that allows exploration of mathematics in ways that are meaningful to students.

For the classroom teacher, difficulty can arise in the shift away from conventional teaching styles of mathematics including lecturing where students transcribe what the teacher prescribes (National Research Council, 1989), to the inquiry pedagogy which values students learning through collaboration. In the US, challenges arose as teachers transitioned to teaching mathematics to meet reform requirements that included having their students work in groups and engaging in discussions (National Research Council, 1989). Importance was placed on students being able to "examine, represent, transform, solve, apply, prove and communicate" ideas with peers (National Research Council, 1989, p. 59). Borko et al. (1997) explained how reform visions of learning and teaching would require teachers to make major changes to their teaching practices. Their research, although focused on aligning assessment with the instructional goals presented by the reform movement, acknowledged that the change would not be accomplished overnight. Makar (2007) reported on the difficulties teachers experienced learning how to teach with inquiry in mathematics. She commented on how teachers needed to "develop their capability with the approach" (p. 69) likening this development as an ability to envision and embrace inquiry. Makar identified a number of supports or connection levers to assist teachers in this process which illustrated a lengthy, and possibly expensive approach to professional development. She highlighted teaching in inquiry as supporting students in decision-making, noting a balance between collaboration and independence. In later research (Makar, 2012) she further noted the importance of incorporating collaboration, as a twenty-first century skill, in inquiry. Collaboration between students in inquiry is a key element of the learning process, and becomes an important element for teachers of inquiry to foster in their classrooms.

Research into the role of the classroom teacher in inquiry considers how the teacher guides and facilitates learning. Yackel and Cobb (1996) describe the role for teachers in inquiry to establish socio-mathematical norms as essential. The phrase *Communities of Practice* was used in 1998 by Wenger to encompass learning as social participation. This social element becomes part of the pedagogical practice of establishing inquiry norms, to guide students to value meaning-making and working collaboratively (Makar, 2012). The notion of *Communities of practice* has already been applied to research conducted in inquiry classrooms (Goos, 2004; 2014) with a focus on the teacher's role in establishing those norms and practices, to foster opportunities for students to learn. A common element in this research (Makar et al., 2011; Makar, 2012; Goos, 2014), is the focus on the teacher and the responsibility to establish socio-mathematical norms in inquiry.

For classroom teachers, questions into their own teaching practices can lead to undertaking action research into their own classrooms. To motivate, direct and sustain such research collaborative structures can provide support through collegial interactions (Jaworski, 1998). As part of my everyday experiences as a classroom teacher I was interested in improving teaching and learning experiences in my own mathematics inquiry classrooms. Collaboration with established educational research assisted me in this process. Teachers-as-researchers undertake research in natural settings to pursue their own research questions and can assist in closing the knowledge gap between researchers and practitioners (Jaworski, 1998). I was able to participate in research projects while undertaking my own research, and gained support to jointly present findings at educational conferences. Collaboration between myself, other teachers and university researchers supported me as teacher-as-researcher to present my own research at conferences. I hope that my own research will contribute to the growth of knowledge within projects I participated in, and the fields of assessment in mathematics, and teaching and learning in mathematical inquiry more generally.

Researcher perspective

Where do the Vygotskian elements of social collaboration and learning as participation appear when you are a researcher in education? Building on the ideas of others and scaffolding the development of ideas are supported through sharing research with a community of mathematics researchers; at conferences and in peer-reviewed publications. English (2008) described the international sharing of research in mathematics education, as a "global effort to improve student performance and make mathematics accessible to all" (p. 6). Worldwide, professional associations exist for those interested in mathematics education research. In Australia, the Mathematics Education Research Group of Australasia (MERGA) aims to promote, share and co-operate on quality research particularly in Australasia (Perry, Lowrie, Logan, MacDonald & Greenlees, 2012). MERGA publish four-yearly reviews to disseminate all Australian research into mathematics education. Productive collaboration between professional communities interested in mathematics education has been recommended to enrich mathematics education and for research development to be more integrated (Goos, 2014). From a researcher perspective, learning as participation in a community of researchers provides the opportunity to have your own ideas about research considered by peers in the field on education.

Questions arise about how to conduct research, and interpret and present findings in ways that align with the philosophical stance of the researcher. Research into mathematics education reflecting constructivist notions, describe it as learning to listen or trying to fathom human thinking (Confrey, 1991; Sfard, Forman & Kieran, 2001). In a guest editorial, Sfard et al. (2001) presented the dilemma of the proper method of inquiry for rigorous research in mathematics education and called for a way that would bridge the relationship between social and individual research perspectives. Criteria borrowed from scientific methodologies may not match the original focus of research and prove insufficient in gaining insight into complex learning environments (Sfard et al., 2001). As research into the field of inquiry mathematics broadens, a kaleidoscope of conceptualisations (Artigue & Blomhøj, 2013) present the need for a more unified structure.

Research into the effectiveness of inquiry contributes to refining understandings about teaching and learning in the inquiry classroom. An aim of this research is to directly influence what is happening in mathematics classrooms. Research conducted at the classroom level by teachers could be used to encourage teachers to think about their own practices. Breen (2003) identified a gap between teacher research and research in academic literature, where teachers were separated from the larger community of researchers. Breen contrasted teacher research to formal, theoretical knowledge about teaching, where the former placed teachers at the centre of the project. Teachers becoming involved in critical exploration of their practice through research and reflection aim to influence change in educational settings (Breen, 2003; Goos, 2008). Collaborative research by Black et al. (2004) into improving formative assessment in the classroom, described collaboration as almost essential when trying innovations; sharing through discussions or mutual observations provided necessary support for groups of colleagues experimenting with innovations. It is with this stance that I continue my own research into assessment, teaching and learning in the mathematics inquiry classroom, in an effort to influence change in educational settings.

Summary: The importance of collaboration

This section extends the Vygotskian idea of the social community as a source of change. In inquiry, students are encouraged to interact with and assist each other. The classroom teacher plays a pivotal role in establishing the social context, and the researcher shares research with a wider community of experts. The openness of problems to solve in inquiry allows all students working collaboratively to access the mathematical content at some level. Similar to professionals working collaboratively in the real-world, inquiry encourages students to share successes and challenges in solving problems and learning is viewed as participation in a community of inquiry. Of course, these relationships are reliant on the teacher's beliefs and values in learning and the social interactions that they support. My own research, conducted through inquiry, is shared and reviewed with the wider education community to encourage feedback. My own learning through participation in this field aims to influence change in educational settings at the classroom level.

Piaget: The importance of doubt and equilibration

In the inquiry classroom

Dewey (1938b) described a view of learning as a continued process of forming doubt and reaching settled conclusion. Although Piaget (1952) described the biological concepts of adaption, he noted the processes of assimilation and accommodation taking place when the mind found difficulty assimilating new ideas with old, having to accommodate the new ideas to fit and make sense. Learning, he described, happens when equilibrium between the processes is reached. Harel and Koichu (2010) use Piaget's description of this conflict between new and old knowledge in their operational definition of learning. They similarly describe learning as the 'essence of change' when perturbations cause a natural inquiry to satisfy or solve a problem and repeated use of an effective process solves future problems successfully. This view of learning resonates with learning in an inquiry classroom. Often, students are perturbed in inquiry when their own beliefs are challenged. The teacher may even design roadblocks to test student plans and insist on evidence to defend their ideas. The dual relationship between provoking a students' intellectual need to learn mathematics through doubt, with ways of understanding and thinking that are developed, is the essence of learning described by Harel and Koichu (2010), and synonymous with classroom practice in an inquiry classroom.

In explaining what inquiry is, Dewey (1938b) described how it was related to doubt, where doubt forms the questions that drive human inquiry. Investigations, probes and inquests into doubts are just part of the thinking process on the way to knowledge, where knowledge terminates inquiry. Skemp (1976) also considered how schemas are never complete, especially when a relational view of learning is applied. He saw ambiguity in the term knowledge and considered knowledge resulting from inquiry as a supposition instead of a truism. Reaching a settled conclusion (as a result of inquiry) would be a continuing process in that there is no guarantee that a settled conclusion will remain settled. Dewey used this notion of removing doubt to reach settled conclusions to describe the continued process of learning through inquiry. An inquiry may not lead only to one particular conclusion; it would hopefully lead to further inquiry as described by Dewey: "The attainment of settled beliefs is a progressive matter; there is no belief so settled as not to be exposed to further inquiry" (Dewey, 1938b, p. 8).

My interpretation of Piaget's work on equilibration (1964; 1977), also included the idea of doubt although the term was not cited in his work. Piaget (1977) described the moments when students solve problems that they previously could not, as the assimilation of new ideas into existing frameworks and accommodating schemes to the special characteristics of the object. More recently, Glassman (2001) described doubt presenting when a situation does not have an easily recognised end-in-view. He noted that the true motivator for learning is an interest in reconciling thinking when conflicts arise. In inquiry mathematics, doubt (or an indeterminate situation that needs solving) can be presented in ambiguities in the overarching question. This ambiguity assists in broadening criteria or opening up more opportunities for students to make inferences from data they collect (Makar, 2007). Doubt can also present to a learner in interactions with peers, or when students become stuck in solving inquiry mathematics problems. It is these moments of doubt in an inquiry classroom that may stimulate learning mathematics in meaningful and personal ways.

Ambiguity in an inquiry question can promote messy situations to explore mathematics in the classroom. Such situations provide contexts or constraints for sense-making, similar to real-life problems (Makar, 2012). Traditional school mathematics, on the other hand, often cues students in to any uncertainties. Neatly presented problems can take purpose and audience away from context-laden reasoning and meaning-making (Makar, 2012). For example, in an inquiry to create the best map (Fry, 2013), ambiguity was presented by the word 'best' as solutions depended on the purpose for which the map was needed. Fry used the operational definition of learning, described by Harel and Koichu (2010), to illustrate mathematical learning for students in this inquiry taking particular note of moments when challenges were pursued, students solved problems, or thinking was reconciled. The need for students in that inquiry to develop knowledge and understandings about the best map arose from perturbations faced by the students through the inquiry. The best map for one person would not always be the best map for everyone. Students needed to agree on what the word 'best' meant, challenging them to generate criteria to describe this. After all, an everyday real problem is not a problem unless doubt questions your own beliefs.

Teacher perspective

Although the work of Vygotsky focused heavily on the role the adult played in the enculturation or education of children, he also saw importance in being able to reconcile thinking when conflicts arose. In Glassman's (2001) comparison of Dewey and Vygotsky and their views on society, experience and inquiry in learning, he described how both theorists recognised the importance of reaching stability when the mind is pushed from comfort into conflict. Vygotsky's major concept of the Zone of Proximal Development (ZPD) incorporated this principle of conflict arising when a student is unable to solve a problem. This partly influenced the model of mathematical learning developed by Steffe and Tzur (1994) which was based on the constructivist views of Piaget and Vygotsky with an emphasis on neutralizing perturbations through social interactions in a mathematics classroom. They concluded it was the teacher's responsibility to induce perturbations within the child's zone of potential construction, to allow the children to experience perturbations for themselves. Stillman et al. (2009) acknowledged the challenge of having students accept discomfort as an opportunity to grow. They turned to targeted scaffolding as a way to support students and enable them to learn beyond their own capabilities.

While it seems that doubt is pivotal to learning, it does not appear to be considered as such by traditionally-aligned assessment practices which measure how much taught knowledge has been replicated correctly in the students' efforts. In this sense, errors or doubt presented by students are disregarded. I again turn to this notion of assessment as part of the everyday perspective of the teacher. Where an aim of education for students is learning, a focus for teachers can be on evaluating the learning that has taken place. Confrey (1991) described the importance of listening, based on constructivist notions that regarded errors and alternate ideas within the framework of an individual’s experience. A constructivist defines the mathematical problem to be solved in relation to the solver (is problematic to them) and listening to students would allow the constructivist teacher to imagine how a student is viewing the problem (Confrey, 1991). Generally, terms used in assessment are generic and applied to all students in terms of curriculum content rather than posed in terms of the learner’s problematic.

In inquiry, each student experiences different aspects of the problem posed depending on the pathways or solutions chosen by the collaborative members of the group. The teacher, rather than compare one child’s learning to a continuum, considers how the mathematics approach chosen was useful to the context and considers student thinking as alternate ideas that are relevant to the solver. Confrey (1991) described how teachers should encourage students to describe their beliefs and challenge teacher assumptions, to gain insight into students’ perspectives. In inquiry, the teacher considers doubt, presented as: non-examples, challenges from or inconsistencies between peers, deductive reasoning that does not present the common understanding or that challenges the expected result.

Researcher perspective

In my own experiences as a teacher of mathematics, I found conflict between how learning was depicted through classroom assessment, and how I saw students as learners in inquiry. How could I begin to find and understand the development of students’ ‘wonderful ideas,’ described by Duckworth (2006) as the “essence of intellectual development” (p. 1)? Design research (Cobb et al. 2003; Confrey, 2006) was a methodology I could use to test and explore my own suppositions about assessment, teaching and learning mathematics through inquiry. The methodology would allow me to systematically study mathematical inquiry experiences as classroom teacher and researcher, engineered to test suppositions depending on my own doubts to instigate the next cycle. The process would ideally result in greater understanding of the inquiry *learning ecology* (Cobb et al., 2003) by incorporating my own inquiry focus on assessment, teaching and learning as elements of this ecology. A learning ecology is considered a complex system, interactive in nature, which involves “multiple elements of different types and levels” (Cobb et al., 2003, p. 9). This description fits with the inquiry classrooms presented in this thesis. Through design research, the design of inquiry classroom elements in this study will allow me to anticipate how these elements function together to support learning. My own doubts about how these elements interrelated would be subjected to iterative phases of testing and revision (Confrey, 2006), seeking to produce theories on how these elements of an inquiry classroom improved student learning.

Views on how students learn mathematics are often reflected in the various pedagogies for teaching mathematics. An operational definition of learning described earlier by Harel and Koichu (2010), acknowledged reaching equilibrium between assimilation and accommodation, and showed promise as a way of describing learning in an inquiry classroom that considered the theoretical standpoints presented in this chapter. Their DNR framework incorporates the moments when students doubt their learning, or struggle with mathematical ideas, as part of the process of learning. This framework (Harel & Koichu, 2010), has previously been used to describe learning in a chapter devoted to the pedagogy of mathematical inquiry (Makar, 2012). An aim of my own research is to refine the phenomenon of learning within the context of inquiry, to include moments of doubt, using Harel and Koichu's framework (2010) as an analytical tool.

Summary: The importance of doubt and equilibration

Throughout mathematical inquiry, there are moments when the solutions students decide upon fail to solve the problem. For instance, data students collect may no longer be useful in answering the original question, or enacting plans might reveal materials that fall short. Once confident, uncertainty prompts students to doubt their unsuccessful strategies. In inquiry, these moments are seen as wonderful opportunities to better understand why a particular strategy did or did not work. Value is placed on moments when students can continue no further as this is an opportunity for learning to take place. In fact, the design of an inquiry question is to present a situation that is doubtful. It is therefore important that the classroom teacher in an inquiry context listens to solutions children present beyond the acquisition of taught knowledge, and beyond notions of being incorrect. When the elements of assessment, teaching and learning do not share similarly aligned values, then a mismatch of practices presents an underestimation of children as learners of mathematics in inquiry. My own research will use the design research methodology to test my own doubts and suppositions about learning in inquiry, with an aim to produce theories on how the elements of assessment, teaching and learning can align to support student learning in an inquiry classroom.

Conclusion

The theoretical perspectives presented in this chapter, draw on three theorists in education: Dewey, Vygotsky and Piaget. The aim has been to present how these perspectives may align with and support the use of the inquiry pedagogy in the mathematics classroom, while being inclusive of a range of constructivist views. In education, when pedagogical choices side with particular theoretical orientations, an imbalance can present in the classroom. In one extreme, this imbalance can result in more emphasis being placed on classroom mathematics that is abstract and that “force-feed(s) inert facts and procedures shorn of any real-life context” (O’Brien, 1999, p. 434). Teacher-centred approaches can prevail as teachers provide the information needed for students to gain knowledge or skills, and then in an effort to teach more in less clock-time, check whether students ‘have got it’ before proceeding to the next step (Hattie, 2009). Likewise, students end up only listening to and watching teacher demonstrations; practicing what was demonstrated using textbook exercises (Goos, 2004). Instead, an inquiry approach can be seen as ‘part’ of a whole approach to teaching mathematics; its implementation used to achieve a balance of teacher- and student-centeredness. I have not meant to singularise each theorist’s particular stance nor polarise teaching mathematics as teacher- or student-centred. I instead hope to highlight the importance of three particular beliefs about education that resonate with the inquiry approach in the classroom, as part of a broader approach to teaching mathematics. A brief summary of these theoretical contributions are outlined below (Table 3-1).

Table 3-1 An overview of the theoretical influences explored in this thesis.

	Inquiry:		
Theoretical underpinnings of the Inquiry pedagogy	**In the inquiry classroom: Student level**	**Teacher perspective**	**Researcher perspective**
Inquiry Dewey (1938a): everyday **activity** and the importance of **experience**	Inquiry experiences attempt to hook students in to the problem, as they are connected relevant and purposeful.	Making judgments through assessment as the everyday activity of teachers in the classroom.	Inquiry into the everyday experiences in an inquiry mathematics classroom.
The importance of collaboration Vygotsky (1978) : learning presupposes a specific **social** nature, inquiry is embedded within **culture** and social interactions, **teacher** as mentor	Learning through participation in a community of inquiry, working collaboratively to solve inquiry problems.	Teacher acknowledges a collective zone of proximal development and scaffolds complex tasks. Assessment values this. Teacher responsibility to build collaborative learning environment.	Collaborating with a community of researchers e.g. conferences, peer review, colleagues.
Learning Dewey (1891, 1938b & 1966): **doubt** as “a state of suspended judgment,**”** and Piaget (1964 & 1977): **equilibration** drives learning, through processes of assimilation and accommodation	Messy problems lead to being stuck and needing to overcome challenges. Multiple pathways to solving problems are valued.	Teacher consideration of misconceptions, listening to errors students make and considering student perspectives.	Engineer inquiry learning experiences to test suppositions about teaching and learning, as part of Design Research.

Finally, I hope to draw on the framework described by Duckworth in the beginning of this chapter, to assist in analysing the interactions of classroom elements in an inquiry mathematics classroom. It seems that the theoretical contributions presented in this chapter by Dewey, Vygotsky and Piaget, to reflect the inquiry pedagogy, show promise as a way to develop the beliefs that Duckworth described. These contributions explain the underlying theoretical principles of inquiry for the learner, teacher and researcher in this thesis. The Discussion Chapter will apply this lens more closely to help consider the findings in each phase of analysis.

Chapter 4 Methodology

Constructivist perspectives guiding this qualitative study informed analysis and interpretation of the relationships between assessment, teaching and learning in the educational setting of inquiry mathematics. As is the nature of this paradigm, the aim of this study is not to claim 'what is/are' the relationships between these three elements, but in this instance, to see if any alignment between these elements and their related interactions can take place. For instance, I imagined it might be possible to identify and position inquiry classroom practices regarding assessment using a continuum from those reflecting principles of scientific measurement through to rich, qualitative illustrations of learning, and respectively for the elements of teaching and learning (Figure 4-1). Where it may be possible to identify assessment practices that reflect scientific measurement of learning, are teacher-centred and focus on an acquisition of knowledge, there may also be a misalignment between these classroom elements in an inquiry mathematics classroom. Prospectively, there would be potential for an alignment of these elements to the pedagogy of inquiry to take place (Figure 4-1). A constructivist lens guided the choice of design research (Cobb et al., 2003) as the methodological approach and grounded theory, as described by Flick (2009), to structure analysis. A goal of analysis through design research was to produce inferences about the ways assessment, teaching and learning were linked and patterned in specific ways, in a local classroom context. These inferences were based on my own interpretations of a purposeful mix of qualitative research methods: participant observation, artefact analysis and interviews (Hatch,

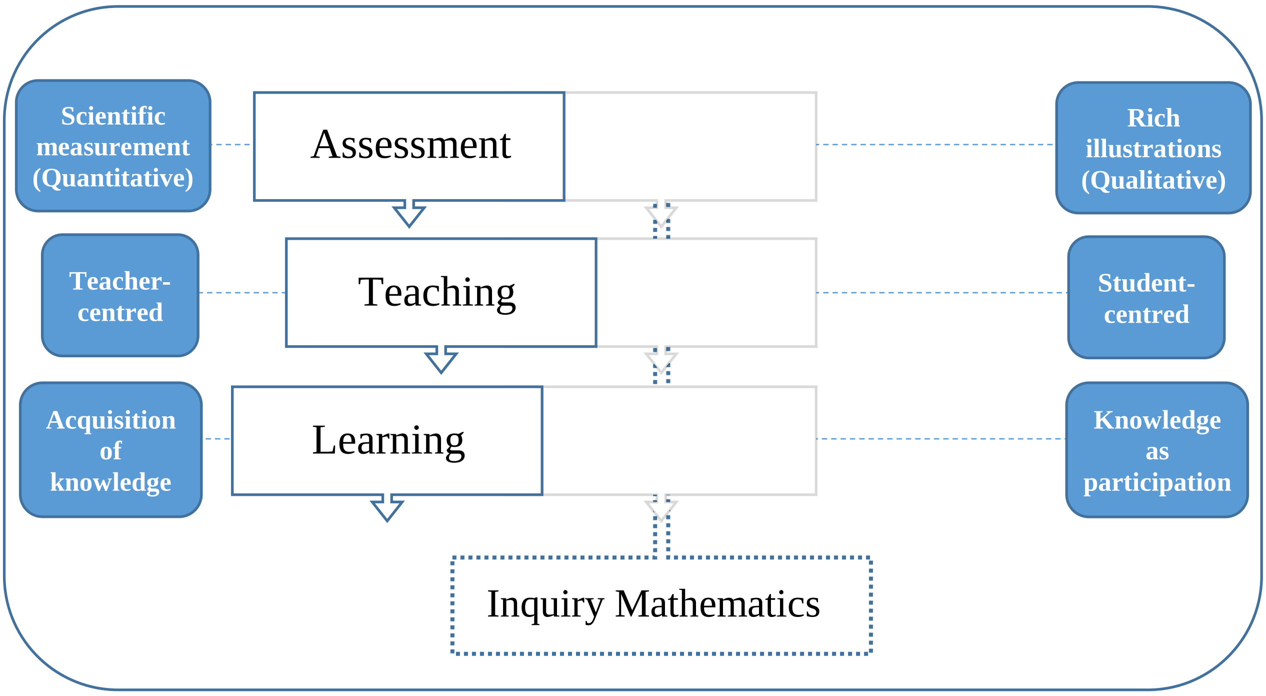


Figure 4-1 The misalignment of assessment, teaching and learning in inquiry mathematics.

2002). Qualitative research approaches are so broad and varied that Hatch justified each study as having its own unique character. He further explained how this character can develop and often change as studies are implemented. Similarly, analysis and discussion of results will be unique to this study, with interpretations attempting to make sense of an inquiry classroom.

There are two roles to consider when presenting the methodology to be used in this study; the author as the classroom teacher and as researcher. Both perspectives will be explained and used to structure this chapter with insights illustrating teacher and researcher views. The first role included the classroom teacher to offer viewpoints that were an original characteristic of this study. Secondly, a researcher perspective allowed me to conduct this research in ways that are systematic and purposeful. The *teacher-as-researcher* role (Jaworski, 1998) brought insight to the research not always afforded by educational research conducted from an outsider's perspective. It also allowed me to indulge as a researcher to address the real classroom issue of trying to align assessment, teaching and learning in an inquiry classroom. A key paper by Cobb and his colleagues describing design research (2003) offered a structure for describing the data, participants and the method of analysis and will be used to guide the structure of this study. The process for analysis will be described using principles of grounded theory (Flick, 2009) and the quality of this research will be judged using trustworthiness criteria (Lincoln & Guba, 2007).

Teacher as Researcher Role

In this study, my dual role of classroom teacher and researcher meant that I was very close to the context being studied and allowed me to present an insider's perspective (Baumann & Duffy-Hester, 2000) when discussing the normative practices of my own classroom community. A social perspective of classroom mathematical practice was presented by Cobb, Stephan, McClain and Gravemeijer (2001) as part of their research into mathematical classrooms. Their work included normative activities of the classroom community, as constituted by interactions between the teacher and their students, as part of the evolving classroom microculture. The study Cobb and his colleagues (2001) presented provided an example of an approach grounded in the work of mathematics educators conducting classroom-based design research, where the classroom teacher was a full member of the research team. Being the classroom teacher in the classroom I was studying meant that I already had in-depth knowledge of normative practices of social norms in the classroom, the students in the class, as well as knowledge around planning and the implementation of teaching units. I was also simultaneously involved in other research projects interested in the pedagogy of mathematics inquiry in classrooms. I had the benefit of participating in collaborative analysis into other inquiry classrooms through field observations and moderation of data. This contributed to my own professional development as a teacher of inquiry, and my own understandings of the theory and empirical studies that informed this approach to teaching. It also offered a variety of methodologies to analyse and interpret a range of classroom data.

As the classroom teacher, I was also mindful that this already-established relationship could also be of disadvantage. Care was taken when considering prior knowledge and conceptions of students' abilities and personalities. Previous notions of student achievement could present as bias in later analysis. In analysis of data, Corbin and Strauss (2008) described how theoretical comparison can force researchers to examine their own biases. Although as the classroom teacher I had previously evaluated the mathematical abilities of my students, it was important through analysis to make comparisons to literature as well as experience. When teachers participate as researchers to develop theories of teaching and learning, investigation and reflection is based on classroom practice. Already a household word in the nineties (Santa & Santa, 1995), *teacher-as-researcher* became associated with language of experimental research, inquiry and intervention (Krainer, 2014), and action research (Kemmis, McTaggart & Nixon, 2014). Teachers inquiring about problems in the classroom and taking action to solve them could suggest that all effective teaching is really a research (or inquiry) activity. The dual role of teachers as researchers in mathematics education was described as *Living on the edge* by Breen (2003). In Breen's analysis he appealed to the mathematics education community to consider the exciting opportunities teacher research provided. He found it crucial that teacher research caused dissonance and unease if education were to improve and challenged teachers to be provocative in reporting findings. Krainer (2014) described the movement from teachers as passive recipients of researchers' knowledge production, to teachers as stakeholders in education research. He considered how this could present more opportunity for practitioner knowledge and theory bases to overlap. My efforts to research mathematical learning in an inquiry context came from my own energies to improve education in my mathematics classroom.

An inquiry classroom context

An inquiry mathematics classroom presents a complex environment in which to conduct research and there are many methodological approaches one may choose. The approach used in this study considered many different and possibly unusual aspects of the classroom environment when searching for evidence of learning; in ways that were open to interpretation and new ideas. A scientific, empirical slant might compare the inquiry classroom to a more traditional classroom. This approach would have difficulty distinguishing the particular differences that make the conditions in an inquiry classroom general to all inquiry classrooms in order to compare these to more traditional contexts. A phenomenological approach could be used to describe the culture of the classroom from a researcher's perspective but this methodology would not suit this study as I was already a member of the learning community, working in a role as both classroom teacher and researcher. Choosing design research (Cobb et al., 2003) as a methodology enabled me as teacher and researcher to engineer an inquiry learning environment in a regular classroom to study. The design experiment framework, as part of design research, allowed me to test and explore different hypotheses or suppositions at each stage of data collection to anticipate how particular elements functioned together to support learning mathematics through the inquiry pedagogy. Suppositions are clearly outlined at the beginning of each Analysis Chapter.

The design research methodology can cope in a less well-controlled environment such as an inquiry classroom. Realities of day-to-day life in a real classroom do make the control of peripheral variables difficult (O'Donnell, 2005). For example, the random selection of students to form a 'typical' classroom for research purposes would impact upon other areas of the participants' education. Working with children in an experimental way can expose them to situations where they may be considered for selection or not, may be forced to work with children, teachers or environments they are unfamiliar with, and may be exposed to teaching practices which are not considered the 'norm' in their own world. O'Donnell (2005) highlighted how variables change during the course of research and how the iterative nature of design research can attempt to characterise messy situations by developing a profile of the design in operation. The appeal of design research was that it explicitly recognised the complexity of an inquiry classroom and the need for multiple dependent measures.

Design Research

This study adopted design research in its methodology to allow qualitative exploration of the nature of learning in two classrooms, exploring three separate mathematical inquiries. As a methodology, design research has also been referred to as design experiments, design-based research methods or design studies (Collins, Joseph & Bielaczyc, 2004; Confrey, 2006). This chapter describes the methodology of design research specific to this study by addressing the five features as identified by Cobb et al. (2003) as being characteristic of the methodology. More than a glimpse, it will describe the participants in each of the three inquiries in a way that appreciates broader aspects of a typical classroom including in particular, critical elements of teaching, learning and assessment. Design research was developed to address the need to go beyond narrow measures of learning (Collins et al., 2004), or the consideration of single variables in a study. This idea suited the complexity and diversity of learning activities in mathematics inquiries, in real classrooms.

Design research provides an analytic approach to finding out how or why something is happening, in a way that values the context in which it is happening in. Results from design research studies are used to characterise the use of and evolution of theory/theories through progressive refinement and consideration of multiple dependent variables as part of integrated systems. Characterisation replaces the strict interpretation of hypothesis-testing in design research (Confrey, 2006) and it is more accepting in the educational research community that "designs in education can be more or less specific, but can never be completely specified" (Collins et al., 2004, p. 17). The methodology entails designing particular learning environments in order to study, develop, test and revise explanations and pedagogical theories; essential if educational improvement is to be a long-term, generative process.

Design experiments are diverse and broadly defined to allow them to adapt to the settings in which they are conducted. An overview of this study is below, incorporating the five features of design experiments as described by Cobb et al. (2003). These include (1) theory development and (2) a highly interventionist approach, designs are (3) prospective and reflective including (4) iterative design to (5) generate theory in context.

Theory development

The purpose of design experiments is to develop theories about both the learning process and the means that are designed to support that learning. This requires the documentation of the learning ecologies at multiple levels. This study did not aim to generate a grand theory such as Piaget's intellectual theories, nor did it aim to find and validate a new category of existence in the world as an ontological innovation. Cobb and colleagues (2003) explained that theories developed in design experiments were relatively humble, targeting on domain-specific learning processes. Similarly, this study relied on established theoretical constructs specific to the domain of mathematics and the pedagogy of inquiry in this domain. Regardless of pedagogy, teachers have complex and multiple agendas to deliver to students, parents and the school community and classroom practice is a hybrid of competing pedagogies that can undermine inquiry and deeper learning.

My broad theoretical goal was to identify and describe how the alignment of assessment, teaching and learning in inquiry mathematics unfolded during the course of everyday classroom teaching. Theory informing these three elements are considered differently in each phase of study, building cumulatively to result in a greater understanding of this learning ecology (Cobb et al., 2003). Insights gained in each phase of study were broadly considered to reflect on the relationships between each phase and contributed to the development of theory in this domain. Analysis of data contributed to refining theories and the profile of inquiry in the primary mathematics classroom.

The 4D inquiry model (Allmond et al., 2010) assisted in the early organisation of data into broad categories that related to the key moments in the inquiry cycle. The first phase of data collection in this study focused on assessment in the inquiry classroom and aimed to understand how mathematical learning was identified and described through assessment. Principles of assessment (identified in the literature) provided a framework to assist with the early conceptual ordering of data collected. The PISA assessment framework (OECD, 2009) showed promise as a way to organise the assessment information revealed in the first phase of study. Theoretical influences in this chapter (Chapter 4 of this thesis) included notions presented by Dewey (1891; 1938a) in relation to capturing learning that is personally constructed through experience.

The second iteration of study included in its design suppositions about assessment practices in an inquiry classroom from the first phase of study. The teacher in the inquiry mathematics classroom provided the focus in this phase and attention turned to the pedagogical decisions the teacher made. Analysis of data aimed to identify characteristics of one inquiry teacher, looking closely at interactions between formative assessment and teaching decisions and how they interrelate. Combined with descriptions of teacher-decisions as 'on the spot' or 'on the fly', the interaction of these elements were examined and systematically integrated to construct an explanatory scheme. Vygotsky's (1978) zone of proximal development theory offered the theoretical framework in which to frame analysis in this chapter.

The third iteration of study built on the insights gained from the previous iterations. Although assessment and teaching frameworks informed initial analysis, there was a need to look more closely at student learning. One framework that aligned with learning in the inquiry mathematics classroom was presented by Harel and Koichu (2010) as the DNR model. Their work was based on Piagetian principles of learning which included the notion of accommodating new ideas to assimilate with the old. These researchers used their framework to carefully illustrate the mathematical learning of particular students (Harel & Koichu, 2010) which offered a way to describe learning in this study that was broad and complex.

Further analysis of how these elements interacted and interrelated lead to the development of theoretical interpretations. For example, formative assessment that considered collaboration between peers, and that planned for students to illustrate reflective thinking that was broad and innovative, informed the teacher how to adapt their teaching to suit the needs of their students.

Teaching decisions could capitalise on moments of being ‘stuck’ by supporting students to seek equilibrium through the environmental resources available as part of the classroom culture (such as peers, the teacher and available learning resources). A deeper insight into how these interactions aligned with or supported each other, through the use of the theoretical lenses described above, contributed to and built on current theories of assessment, teaching and learning, to align these notions with the inquiry approach to teaching mathematics.

An interventionist approach

In this study, I was concerned with the dominance of traditionally-aligned assessment practices in inquiry classrooms to describe learning. The context of inquiry mathematics was explored as “a test-bed for innovation” (Cobb et al, 2003, p. 10), to investigate the possibility for educational improvement through an alignment of assessment, teaching and learning. In design research, forms of learning are designed to test innovations that draw on prior empirical research and resulting theoretical results (Cobb et al. 2003). In contrast to naturalistic investigation of a typical mathematics classroom, inquiry was chosen as the form of learning to focus on in this study. Inquiry teaching and learning experiences were designed to test and revise evolving findings about assessment, teaching and learning, specific to inquiry mathematics. The pedagogy of inquiry is becoming a more accepted approach to teaching mathematics, included in the rationale for the recently implemented Australian Curriculum (ACARA, 2012). Research into the pedagogy of inquiry will assist teachers and schools with the implementation of this approach to teaching mathematics.

In a practical sense, data were initially contextualised by the inquiry unit it was generated in. Phases of intervention are highlighted below to highlight the mathematical classroom inquiry being explored in each phase and the related focus for analysis, and were designed by myself as is the interventionist nature of this approach (Figure 4-2). A difficulty with the intentionality of design research interventions is knowing how to consider the many challenges associated with design as implementation. Collins and colleagues (2004) noted how the actions of participants require “constant decisions about how to proceed at every level” (p. 17) which can undermine the way designs are enacted. Hence, descriptions of each phase will be elaborated in each subsequent analysis chapter.

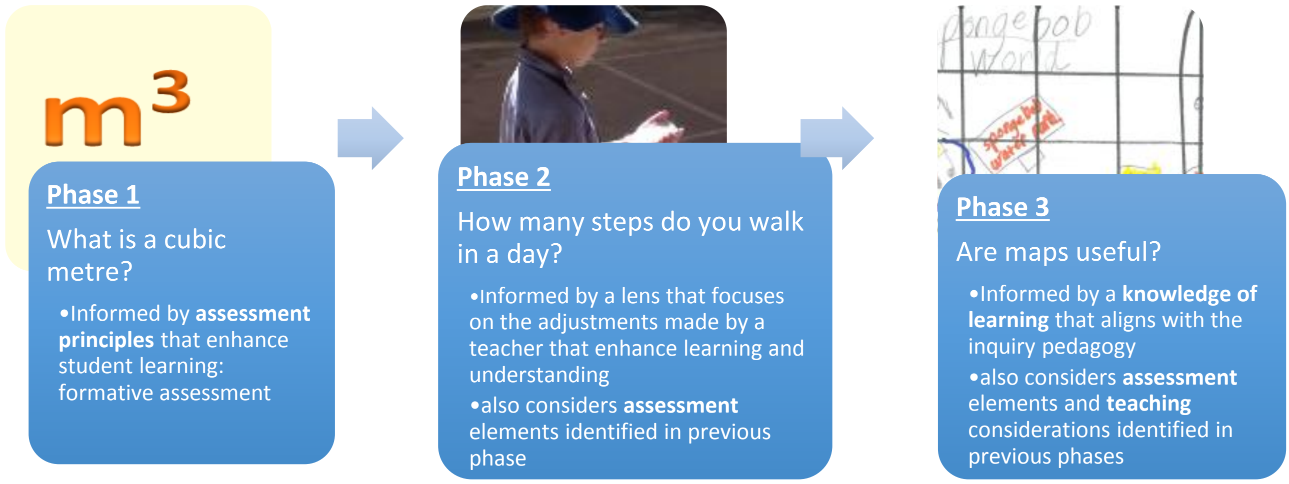


Figure 4-2 The mathematical inquiries explored in the three iterations or phases of data collection and a brief summary of the related focus of analysis.

Choices made around assessment, teaching and learning in a mathematics classroom may all be informed by research and established theories. In inquiry, these choices may not necessarily align in ways that enhance and support learning (Figure 4-1). Novel theoretical constructs are sought in the field of mathematical inquiry in primary classrooms, to provide explanatory power for the development of deep mathematical understandings in students (Confrey, 2006). In this study, insight was provided through reflections on classroom practice from my own perspective as the classroom teacher, and analysis considered how planned assessment opportunities support teaching and learning in this context. Although findings were used to develop theoretical constructs around assessment that support this pedagogy, a teacher lens considered the practicality and usefulness of findings, relevant to day-to-day classroom routines.

Examples of interactions that valued the process of student learning through inquiry were of most interest. Data collected from interventions studied in this thesis offered examples of interactions between the teacher and students and between students, and artefacts that illustrated learning taking place. Planned formative assessment opportunities, as part of the interventionist approach, were used to 'set up' these interactions. These experiences included: classroom discussions and dialogue between speakers, small group discussions between peers and with the classroom teacher, one-on-one interactions between the teacher and a student, or between a student and another student. Particular interactions that illustrated a change in teaching direction or student thinking were selected for close analysis to consider how they might contribute to learning mathematics through inquiry.

Prospective and reflective

Another key feature of design research incorporates the above two elements of theory development and an interventionist approach to engineer conditions that place developed theories in harm's way (Cobb et al., 2003). Prospectively, and somewhat deductively, designs implement theory with hypothesised processes and supports (described as suppositions, insights or findings). Reflectively, analyses can refute, contradict or contest previous findings, or support previous suppositions. Reflections on each phase of intervention offer opportunity to form further potential processes and supports, or to make alternative suppositions for future phases. Inductively, opportunities may highlight unforeseen events that lead to generating new, more specialised conjectures to test (Cobb et al., 2003).

In this study, my own suppositions included the belief that deep mathematical understandings were being developed in inquiry classrooms. Little was known about how assessment principles supported and illustrated the learning process in this context. Through my own personal experience, I had observed richness in the interactions between peers in this context, different to the peer interactions when teaching mathematics with a more traditional or 'school' mathematics approach. When classroom interactions challenged expectations during the conduct of the design study, reflection offered opportunity to revise and test more specialised findings

Characteristic of design research, prospective and reflective conjectures are informed by the extensive records which result from each phase of data collection (Confrey, 2006). In this thesis, each analysis chapter foregrounds the research aim/s for that phase, followed by my own suppositions about what I assume could be the answer. Reflection on analysis at each phase of study (Chapters 5, 6 and 7) is presented as retrospective analysis, to consider the research aims in relation to the findings from analysis in that chapter and to generate potential innovations or more specialised conjectures to consider in the next phase of study. Alternative and refined conjectures rely on the ongoing cycle of interaction and reflection, based on the extensive and varied amount of data collected (Cobb et al., 2003). In this study, the classroom design in each phase of study was engineered to test and confront findings or suppositions, in an effort to refute or strengthen such suppositions. Cycles of invention and revision required systematic attention as vast amounts of qualitative data were collected. Each phase was informed by theory and by reflection on previous phases. Findings were used to contribute to understandings of inquiry as a pedagogy for teaching and learning mathematics.

Throughout analysis I found it useful to keep a personal journal to record questions I posed, decisions I made throughout the process, and the reasons why I made them at the time. Keeping a personal journal during the analytic processes is recommended by Corbin and Strauss (2008) as a way of recording thoughts, actions and feelings aroused in different stages of the process. Using this analytic tool allowed me to reflect on the shaping of my ideas explored in this study. The data collected in each phase of study was complex and varied. Later consideration of these questions and ideas often generated a whole new set of ideas about a phenomenon and focused attention of refining these ideas. For example, in analysis of the first iteration I noted difficulties with differentiating between codes such as *inquiry skills* and *teaching*. Was *brainstorming* a property of *inquiry skills* or of *teaching*? After all, a teacher had to design opportunities for students to brainstorm into planning yet *brainstorming* was a skill necessary in inquiry. These reflections helped me to refine the properties and dimensions of particular categories and track the decisions I made. I referred often to these reflections to consider questions I had posed at the time.

Iterative design

A result of the prospective and reflective characteristics described above is the iterative nature of design research. New conjectures are developed and tested in an iterative design process which features cycles of invention and revision (Cobb et al., 2003). Evidence about learning specifies expectations that become the focus of investigation during the next cycle of inquiry. Collins et al. (2004) considered design research as a formative kind of research that placed designs into the world to test and progressively refine. Design research includes a process of continued induction and deduction as hypotheses or suppositions being generated by the researcher are tested in real classroom environments, or put into harm's way (Cobb et al., 2001). In this study, inquiry units of work were engineered to identify and explore relevant factors that contributed to assessment, teaching and learning. These were examined to highlight interrelations between these critical classroom elements.

Although in design research conjectures are developed around possible learning trajectories, Cobb et al. (2001) noted the difficulty in trying to develop the trajectory of each and every student's learning. They instead regarded the mathematical development of the classroom community through the theoretical notion of collective mathematical learning. This study will consider interactions that illustrate learning when that learning differs to the general classroom community. Of interest was how particular students differed from other students in the class collectively, such as many students who presented routine solutions to inquiry problems whereas others who showed innovative, creative, or unhelpful solutions. Why this was, or how the students came to think that way in relation to a general reflection of their classmates are questions that I considered through analysis. I did not intend to generate learning trajectories around particular mathematical content but to look closely at the classroom community and the interactions that take place within it. More generally, innovations or findings from each phase of this study contributed to a suite of mathematical practices that support and organise the emergence of each innovation, contributing to a greater understanding of inquiry mathematics.

Theory generation in context

An aim of design research is the development of theories that are accountable to the activity of the design and must do real work in the context in which they were created (Cobb et al., 2003). Theory development and testing occur simultaneously. In this study, the aim was to develop theoretical relationships between assessment, teaching and learning in an inquiry mathematics classroom. What I learned in each phase formed interim theories. Theories related to assessment, teaching and learning in the mathematics classroom informed the prospective design of this study and contributed to the engineering of intervention phases. Design research locates the theoretical scope of this study between a narrow account of a traditional mathematics classroom and a broader account of mathematical inquiry (diSessa, 1991, as cited in Cobb et al., 2003) and specifies the circumstances into which these representations might be projected.

Design experiments intend to articulate two related concepts at a broader grain size: a *conceptual corridor* and a *conceptual trajectory*. The notion of inquiry mathematics is the theoretical construct, or *conceptual corridor*, that framed all phases of this study. Articulation of the *conceptual corridor* of inquiry mathematics will describe the possible space to be navigated successfully to learn conceptual content (Confrey, 2006). The three units of work analysed in this study illustrate *conceptual trajectories*, or the possible pathways in which the students (and the teacher) traverse, through the *conceptual corridor* of inquiry mathematics. Although this study did not describe all possible *conceptual trajectories* it did aim to seek novel theoretical constructs surrounding assessment, teaching and learning in a classroom community of mathematical inquiry. These theoretical constructs will contribute to the body of knowledge which is the pedagogy of inquiry mathematics, to guide instructional decision-making towards improved student learning.

Design research: A summary

Design research places value in conducting classroom-based research to develop understandings of phenomena influencing classroom practice. Findings will be framed as a means of supporting inquiry as a broader class of phenomena (Cobb et al., 2003). The cross-cutting features of design research, described by Cobb et al. (2003), were elaborated above in relation to this study to clarify the methodological approach. This methodology complements the nature of this study as experimentation develops theories about both the learning process and the means that are designed to support that learning. The approach is highly interventionist and units of work were created to test conjectures, which in turn were then placed in harm's way both prospectively and reflectively. Iterations allowed for cycles of invention and revision as humble theories were developed describing how elements of teaching, learning and assessment aligned and supported each other. The appeal of design research was that it explicitly recognised the complexity of an inquiry classroom and the need for multiple dependent measures.

Method

This section will describe the method of inquiry for this study, determined by the methodology outlined above. Although related to research into literacy, Baumann and Duffy-Hester (2000) described *method*s in teacher research as including tools and procedures, mechanisms for synthesising data and analysis techniques. The structure used in this study firstly presents the context in which each phase was set and the participants. Then an overview of each inquiry, designed for each of the three different phases of study, is included. Finally, data collected in each phase is presented. Elaborations can be found in the subsequent analyses chapters for each phase. Analysis is then outlined using two lenses: a design research lens and one that relies on elements of grounded theory.

Contexts and participants

The study initially aimed to formalise formative assessment tools which promoted the development of mathematical literacy through an inquiry approach to teaching. This aligned with the participant school's DETA priority[1] of laying strong educational foundations, contributing to every child and young person being well-prepared for life success through learning and education. The focus on formative assessment was later extended as the study progressed, to consider how such practices aligned with teaching and learning. Principles of formative assessment were implemented into the design of inquiry units of work in this study, with an aim to uncover what the learner in inquiry knows and learns. Many characteristics of divergent assessment described by Torrance and Pryor (2001) aligned closely with the definition of inquiry used in this study such as the social

[1] Department of Education, Training and the Arts (DETA) sets broad priorities from which schools align their programs.

constructivist view of learning it supported, a focus on flexible planning and analysis from the point of view of both the learner and of the curriculum. With similar characteristics to the openness of the inquiry approach to teaching and learning mathematics, divergent assessment practices were implemented to support student learning at the classroom level, and to assist in making learning visible for research purposes.

The participants involved students of a large, suburban, government primary school setting in Queensland. This co-educational school had grown over the last five years to the mid-800 range from the mid-500 range. There were a small number of teachers in the school who used inquiry teaching in their mathematics classrooms. The school described itself as having a diverse community. Further characteristics of the community described by the school included students coming from diverse cultural backgrounds, with a variety of languages and cultural heritages, and strong support from members of the community.

Participants were the students who I taught in Years 6 and 3 while completing this study. Please note that the names of student participants used in this thesis are all pseudonyms. Throughout the study the students engaged in activities that were part of the normal process of teacher-designed and state administered activities and assessment practices. These practices were supported within the inquiry-based classroom and units of work were not considered an add-on to the curriculum. The study was aligned to school practices with the aim to improve, record and reflect on assessment practices during mathematical inquiry.

The Year 6 class included 28 students. Around one-fifth of the students spoke English as their second language, none of whom received additional classroom support in the school year. The school in general provided specialised education for students with Special Needs and about 10% of students in this class received support. Another fifth of the students were either identified as having either Attention deficit hyperactivity disorder or notable learning difficulties (not identified as having any particular disorder). This picture painted a general description that would fit many classrooms in Australia. It has been included here to explain the ways that these students similarly represented the general population of students of this age, in a large, suburban, government primary school.

There were 26 students in the Year 3 class. Almost a third of students in this class spoke languages other than English (LOTE). LOTE was an acronym commonly used in Queensland classrooms to identify a broad range of children with other language backgrounds. Because of their young age, LOTE students often received a small amount of additional teacher aide support in the classroom which often occurred during the times the class explored mathematics through inquiry. This assisted in providing additional English support. Around 20% of the students in this class received Special Needs support in and out of the classroom each day. In-class support was provided for a range of physical and cognitive disabilities and to support these students socially. In contrast to the Year 6 class, there were also a very small number of students who displayed gifted tendencies.

Brief descriptions of each class have been provided to illustrate two different yet typical classrooms with a balanced mixed of students of different needs and abilities. The planning and design of mathematical inquiries required consideration also of these students' needs, in terms of assessment, teaching and learning. The inquiry pedagogy lends itself particularly to these different classroom environments as the learning experiences are open-ended encouraging student-centred approaches to solving problems. It is worth noting that I did not describe the attributes of individual participants: my research focus was on the development of assessment practices that could capture and support learning in an inquiry classroom, not what each student learnt. Student diversity would make it very difficult to rely on generated trends or patterns if I chose to describe individual students. Just as assessment in inquiry needs to be broad and flexible, analysis needed to be broad and encompassing of student learning if understandings about inquiry mathematics were to be generated.

Inquiry units and further considerations

Three inquiry units of work were selected for analysis in this study and each took place at different stages in the year. The first inquiry exploring the cubic metre took place in a Year 6 classroom (10-11 year olds) in the fourth and final term of the school year. The children had participated in two other inquiry units of work and had experience with having to work collaboratively on mathematical tasks. The second inquiry, which explored the concept of numbers to 10 000, was in a Year 3 classroom (7-8 year olds) and was the first inquiry that these students had encountered in their years of primary education. The third inquiry about maps was with the same participants. This inquiry took place in the fourth and final term of the year after students had been exposed to two other short inquiry mathematics units of work. The amount of exposure students had to the inquiry pedagogy needed to be considered when identifying and describing learning for these students. This is considered in the Results Chapters (Chapters 5, 6 & 7) in terms of analysis of the data collected.

The three inquiries chosen to be used as data for this thesis were chosen because they reflected most closely the characteristics of a mathematical inquiry outlined by Allmond et al. (2001) as the 4D inquiry framework. Literature about how to teach inquiry, or the important phases of inquiry that students go through, needed to be reflected in the inquiry units of work developed for the study. This was to ensure consistency when comparing and analysing findings across the units, and to offer initial categories with which to organise data. The summary provided in Table 4-1 does not elaborate on specific teaching strategies or assessment practices, nor is it intended to do so. It offers a brief précis of the three units and their key characteristics, aligned with the 4D inquiry framework (Allmond et al., 2010).

The inquiry units were developed to align with the curriculum requirements at that time (outlined in Table 4-1). Planning also considered students who had never been taught by a teacher using inquiry pedagogy to teach mathematics and were unfamiliar with the approach. This meant that some planning included an explicit focus on the phases of inquiry, and the vocabulary associated with each phase. Inquiry in the classroom can vary in length from a single lesson to a series of lessons that span several weeks. The time frames for each of the units in this study are highlighted in Table 4-1. Finally, each unit carefully considered the elements of assessment, teaching and learning as part of the aim of the researcher.

A further classroom influence that occurred during the phases of data collection was the change in curriculum and school requirements, as reflected in Table 4-1. Content objectives for each phase were informed by two different curricula: the Essential Learnings (Queensland Studies Authority, 2007a) and the Australian Curriculum: Mathematics (ACARA, 2012). The development of the Australian Curriculum had a great impact upon planning in 2012 as the mathematics curriculum was implemented in all Education Queensland schools. It became apparent in 2012 that there were huge shifts, vertically, in the content teachers were expected to teach and each year level was required to raise their standards very quickly. At the classroom level, a State interpretation of the Australian Curriculum was developed known as Curriculum into the Classroom (C2C) (The State of Queensland, 2012). Five-week units of work were developed by Education Queensland to ensure that all content descriptions in the Australian Curriculum would be covered throughout the year. Although the structure of the classroom units (in Phases Two and Three of this study) met the definition of inquiry, the influence of C2C (The State of Queensland, 2012) on planning is reflected in my reflections and planning.

In each of the three inquiries, experiences that captured teaching and learning moments that were surprising, or highlighted patterns of evidence, or that matched pedagogical categories were of particular theoretical interest. The inquiries included *How much is a cubic metre?* (Term 4, Year 6, 2010), *How many steps do you walk in a day?* (Term 2, Year 3, 2012) *and What is the best map?* (Term 4, Year 4, 2012) (Table 4-1).

Table 4-1 A summary of the three inquiries from each phase of teaching and learning.

.	**Phase One:** **Inquiry: How much is a cubic metre?** **Year 6 classroom**	**Phase Two:** **Inquiry: How many steps do you walk in one day?** **Year 3 classroom**	**Phase Three:** **Inquiry: Are maps useful?** **Year 3 classroom**
Curriculum requirements	*From* **Essential Learnings** by the end of **Year 7**, (Queensland Studies Authority, 2007a): **Strand: Measurement Knowledge and Understanding:** Relationships between units of measure and the attributes of length, area, volume, mass, time and angles are used to calculate measures that may contain some error. **Elaborations:** • Estimation strategies are used to identify a reasonable range of values for a measurement • Measurement involves error, which can be reduced through the selection and use if appropriate instruments and technologies.	*From* The Australian Curriculum (ACARA, 2012) **Strand: Number and Algebra** **Sub-strand: Number and place value** **Content description:** • Recognise, model, represent and order numbers to at least 10 000 (ACMNA052) • Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems (ACMNA053)	*From* The Australian Curriculum (ACARA, 2012) **Strand: Measurement and Geometry** **Sub-strand: Location and transformation** **Content description:** • Create and interpret simple grid maps to show position and pathways (ACMMG065)
Time frame	4 lessons, 4 hours, over 1 week	7 lessons, 11 hours, over 3 weeks	6 lessons, 6 hours, over 2 weeks
Inquiry phases: ***Discover***	- Pre-assessment task to assess content knowledge. - Activity and classroom discussion around 'really big things – how much do they weigh?' - Introduce question and initial thoughts are shared.	- Introduce question and initial thoughts shared about the mathematics required to answer the question and the tools that may be required. - Students predict how many steps they might walk in one day. - Pre-assessment of students' knowledge about place value to 10 000	- Students connect to prior knowledge of maps. - Students identify the different purposes for maps and contexts where maps are used. - Introduce question and initial thoughts are shared.

Devise	- Students work in groups of three [informed by traffic light cards (Wiliam, 2007)] to plan how they will create their cubic metre, to meet particular context requirements. - In groups, students reach consensus about how much 1 cubic metre is.	- Students work in pairs to devise a plan to answer the inquiry question. - Three students wear pedometers for the lunch break to see how many steps they take during lunch. - Students share their plans and revise as necessary.	- Students examine different maps in groups of three to identify what features are useful or not. - Students discuss what features they will include in their map design, to make their map the best. - Students have a go at making their own maps.
Develop	- Students share their plans and findings so far. - Classroom discussions to share efficient strategies as well as ideas that do not seem productive. - Students follow and revise their plan to make their cubic metre.	- Students collect data using the tools they selected. - Students identify the number of steps to different pathways within the school. - Students follow their plans to answer the question. - Classroom discussions to share efficient strategies as well as ideas that do not seem productive.	- Students create their own maps, which are purposeful for the context they design. - Classroom discussions to share useful features as well as ideas that do not seem productive.
Defend	- Students present their cubic metre to the class and justify why it is as much. - The class discusses challenges and successes they met while making their cubic metre.	- Students present their answer to the class and justify why it is so. - The class discusses challenges and successes they met in reaching solutions.	- Students share their maps and justify why their map is the best according to the features they have included. - The class discusses challenges and successes they met in reaching solutions.

Data

Much data were collected to capture the many different aspects of the classroom culture in the inquiry mathematics classrooms with the research focus particularly on the classroom elements of assessment, teaching and learning. Data included the designed formative assessment opportunities including interactions between the teacher and students, and between students. Also included were artefacts generated by students that responded to planned assessment and illustrated learning, and teacher-generated artefacts that illustrated assessment and decisions around teaching and student learning. Documented data that includes a range of illustrative examples is characteristic of teacher researcher methods (Baumann & Duffy-Hester, 2000). In their analysis of methodology in teacher research, one category they identified as common in writing and reporting of classroom inquiry was how it was illustrative. They describe illustrative data as including excerpts of transcripts and interviews and student artefacts. Similarly, a range of illustrative data were collected in this study to assist the reader to interpret this research. Core characteristics of each inquiry have been provided (Table 4-1) to contextualise the data within the planning and curriculum requirements already existing in the classroom.

Data represented teacher and learner perspectives in the inquiry classrooms. Data also reflected the day-to-day 'runnings' of the classroom including work generated by students, conversations and shared presentations, feedback generated by the teacher and assessment programs already in place. Lesson planning and resources are also been included. Teacher reflections were recorded after each lesson to present an insider perspective (as described earlier in the Teacher as Researcher section). Particular reflections were bolded, italicised or included coloured text and comments to highlight moments which I thought were of particular research interest.

Student artefacts included those which traditionally inform a teacher about student learning including; students' workbooks, formalised assessment tools and products of learning (individual). These were used to help identify learning related to the processes and pedagogical practices of inquiry. Other student artefacts included a wide variety of less traditional products of work such as posters or concept maps which demonstrated collaboration of ideas between small groups of students. Often, students created a 'product' of work which could only be considered useful in that particular context. For example, in an inquiry unit where students investigated the height of a typical Year 3 student, some children had glued a measuring tape to paper, which was stuck on the wall or floor, and used this to record heights of students. The resulting 'product' was up to two metres long and contained all of the data the students used to solve the problem. An iPad made it easier to capture these artefacts through photographs and video recordings, in the context designed by the students.

In most cases, students in the inquiry classroom worked in small groups and their interactions were also of interest in this study. In general, a teacher would be unable to observe the learning that takes place in each of these interactions unless they are able to sit, momentarily, with one group of students. As teacher-researcher, I used the iPad to record conversations of particular groups of students. Initially these groups were selected non-systematically but after reflection each day, groups of students who were more articulate became the focus of this data collection. Recorded conversations would allow insight into the interactions between students learning in group situations, without intervention by the teacher/researcher.

Formative and summative assessment tasks were also collected and considered as a way to track and describe student learning in relation to the inquiry. In each phase of exploration (Chapters 5, 6 & 7), common summative assessment tasks that informed the design of each subsequent inquiry are included. Formative assessment tasks encouraged students to record their thinking and understandings in writing. These data were collected to broaden the scope of student learning beyond the restrictions of the design of summative assessment tasks.

Teacher as researcher observations were also collected to assist in identifying initial pedagogical categories. A research journal recorded field notes and initial reflections on the data collected; close in time to events taking place. A personal online blog recorded ongoing reflections about the process of analysis; summarising emerging theoretical considerations. This assisted in defining issues and in keeping track of evolving conjectures throughout the project. Similar to field notes, the research journal contained reflections that were useful in illustrating moments where learning seemed to occur. From a researcher's perspective this proved most useful as the reflections included the events that happened prior to learning moments, during and beyond. This insight might not be gained where the researcher only visits a classroom at scheduled times, in a refined environment.

Analysis

This study is qualitative in nature and data are analysed from two perspectives: design research (as described in the design research section of this chapter) and the analytic frameworks of grounded theory methodology (Corbin & Strauss, 2008). Qualitative content analysis (Flick, 2009) focused particularly on the three classroom elements chosen as the focus of this study as assessment, teaching and learning, and the ways that these classroom elements interrelated and aligned to support the learning of mathematics. These elements could be considered at the basic-level (Corbin

& Strauss, 2008) to provide a framework in which a more open and generative approach to defining categories can be made. The coding process of grounded theory is presented in relation to design research as many of the stages of analysis were informed by the engineered phases of study. In each phase of study, the process of grounded theory was utilised to generate properties and dimensions of the three particular classroom elements of assessment, teaching and learning. Comparative analysis compared events in the data to form higher level concepts or categories. Iteratively through analysis, these categories were related to generate characteristics of assessment, teaching and learning in an inquiry mathematics classroom.

Although the research questions were defined in advance, analysis began with identifying properties and dimensions of the three classroom elements being explored in this study based on theoretical frameworks outlined in Chapter 3. These categories were modified as necessary. Each phase built on the different theoretical frameworks informing previous phases and resulted in more refined properties and dimensions of these classroom elements. Analysis of each phase of data collection was varied and an in-depth explanation for each phase can be found in the related analysis chapters (Chapters 5, 6 and 7). Outlined below is a general description of analysis which guided this process each time. Central to the process of grounded theory, the process of coding can be controversial and Flick (2009) suggested that a researcher could use an eclectic approach, selecting concepts and procedures from a variety of approaches. Analysis of data in this study drew on the approach described by Corbin and Strauss (2008), summarised by Flick (2009) as open coding, axial coding and selective coding.

Audit of data

An 'initial sweep' organised the data at a whole-text (or artefact) level by classifying it by: the inquiry it was from, the phase of data collection, the medium or type of data, a description of the mathematical process and temporal-related information. This involved auditing all of the data, at the completion of each phase, to describe it in ways that were useful to identify and analyse. It is useful to keep lower-level concepts in any explanation of higher-level concepts to ensure analysis is not removed from the data (Corbin & Strauss, 2008). Table 4-2 illustrates details of the data collected, and how each item was initially organised. Language was descriptive and used mainly in an organisational sense such as formative or summative assessment tasks.

Table 4-2 Initial defining and organisation of data collected. This assisted in the sorting of data and the later generation of codes at the open-coding level.

Inquiry or Interview	Medium	Stage of Inquiry	What maths are we doing?	Date file modified	Artefact number (if more than one on this date)
C – How much is a cubic metre? **S – How many steps?** **M – Are maps useful?**	P – Photo of Ss working Ph – Photo of teaching artefact V – Video A – Audio S – Student artefact (including photos of their work) T – Teacher comments (Assessment data) R – T Reflection U – Un-used resource Res – Teaching resource B – Rubric Te - Test	0 – Pre assessment 1 - Discover 2 - Devise 3 - Develop 4 - Defend 5 – Diverge 6 – All phases 7 – post assessment () – multiple phases	PA – Pre-assessment F – Find the math S – Share the math D – Do the math P – What problems might we encounter Pr – Predictions A – All phases R – S reflection F – FA task SA – Summative assessment task () – multiple Dev – devising a plan	Eg 240412	a b c *Etc.*

Through constant comparison, the data were revised again and again to further classify, describe it more deeply, and to organise it in ways that make it easier to access (Corbin & Strauss, 2008). In the example of analysis of Phase One data, I generated a short summary of each artefact. For example, the item described below (Table 4-3) is from the first phase of data collection (inquiry 'How much is a cubic metre?') and is a student artefact (includes a photo of the traffic light cards, sorted into groups). It also includes the teacher's reflection of all phases of the inquiry and describes when the file was generated (when the artefact was created). These descriptions contained no theoretical abstraction at this stage and mainly contained notes to myself describing each artefact.

Table 4-3 Further organisation of data including description.

Data	File name/location	Description
CSR6A-xxxxxx	xxx	- Photo of traffic light cards, sorted into groups - Teacher reflections xx/xx/xx and next 3 lessons, xx/xx/xx and notes for next lesson

As each artefact represented more than the description I ascribed to it, sensitising questions helped to tune me in to what the data might be indicating (Corbin & Strauss, 2008). These questions assisted in establishing relationships between the classroom elements of assessment, teaching and learning (What was the purpose or intention of this artefact? Who was it intended to be used by?). Although not coding families, the formation of categories contributed to the properties of each artefact. For example, Table 4-2 illustrates how I reviewed my descriptions and decided to use different colours to highlight who the artefact belonged to or had been generated by. The colours were used to classify items in the 'Medium' column (Table 4-2) in three ways: artefact descriptions were coloured green if it described an artefact generated by a student; blue was used to indicate artefacts created by the classroom teacher, often including a researcher perspective; and artefacts that were observations (photos and video observations) were highlighted using red text. Subsequently, descriptions of particular artefacts (Table 4-3) highlighted these perspectives also. This visually represented the data as a sea of blue, green and red enabling me to direct my focus on the perspective being represented in that data. These different perspectives also highlighted that there were interactions between the classroom elements of assessment, teaching and learning.

Open coding

Open coding is described as breaking down data into segments (Flick, 2009). However, Flick explained how this process could be applied in various degrees of detail. Corbin and Strauss (2008) liken coding to mining data in terms of their properties and dimensions. During this stage of analysis, I thought more about the purpose or intention behind the artefacts and how it might represent learning. For example, the mathematical content illustrated in an artefact reflecting student learning did not necessarily align with the teaching intent of that learning episode. The research focus was on the relationships between the elements and if alignment existed so further description seemed necessary. I generated properties of Teaching (T), Learning opportunities (LO) and Learning (L) to further describe each artefact (Table 4-4). The T property related to the teaching intent behind the artefact, task or learning activity; why this activity was planned and what 'big ideas' the teacher intended students to learn. LO identified the mathematical content students might need to complete the task described. It is worth noting here that rather than describe LO relative to each student's prior knowledge, descriptions of LO were in relation to the intentions grounded in the curriculum - related to general processes or proficiencies used in the mathematics classroom. Finally, I thought it would be important to understand what student L the artefact captured (or described, or was evident) either through the use of assessment or as evident in the artefact. This became a challenging task as the distinctions were not always clear or obvious. Questions I noted in my journal considered the planned use of assessment in the inquiry: Were the LO always describing mathematical content? Wouldn't the LO be the same as the T? If so, would L be similar to T if what was intended to be taught each time was actually what was learnt and evident in the artefact? Also, descriptions of mathematical content were always broad and interconnected for example. This was a key characteristic of inquiry. Comparing incidents helped to focus and refine classifications of data and annotations contributed to the properties and dimensions of higher level concepts, or categories.

Table 4-4 Further organisation of data using the categories of Teaching (T), Learning opportunities (LO) and Learning (L).

Codes	File location	**Teaching (T)** – Intent evident in data **Learning opportunities (LO) -** Content and processes/proficiencies **Learning (L)** – captured or assessed	Annotations
CSR6A-xxxxxx	xxx	Photo of traffic light cards, sorted into groups (T) - to gain an understanding of students' confidence about measuring volume, cubic metres and its relationship to metric tonnes (LO) - Self-assessment of confidence in understanding how cubic metres are used to measure volume (L) - photo demonstrates students' levels of confidence Teacher reflections xx/xx/xx and next 3 lessons, xx/xx/xx and notes for next lesson (T) - understanding of one cubic metre (size, equivalent to, how to make) (LO) - Understanding of cubic metres is multiplicative - To create $1m^3$ to better understand size - To calculate the volume of a cylinder (L) - the difference between area and volume - how much is half a cubic metre	Self-assessment Pre-assessment Content knowledge: measuring volume using cubic metres, the relationship between m^3 and 1t/1000L Link to prior knowledge Inquiry skills: collaboration, questioning, creating, brainstorming, communicating, justifying, making errors, relate maths to everyday life

Please note that the descriptions of T, LO and L were my own notes, for my own consideration (Table 4-4). They have been included for the benefit of the reader to indicate how I thought about the relationships between teaching, learning opportunities and learning, including assessment, of each artefact. Now that the data were organised by type and inquiry, and by perspective, I began to express this in the form of concepts (Flick, 2009). In the first phase for example, literature and analytic frameworks informing the concepts of Teaching, Learning Opportunities and Learning (Table 4-4) contributed to properties of the phenomena of assessment being studied in this phase. Inductively, more than 50 codes were generated from the data (described as *in vivo* codes by Flick (2009) and were grouped by phenomena. One property of Learning for example included the codes 'I do not understand,' 'more complex solutions' and 'shift in thinking.' Dimensions and properties belonging to the lower-level concepts of assessment, teaching and learning were further developed as more abstract codes were identified and linked to generate higher-level concepts of categories. Questions were generated regularly in response to the data (Flick, 2009) which were recorded in my personal journal (described in the Data section above) including: What will formative assessment reveal? How easy is it to identify this information? What did the teacher know about her students? Was students' 'prior knowledge' incorrect/correct?

Axial coding

A complex process of inductive and deductive thinking, the process of axial coding relates subcategories to categories most relevant to the research question (Flick, 2009). The process of analysing the relations between the different axial categories meant I became immersed further in the data for a second and third time. Flick (2009) described the aim of qualitative content analysis as reducing the amount of material collected, especially when a wide range of different textual materials (such as in this study) is to be analysed. Aims and research questions directed analysis, yet axial coding was used to reflect a more abstract way of thinking about the rich descriptions of each artefact, and consider the multiple perspectives it represents in relation to other categories.

In the example above, the fourth column was added (Table 4-4) showing short phrases or single-word codes that were generated and how these began to further describe properties and dimensions of the concepts being explored in each cycle of study. Artefacts showed L related to thinking processes such as questioning, creating and brainstorming. These processes were organised using the heading *inquiry skills*. A general picture was being illustrated as these annotations were used frequently, developing a larger profile or picture of inquiry. As with other taxonomies, thinking skills such as those evident in this data could be sorted into hierarchies to show the level of thinking they presented. These kinds of processes were described by PISA (OECD, 2009) as competencies, further organised into clusters; a matrix or taxonomy organised by levels of thinking ability. Similarities evident between the thinking processes students used in inquiry and competencies described in the PISA assessment framework (OECD, 2009) seemed to show promise that the latter could be used to describe learning in an inquiry classroom (Table 4-4). For example asking questions is a term used by PISA (OECD, 2009) to describe the competency of Thinking and reasoning, and communicating and justifying would similarly contribute to the development of the competency of Communication.

Selective coding

Continuing axial coding at a higher level of abstraction, selective coding "elaborates the development and integration of it in comparison to other groups…focusing on potential core concepts or core variables" (Flick, 2009, p. 312). Comparative analysis (Corbin & Strauss, 2008) allowed me to analyse the properties and dimensions of the classroom elements of assessment, teaching and learning, and the higher level concepts that were generated, in relation to each other.

This process of constant comparison had the potential to "bring out aspects of the same phenomenon" (Corbin & Strauss, 2008, p. 74) highlighting particular cases to further investigate and compare. As outlined in each analysis chapter (Chapters 5, 6 and 7) patterns in the data and the conditions under which these applied contributed to theory development of the alignment between assessment, teaching and learning in an inquiry context. Data that showed patterns were selected for closer analysis and used to illustrate interactions between relationships. Theoretical sampling is also a strategy for qualitative data analysis. Asking questions of the data opens up the line of inquiry to direct theoretical sampling, helping "with the development of the structure of theory" (Corbin & Strauss, 2008, p. 72). Data were then selected for analysis to maximize opportunities to develop properties and dimensions of concepts, and to consider relationships between such concepts.

Criteria for Assessing Quality of This Study

Trustworthiness will be used to explain the quality and methodological rigor of this study. Credibility, transferability, dependability and confirmability are the criteria outlined by Lincoln and Guba (2007) to assess the trustworthiness of qualitative research; developed in response to the paradigmatic nature of qualitative research to parallel rigor in positivist experience. Traditionally, more emphasis has been placed on research in education that was scientifically based, relying more on quantitative data for causal explanations. Even in 2002, rigorous, sustained and scientific research was recommended in education by the National Research Council (2002). In a later response by Confrey (2006), design studies had to evolve in order to qualify as a methodology; especially when many examples could then be provided. She elaborated on three ways methodological rigor could be evident in design studies: in the conduct and analysis of the experiment; how robust the claims were relative to the data and theory and if they were subject to alternative interpretations; and the explicitness and feasibility of the claims it made in regards to educational practice.

Credibility, as the first criteria of trustworthiness, is the term used to address notions of internal validity. In a chapter on methods for evaluating educational interventions, O'Donnell (2005) explained how in design research, continuous change of various aspects of the research makes internal validity very difficult. This study seeks to describe in detail, the complex interactions in an inquiry classroom. Three phases of inquiry in practice are studied with data being collected over two school years. These extended periods of time are illustrated in this thesis not to find a universal law to describe learning in particular classroom settings which rely on causal claims, but to consider how particular interactions in an inquiry classroom contribute to mathematical learning. Prolonged engagement in the field and persistent observation over this period of time support the credibility of this study by allowing the researcher an in-depth pursuit of the research aims and questions. Lincoln and Guba (2007) also referred to triangulation of data to ensure credibility. In this study, a large amount of data were collected to represent various perspectives in the classroom. This enabled analysis to compare: products of students' work, to classroom teacher reflections, to formative and summative assessment tasks, to peer and self-evaluations and often to video observations and audio recordings. Cross-checking data in this way increased the probability that analysis was credible and trustworthy.

In this study, transferability as a measure of quality is considered instead of external validity. Similar to a classroom teacher considering how to cater for many different elements in planning for teaching, educational research that is qualitative, prolonged and inclusive of day-to-day classroom requirements makes issues of external validity extremely difficult to consider. Shenton (2004) recognised how it is impossible to apply findings from small, qualitative research projects to other situations and populations. For transferability, Lincoln and Guba (2007) suggest developing thick descriptions of data so that others may judge the degree of fit when applying findings from this study elsewhere. The three inquiries presented in this study, as three phases of classroom exploration, are characterised carefully and thoroughly. This study has been designed to contribute to the field of study concerned with mathematical learning through inquiry, and results may assist researchers in making sense in similar situations.

Judgments about trustworthiness will question dependability and confirmability of this study (Lincoln & Guba, 2007). Positivist influences on criteria for quality include a desire about the precision of conclusions and the objectiveness of conclusions reached. As is the nature of design research, accounts include contextual detail and representation of the voices of the participants (Hatch, 2002) that build 'explanatory power' of findings. Events that confirm and disconfirm suppositions will "make explicit the case for the explanatory power of (the) proposed construct, over time, across circumstance, and across students" (Confrey, 2006, p. 147). Closely tied with credibility, dependability can be somewhat demonstrated in the operational detail of method presented earlier in this chapter (Shenton, 2004). Located in time and setting, contextual elements must be considered as part of the findings from this study including the researcher's role as classroom teacher. This role has been explicated clearly to assist the reader in making judgments about confirmability; and detail in the methodological approach can enable "any observer to trace the course of the research step-by-step via the decisions made and procedures described" (Shenton, 2004, p. 72).

Given the constructivist rationale for this study, the theoretical underpinnings also of inquiry in the mathematics classrooms, and the choice of design research to undertake this qualitative study, findings describe complex, relatable interactions that are specific to this study. A constructivist goal of studies in mathematics classrooms "is for the researcher to learn the mathematical knowledge of the involved children and how they construct it" (Steffe, 1991, p. 178). The trustworthiness of generalisations will be judged by questions of credibility, transferability, dependability and confirmability. The theoretical stance, choice of methodology and subsequent analysis will be made transparent throughout this thesis to allow the reader to evaluate how they may apply the findings to their own classroom practice or educational research.

Chapter 5 Phase One: A Focus on Assessment

How much is a cubic metre? The students pondered the question on the interactive whiteboard. The class had already been working with volume and relating units of mass, yet the students knew that their teacher expected them to be challenged to explore this question further. While the teacher called out the names of students who would work together, other students made their way across the classroom towards their classmate, begrudgingly if it was girl and the student was a boy, and vice versa. But once they arrive together, and quickly accommodate their workspace to fit the blank A3 sheet of paper handed to them by the teacher, they reconsider the question. "So we have to make a cubic metre, like, really make it?" In conversations, students consider what things could be made that would be about this size. The teacher purposefully moves between these conversations for different reasons: to join the discussions, to check who is on task, to stimulate conversations, to question ideas. One group considers a touch pool fish tank. They talk about aquariums and their previous visits to Underwater World. "I wonder how deep that would need to be if you wanted to reach the starfish at the bottom of the tank, without falling in," the teacher deliberates. The students resume their conversations and note their ideas down on the A3 page. "Bob the builder," and, "a tray on the back of a ute carries sand," are bits of conversations that can be heard. "How cute! A pig in a mudbath!" squeals one group of girls. The volume begins to increase as the students think of different ways to make a cubic metre and what resources they will need. "Do we have to bring the cushions from home?" "That kennel would be way too big for a Chihuahua!" One student, Seamus who was notorious for blurting out his thinking, shouts atop the din, "Is a cylinder that is 1m tall by 1m around and 1m wide a cylindrical metre?" The teacher suggests that this could be a logical assumption. She quickly encourages Seamus and his partner to 'jump' onto the laptop they are using to Google the question. The teacher takes a quick mental note to follow up on this later and picks up a 1m tape measure to remind her. This will be a great idea to test with the whole class, as well as being a worthwhile thought to write down about Seamus' thinking. She holds the tape measure in her hand while she draws everyone's conversations to a close, and the students come together to bring their focus upon the teacher.

Research Aims

This chapter will preface with a brief description of the context of this inquiry as classroom characteristics change in each phase of data collection. This description will help link the first phase of data collection to the research questions refined for this phase of data collection, regarding assessment. Research questions and suppositions that provided the focus for analysis of data in this phase included:

Research question one: How is mathematical learning in one inquiry classroom assessed currently and does this align with learning in this inquiry context?

Supposition: Traditional assessment techniques will not fairly nor adequately capture and describe learning of particular mathematical concepts in an inquiry classroom. Current beliefs surrounding assessment will reflect learning as an ability to answer questions correctly and to follow taught procedures accurately.

Research question two: Are there characteristics of assessment that support learning in inquiry and can illustrate insight into what students know?

Supposition: Literature supports the notion of formative assessment enhancing learning in the classroom. Formative assessment provides opportunities for students to reflect on and share thinking generally. This may provide insight into other aspects of what children know and can do in an inquiry mathematics classroom.

Research question three: What understandings, skills and procedures are developed by students through inquiry as they learn mathematics?

Supposition: Other knowledge and skills developed in the inquiry classroom are valid, even when they are not considered by traditional assessment such as rubrics or tests. Inquiry fosters the development of understandings, skills and procedures that are valued in the real-world. PISA (OECD, 2009) described how citizens are confronted every day with tasks involving a myriad of mathematical concepts and competencies: information in the form of tables and charts are commonly found in media outlets; interpreting bus timetables or successfully carrying out money transactions, and so on. Solving real-world problems demands an ability to apply mathematical understandings, skills and procedures in less structured contexts, often involving unclear instructions, where one makes decisions that are relevant and useful (OECD, 2009). Inquiry as a pedagogy to teach mathematics could foster such a learning environment.

These three research questions informed the careful consideration of formative assessment practices in the planning of the inquiry learning experiences in this first phase of study. Each question considers how assessment may complement teaching and learning in one inquiry classroom and aims to reveal characteristics of these assessment practices. Literature reviewed in the second chapter of this thesis considered learning from a social and constructivist perspective (Confrey, 1991; Sfard, 1998a;), beyond the description of mathematical content (Duckworth, 2006), and that values learning as a continuum of disequilibrium-equilibrium phases (Harel & Koichu, 2010). Assessment literature highlighted a purpose of assessment as finding out about student learning.

Traditionally, language in policies and research on classroom assessment has included terms such as performance, competency, comparative information and achievement, generally in relation to common teaching and learning goals informed by the curriculum and at the school level (Masters, 2013b; National Research Council, 2001a; Wiliam, 2011a). These kinds of policy goals do not seem to 'fit' with teaching and learning in an inquiry classroom which encourages students to think broadly about solving inquiry questions, and values students learning beyond what is intended. Contexts in which mathematics is learnt in inquiry are personally constructed by students as they make connections between mathematics and the world, a primary tenet of Dewey's (1891; 1938a) definition of thinking and his description of progressive education. Theoretical analysis of assessment in this phase of study was informed by the work of Dewey and his notion of knowledge in relation to the learner, to stimulate thinking about the properties and dimensions of assessment that align with teaching and learning using this approach (Corbin & Strauss, 2008).

Chapter Outline

This chapter will describe the preparation and design of this phase of the study. A piece of classroom assessment belonging to a more traditional approach to teaching mathematics is first examined. Characteristics of this type of assessment were analysed using PISA's (OECD, 2009) assessment framework. In contrast, learning in the inquiry unit exploring the same mathematical content was comparatively analysed (Corbin & Strauss, 2008) using the same assessment framework to aid refinement of the characteristics of assessment in inquiry, and to analyse the mathematical literacy of students learning in the inquiry context. Data including formative assessment experiences, informed by assessment literature in Chapter 2 of this thesis, was explored to consider what assessment information it revealed. Further analysis of this data through selective coding (Corbin & Strauss, 2008) considered the misalignment of assessment and learning in inquiry. Finally, analysis of assessment for one student provided further insight into what assessment revealed about their learning in this inquiry. Theoretical analysis was informed by Dewey's (1938a) relational view of learning that posits learning based on personal experience. Retrospective analysis generates potential innovations to consider in the next iteration of data collection, as part of the design research process (Cobb et al., 2003).

Preparation and Design: How Much is a Cubic Metre?

The first phase of data collection involved working with a class of 28 students in a Year 6 (10-11 year olds) classroom where each child had in-class access to their own laptop. These students were answering the inquiry question of *How much is a cubic metre*? Prior to the inquiry, the students had participated in multiple learning experiences exploring this concept to complete a common teaching unit agreed upon by the year level, designed by a commercial mathematics education series used by the school. The unit of work included five lessons exploring volume and relating units of mass (Irons, Burnett & Turton, 2005). During this time the students had discussed, explored and completed written bookwork on cubic centimetres and how it related to one kilogram or one litre. The Core Learning Outcome (Table 5-1, left column) was identified on the cover page of the resource and had been selected from the Mathematics Years 1 to 10 Syllabus (Queensland Studies Authority, 2004). A common summative assessment task, designed to support the commercial unit of work, was implemented and the year level had agreed that a student would need to correctly calculate the problems (Figure 5-3) involving converting grams to kilograms, to gain an "A" level grade. Results for students in my class were high but I was discontented with the assessment task as I felt it did not capture conceptual understandings of cubic metres that reflected related classroom teaching. It was not until my students were presented with the inquiry question 'How much is a cubic metre?' that these notions were challenged.

Table 5-1 Core learning outcome and related curriculum content.

Core Learning Outcome (Queensland Studies Authority, 2004)	Curriculum intent (Queensland Studies Authority, 2007b).
M 3.1 Students identify and use equivalent forms of standard units when measuring, comparing and ordering, and estimate using a range of personal referents.	Knowledge and Understandings: - Relationships exist within the International System (SI) of measures, including between mm, cm, m and km; kg and t; cm^2 and m^2; cm^3 and m^3 - Appropriate instruments, technologies and scale are used when exploring measurement of length, area, volume, mass, time and angles where not all of the graduations are numbered - Measurement involves error, which can be reduced through the selection and use of appropriate instruments and technologies - Estimation strategies are used to identify a reasonable range of values for a measurement

The inquiry, *How much is a cubic metre?*, was designed to fulfil the new curriculum requirements (Table 5-1, right column) of the Queensland Studies Authority (2007b). Learners were asked to make a cubic metre for a real-life context, with encouragement to use difficult measurements (as opposed to simple measurements such as 1m x 1m x 1m). This inquiry was conducted over four lessons reflecting the phases of inquiry explained in the Literature Chapter of this thesis (Allmond et al., 2010). In the Discover phase students explored concepts related to volume and capacity, sharing their pre-existing knowledge of this content. Next, they collaboratively planned (in small groups generally of 2 or 3) how they might create a cubic metre in the Devise phase by considering how the mathematical concepts were relevant to the real world. The formation of groups was informed by student feedback gained using traffic light cards (a formative assessment strategy outlined by Wiliam (2011a)) to ensure that groups included a mix of confident and less-confident students. Students refined their plans in the Develop phase and created models of a cubic metre based on decisions reached through consensus in groups. Finally in the Defend phase, students presented the cubic metres they created to their peers, and considered the methods and procedures that others presented.

Assessment decisions made in the planning and implementation of the inquiry unit were informed by literature (see the Assessment section of Chapter 2) and centered on formative assessment practices. These practices (Table 5-2) in the inquiry classroom offered insight into the ways students moved forward in their learning, acted on feedback from myself or one another, and revealed when these opportunities occurred. Of interest were the knowledge and skills, both mathematical and beyond, being developed through the inquiry and made evident through the use of formative assessment.

Data and Analysis

A range of artefacts were collected and used as data in this first iteration of the study. These included: reflections recorded in my own teacher journal which provided feedback about teaching and learning; feedback generated through planned formative assessment which included student reflections, collaborative planning and brainstorming; analysis of the summative assessment task which accompanied the commercial unit of work that students completed prior to the inquiry (both the commercial resource and the inquiry were designed to develop understandings of cubic metres); and filmed classroom observations.

The openness of the inquiry approach to understanding cubic metres presented the students with challenges that conflicted with their own understandings, offering valuable feedback as assessment information. Reflections in my teacher journal recorded anecdotal notes commenting on particular classroom moments where students' understandings appeared to be challenged (Figure 5-1). These kinds of comments were not recorded in common assessment tasks, nor did the commercial teaching resources request to watch for these opportunities. This fueled my interest in exploring the relationships between the classroom elements of assessment, teaching and learning in inquiry mathematics classrooms.

Gavin and Harry (Seamus was away) were stuck on making a ute tray to hold half a m3. They sketched a cube .5m x.5m x .5m and assumed it would be half a m3 as the sides were half the length of the m3. I asked them to visualise how many cubes like this would fit into our big m3 and they decided eventually on 8. They were shocked to see that this cube was not half a m3 but one-eighth!

Figure 5-1 Reflection recorded in the teacher journal describing students' thinking during the journey.

Analysis in this first iteration of study focused on the concept of assessment to generate properties and dimensions particular to the inquiry mathematics pedagogy. Literature revealed how formative assessment, when used effectively, could improve learning for students. Subsequently, formative assessment strategies were implemented into the design of this iteration of study to see how learning was supported in this context, to analyse what information about learning was revealed. Feedback generated as part of the process of formative assessment offered insight into student learning in three ways: what is to be learnt, what one has learnt so far and how one could learn the intended goal/s.

Initial analysis of the assessment task, accompanying the commercial unit of work, characterised the qualities of learning and assessment that the task seemed to support. Although not designed for assessing mathematical learning in an inquiry context, the PISA assessment framework (OECD, 2009) offered a broader view of learning than content alone, in the context of solving open-ended and complex real life mathematics problems. It identified eight cognitive competencies required by students to engage successfully in mathematisation (Thinking and reasoning [1], argumentation [2], communication [3], modelling [4], problem posing and solving [5], representation [6], using symbolic, formal and technical language and operations [7], and use of aids and tools [8]), to be categorised according to three different levels of mastery (Figure 5-2). The levels, referred to as Clusters, described the cognitive activities students undertake when solving mathematical problems. Just as the clusters described student thinking as they solved mathematical problems on a continuum from low-level, routine problem solving to high-level, complex methods, the clusters also provided a framework for analysis in this chapter.

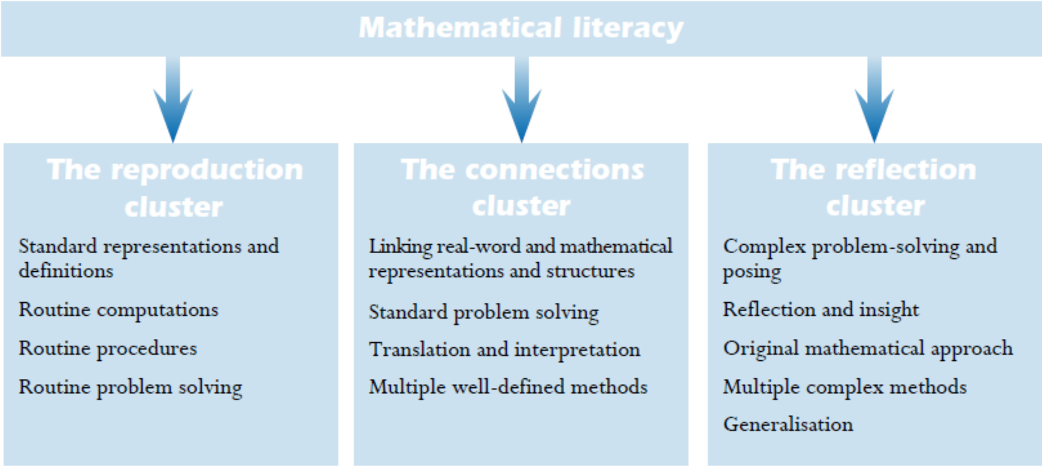


Figure 5-2 PISA Competency clusters to assess mathematical literacy (OECD, 2009, p.115).

In this phase of study, data were collected from the first three phases of the inquiry: the Discover, Devise and Develop phases. Analysis focused on three main artefacts reflecting formative assessment, as they offered feedback about learning. These artefacts included: (1) reflections recorded by the students in their digital journals and other products of their work; (2) planning and brainstorming completed in collaboration with peers; and (3) teacher reflections about lesson progress and student responses, in a teacher journal. Similar to field notes, these reflections were recorded to highlight events and thinking that took place in the classroom. This proved useful for dis/confirming thinking about particular events from a teaching perspective, close in time to the event happening. In addition, the first two lessons were filmed and video footage was used to further support the observational data collected.

Initial analysis of all assessment data collected in this phase of study, including the summative assessment task, denoted four main concepts (Corbin & Strauss, 2008). Three of the concepts were informed by the PISA assessment framework and generated inductively. These included the three clusters outlined in Figure 5-2: reproduction, connections and reflection. The definitions offered by PISA were useful in helping to identify instances in this inquiry classroom, yet analysis adapted the properties of connections and reflection to reflect the inquiry context more closely. These properties characterised the first three concepts related to assessment in this inquiry classroom:

1. Concept: Reproduction
    a. In terms of summative assessment included in the inquiry, designed to support the commercially-produced assessment task: competencies reflecting the reproduction cluster characterised learners who could perform routine and practiced procedures correctly. Properties of this concept included situating mathematics in contrived contexts that were not purposeful, with an assessment focus on the individual learner.
    b. In terms of formative assessment strategies planned as pre-assessment: competencies reflecting the reproduction cluster were characterised by what learners already knew about the topic, to activate thinking about the topic.
    c. In terms of formative assessment strategies planned generally to support student learning in this classroom (as supported by the inquiry): competencies reflecting the reproduction cluster characterised learners being able to articulate developing ideas. This makes learning visible in the sense of developing understanding of mathematical knowledge and procedures.
2. Concept: Connections
    a. In terms of formative assessment strategies planned generally to support student learning in this classroom (as supported by the inquiry): competencies reflecting the connections cluster characterised learners relating mathematics to less familiar, real-world contexts that were relevant mathematically; connecting solutions to other mathematical topics in order to solve more complex, multi-step problems; and focusing on the learner more socially, who considered or acknowledged peer contributions in their responses.
3. Concept: Reflection
    a. In terms of formative assessment strategies planned generally to support student learning in this classroom (as supported by the inquiry): competencies reflecting the connections cluster characterised learners responding to formative assessment opportunities to articulate thinking and learning; reflecting on prior knowledge of mathematics to connect to other mathematical topics; changing thinking and learning upon reflection of peer interactions; and reflecting on meaningful inquiry experiences.

Reproduction of taught knowledge and skills was evident in the inquiry but these instances were situated in contexts that the students had designed, whilst the concepts of connections and reflection were not evident in the summative assessment task. In the inquiry, reproduction of knowledge and skills were purposefully implemented by the students to aid in answering the inquiry question. Evidence of learning in the inquiry articulated individual learning progress but only in relation to learning through participation with peers.

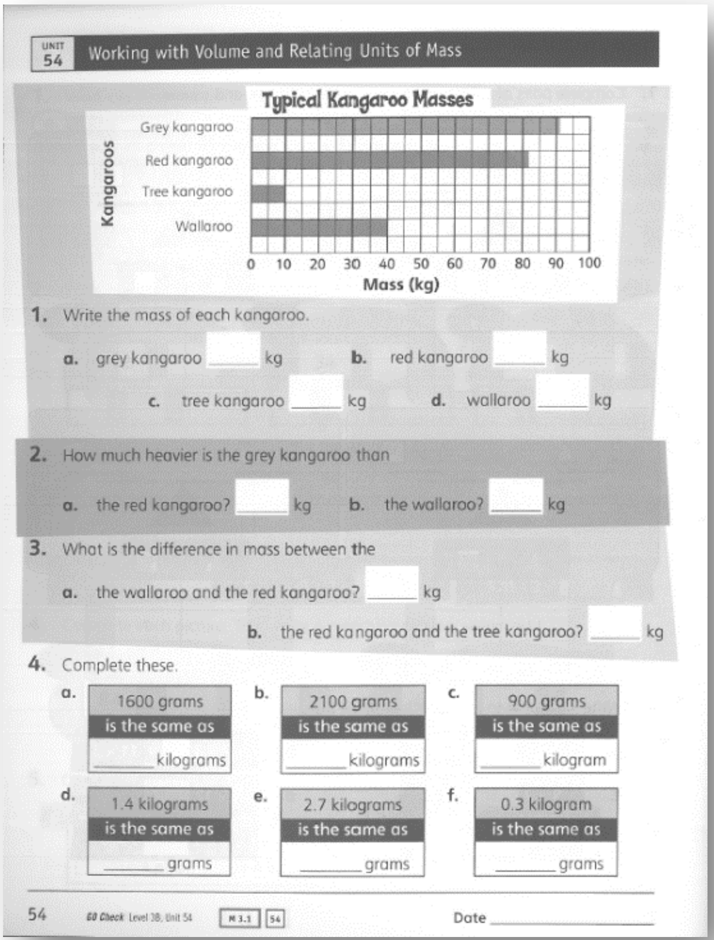
The fourth assessment concept to be characterised in this inquiry classroom was the element of frequency. Formative assessment opportunities were planned for well in advance of the event taking place, and also occurred 'on the spot' during teaching and learning. Feedback was generated in every lesson to offer insight into thinking and learning through interactions with others, such as conversations, posing or responding to questions. Interactivity, as a dimension of frequency, highlighted the opportunities for multiple interactions to take place in this classroom. A further dimension to characterise frequency was the reflexivity involved with generating short feedback loops; 'on the spot' feedback. Often insights gained through this feedback reflected change and/or development in learners' thinking. Multiple formative assessment opportunities considered learning individually and through collaboration with peers. The inquiry was frequent and reflexive, formative assessment.

It is interesting to note how in a research sense, the various forms of assessment captured different levels of thinking in the inquiry classroom, as described by the PISA assessment framework (Figure 5-2). Theoretical sampling highlighted the following illustrations for further analysis, to refine dimensions and properties of assessment that contribute to higher-level concepts pertinent to one inquiry learning context. Theoretical analysis was informed by Dewey's (1891; 1938a) notion of learning that is intimate and comes about through the contextual experience of inquiry.

Thinking in the reproduction cluster: Lessons learned

Firstly, it is important to consider what the learning and assessment focus of the commercial unit of work was. There is no intention here of critiquing this resource as the school had already reviewed its usefulness and it had been agreed upon that it would support and align with the curriculum at the time (Table 5-1). According to the assessment criteria, upon completion of the unit students would be able to: count cubes to calculate the volume of prisms, solve problems involving kilograms, write grams as a decimal fraction of a kilogram, and solve problems involving grams. This was how learning was described in the commercial unit of work and students would have to reproduce this mathematical content correctly to show that they had learnt it.

The commercial unit offered five lessons to explore the mathematical content, which was a common characteristic of these resources. In the first lesson students built a cubic decimetre to relate units of capacity (litres and millilitres) to volume measured in cubic centimetres. Students had an opportunity to view a wooden thousands block (a classroom resource which is a wooden cube showing the dimensions 10cm x 10cm x 10cm, also known as a thousands block) and to create their own out of paper in order to measure its capacity using centicubes (small wooden blocks with the dimensions 1cm x 1cm x 1cm). In the second lesson students used the paper cubic decimetre to discuss the metric units that related to it. For example, students filled a plastic cubic decimetre with one litre of water to relate the mass of water to one kilogram. The remainder of the five lessons focused on solving problems concerning mass, including problems involving tenths of a kilogram. There was no reference in lessons three, four or five to cubic metres or even to the decimetre students had created earlier.


UNIT 54 Working with Volume and Relating Units of Mass

Typical Kangaroo Masses

Kangaroos: Grey kangaroo, Red kangaroo, Tree kangaroo, Wallaroo

0 10 20 30 40 50 60 70 80 90 100

Mass (kg)

1. Write the mass of each kangaroo.

a. grey kangaroo ______ kg b. red kangaroo ______ kg

c. tree kangaroo ______ kg d. wallaroo ______ kg

2. How much heavier is the grey kangaroo than

a. the red kangaroo? ______ kg b. the wallaroo? ______ kg

3. What is the difference in mass between the

a. the wallaroo and the red kangaroo? ______ kg

b. the red kangaroo and the tree kangaroo? ______ kg

4. Complete these.

a. 1600 grams is the same as ______ kilograms

b. 2100 grams is the same as ______ kilograms

c. 900 grams is the same as ______ kilogram

d. 1.4 kilograms is the same as ______ grams

e. 2.7 kilograms is the same as ______ grams

f. 0.3 kilogram is the same as ______ grams

54 Date ______

Figure 5-3 Assessment supporting the commercial teaching resource.

Assessment for this unit consisted of a one-page black line master titled *Working with Volume and Relating Units of Mass* (Figure 5-3). In this test students were required to correctly read the masses of different kangaroos from data on a horizontal bar graph and calculate differences between them. Finally, students had six opportunities to convert measurements written as grams, to kilograms, and vice versa. Any reference to cubic metres or decimetres was not included. A student could get 100% correct on this test without needing to know anything conceptually about a cubic decimetre or a cubic metre. Results were used as a pre-test for the inquiry unit of work.

Using definitions from the PISA assessment framework (OECD, 2009), the cognitive demand of students required to complete this test appeared to lie within the range of competencies described in the reproduction cluster. The test required students to perform routine computations and procedures such as interpreting the masses of different kangaroos. Calculating differences in masses was routine problem solving already practiced in the lessons, and representing 1 600 grams as kilograms would be described in PISA as a standard representation. They were required to communicate these understandings in only one way, reproducing practiced standard problems in closed ways (where there is only one correct answer). All of these cognitive activities fitted within the reproduction cluster described by PISA criteria in Figure 5-2.

Incorporating typical kangaroo masses into the test was an attempt to link the mathematics to the real-world, although there was no attempt to situate this idea in a broader context. For example, there was no explanation as to who might collect or use this information or what it would be used for. The application of problem solving to the context of kangaroo masses was not routine practice in the classroom. If students were able to solve these problems, in this context, then there may be some element of mathematical literacy within the connections cluster although there was no need to know that the masses represented kangaroo masses in order to get a correct answer.

To continue, it could not be said that converting grams to kilograms and vice versa is a complex problem-solving and posing exercise involving reflection and insight, or multiple complex methods (as evident in the mathematical literacy of someone working in the reflection cluster). This skill had been practiced repeatedly in the classroom lessons as students solved similar tasks. The competencies students would rely upon to complete this test correctly reflected those in the reproduction cluster, defined by PISA (OECD, 2009) as “reproducing practiced material and performing routine operations” (p. 106). Ideally, thinking beyond the reproduction of taught mathematical knowledge and skills would be an aim for students.

The ability to replicate taught rote procedures, to correctly solve examples of routine computations, is considered a valuable part of mathematical learning. In this instance, the assessment task focused entirely on accuracy of calculations and following taught procedures correctly, relating to mass. The teaching unit that was designed to support this assessment task had focused on concepts related to cubic metres. I felt that a problem with this assessment task was that it was too narrowly focused on concepts of mass and could not reflect what students could and couldn’t do when working with and using cubic metre measurements. Regardless of the commercial branding, the characteristics of this assessment task and related resources were generally valued by teachers at the school. Its commercial success supported the notion that to many schools, these characteristics demonstrated a valid pedagogy for teaching and learning, and assessment. The focus of this assessment mirrors the description of traditional education offered by Dewey (1938a). In this task, curriculum standards are imposed upon the students as objectives or learning to be acquired; in this case, the students need to be passive learners and no need for an intimate experience with the content is required. Impersonal learning in this instance seemed superficial as students only needed to retain these understandings up until the point of needing to correctly answer problems that fitted within the scope of problems practiced in the classroom (as part of the regular classroom practice associated with the commercial tests). Would the learning that results be 'surface' only, not complex, nor requiring reflection and insight? If the students had to transfer their understandings of volume and capacity to a more complex problem, would they reflect on this learning, connecting different mathematical areas to generalise information based on their investigations? I developed the inquiry unit of work to explore the concept of a cubic metre further and to push thinking beyond the reproduction of what had been taught. I was eager to find out what other insights inquiry would allow assessment to capture and how it would align with the inquiry approach to teaching and learning mathematics.

Opportunities for assessment in making a cubic metre: Interactions as data

Frequent opportunities for formative assessment contributed to the highly interactive and reflective nature of learning in this classroom. Table 5-2 summarises of all of the formative assessment practices evident in the inquiry. Formative assessments recorded in writing by students, were kept in student reflection journals and are marked with an asterisk (Table 5-2). This was a digital journal in that reflections were typed into a Word document and saved each time. Please note that the journal is not added as a separate formative assessment practice in table 5-2. The sheer number of assessment opportunities far outweighed the sole, summative assessment task from the commercial unit of study. Each item represented an interaction between teacher and student/s, or between students. These interactions were designed into the inquiry as formative assessment, and identified in analysis through the open coding process to be grouped within the concept of assessment. Feedback gained through the formative assessment revealed insight into student thinking about mathematics, in an inquiry context. The thinking of particular students will be explored further below (see Laura, Paul and Mary).

Table 5-2 List of formative assessment practices evident in the cubic metre inquiry and length of time feedback was received by the teacher or learner.

How soon this information was utilised	Assessment informing the teacher	Assessment informing peers	Assessment informing self (students)
Short term response/ feedback loop	Traffic lights * Pre-assessment questions Questioning to prompt Questioning to activate prior knowledge Questioning student responses to guide understanding * Quickwrites (Sentence starter prompts) * Posing questions where the answer is $1m^3$ Sharing with the class Class discussions Small-group discussions (where the teacher is involved in the conversations and discoveries) Listening and providing time	Pre-assessment questions * Student responses to teacher questions * Posing questions where the answer is $1m^3$ Small-group discussion Class discussions Sharing with the class Restating what another student said A3 sheets of paper showing collaboration between peers	Traffic lights Student responses to teacher questions * Quickwrites (Student reflections) Sharing with the class Small-group discussions Class discussions A3 sheets of paper showing collaboration between peers
Longer term response/ feedback loop	* Quickwrites (Student reflections) A3 sheets of paper showing collaboration between peers		* Pre-assessment questions *Posing questions where the answer is $1m^3$

The large quantity of interactions reflected assessment in this classroom that considered learning that was both personally relevant, and constructed in collaboration with peers. Each day, students recorded in a learning journal their thoughts, sketches and ideas relating to their inquiry. This became an important artefact to consider for analysis as it revealed evidence of student thinking over time. Assessment practices recorded here included Quickwrites (including sentence start prompts and student reflections), posing questions where the answer is $1m^3$, answers to pre-assessment questions and student responses to teacher questions. These have been highlighted in Table 5-2 with an asterisk. As students collaborated throughout the inquiry, each group recorded their ideas on an A3 sheet of paper (Figure 5-4 below is an example). Analysing both the individual and collaborative efforts of students in this way would consider how student thinking might change, or be influenced by the collaborative environment of inquiry. In inquiry, it is important to consider the social environment when describing an individual child's learning (Goos, 2004; Makar, 2012). I began to question whether it made sense methodologically to singularise the mathematical thinking of one student without considering influences developed through collaboration with peers, adding a social dimension to the concepts of connections and reflection.

A further dimension of formative assessment in inquiry reflected the purposefulness of the feedback loop. A closer look at the assessment interactions listed in Table 5-2 prompted questions of who utilised the assessment information, or feedback. These interactions were categorised to illustrate who used or who was informed by the feedback and the length of time it took to close the feedback loop; literature noting how a shorter or more timely feedback loop is more effective (Furtak & Ruiz-Primo, 2008; Hattie & Timperley, 2007; Shute, 2008). Yet an important aspect of this inquiry was in the feedback that purposefully provoked student thinking due to having a longer time to close the response loop. This included the responses to pre-assessment questions recorded by students in their learning journals. In the classroom, I had purposefully chosen not to provide correct answers to the pre-assessment questions. It would not be until students presented a peer-reviewed and accepted method or solution to the inquiry problem that they would know how their previous responses (individual thinking) were acceptable ways of working with volume. This was a long feedback loop used to encourage students to evaluate (during the Defend phase) their early thinking about volume.

A rubric was created (Table 5-3) to consider the mathematical thinking of students, evident in the reflections recorded over the time of the inquiry, in their learning journals. Early exploration of these artefacts was messy in an attempt to reduce the large amount of material I had collected. Modification of the PISA assessment framework (OECD, 2009) showed promise as a way to rate or evaluate assessment interactions that reflected learning and to show the levels of thinking illustrated. This would assist in making judgments about the organisation of data in terms of competency clusters. For example, *thinking and reasoning* is described by PISA (OECD, 2009) as an ability to pose questions characteristic of mathematics, and knowing which kinds of mathematical statements or understandings could offer answers. To get a score of 2 (the connections cluster), a characteristic response would need to show understanding of the corresponding kinds of answers in contexts slightly different to those already practiced.

The same rubric was also used to evaluate learning reflected in the collaboration portrayed on the A3 sheets of paper (Figure 5-4). An example of a score in the connections cluster is illustrated in the thought bubbles in Figure 5-4 where the group considers a range of contexts (sand pit $1m^2$, boxes, dog home, dimensions of a dog/to make a doghouse) and then select the most suitable; a dog kennel. Artefacts were rated according to each competency cluster: reproduction, connections, or reflection with a 1, 2, or 3 respectively. In the PISA assessment framework (OECD, 2009), there is considerable overlap in the processes or competencies students use to work mathematically, as is common when working through mathematical inquiries in the classroom. Potentially, each artefact could receive a maximum score of three for each of the 8 competencies, to achieve a total score of 24.

Table 5-3 An example of the completed rubric used to rank the use of competencies by Laura, Gus and Chris, evident in their reflection journals.

Description	Competency	Cluster: Reproduction – 1 Connections – 2 Reflection – 3
Thinking and Reasoning	1	2
Argumentation	2	1
Communication	3	2
Modelling	4	1
Problem posing and solving	5	1
Representation	6	1
Using symbolic, formal and technical language and operations	7	1
Use of aids and tools	8	1
	Total score:	**10**

Evaluating written entries in the students' learning journals using the PISA-based rubric would assist in analysing thinking and learning for individual students that was personal (See Figure 5-5 as an example of a journal entry). Looking at Fleur's scores as an example (Table 5-4) we can see that she received a total score of 4 out of 24 (where 3 is the highest possible score in each of the 8 competencies to equal 24). Written entries by Fleur have not been included in this thesis as illustrative examples although I expand on her efforts here to explain her score of 4. Her learning journal included reflections on decisions she and her partner had made about how to represent their cubic metre (Thinking and Reasoning and Communication scores of 1). Her reflection also included an algorithm for calculating volume (Language and operations [derived from the category Using symbolic, formal and technical language and operations, OECD, 2009] and Problem posing and solving scores of 1). Further, focusing on the competency of argumentation (OECD, 2009), her journal did not involve any justification of mathematical processes she used and so she received a score of 0 out of 3. If there was no evidence of the competency, then a score of 0 was allocated.

Table 5-4 Scores based on one entry, a Quickwrite, recorded digitally in learning journals.

Student	1 Thinking and Reasoning	2 Argumentation	3 Communication	4 Modelling	5 Problem posing and	6 Representation	7 Language and operations	8 Use of aids and tools	Total Score	Context
Laura	2	1	2	1	2	2	1	2	13	Dog Kennel
Fleur	1	0	1	0	1	0	1	0	4	Concrete slab for a shed
Scott	1	1	1	1	1	0	1	0	6	Fish bowl (developing ideas)
Brianna	1	1	1	0	1	0	1	0	5	Concrete slab for a shed
Suzie	0	1	2	1	0	0	1	1	6	Crate filled with cans
Robbie	0	1	2	1	0	0	1	0	5	Garden cubic metre/bed
Ophelia	2	1	2	2	1	0	1	0	9	Touch pool for starfish
Paul	2	1	2	2	2	0	1	2	12	Crate filled with cans
Patrick	1	0	1	0	1	1	1	1	6	Crate filled with cans
Chris	1	1	1	1	1	0	0	0	5	Dog Kennel
Mary	1	1	1	2	1	1	1	2	10	Cushions
Calvin	0	0	0	0	0	0	0	0	0	Touch pool for starfish
Naomi	1	2	1	1	1	1	1	1	9	Touch pool for starfish
Gabriel	1	1	1	1	1	0	1	2	8	Spa holding one cubic metre
Francis	0	0	1	0	0	0	0	0	1	Spa holding one cubic metre
Louise	0	0	0	0	0	0	0	0	0	Spa holding one cubic metre
Gavin	1	0	1	0	1	0	0	0	3	Ute that can hold one cubic metre
Natalie	1	1	1	1	1	1	1	2	9	One cubic metre tidy tray
Andrew	0	1	1	0	1	0	1	1	5	One cubic metre tidy tray
Average	<1	<1	1.16	<1	<1	<1	<1	<1	6.11	[2]

Dewey's (1891) explanation of thinking informed further consideration of this evidence. He described how thinking included aspects of conception, judgment and reasoning, each of which could not occur without each of the others. Dewey's processes of conception included abstraction where the mind *seizes* upon one aspect or element of the perceived concept, by actively relating qualities to past experiences (apperception). The scores that resulted from the PISA evaluation (Table 5-4) seemed low overall although the learning experience of inquiry offered students an opportunity to make abstract through apperception, mathematical concepts of volume and measurement. I had not predicted that learning in an inquiry classroom would reflect lower-level thinking as outlined by PISA (OECD, 2009), particularly given the research reporting into this pedagogy (Fielding-Wells, 2010; 2014; Fry & Makar, 2012; Makar, 2012). Instead, I considered the evidence here as part of a learning journey, where students were beginning to develop vital

[2] Results for nine students are not included in Table 5-4. This was either due to not gaining participant permission, or because students were absent or did not complete written reflections during the inquiry.

experiences of judgment and reasoning. Throughout Dewey's description of thinking he used the language of development included here to explain the thinking of students evident in their learning journals. Research by Makar (2012) similarly presented this notion of learning taking place in inquiry that is apperceptive. The teacher she interviewed realised that what she perceived as 'failed' experiences from one inquiry in fact had transferred to the next inquiry and had a strong impact on students' developing mathematical understandings. Although the PISA assessment framework seemed to measure learning captured through assessment broadly, the results in Table 5-4 are not enough to consider learning for these students. This analysis could only offer a general sense of the level of thinking which relied upon a student's ability to articulate their thinking, in writing. It did not consider the idea of thinking generated through participation in discussions, or during the quiet individual moments of pondering. Further analysis needed to consider the social nature of inquiry pedagogy.

Thinking in the connections and reflections clusters: How much is a cubic metre?

This inquiry was designed for students to explore cubic metres and to measure volume in contexts that were real to them, hoping to extend student thinking to the connections cluster by fostering a link in students' mathematical thinking with real-world representations. Initial analysis of reflections in students' learning journals (Table 5-4) provided a general overview of the level of thinking exhibited by students in the class. Ratings revealed no evidence of students working in the reflection cluster (no scores of 3 were allocated) although the score totals did show evidence that some students were making connections (scores of 2). Only three students in the class received a total score higher than ten, and of these, the highest score was 13 (out of 24). The scores for Laura, Paul and Mary (Table 5-4) warranted further investigation. To gain a broader picture of learning for these students beyond reproduction, attention turned to artefacts that displayed group or collaborative efforts linked to Laura, Paul and Mary.

Laura: Collaborating to make connections

A sense was building that students in this class were still working mainly at the reproduction level (Table 5-4) although the journals of some students indicated working beyond this in the connections cluster. Laura was one such student. Using PISA, her learning journal received a higher ranking than her peers. In each statement she made she referred to Gus and Chris with whom she had worked. You will note that there is no score in Table 5-4 for Gus. This was because he had not recorded any reflections or workings in his reflection journal. For Gus, bookwork was a difficult task and generally, his classroom efforts were recorded as audio. A closer look at the work produced by Laura's group (A3 sheet of paper), showed evidence of thinking in the connections cluster in PISA (OECD, 2009). Their collaborative ideas are expressed below (Figure 5-4) where they represent one cubic metre as an object in the real-world, a dog kennel. They initially considered representing a square metre as a sand pit, then in thinking about 'people who would like to store things' have sketched boxes as a structure. These reflections show how the students' reflections shift from initial ideas about area to representing volume, through the size of the dog kennel. This matches descriptions of thinking in the connections cluster, of using mathematics in situations not routinely practiced in the classroom but that are familiar (OECD, 2009). This also related to the theoretical process of conception used by Dewey (1891) which included the process of comparison; alongside abstraction, the learner goes from the isolated idea to the idea as connected with other objects. This becomes purposive as Laura, Gus and Chris continue to reproduce principle understandings of cubic metres in more than one way ($1m^3$, 3D, cube, 1 tonne [tone], volume, dog kennel). They also translated the mathematical concept from a sketch of a cube, to a sketch of a dog kennel showing hidden edges, to a more 'real' representation of the dog kennel they wanted to produce. These characteristics of thinking translated to working beyond the reproduction cluster, to the connections cluster as described by PISA in Figure 1, and supported a relational aspect of learning (Dewey, 1938a).

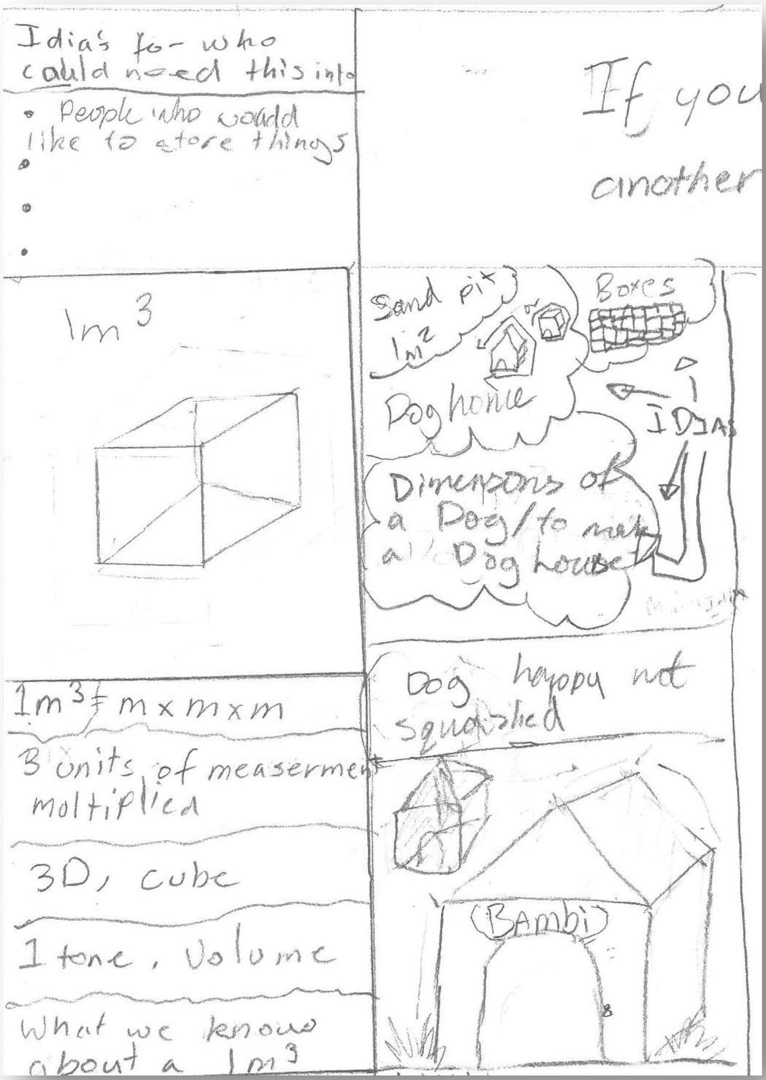


Figure 5-4 Laura, Gus and Chris making connections.

Paul: Thinking in the reflection cluster

The PISA assessment framework (OECD, 2009) adopted in this iteration of study, to characterise what assessment revealed about learning, was designed to assess 15-year olds. Participants in this study were much younger. Looking more closely at the criteria in the reflection cluster, I searched for evidence of students exhibiting the knowledge and skills it described, at a level higher than their peers. Notes on one such student, Paul, also appeared in the reflections in the teacher-journal. This prompted me to look closer at his work.

Paul's written responses were more articulate (contained higher instances of coding) than other students, and closer analysis of his written thoughts revealed more about his learning in the inquiry, supported through formative assessment. The inquiry had already shown support for some students to use competencies in the connections cluster (see Laura, Ophelia and Paul in Table 5-4) and further evidence was required to show if higher-level thinking was used in mathematical problem solving in this inquiry, as part of the reflection cluster.

Paul collaborated with Patrick and Suzie on this inquiry task. He had ‘flagged’ his confidence in understanding volume as green, whereas Patrick and Suzie had both used orange traffic light cards (Wiliam, 2011a) to indicate that were not as confident. In the first inquiry lesson, Paul’s responses to the pre-assessment questions indicated he already understood the mathematical topic being explored (Figure 5-5). His first answer showed that he had already learned or achieved the outcomes or goals for the original, commercial unit of work. If students are only tested for knowledge acquisition (Sfard, 1998a) at the end point of a series of lessons on a particular topic, then an end point in learning has already been defined: there is no reason to continue further. The inquiry would offer Paul the opportunity to connect his understandings with other mathematical strands, to solve more complex problems that he himself would pose, set in unfamiliar or unrehearsed contexts. I analysed Paul’s PISA ratings more closely as an alternative way of describing what new understandings he developed in the inquiry from this point forward.

The inquiry challenged Paul to continue. Expectations in the inquiry classroom were already established which challenged students to consider real contexts in which to explore mathematics. After meeting with his group and brainstorming ideas for answering the inquiry question, Paul explained in a Quickwrite (Figure 5-5) how they were going to represent the cubic metre. Together they had made the decision to calculate and show how many 375mL drink cans fitted onto a crate that held one cubic metre. They had decided on the dimensions of a crate and how to represent this digitally. Paul’s reference to the real life example showed a deeper understanding of the concepts of volume and mass, reflecting his thinking in the connections cluster. I am unable to separate Paul’s thinking from his peers. The ideas he recorded were generated in collaboration with his group members and so I refer to all three students in this analysis.

> Today Miss Fry said that we needed to answer the question How much is 1 cubic metre? … our group with *Patrick*, *Suzie* and I chose that we would present this by showing how many 375mL drink cans can fit into the crate. We decided as a backup plan if we did not have enough cans we would show it digitally by place photos of cans inside the 4m by 0.25m by 1m crate. We thought this was like real life because companies who package cans would need to know how many cans they can ship in the least amount of crates to save money. My guess for this will probably be about 900-1000 cans.

Figure 5-5 Paul's Quickwrite

Solving this problem would require “a chain of reasoning and sequence of computational steps” (OECD, 2009, p. 110) that were not fostered in the book-unit of work. PISA would describe Paul’s task as: requiring higher cognitive-demand than the mathematical processes in the reproduction cluster to solve it; containing more elements and being more original than problem settings in the connections cluster; and making connections between different mathematical areas through reasoning beyond his peers in the reflection cluster. The cubic metre they made required the three students to firstly measure the dimensions of one can, to calculate how many cans would fit on the first layer of a crate with the dimensions 4m x 0.25m x 1m. Then they considered how many layers of cans could fit on their crate. A can is cylindrical, so they also became interested in how much space would be left between each can (or wasted) as it was stacked on the crate. The group of students was interested in whether it was an advantage economically to sell drinks in cans or as rectangular prisms, and the overall difference between what the crate could hold and how much soft drink was actually stored on the crate. After calculating how much soft drink the crate could hold, the group posed further the second part of the problem they identified: “how many cans they can ship in the least amount of crates to save money.” Knowing which mathematics was involved in these processes started to illustrate Paul’s thinking and reasoning in the reflection cluster. Modelling this digitally (as Paul suggests in Figure 5-5) translated the problem to a context that was different from what students had already practiced. Depicting the cubic metre as soft drink cans on a pallet, and considering the cost efficiency of this, involved elements of ‘reflectiveness’ as described by PISA (OECD, 2009).

When applying PISA’s (OECD, 2009) definition of mathematical literacy (Figure 5-2) to Paul’s written response (Figure 5-5), it reflected thinking in the reflection cluster, although it must be considered that Paul was younger than the 15-year olds PISA intended to assess. Working in the reflection cluster was described using the key descriptors of “advanced reasoning, argumentation, abstraction, generalization and modelling applied to new contexts” (OECD, 2009, p. 112). Paul had articulated how to explore the concept of cubic metres where success would require on these characteristics. His description illustrated thinking and reasoning beyond that of his peers (including Patrick and Suzie’s).

I would argue that Paul had already abstracted the concept of cubic metres in the Deweyan (1891) sense of the process of conception. He had connected the abstract idea of a cubic metre to ideas of a packing company storing and moving soft drink cans, recognising qualities of the problem associated with money. The *organic unity* Paul presented in Figure 5-5 was his synthesis of the concept of cubic metres (Dewey described synthesis as an organic unity). The concept of cubic metres, as an aspect of thinking, was not intended to be considered in isolation to other aspects of thinking, yet has been separated here to illustrate how in this classroom, inquiry supported the processes of thinking and learning made visible through formative assessment.

Insights into the reflections and thinking of students like Laura and Paul described thinking beyond low-level, routine problem solving required in the commercial assessment task. Formative assessment offered support for student learning (Hattie and Timperley, 2007; Wiliam, 2011a) by encouraging connection and reflection, and allowed insight into what students were thinking (Furtak and Ruiz-Primo, 2008). From a design research perspective, these opportunities were carefully planned for and informed by assessment literature to ensure learning was captured in multiple, flexible ways, supporting the highly interactive nature of teaching and learning in this context. The assessment opportunities also revealed aspects of learning that were not considered assessable information in the commercially produced assessment task. These included the connections made by students that supported abstraction, comparison and synthesis as part of the process of conception (Dewey, 1891). Opportunities to reflect had offered students moments to synthesise their ideas and thinking. In a Deweyan sense, opportunities to abstract, compare and synthesise collectively contribute to the process of thinking.

Mary: An insight into learning beyond right and wrong

Using axial coding, properties and dimensions of the category of learning were identified, including *being stuck* and *overcoming challenges*, considering *incorrect ideas* (based on the learning goals of the inquiry) in relation to *correct application* of taught procedures. These properties reflected possible moments in the process of students fitting certain experiences into certain thoughts, to move forward in learning (Duckworth, 2006). An example was found in Mary's thinking. Mary's learning journal had reflected the third highest score of the journals assessed using the PISA rubric (Table 5-4). I compared her learning journal, to teacher reflections about Mary, as well as to evidence of her input in group work (A3 sheets reflecting collaboration). Her initial answers to the pre-assessment questions were incorrect. She stated that grams and kilograms were the units of measurement to describe the volume of solids and explained how one tonne of feathers would be heavier than one tonne of bricks as there would be "more feathers than bricks." She explained further how she was unable to think of any problem to match the answer $1m^3$: she was *stuck*. She had also already indicated to the teacher that she lacked confidence in her understanding of volume by submitting a red traffic-light-card. Yet after discussing with her group how to approach the inquiry question, her journal reflected a change in her thinking about cubic metres. In response to the inquiry question she recorded ideas such as finding out 'How many cubic metres am I?' and 'How many cubic metres would fit (in our) class?'

Development of her understanding about using cubic metres to measure volume was evident in the following lesson where the class was asked to calculate volume using given measurements.

Independently, Mary calculated an answer to the problem posed by the teacher of 1.3m x 1.5m x 75cm (Figure 5-6). She explained how to do this by multiplying the three measurements together. The answer she calculated was incorrect though. The error she made was not converting the 75cm to 0.75m before multiplying. The question was devoid of context which made it difficult to check the reasonableness of her answer. The students were asked to compare their answers with the other members of the group to check what the correct answer could be. After the group discussed and shared their answers a *shift in* (her) *thinking* occurred and Mary changed her answer to fit with the thinking of her peers; 'Our groups answer' was capitalised and bolded and showed the agreed answer to the problem as 1.4625cm^3 which was correct (Figure 5-6). Duckworth (2006) described these transitional moments of children moving ahead as Piagetian notions; perturbations (being stuck) are overcome when equilibration takes place (in my early analysis coded as a *shift in thinking*). Mary continued to explain her answer (Figure 5-6) with the inclusion of a table showing the answer converted to other units of measurement including cubic centimetres, cubic yards, cubic feet, cubic inches, U.S. gallons and litres. This information was cut and pasted from a website the group had researched. Finally, Mary's learning journal concluded with a Quickwrite confirming a plan of using square cushions (from home) to make the cubic metre to show in class incorporating concepts familiar to cubic metres. Mary also provided a prediction of the number of cushions they would need to 'bring in' to complete the cubic metre before making any calculations (Figure 5-6).

146.25

1. You would multiply 1.3 x 1.5 x 75cm
2. the answer would be 146.25
3. then you

<u>OUR GROUPS ANSWER:</u>

Volume	= 1.4625 cubic meters
	= 1462500 cubic centimetres
	= 1.9129 cubic yards
	= 51.648 cubic feet
	= 89247 cubic inches
	= 386.35 U.S. gallons
	= 1462.5 litres

67 cushions

Today we were thinking about 1m3 we are going to use square cushions to make our 1m3. I am going to bring in 10 square cushions and *Wendy* and *Molly* are going to bring in 1.

Figure 5-6 Mary's journal entry.

Contributions to the properties of learning identified in Mary's reflections included: *stuck, shift in thinking, incorrect, correct, assessment* and *learning*. Traditional assessment would only value the correct final answer and seek evidence of the ability to follow taught procedures correctly. Formative assessment principles and the collaborative nature of inquiry allowed opportunity for further insight into developing understandings. Her answers to the pre-assessment questions were incorrect and attempts appeared to be based on previously taught procedures. The tasks had not yet challenged her to think beyond the textbook-type ideas already practiced. After discussing her answer and ideas with the group we see that she changed her answer, using the title 'Our groups answer.' Mary's journal entry after this (Figure 5-6) revealed what her group would do next (Devising a plan) to answer the inquiry question. In her reflection (Figure 5-6) she actually uses the word 'thinking.' I wondered if thinking wasn't required in the first instance to complete the procedural-based task, yet the conversation with her peers allowed her to do so.

An aim of this study was to find out how, through assessment, to identify and describe learning in ways that are well-informed and respectful of students as learners of mathematics in inquiry. Moments where students were *stuck,* and when ideas reflected a *shift in thinking,* might be vital in the development of the concept of cubic metres yet were not considered in the commercial task as assessable information. As described by Sfard (1998a), learning is the constant flux of doing. Only the correct or incorrect answer would be of interest if knowledge acquisition was most valued (Sfard, 1998a). Compared to Paul (mentioned above), traditional assessment would have revealed that he had learnt nothing through inquiry, as he already displayed correct answers to the pre-assessment questions. It seemed that current assessment practices were not respectful of learning in this inquiry classroom.

Retrospective Analysis

Analysis in this iteration of study focused on assessment practices evident in one inquiry classroom, to provide a window into the mathematical learning being supported. Firstly considered was the summative assessment task that students were required to complete, as part of general classroom assessment practice. Clusters identified by PISA (OECD, 2009) were used to characterise the levels of thinking it supported as reproduction of taught procedures and knowledge. Completing the summative assessment task did not require any intimacy with the mathematical content, nor to connect learning to any personal referents. Secondly, Table 5-2 offered a list of all of the formative assessment opportunities planned for in the inquiry *How much is a cubic metre?* which presented assessment in inquiry as frequent, highly interactive, reflexive and striving to know what students are thinking. Assessment considered student learning personally and in collaboration with their peers. Feedback loops were purposeful and designed to encourage thinking. Next, thinking was elaborated on using Dewey's relation notion of learning and considered PISA's connections and reflection clusters as vital to students' conceptions of mathematical ideas. Formative assessment offered opportunities for apperceptive abstraction of student ideas, comparison of these ideas and ultimately, to synthesise thinking and conception as part of the journey of mathematical learning. Finally, this journey for one student reflected properties of learning including *being stuck, overcoming challenge*, *incorrect ideas* and *correct application* of taught procedures. The learning journey for students was not completed by the end of this one inquiry alone and assessment would need to consider learning beyond the inquiry, into future inquiry experiences (Makar, 2012).

Presented in this chapter were two different ways to assess mathematical learning. The first was outlined through the commercial unit of classroom work and the related method of assessment (Figure 5-3). In a traditional sense, the assessment in this unit of work was designed to measure the learning outcomes identified (Table 5-1) and focused specifically on an ability to replicate taught rote procedures correctly to solve examples of routine computations. The inquiry into *How much is a cubic metre?* offered an opportunity for students to have a more intimate, personal experience with the mathematical content to be explored (Dewey, 1891; 1938a) as the students decided which context to explore. Principles of formative assessment also considered learning that took place in collaboration with peers. Through formative assessment, a picture of learning in inquiry was revealed as broad and complex. In terms of student thinking, the conception of mathematical ideas was supported by the many opportunities to revisit mathematical content and to make personal connections. This would seem to indicate that learning in inquiry is only part of a learning journey, based on personal experience. Unfortunately, this view was not supported by the commercially-produced assessment task.

Research question one: How is mathematical learning in one inquiry classroom assessed currently and does this align with learning in this inquiry context?

Assessment designed to measure learning in the commercial unit of work leading up to the inquiry (Figure 5-3) may have been well-suited to the pedagogical choices aligned with that teaching and learning resource. Evaluated using a modified PISA assessment framework (also used to evaluate students' learning journals, Table 5-3), this assessment task required students to use the lowest level of cognitive activity to answer the questions (working in the level of the reproduction cluster). The inquiry was able to extend student thinking into the connections cluster, and beyond for some students as they reflected on other areas of mathematics to solve problems they generated.

Supported by formative assessment, students were able to generate purposeful connections between mathematical content through the experiences and contexts generated by the students to solve the problem. Dewey considered when connections between trying something out and its consequences result in further finding out, we learn (Dewey, 1966). In this inquiry, students connected mathematical learning to situations that were not always routine (OECD, 2009), encouraging children to have a more active role in constructing their own knowledge (Gravemeijer, 2014). The commercial assessment task (Figure 5-3) was not suited to describe learning in the inquiry as the task did not consider student use of cognitive skills in the connections and reflection clusters identified by PISA (OECD, 2009), or learning that took place in collaboration with peers, as part of each student's learning journey.

The commercial assessment task did not consider students' prior knowledge, exploration of topics, mistake-making, being stuck, and shifts in thinking; all properties of learning identified through analysis of data in this study. These properties mirror Piaget's (1952) notion of intellectual adaptation, where external realities that don't seem to fit with a subject's activity are accommodated for into progressive equilibrium. It is worth noting however, that if the classroom teacher does not consider the moments illustrated in this chapter as learning, planned assessment will also not consider these insights. A lack of acknowledgment of inquiry skills and processes could further be unsupportive of learning in this context.

Research question two: Are there characteristics of assessment that support learning in inquiry and can illustrate insight into what students know?

My own response to the first research question introduced a notion that learning mathematics in inquiry was in contrast to reproductive thinking as described by PISA (Figure 5-2). Only reproductive thinking was measured by the commercially produced assessment task (Figure 5-3). Engineering the artefacts for the inquiry unit in this phase relied on literature in formative assessment (Black & Wiliam, 1998a; Furtak & Ruiz-Primo, 2008; Wiliam, 2011a) for analysis into student learning (Table 5-2). Analysis of these artefacts illustrated that some students in the inquiry setting, were working in the connections and reflection clusters while considering learning as participation in a classroom context (Goos, 2004; Sfard, 1998a; Vygotsky, 1978). For assessment to *support* learning in inquiry, it needs to *value* learning in inquiry, possibly aligning with descriptions of competencies in the connections and reflections clusters (OECD, 2009). When students are offered the chance to abstract, compare and synthesise thinking by connecting ideas to other ideas, the process of conception can take place (Dewey, 1891). Future phases of data collection will continue to include formative assessment practices in the engineering of the inquiry to support student learning, and to continue to offer insight into student thinking and learning.

The planned use of pre-assessment could support learning in inquiry. In this study, it was intended that students would use their responses to the pre-assessment questions to inform them of their own learning. Pre-assessment, as a formative assessment technique, generally makes gaps in learning obvious to students when teachers offer feedback on this information. Pre-assessment was purposely designed into this inquiry learning experience to capture student thinking before the inquiry unit, and after teaching and learning in the commercial unit of work had taken place. The feedback loop would take time to close but this was purposeful and intended to provoke thinking. It encouraged the students to consider their own responses throughout the inquiry process. Their responses were read each day as students recorded new ideas in their reflection journal. It was hoped that the students would see a difference in their thinking themselves that did not include a teacher's interpretation of wrong or right, knowledge acquired or yet to acquire. Pre-assessment will be a formative assessment practice to be developed further in the next phase of study.

Research question three: What understandings, skills and procedures are developed by students through inquiry as they learn mathematics?

What kind of mathematician was illustrated in the assessment in the commercial unit of work? Mathematicians in this class were quickly and briefly introduced to mathematical concepts, presented without careful consideration of real-life contexts that were relevant to the students themselves. The teacher deemed what mathematical knowledge was important and measured acquisition of this knowledge as a test score. Correct answers were valued and incorrect answers would require further remediation although the topic may not be revisited again until later in the year. Success depended upon routine practice of basic procedures. These characteristics hark back to Dewey's (1938a) definition of traditional education considered unsuitable for preparing students for the twenty-first century (Gravemeijer, 2014). Students had until the end of the unit to learn or acquire mathematical knowledge and one opportunity to demonstrate it correctly. It was not necessary for students to use the learnt mathematical knowledge in any unfamiliar context or to consider how it related to other content areas. Nor were students required to ponder the purpose behind the acquired knowledge or how it related to the real world. This describes assessment that was typical practice across the year level and student results were compared across classes in order to allocate letter-grades to students for reporting purposes. Would students learning in this way become numerate beings of the future?

Regarding levels of thinking, ideally the challenge was to situate students' use of mathematical competencies in the reflection cluster through inquiry. Collaboration was an important element in this inquiry also. A closer look at evidence of learning for Laura, Paul and Mary considered their participation in a community of mathematicians who: were encouraged to consider and use their prior knowledge to make connections (what kind of dog would fit in a kennel with a volume of $1m^3$?); reflected on real-world applications that are complex (how can we economically pack and send this soft drink?); benefited from mistakes (comparing, listening to and defending answers among peers); and contemplated the ideas of peers to form new understandings (Why is my answer different? Am I wrong?). Using PISA, mathematicians in this classroom used higher levels of thinking about the mathematics they were exploring.

Potential Innovations to Consider

Retrospective analysis of this first phase of study was intended to either confirm or refute suppositions, or to propose alternative suppositions for the next phase of analysis (Confrey, 2006). Frequent formative assessment experiences in this inquiry fostered the chance for students to abstract, compare and synthesise mathematical thinking, contributing to mathematical learners being active. Assessment in future iterations of study would need to be adaptable to consider the range of mathematical topics being explored. For Laura and Paul this would include: knowledge of cylinders and ways to measure them, rectangular prisms and how they could be stacked, volume of triangular prisms, 3D shapes that are regular and irregular, calculating volume of 3D shapes, and concepts such as the most cans that could fit on to a crate to make it economically viable for transporting. This content was beyond that which was assessed by the commercial unit of work (Figure 5-3). Understandings of volume presented through formative assessment practices in the inquiry were much broader than the requirements of the assessment task. For instance, Paul, Patrick and Suzie chose to explore cubic metres in a way that was very different to Laura, Gus and Chris. In this inquiry, the number of opportunities for students to express their thinking, through the use of formative assessment principles, assisted students in making connections and reflecting on other areas of mathematics. When assessment requires students to demonstrate only reproductive thinking, why would a classroom teacher choose inquiry pedagogy to teach mathematics? In this phase, the inquiry was the means of assessment of students' conceptual understandings of a cubic metre.

Effective feedback has a critical influence on student learning (Hattie & Timperley, 2007). Assessment opportunities in this inquiry presented a range of teacher-student and student-student interactions (Table 5-2). Feedback was generated in multiple and different ways throughout the learning process. A closer look at these interactions, and the resulting feedback, offered insights into student thinking that revealed further information about the connections they were making to other areas of mathematics and personally relevant contexts. It revealed students being stuck and overcoming challenges as opposed to a neater process of not knowing then knowing. Frequent formative assessment practices were engineered to gain insight into student learning in inquiry. In the classroom, feedback gained requires the teacher to make on-the-spot decisions making inquiry teaching difficult to plan for. The majority of formative assessment practices in this classroom had a short feedback loop (Table 5-2), with much feedback generated on-the-spot. A teacher's ability to frequently and reflexively offer feedback to learners seemed an important feature of this classroom. If the inquiry in this phase (as a means of assessment) demonstrated students' understandings then as a teacher, this information must be used to inform teaching practice. Future phases of this study would need to look more closely at how teacher decisions were informed by feedback gained through formative assessment. These 'split-second' decisions may interact with and influence student learning in ways that offer further insight into student thinking.

Feedback is most effective when the information is used to adjust teaching to match the present understandings of the students, closing the gap between where students are and where they aim to be (Hattie & Timperley, 2007; Sadler, 1998). The difference between where students are in an inquiry (Vygotsky's actual developmental level), and where they need to be (level of potential development), reflects Vygotsky's (1978) zone of proximal development (ZPD). In inquiry, feedback is important if it is being used by the teacher and students to inform where students are in the learning, so that adjustments to teaching can be purposeful and respond to learning needs, a major premise of Vygotsky's ZPD. If we consider the response to the previous research question that mathematical learning in inquiry is broad, adjustments to teaching would also need to be broad and flexible to assist students in completing difficult tasks, or moments when they were stuck. This is different to how teachers respond to learning in a traditional classroom, where neat problems are designed to practice taught methods. Identification of where students are in their learning in a traditional classroom might consider only whether the student can perform the routine correctly or not.

Assessment practices that reflected a school mathematics tradition did not fully consider student learning in this inquiry classroom. In this inquiry classroom learning was connected to personal and contextual experiences and reflected a broad range of mathematical understandings, to select which was relevant the problem (Dewey, 1966; OECD, 2009). Feedback from interactions informed teacher planning and the teacher considered how learning developed in collaboration with peers (Goos, 2004; Makar, 2012). Formative assessment strategies (categorised by Wiliam, 2011a; b

[Figure 2-1]) will continue to be used to support learning in inquiry in the next phase of study; learning described by PISA (OECD, 2009) in the connections and reflection clusters. Formative assessment practices offered insights into student thinking beyond a raw score, and highlighted properties of learning so far. Of interest in the next phase of study is how the teacher uses the information gained through formative assessment about her learners, to inform her teaching.

There are two practical points of concern when considering multiple assessment opportunities throughout phases of participation in learning in inquiry: what to assess at each point and the time it requires a teacher to read and evaluate each and every child's thoughts, ideas, conversations or *scribblings* of inchoate ideas. It is these expressions of learning that may offer insight into the kinds of beliefs are fostered through inquiry (Duckworth, 2006). If formative assessment is feedback that influences learning and teaching, then inquiry is analogous with formative assessment as the classroom teacher listens and responds to feedback about learning, changing their teaching to suit. How the teacher reacts to learning opportunities in inquiry, supported by formative assessment, determines the importance of the information they capture. This in turn communicates to students what is most valued in the classroom.

Figure 5-7 is my impression from this first phase of study, of the difficulty in trying to align the classroom elements of assessment, teaching and learning with the inquiry pedagogy. Considerations from this phase might place assessment more centrally (than in Figure 4-1), between the extremes of scientific measurement and rich, qualitative illustrations of learning. In this phase, formative assessment strategies were able to capture and describe learning qualitatively, to consider the reproduction of taught of mathematical knowledge and higher levels of thinking about mathematics. Yet what will be the implications of this in the subsequent phases of study in terms of teaching and learning? How might teaching practices in the next mathematics inquiry classroom be identified and analysed in ways that support the implementation of these assessment practices? The next phase of data collection will highlight the pivotal role the teacher plays in the inquiry classroom, with regards to assessing learning in this context in ways that respect and support the culture of inquiry.

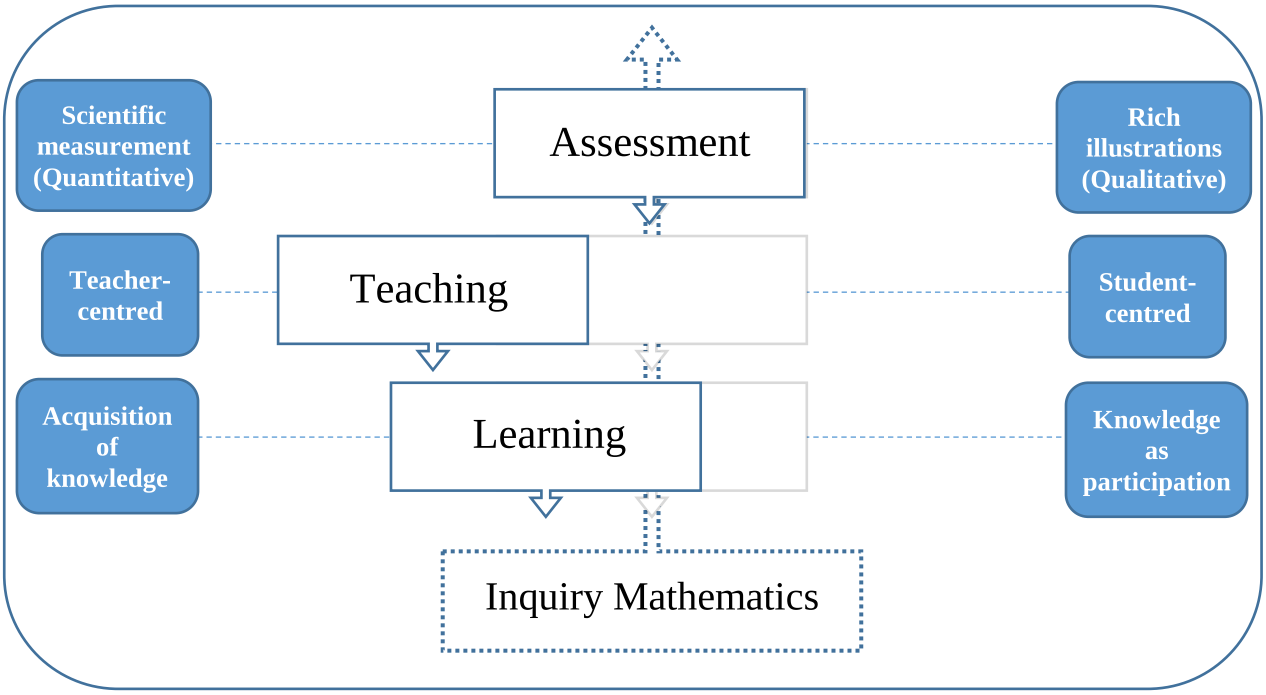
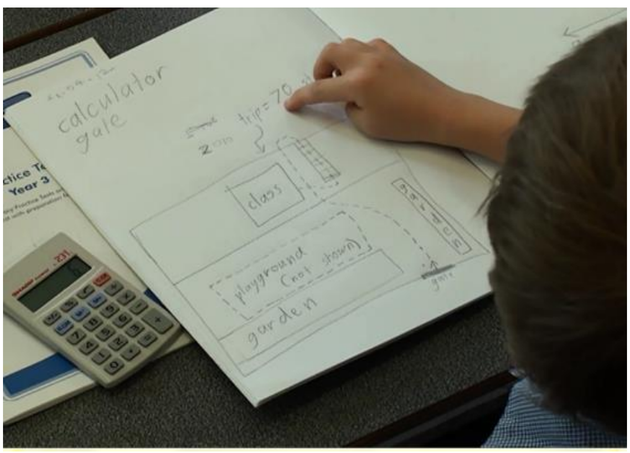


Figure 5-7 An impression of findings: Difficulties in trying to align classroom elements with inquiry.

Chapter 6 Phase Two: A Teaching Focus

How many steps do you walk in a day? The whole class sits on the carpet facing the teacher's small whiteboard which displays the question written on a poster. After a moment to consider this by themselves, one student exclaims that it would take a long time to count. The teacher stops introducing the inquiry and promptly records 'count' on the poster. Quickly, another student rejects this idea of counting and suggests using a pedometer. Partly to enable the teacher to finish recording their ideas, she asks them to explain what a pedometer is and how they would use it. In trying to bring the focus to the mathematics in the inquiry question, other mathematical suggestions are quickly recorded and organised by the teacher, and ideas rephrased for the whole class to hear. Measurement terms such as metres, paces, steps, hours, and days are recorded; mathematical processes of multiplication and addition are noted also. Students also consider how they might walk different amounts each day depending on the activities they did. While the students eagerly consider how many steps they might walk in a day the teacher directs them to record their predictions in their books. Already students begin to differ in the pathways they will take on their journey to answering this question. One student asks if it is ok to record up to five predictions, for different days of the week. A friend agrees and while some record their ideas quietly, these two students continue to discuss the regular activities they participate in on different days. The teacher explains that soon they will be solving the question and in pairs, students start planning how they might collect their data. Noisily, the children begin sketching their "mud maps," rough maps not drawn to scale, of areas in the school that they will walk to; they make decisions about how they will record the information they collect and the different jobs each person will have, such as recording, counting, walking and even skipping. They soon share these decisions with the whole class, encouraging other students to think differently about the process of answering the question and to explain the counting process they will use.



Research Aims

The research question that provided the focus for planning and analysis of data in this iteration of study, turned the focus of research to teaching in one Year 3 inquiry classroom, and how it was responsive to feedback gained through formative assessment:

Research question: How does one teacher of inquiry mathematics respond to feedback gained during formative assessment, to guide student learning towards particular learning goals?

> *Supposition:* An inquiry pedagogy presents a messy approach to learning mathematics. Constructivist notions of personal learning and variety in mathematical approaches are encouraged. Although the classroom teacher guides students towards the learning of mathematical goals, the directions students take are often varied as they attempt to answer inquiry problems in a variety of ways. The classroom teacher in an inquiry classroom needs to be flexible if they are to adjust their teaching as a result of feedback gained through formative assessment; flexible in response to the learning and social needs of their students, in response to the mathematical content they are required to teach, and in their response to the mathematical thinking or strategies that their students use.

Findings from the first phase of study revealed that through inquiry pedagogy, children can think broadly about mathematics. Research in this phase aimed to consider the teacher actions to push learning in an inquiry classroom, leading to deeper thinking. Informing this phase of analysis was the literature on the classroom environment in an inquiry classroom, as summarised in the theoretical framework of this thesis. Literature informing this phase of study included social constructivist ideas and principles of formative assessment. A key characteristic of a constructivist teacher is listening to students to consider how they are progressing towards solving a problem (Confrey, 1991). Feedback gained through formative assessment can inform the teacher about what learners know, understand or can do (Hattie & Timperley, 2007). A primary tenet of Vygotsky's work (1978), scaffolding includes adjusting teaching to support learning (the process of the teacher modelling and supporting the social interactions that support learning (Smit et al., 2013; Vygotsky, 1978). Feedback can provide information about how to adjust teaching to scaffold learning within the ZPDs of students in the class (Torrance & Pryor, 2001). Previous research has explored the interweaving of students' ZPDs as part of the mathematical culture of inquiry (Goos, 2004).

Analysis in Phase Two of this study will consider how the teacher in this inquiry classroom responded to the collective feedback about her students' various ZPDs, and how teaching was adjusted to support student learning that was personal, through the processes of scaffolding. In a general mathematics classroom this may seem like a typical process; feedback is generated through formative assessment to see how students are progressing towards the learning goal/s and teaching is adjusted to suit the learning needs of students. The difficulty in teaching inquiry can be in guiding students towards previously defined, shared learning goal/s when students are encouraged to begin from very different starting points in their learning, in regards to the prior knowledge they bring or the connections they make. The research focus in this phase considered the complexities of the ZPDs of learners in one inquiry classroom, made explicit in the feedback gained through the use of formative assessment, and how the teacher responded to this information to guide students towards mathematical learning goals.

In the retrospective analysis of this chapter, the mathematical focus of learning in this inquiry will be expanded upon to include Duckworth's (2006) beliefs framework. Findings from the previous iteration of study illustrated how mathematical learning for students in inquiry can move beyond the reproduction of taught skills and procedures. Students make personal and meaningful connections through the experience of inquiry and may reflect on many different areas in mathematics to find their answer. While a focus of this study is capturing and describing learning through assessment that aligns with the inquiry approach to teaching and learning, there underlies my interest in knowing 'what else' is learnt in an inquiry classroom that traditional assessment seems unable to capture. Rather than narrowing descriptions of learning to particular mathematical content (as described in relevant curriculum), Duckworth (2006) preferred stating learning aims as beliefs which presents an opportunity to consider learning more broadly. This may provide a new lens for analysis of mathematical learning in an inquiry classroom in this second phase of study. The four kinds of beliefs she exemplified in her work, to characterise all things to be learned, were elaborated in Chapter 3 of this thesis to include:

1. '*The-way-things-are*' (already known in the world),
2. '*It's-fun*' (interest),
3. '*I-can*' (self-confidence), and
4. '*People-can-help*' (sharing and calling upon other resources) beliefs.

Duckworth's (2006) framework will be used to clarify the learning goals that are characteristic of this inquiry, beyond the scope of the curriculum. These characteristics will contribute to the greater profile of the inquiry pedagogy to teach mathematics.

Chapter Outline

This chapter illustrates the inquiry classroom in this iteration of study. A summary of the preparation and design of this phase of study will be included to consider how particular factors influenced the design of the teaching and learning episodes. Key characteristics of the inquiry 'How many steps do you walk in a day?' will be outlined to situate the illustrations of teaching and learning explored through analysis. Analysis will consider opportunities for formative assessment that generated feedback, through four illustrations. Firstly, pre-assessment will be explored to illustrate the collective understandings of students in the class. This will take into consideration the intended learning goals of the inquiry. Second, data that reflected the teacher responding to this feedback will be analysed. Theoretical sampling offered a way to select data from conversations between students that reflects identified themes, or offered responses to theoretically-based questions of the data (Corbin & Strauss, 2008). Theoretical sampling involves defining samples or cases during the process of collecting and interpreting data, step by step, and emerging theory controls the process of data collection (Flick, 2009). One particular teaching episode (constructing a dot plot) was analysed to consider how the teacher considered the ZPDs of her students while guiding learning towards a shared mathematical understanding. Third, the feedback loop will be continued as students contribute to classroom discussions. Fourth, understandings developed during the classroom discussion will be made explicit in three student responses. Finally, retrospective analysis will summarise characteristics of teaching in this inquiry classroom and explore the conflicts that arose from each of the four illustrations as potential innovations to consider in the next iteration of data collection (Cobb et. al, 2003).

Preparation and Design: How Many Steps Do You Walk in a Day?

The classroom in this second phase of study was very different to the classroom in the previous cycle. I will firstly reflect on the findings from the previous chapter. The participants and context in this phase of study will be outlined next, including the mathematical content required to be taught and other classroom constraints influencing the particular design of this inquiry. Finally, the inquiry will be summarised including the design of formative assessment processes highlighted in the previous analysis chapter, specifically incorporating feedback principles.

Findings from the first iteration of study highlighted how principles of formative assessment in an inquiry context can support the development of students' mathematical thinking when it is adaptable, values learning broadly, and is respectful of learners in inquiry. Formative assessment encouraged students to express learning in personal ways; to move beyond the boundaries seemingly set by the more traditional types of assessment currently in place. Formative assessment opportunities were embedded into the design of key phases throughout this inquiry, to make learning visible, and to generate feedback for the classroom teacher to respond to, and for analysis (Wiliam, 2011a). Instances in the classroom of formative assessment guided the initial selection of data to be analysed. Properties of learning identified in the previous phase of study also emphasised the social nature of learning in inquiry from a constructivist perspective (Confrey, 1991; Sfard, 1998a) and continued to apply in this chapter. Exploration of teaching moments that illustrated a change in teaching direction, as a flexible response to feedback on student learning, will be analysed in this phase of study to articulate characteristics of teaching in one inquiry classroom.

This iteration of study illustrates an inquiry in a Year 3 classroom, in the same school. This classroom consisted of 26 students (7-8 year olds) with diversity that would be representative of many classrooms in the region of Queensland in which it was situated. The inquiry was designed to meet the curriculum at the time with the mathematical focus on large numbers (Figure 6-1). A common assessment task also had to be considered in the design of the inquiry (Figure 6-2). An inquiry presented one way to explore this mathematical content. The design was inspired by a unit published for 8-10 year olds called *10 000 Steps* (Allmond et al., 2010).

Year 3 Content Descriptions
Recognise, model, represent and order numbers to at least 10 000 (ACMNA052)
Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems (ACMNA053)

Figure 6-1 Selected content from the Australian Curriculum informing this inquiry (ACARA, 2012).

In this phase of study, the inquiry 'How many steps do you walk in a day?' was designed for students to use large numbers purposefully. Students worked collaboratively in pairs of mixed abilities and behaviours, determined mainly by physical proximity but also by the teacher. The inquiry question was designed to be ambiguous; a key characteristic of inquiry (Makar, 2007; 2012). Students needed to define terms such as a day, a step and walking, and decide which mathematical resources might prove helpful in answering the question. Student plans to answer the question were considered collaboratively and when it was discovered that there was variability in the number of steps taken to different locations by different people, students shared the data they collected. This was an important moment to consider in the inquiry and further consideration of how the teacher included this notion of variability in her teaching is discussed in the analysis below. A brief outline of the inquiry is provided in Table 6-1. A full description can be found in Table 4-1.

Table 6-1 A summary of classroom experiences in the inquiry 'How many steps do you walk in a day?

Inquiry Phase (Allmond et al., 2010)	Classroom experiences
Discover	- Introduced question and shared initial thoughts about the mathematics required to answer the question, and the tools that may be required. - Students predicted how many steps they might walk in one day. - Pre-assessment of students' knowledge about place value to 10 000.
Devise	- Students worked in pairs to devise a plan to answer the inquiry question. - Three students wore pedometers for the lunch break to see how many steps they took during lunch. - Students shared their plans and revised as necessary.
Develop	- Students collected data using the tools they selected. - Students identified the number of steps to different pathways within the school. - Students followed their plans to answer the question. - Shared efficient strategies in classroom discussions as well as ideas that did not seem productive.
Defend	- Students presented their answers to the class and justified why they were so. - The class discussed challenges and successes they met while working out how many steps they took in one day.

Data and Analysis

In this second iteration of study, four forms of artefacts were collected as data: pre-assessment of student knowledge, reflections recorded in my own teacher journal which provided feedback about teaching and learning, filmed classroom observations and audio recorded using an iPad to capture classroom discussions, and finally written efforts recorded by students in their workbooks/scrapbooks. Prior to the inquiry commencing, the students completed the common school-based assessment task (Figure 6-2) intended for measuring learning at the end of the unit of study. As pre-assessment, this highlighted what students already knew about the mathematics to be learnt. As the classroom teacher, I recorded reflections on teaching and learning each day (Figure 6-3). These were recorded in a Word document, saved each day with a new date extension. The reflection journal contained: descriptions of the lessons including the mathematical content explored, planned formative assessment practices, explanations of what the teacher and students actually did, conversations and movements, and reflections that considered adjustments to teaching in the following lessons. Lessons were filmed to capture classroom interactions where oral feedback was generated, and offered insight into learning for comparative analysis. When filming was not possible, an iPad was used to record audio in lessons. Throughout the inquiry, the children were given many opportunities to express their thinking and learning in their scrapbooks (Figures 6-5, 6-6 and 6-7). These were often guided formative assessment processes but also included *scribblings* of inchoate ideas as students attempted to answer the inquiry question.

Analysis aimed to consider how teaching was influenced by teacher engagement with feedback, generated through formative assessment. Initial coding of data focused on concepts related to the classroom element of teaching as a lower level concept. I wanted to combine related dimensions and properties of the classroom elements of formative assessment (Chapter 5, Data and analysis) and teaching, to generate higher-level categories to depict formative assessment that includes the teacher's role in an inquiry classroom. Based on principles of theoretical sampling (Flick, 2009), four key events in the data were identified to explore how properties of teaching related to themes of formative assessment from the previous cycle of study, to contribute to the refinement of the element of teaching in an inquiry mathematics classroom (Corbin & Strauss, 2008; Flick, 2009).

Illustrated below are these four events. The first illustrates how the classroom teacher considered feedback gained through formative assessment to understand her students' ZPDs collectively. She then guided student learning through co-construction of a dot plot. Next, teacher reflections on one particular lesson showed how the classroom teacher respectfully guided her students towards a shared understanding of the intended learning goals. Scrapbook entries (written reflections) for three different students highlighted how with teacher guidance, student understandings of the taught idea could be varied and personal. Theoretical analysis of these events, using Duckworth's belief framework (2006), combined concepts and generated themes for consideration in the third phase of study.

Feedback revealing a collective notion of ZPDs of students in the class

This section establishes the *actual developmental level* (*ADL*) of the class in general, to determine the developmental level of students (Vygotsky, 1978) prior to the inquiry. This, along with knowledge of the *level of potential development* (*LPD*), helped to define the ZPDs of students in the class offering a tool through which learning progress could be understood in this classroom. Explicating the ZPDs of students in the class is included in the analysis, to refine dimensions of teaching in inquiry that include teacher scaffolding and guiding students in their learning. Analysis considered how the teacher adjusted teaching to suit the *LPD* of her students in this inquiry, articulated in the feedback gained through formative assessment. In a school mathematics tradition, the *LPD* might generally refer to the mathematical content which has to be taught; the learning goals. I argue that in inquiry, the *LPD* for different students, or groups of students, shifts as they move towards different potentialities. In this inquiry for example, some students chose to explore the different types of physical activities undertaken daily without having to rely on a need to understand large numbers. Constructivist notions in inquiry value that knowledge is constructed in the mind of an individual, in their own mathematical activity (Cobb & Steffe, 2011; Confrey &

Kazak, 2006; Lerman, 1989). The *LPD* shifted for students as they explored the relationships between fitness and the amount of physical exercise peers participated in, rather than the explicit learning goals (curriculum intent) for this inquiry.

This inquiry aimed to generate a purpose for using large numbers by exploring how many steps one might walk in a day. Although the content (Figure 6-1) clearly described what students needed to know, another factor influenced the inventive engineering of this inquiry: the common assessment task used by the school (Figure 6-2). The term ‘engineering’ has been used synonymously in this thesis to describe planning, designing, preparing or development.

The common assessment task was the summative assessment of learning for students in this year level. Results from this test were compared across the year level and contributed to final semester grades. The school and year level teachers had decided that to demonstrate successful learning of the prescribed mathematical content (Figure 6-1), students would need to correctly answer the questions posed in this task, including ordering large numbers on a number line. I include this in analysis to articulate the *LPD* expected by the school to be reached by all students in the year level.

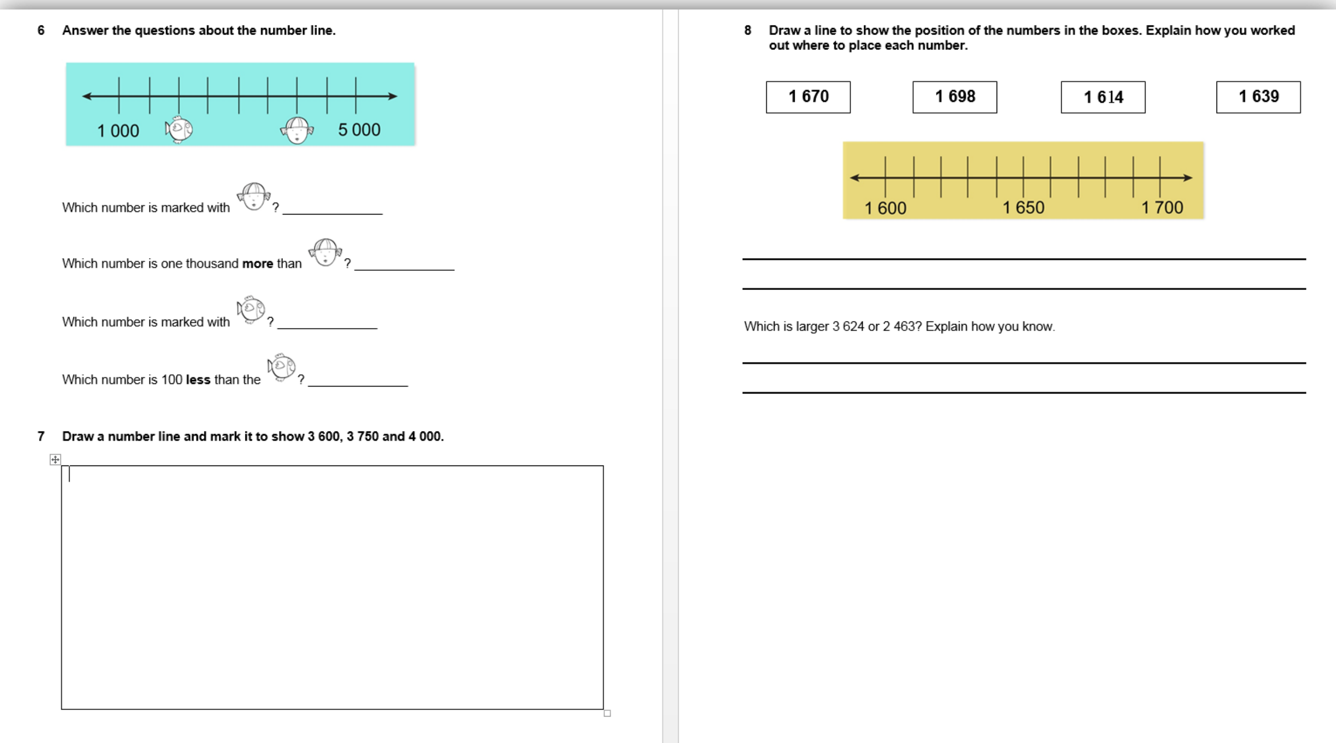


Figure 6-2 Questions from the standardised, curriculum-designed assessment task relating to ordering numbers on a number line (The State of Queensland, 2012, pp. 3&4 of 6 pages).

In this classroom, the common task was used formatively to pre-assess what the students already knew about number lines. Their responses (Table 6-2) indicated little confidence with no more than

53% of the class answering correctly on each of the items related to number lines (questions 6, 7 and 8 in Figure 6-2). Table 6-2 only shows the students' results that related to the number line questions. Question 6a refers to the first response to question 6, 6b the second response and so on. A hyphen shows that the student did not attempt the question. N shows an incorrect answer and Y is a correct answer.

Table 6-2 Student results from the pre-assessment task, indicating student achievement of number-line concepts.

Student	6a	6b	6c	6d	7a	7b	7c	8a	8b	8c	8d	8e	8f
1	-	-	-	-	-	-	-	Y	Y	Y	Y	-	-
2	N	N	N	N	-	-	-	Y	N	N	Y	-	N
3	N	N	N	N	-	-	-	Y	N	Y	N	-	Y
4	-	-	-	-	-	-	-	Y	Y	Y	Y	Y	Y
5	Y	Y	Y	Y	-	-	-	Y	Y	Y	Y	Y	Y
6	Y	Y	Y	N	-	-	-	Y	Y	Y	Y	N	Y
7	Y	Y	Y	Y	-	-	-	Y	Y	Y	Y	Y	Y
8	N	N	Y	N	-	-	-	Y	Y	Y	N	-	-
9	Y	Y	Y	N	-	-	-	N	N	-	-	-	-
10	-	-	-	-	-	N	N	-	-	-	-	-	-
11	Y	Y	Y	N	-	-	-	N	Y	Y	Y	-	Y
12	N	N	N	N	-	-	-	-	-	-	-	-	-
13	N	Y	N	N	-	-	N	N	N	N	N	-	-
14	N	N	N	N	-	-	-	Y	Y	Y	Y	N	Y
15	N	Y	Y	N	-	-	-	Y	N	Y	Y	N	Y
16	Y	N	Y	N	-	-	-	N	N	N	N	N	Y
17	N	Y	N	Y	-	-	-	-	-	-	-	N	Y
18	-	-	-	-	-	-	-	-	-	-	-	-	-
19	N	Y	Y	N	-	-	-	-	-	-	-	-	-
Total correct responses	6	9	9	3	0	0	0	10	8	10	9	3	10
% correct	32%	47%	47%	16%	0%	0%	0%	53%	42%	53%	47%	16%	53%

Overall the class seemed to struggle with creating number lines; only two students attempted to draw a number line in question 7. This illustrated a collective sense of the *ADL*s of students' ZPDs (or actual understandings of this topic) as varied beginning points in a learning journey towards intended learning goals. This feedback offered the teacher information to guide her approach to teaching the mathematical content, that reflected learners starting at different levels, or bringing varied knowledge to the task they were about to begin. Applying Vygotsky's (1978) notion of the ZPD in learning to inquiry, the *LPD* established through content and pre-assessment would be at an emergent level only. The emergent level relates to a sense of beginning, where the design of inquiry experiences included mathematical content informed by the curriculum as the unit focus. Using formative assessment to inform teaching about student learning is not a new concept and learning goals are established in all mathematics classrooms. Yet reaching the end goal in this inquiry involved variations along the journey, to include a broader sense of what was to be learnt, as inventively engineered by the classroom teacher (I use the term inventive to encompass how a classroom teacher of inquiry has to integrate core content and common assessment tasks with knowledge of her students and context). This is the sense of being emergent that contributes to the emergent *level of potential development (LPD)* in an inquiry classroom.

Teacher response to feedback: Constructing a dot plot

The following learning experience has been included to articulate scaffolding as a dimension of responding to feedback when teaching in inquiry. A response to feedback gained through pre-assessment was to engineer a purpose for ordering numbers on a number line into the inquiry, to guide students towards success on the common assessment task, used post-inquiry. The teacher provoked students to think about the answer to the inquiry question in ways that considered all students in the class. When collectively considering data representing the number of steps to a particular location, the variation in responses presented a perturbation. An informal sense of average to describe a typical number of steps was a potentiality to consider to provide support for students' answers, when applied to the terms 'number of steps' and 'each day.' This potentiality became a new *LPD* for the teacher to consider, in a sense less immediate than the emergent level described by content in the curriculum, but concomitant or contributing to it. To aid students in visualising a sense of average, a dot plot was a model which could represent variation in data collected by the students. First, teacher scaffolding included modelling how to construct a number line to first guide students towards the *emergent LPD* as articulated by the mathematical content, then co-constructing a dot plot with students to assist students in doing similar later by themselves, described as scaffolding by Vygotsky (1978). The challenge for the teacher was in scaffolding all students towards the *emergent LPD* to reflect constructivist notions of multiplicity of meanings of mathematics (Confrey, 1991).

Analysis focused on the teacher journal which included reflections on lesson progress. Audio of this lesson was also recorded using an iPad. This recording elaborated on claims made in the teacher reflections. My research focus was on moments when changes in the direction of teaching took place. Figure 6-3 illustrates the lesson when the teacher introduced the task of constructing a dot plot to students in the class. The column on the left in the teacher journal described what happened during the lesson. The right column was a space to record reflections as evaluations of teaching practice, of student learning or of particular insight. Reflection on this lesson stood out from other lesson reflections as evident by the bolded text used (no other lesson reflections were bolded [Refer to Methodology]).

Tuesday xx/xx/xx (Week Three, Term Two) Lesson 3: How many steps do you walk in a day? 11:30-1pm ***Recorded on iPad also***	
Reflect on yesterday's findings – did everyone have a number written down to show how many steps to the old library? Are these numbers the same? Why not? **Students then came to the carpet with their scrapbooks to share the number of steps taken to walk to and from our classroom to the new library. *'I need to represent these numbers in a way that shows me all of the numbers, so that it is easy to see the range or how many numbers there are and I think the best way to do that is on a number line. First, we will have to know how big the number line has to be.'* I described that one way we could represent these numbers to see them clearly is through the use of a number line. It became apparent that the students had already discussed the range required to show on the number line based on the least and greatest amount of steps taken. I used Easiteach *(an educational software program)* to create a number line, with intervals of 10, from 60 to 180. Students proceeded to call out their counts while I 'dot plotted' this information on the number line.**	Initial question – *How many steps do I take from our classroom to the library?* Brianna wanted to change the question to *How many steps do I take **to and** from our classroom to the library?* *From year 7 eating area to our classroom?* William – some people have big legs Amelia – Size of their steps Lots of *'I need...'* ☹ There was a good discussion about how to make the intervals on the number line.

Figure 6-3 Reflection on inquiry lesson from teacher refection journal.

Students had explored the inquiry question in small groups and considered aspects of the inquiry that were important to them: on which days to count steps, what does a step look like, how will we record this. Teacher reflections noted this idea that the number of steps taken was different for each student. Success in making relevant to students the concomitant potentiality of an informal sense of a need for average, was illustrated in some of the students' comments: that some people have big legs, and the size of steps for each person is different also. Students were guided towards the emergent potentiality of understanding the value of large numbers, through the concomitant potentiality of an informal sense of average when the teacher posed questions to provoke thinking about the need to represent numbers on a number line: "*Are these numbers (of steps) the same*?"; and explicitly modelled how to construct a number line: "*I need to represent these numbers in a way that shows me all of the numbers, so that it is easy to see the range or how many numbers there are*," "*I created a number line with intervals of 10*" and "*I dot-plotted this information*." Scaffolding learning in this way involved inventive engineering that was encompassing of students' responses to the inquiry question so far, while guiding them towards the *emergent LPD*; a shared understanding of number lines.

Reflection on this learning experience highlighted a lesson where the teacher changed the direction of learning for the whole class, to include potentialities she engineered based on feedback gained through formative assessment. Constructing a dot plot was not what students were required to do in the common assessment task, neither was it stated in the curriculum. This learning experience was engineered to guide students towards learning of the concomitant potentiality of building a sense of the need for typical. This task also related to the *emergent LPD* where students were required to order large numbers on a number line.

The classroom teacher was responsive to the needs of her students, inventing a need for learning the mathematics she had to teach as opposed to students just learning the intended content. She sought and responded to feedback about student learning, acknowledging and encompassing the many potentialities her students presented, and modelled the mathematical task in an inquiry setting. Emphasis was placed on organising the students' collective thinking using a dot plot, to generate a sense of average in response to the perturbation variability presented. By focusing on the shape of organised data, students could consider the distribution of data and concepts related to average in a more intuitive way (Confrey, Makar & Kazak, 2004). Empirical research on student development of informal inference used dot plots as a method of looking for and describing patterns in data (Fielding-Wells & Makar, 2012; Makar & Rubin, 2009; Pfannkuch, Budgett, Parsonage & Horring, 2004). Finally, this learning experience supported students in being able to successfully create and interpret their own number lines in practice for the common assessment task that they would soon complete.

Continuing the feedback loop

In this inquiry, the *emergent LPD* was informed by the curriculum and the common assessment task. Concomitant potentialities engineered by the teacher were the attempt to close the gap between what students could already do by themselves (*ADL*), and the *emergent LPD*. This section presents further analysis of interactions that continued the feedback loop, to see how the teacher guided her students, in ways that were open and allowed for personal construction of mathematical understandings. Scrapbook entries written by three students allowed further consideration of their individual development towards the concomitant potentiality of a need for a sense of average.

I return to the previous lesson describing the joint construction of a dot plot to organise the data students collected. Once all the data had been added to the dot plot (Figure 6-4), the teacher engineered a classroom discussion for students to consider this data further. Classroom discussions in inquiry can be a daunting experience for the classroom teacher as it is often unknown how the discussion might unfold; teachers need to cope with uncertainty in inquiry (Makar, 2008). Video observations of this teaching and learning experience began with an open question to initiate discussion:

Teacher: Have a look at that (Figure 6-4). What could you tell me about all the data we collected?

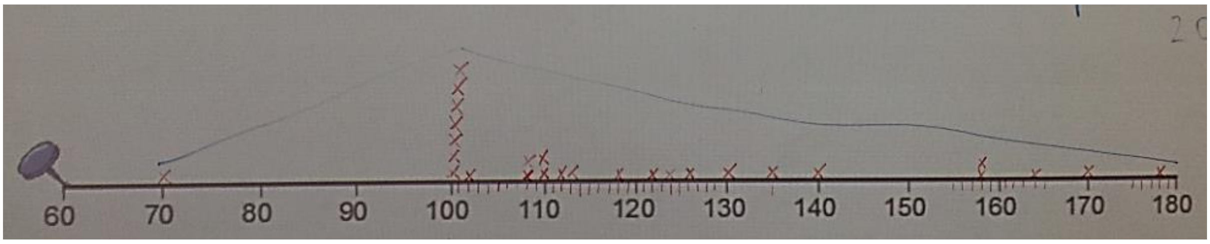


Figure 6-4 Dot plot depicting number of steps to or from the Year 7 eating area.

It is interesting to note the number of students who offered 100 as the exact number of steps they recorded (Figure 6-4). These students may have lost count or not counted at all while the class walked together. Instead these students may have decided to copy the number of steps presented by a peer. This was fortunate in a sense as it presented an anomaly in the data to consider. The entire discussion has not been included here, yet sections of the transcript have been included to reflect different student responses.

Georgia: It took more people 100 steps to get there and back.

Teacher: Good girl. (Repeats loudly) It took more people 100 steps to get there and back. What else?

Dan: Well I think I know the average. I think the average is going to be about 133 because, wait… it's nearly in the middle and there are more on the left. So it tells us… (Another student interrupts)

Teacher: Hold on, sorry, you can be next. (Returns to previous student) So, so what is it, this average? What does average mean?

The term 'average' had been used by another student on the first day of the inquiry yet had not been explored further with the class. In both inquiries presented in this study so far, purposeful classroom conversations were a formative assessment technique that could reveal mathematical concepts needing further exploration and conceptions that were under-developed. Classroom discussions were already an accepted norm in this classroom community and all students were expected to contribute and to think actively about the ideas of others (Goos, 2004). Goos (2004) described how a teacher can create a variety of ZPDs in an inquiry classroom through the sociocultural practice of classroom discussion. She provided elaboration specifically on withholding judgments "to maintain an authentic state of uncertainty" (Goos, 2004, p. 282) to orchestrate discussion. In this example, the classroom teacher had withheld earlier judgment of Dan's reference to average, and now engineered his ideas into the discussion to carefully guide learning about this topic in a shared forum.

Teachers choose when and how to respond to student thinking when it is made visible; whether to define, offer an answer, disregard, challenge, test, connect it to other experiences or to explore the idea further. In inquiry, students are encouraged to make connections to the mathematics they know to form a web of mathematical connections, which when combined with the connections made by others, can encompass a complex tangle of ideas. Duckworth (2006) described how teachers must strive to understand the meanings any particular experience holds for their students and not assume that planning an experience that has a clear meaning to the teacher will result in assimilation into the meanings held by their students. In a traditional classroom where the teaching focus is narrower, it may be essential to address incomplete understandings as they arise. Yet in this classroom, the teacher had not commented on the use of the word *average* considered previously, even though it was considered by her as a concomitant potentiality.

Dan: It means… like middle.

Teacher: Middle?

Dan: It kind of means that you add them all up and then um… no it doesn't really…

Teacher: Ok, all right then

Dan: it's about in the middle

Dan had heard of the word 'average' before and knew to apply it to this situation, but he was unable to clearly explain what it meant. I believe he was trying to describe average as the formula for mean when he elaborated on how to add all the data up; part of the process of calculating mean. The teacher in this classroom artfully guided classroom discussions to capitalise on moments of unsureity (a code I included in data analysis), prompting students to reason about their thinking and organise their internal thoughts. The vocabulary of average had not yet been presented by the teacher and other students may not have considered this word before, nor developed a sense of average. Just as an artist combines all the elements of art to create an aesthetically pleasing work of art, I use the term 'artful' to describe the complicated process for a teacher of guiding a classroom discussion, in a way that is pleasing to all participants. Artful engineering by the teacher meant that the conversation remained within the varied ZPDs of learners in the classroom. Dan was left to ponder his notion of average, to organise his thoughts around this idea.

The conversation continued and the teacher attempted to change the direction of the conversation to encourage other students to join in, to encourage them to think about the data in terms of a range of data points, returning to the ZPD generally of the classroom community. In classroom discussions, the teacher has a central position "in assisting students to appropriate mathematics as cultural knowledge" (Goos, 2004, p. 282).

Teacher: Between what numbers did most people walk? Between what numbers? (Pauses) Come on Simon, between what numbers did most people walk? Priya?

Priya: 100?

Teacher: Between. Give me two numbers. Between this and something. So between 100, 100 and…

Daisy: (Calls out) 140

Teacher: What would you say Priya? (Pauses)

(Another student calls out 100.)

Priya: …140?

Dale: No way

Teacher: Well what would you say then Dale? Between 100 and what did most people walk?

Dale: I would say 120

Teacher: Pardon? (Other students are chatting about the previous question)

Dale: I would say between 120, (Pauses) 140 then

Teacher: (Repeats loudly) So you would say between 100 and 140? Most people took about that many steps. (Pauses)

(Dale agrees)

Teacher: Well, what about 132? It is in that range (recalls Dan's earlier guess of 133).

Dan: average

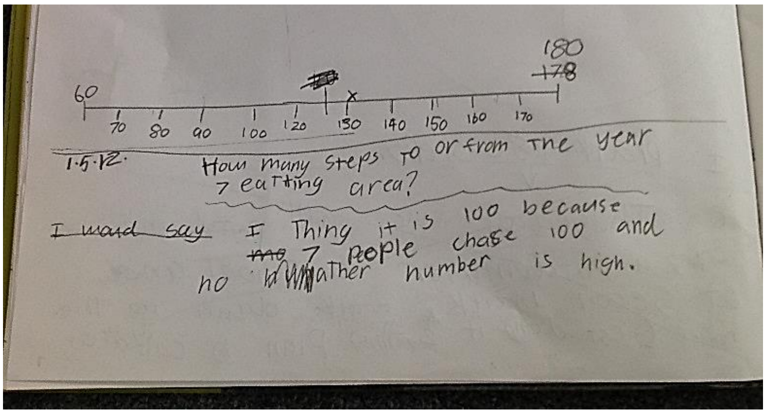
Now the students began to consider their answer to 'How many steps to the Year 7 eating area?' as an interval, or a range of data points. Ponderings were not judged as incorrect as they were heading toward the concomitant potentiality of a need for a sense of average that included intervals. In the discussion, Dan 'jumped' on the opportunity to use the word 'average' again, possibly as the teacher artfully referred to his previous comment to link the two responses.

David, another student, suggested an interval estimate of 60 to 180 as the number of steps. When the class disagreed he narrowed this interval to between 100 and 180. The teacher modelled student responses on the board, by drawing number lines to represent the two intervals suggested by Dale (in the section of conversation above) and David. Few students seemed to want to add to or challenge the intervals presented by the boys. A challenge to these responses was engineered by the teacher to encourage students to reason about their own ideas. A new interval was added to the board showing an interval estimate of 100 to 120. To model reasoning the teacher explained that this interval included 15 students out of the 24 in the class; more than half the class (Figure 6-4). The teacher noted however, that all three intervals could be correct. Other students had not yet offered a response to the data. The ZPD of learners in the classroom who had not yet contributed orally to the discussion was not yet visible. Goos (2004) defined scaffolding as the "interactions where the teacher structured tasks to allow students to participate in joint activities that would otherwise be beyond their reach" (p. 262). It had been established that there were different ways to interpret the data and the next task would scaffold students' reasoning about interval estimates of the typical distance walked.

Student responses as feedback: More than one correct answer

Now the potentiality of an interval was a new concept, concomitant to the ZPDs of student in the class. To explore this, learning for three students was considered through analysis of the scrapbook entries they recorded, in conclusion to the classroom discussion just analysed. I considered the title for this sub-section as *Closing the feedback loop* but could not define when this closure took place. Each section of this chapter is a further response to feedback.

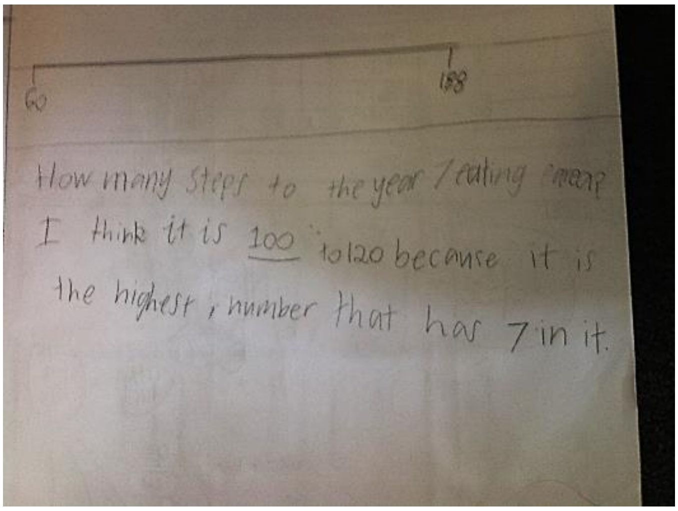
Vygotsky (1978) emphasised how a child's mental development can be determined in the independent activity of children. He described how an aim of trying to describe "the internal relations of the intellectual processes awakened by school" was analogous to the use of x-rays (Vygotsky, 1978, p. 91). Student reasoning assisted in making learning visible and planned formative assessment encouraged students to reflect upon the class discussion, in writing, in their scrapbooks (as part of the same lesson). A sentence starter was written on the board (I think…) to prompt students to complete a statement about the data. Students were encouraged to draw, write and/or explain their ideas in ways that they found easiest and were reminded that they could refer to the range of responses already on the board. The three responses below (Figures 6-5, 6-6 and 6-7) illustrated complete ideas from three different students. The responses varied in mathematical content and evidence to support the statements they made.



How many steps to or from the year 7 eating area?

I thing (think) it is 100 because 7 people chose 100 and no other number is high.

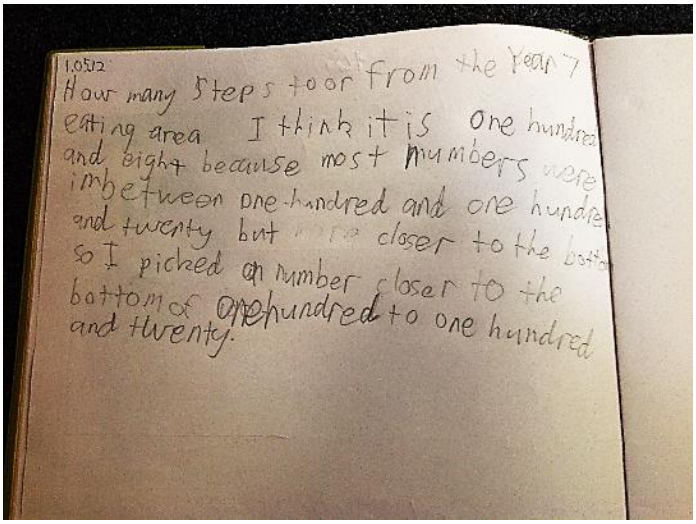
Figure 6-5 Priya's response describing mode.



How many steps to the year 7 eating area?

I think it is <u>100</u> to 120 because it is the highest, number that has 7 in it.

Figure 6-6 Georgia's response referring to an interval and then selecting mode as her answer.



How many steps to or from the year 7 eating area?

I think it is one hundred and eight because most numbers were im (in) between one-hundred and one hundred and twenty but closer to the bottom so I picked a number closer to the bottom of one-hundred to one hundred and twenty.

Figure 6-7 Dan's written response including an interval to support his estimate.

Analysis now turned to the statements about data written by Priya, Georgia and Dan and how their ideas had developed around average, and their estimates of average: these notions were now potentialities within the students' ZPDs. The statements offered insight into student thinking through their reasoning of ideas, guided by the teacher through classroom discussion. These statements signified another step in the feedback loop that would again inform the teacher about how next to approach teaching and learning in this inquiry. Evaluation might consider *how* each student was thinking about the problem: Priya relied on the number of steps that was most common; Georgia firstly consider her answer as an interval of 100 and 120, then underlined 100 as the most common number of steps; and finally, Dan elaborated on his own thinking about the data. He has not included the word average in his response but has articulated his sense of average to include an interval and a mid-point within that interval, although slightly skewed towards 100. All three responses referred to the potentiality of a sense of average; a concomitant potentiality of this inquiry.

An important note to make here regards how bold these statements are for students to make. Traditionally in mathematics classrooms there is one right answer (or a small number of responses) which is to be acquired (Cobb, Yackel & Wood, 1992; Makar, 2012; Sfard, 1998a). In the traditional school mathematics classroom, written responses such as these are often checked by the classroom teacher and an evaluation made of the work as part of *quality control* (Leahy et al., 2005). In the classroom discussion above, there was no mention about contributions being wrong or incorrect. The openness of the conversation and how the students responded meant that courageous students were not seeking an evaluation from the teacher, only an opportunity to share their interpretations of the data in ways that were meaningful to them.

Retrospective Analysis

I would first like to expand upon the mathematical focus of this inquiry using Duckworth's (2006) beliefs framework. This analysis aids in broadening the scope of learning beyond narrow content descriptions, as is the nature of learning in inquiry to consider a broad range of mathematical topics to solve an inquiry question. Little research has been able to operationalise what else students learn through inquiry beyond mathematics. It has been a focus of this study to try and capture what else students learn through inquiry that assessment doesn't foster, and this framework may offer one way to consider this. Retrospective analysis will then consider the conflicts that arose from each of the four situations illustrated in this chapter. A response to the research question posed in this chapter will be presented next, to incorporate theoretical perspectives from this phase of study (Vygotsky, 1978). Finally, potential innovations to consider in the next phase of data collection are articulated as part of the design research approach in this study (Cobb et al., 2003). This is included as an illustration (Figure 6-8) depicting the classroom elements of assessment, teaching and learning in this inquiry classroom. It depicts students beginning their learning journeys at different starting points (regarding for example, mathematics already known or understood, personal connections and problem-solving approaches), revealed through formative assessment engineered by the teacher. Potentialities are revealed throughout the inquiry and engineered as concomitant to the emergent level of potential development. Rather than a particular learning trajectory, both concomitant potentialities and the emergent level of potential development are equally considered as important to learning. Figure 6-8 builds on the illustration offered in the first iteration of study (Figure 5-7).

Mathematical focus: Beyond content

Duckworth (2006) critiqued how assessment and subsequent classroom experiences were commonly concerned with the content required to teach. Described as '*the-way-things-are*' beliefs, Duckworth added how testing of these types of beliefs had traditionally been the judge of success. In this inquiry, these types of beliefs could be described as: the belief that you can demonstrate understanding of large numbers when you can order large numbers on a number line, and there can be a need for a sense of average when a more general answer to the problem is required. Relating this concept to analysis of teaching in this inquiry classroom using Vygotsky's notion of ZPD, '*the-way-things-are*' beliefs would include the emergent and concomitant potentialities within students' *LPD*s. As part of the theory of ZPD, the pre-assessment task was used formatively to make students' *ADL*s visible. Neglected by '*the-way-things-are*' beliefs is the consideration of any other kinds of beliefs students might develop through inquiry pedagogy.

Applying ZPD to analysis of teaching in this inquiry mostly considered beliefs that have to do with knowledge of the world. Little regard for other kinds of beliefs was taken. Duckworth offered three other kinds of beliefs to characterise most of what we would like children to learn: those that characterise student interest, of being able to do something, and of sharing knowledge (or knowing when to call upon the help of others). I would like to apply Duckworth's belief framework to further characterise dimensions of teaching in this inquiry.

The second of Duckworth's (2006) four kinds of beliefs to characterise all that is to be learnt, '*it's-fun*' beliefs, had to do with student interest. During the classroom conversation (Continuing the feedback loop section), the teacher guided students towards the concomitant potentiality of a need for a sense of average or typical. The teacher engineered this opportunity to provoke Dan's thinking about average, as well as other students in the class, by relating mathematics to experiences that were important and personal to Dan. This could encourage an interest in finding out ('*it's-fun*' beliefs) as he continued to return to his concept of average, interested in finding out more. The classroom teacher was the artful engineer in this illustration. This included the dimensions of being respectful and inclusive of beliefs that were important to students, to encourage students to continue on their own personal learning journeys, as fun or interesting endeavours.

The third of Duckworth's beliefs characterised an ability to be able to do something ('*I-can*' beliefs). Each day, the class got closer and closer to answering the inquiry question *How many steps do you walk in a day?* Inquiry questions are ambiguous by nature to open pathways of exploration for students (Makar, 2012). The teacher guided students towards the *emergent LPD* by modelling how to construct a dot plot and responding to student feedback during this process. Informed by feedback gained through formative assessment, this teaching and learning experience offered students an opportunity to engage with taught ideas, to internalise and organise their own thoughts and to continue solving the problem (Figures 6-5, 6-6 and 6-7). I considered teacher scaffolding in this inquiry as fostering '*I-can*' beliefs: that every student in the class can find out. Even if students were unsure of how to proceed during the inquiry, the teacher scaffolded students' ideas that could prompt them to continue. A dimension of teaching in this inquiry included engineering inquiry experiences that would encourage students to move forward to answer the inquiry question.

The fourth kind of belief that Duckworth characterised of all things to be learned is '*people-can-help*' beliefs. She described how this belief had to do with sharing knowledge, and knowing when to call upon other resources. The teacher engineered a classroom discussion for students to share their thoughts about the data (*Constructing a dot plot* section). For the students this involved pondering the ideas shared by others as students articulated their own responses to *How many steps to or from the Year 7 eating area*. The belief that knowledge is shared and learning is through participation with peers in a community of practice was already a key feature of the inquiry pedagogy (Goos, 2004; Makar, 2012). Teachers as engineers in inquiry are aware that individual learning is constructed through interactions with peers, through participation in the classroom community of learners. This included dimensions of establishing the classroom environment as a community of inquiry, by including individual student ponderings for the whole class to consider.

Teaching in conflict: The conflict zone

Theoretical sampling in this cycle of study has resulted in four particular classroom situations to be illustrated in this chapter for comparative analysis of teaching in this classroom. In order to generate higher levels concepts or characteristics of teaching, theoretical conflicts were considered comparatively (Table 6-3). These conflicts were identified in each of the data analysis sections in chapter 6, above. The column titled Conflicts for Consideration acknowledges that these characteristics of teaching and learning experiences this inquiry classroom, contribute to the engineering by the teacher, of those experiences. In a mathematics classroom where inquiry pedagogy is not in use, these conflicts may not be wholly incorporated so that each experience informs the next in a feedback loop. For example, a classroom teacher may build a general sense of the need to learn a particular mathematical concept, yet only one response may be accepted as valid in completion of the learning experience. A summary of the characteristics of teaching in this inquiry classroom will highlight how teaching in a community of inquiry can be challenging and complex.

Table 6-3 A summary of the analysis of teaching-learning experiences in the inquiry 'How many steps do you walk in a day?'

Experience	Teaching Purpose	Conflict for consideration	Theoretical implication
1. Feedback revealed a collective notion of ZPD of students in the class (pre-assessment task).	To establish the collective learning of the class as the ALD.	Students began their learning journeys at varied starting points that were personal.	The *LPD* (as defined by content and the curriculum) was emergent. It provided the initial focus for teacher planning yet needed to be adaptable to consider variations in students' approaches to learning.
2. Teacher response to feedback: constructing a dot plot.	The teacher incorporated feedback into the design of this learning experience.	The teacher considered how to build a sense of the need for *LPD* to scaffold learning.	Concomitant potentialities were developed that built a sense of the need for the *LPD*.
3. Continuing the feedback loop	Classroom discussion continued the feedback loop, to guide students towards the *emergent LPD* and concomitant potentialities.	The teacher integrated varied knowledge of students' thinking into teaching and learning experiences.	Concomitant potentialities encompassed a complex tangle of ideas and drew on students' experiences of being unsure.
4. Student responses as feedback: More than one correct answer.	Students pondered the ideas of others, and considered their own ideas. Thinking is articulated.	Varied responses reflected how students reached concomitant potentialities that were personally constructed.	Students developed understandings about concomitant potentialities. These understandings contributed to the *emergent LPD*.

Analysis of data in this iteration of study has drawn on the theoretical framework of learning and development by Vygotsky (1978) to highlight properties and dimensions of teaching in this inquiry. The framework was used to articulate how the teacher scaffolded learning to guide students towards the *LPD*, identified as emergent in this chapter. Teaching was responsive to feedback gained through formative assessment as learning was made visible. Vygotsky's ZPD theory was modified to consider teacher scaffolding in inquiry when the teacher used feedback to adjust teaching. Concomitant potentialities were explored and integrated into lessons rather than the teacher continuing on a planned path towards learning particular content. Properties of teaching were also retrospectively analysed using Duckworth's belief framework to characterise learning other than the *LPD* in a Vygotskian sense.

Focusing further, I considered the properties and dimensions extracted through theoretical analysis using Vygotsky's ZPD and Duckworth's belief framework. Drawing on these findings, I considered abstract terms that could organise the properties and dimensions more broadly. These higher level concepts gave meaning to events that might otherwise seemed obscure (Corbin & Strauss, 2008). This contributed to four broad categories, or higher level concepts, underlying the classroom element of teaching in this inquiry:

1. Encompassing and comprehensive: The teacher integrated students' thinking into the collective ZPD of the class. The pathway from where students began their learning journey, to the *emergent LPD*, included the design of concomitant potentialities informed by students' ideas.
2. Respectful: The teacher valued variety in approaches used by students. Student responses were considered broadly, beyond narrow concepts of right or wrong. Solutions were considered as potentialities to re-engineer into the inquiry.
3. A thirst for knowing: The teacher pursued feedback about students' learning.
4. Aware and responsive: The teacher listened to moments when students were stuck, made mistakes or shifted their thinking about the topic. She was aware of the collective ZPDs of students in the class and engineered concomitant potentialities into teaching and learning experiences. She responded to feedback about learning in purposely ways.

These four categories illustrate the extensive demands placed on a teacher of inquiry as the engineer. Considered together, an overarching characterisation presents the teacher of inquiry as an artful and inventive engineer. This synthesises the categories generated through analysis of this data, to humbly elaborate on the learning goals one classroom teacher of inquiry considered for her students, beyond the narrow descriptions offered to her to report on student progress.

Research question: How does one teacher of inquiry mathematics respond to feedback gained during formative assessment, to guide student learning towards particular learning goals?

The research focus in this iteration of study was on how a teacher responded to the complexities of learning in an inquiry classroom. Principles of formative assessment were included in the design of the inquiry presented here, to assist in making learning visible and as a tool for scaffolding learning. Theoretical analysis of teaching as scaffolding was based on ZPD theory, presented by Vygotsky (1978). The teacher scaffolded learning by firstly considering both the collective *ADL* of students in the class (informed by feedback gained through pre-assessment) and the *LPD* as an emergent potentiality. This second level was informed by curriculum intent and the common assessment task designed to measure learning of this intent. For students to reach the *emergent LPD*, the teacher designed concomitant potentialities to be integrated into the inquiry to build a sense of the need for the mathematical content to be explored. This was informed by the context of the inquiry and the feedback gained about student thinking. The level of potential development in this sense was emergent and the ZPD had to flexibly incorporate potentialities presented in students' solutions. The teacher artfully engineered a purpose to guide students towards the emergent potentiality of ordering numbers on a number line, by developing a sense of the need for typical when considering variability in data collected by students. Rather than shifting the *emergent LPD,* developing notions of average were the concomitant potentialities. Further to this, the teacher further engineered learning to include a notion of intervals as a way to consider typical. These artful changes to the direction of teaching in this classroom reflected the complexities of teaching mathematics through inquiry; that is informed by an awareness of the varied ZPDs of students in the class made visible through formative assessment. The teacher was responsive to learning needs and scaffolded this journey through the artful engineering of concomitant potentialities.

Duckworth's (2006) belief framework also elucidated the role of this teacher as an artful engineer. Theoretical analysis informed by this framework considered learning beyond an *emergent LPD*; in a traditional sense, beliefs other than those that have to do with knowledge of the world. Shepard (2000) also considered the challenge of implementing constructivist pedagogies into classrooms while conforming to behaviorist assumptions. When *emergent LPDs* are the sole focus, feedback from formative assessment is considered only in terms of whether or not learning is still 'on track', as defined by the curriculum. Other learning beliefs were also held in this classroom community which contributed to the artful dimension of teaching. The teacher linked learning to the personal lives of students in respectful and inclusive ways to encourage an interest in finding out. She did this by engineering opportunities throughout the learning journey of students, to make explicit their ALDs using formative assessment. A variety of practices contributed to developing '*I-can*' beliefs in her students: through modelling of mathematical methods, engineering open discussions to interpret data, providing prompts to scaffold reasoning and an open approach to solutions. The teacher encouraged her students to move learning forward, towards answering the inquiry question. Sharing knowledge and pondering the ideas of others was also artfully engineered by the teacher. She offered three interpretations of the data plotted during the classroom discussion, for example for students to use in their own interpretations if they wished. Not all of Duckworth's kinds of beliefs are valued in a traditional sense (not commonly assessed [Duckworth, 2006]) but could characterise other beliefs about learning in this inquiry.

Potential Innovations to Consider

In education, *levels of potential development* are generally taken to mean the content articulated in the curriculum and classroom assessment are used to measure learning progress of this content. The ZPD then becomes the gap between what students know and the curriculum intent. Inquiry broadens this scope as it considers content to be taught as an emergent level. The openness of the inquiry question presents other potentialities, and inventive engineering by the teacher introduces concomitant potentialities that link curriculum content to learners in the classroom.

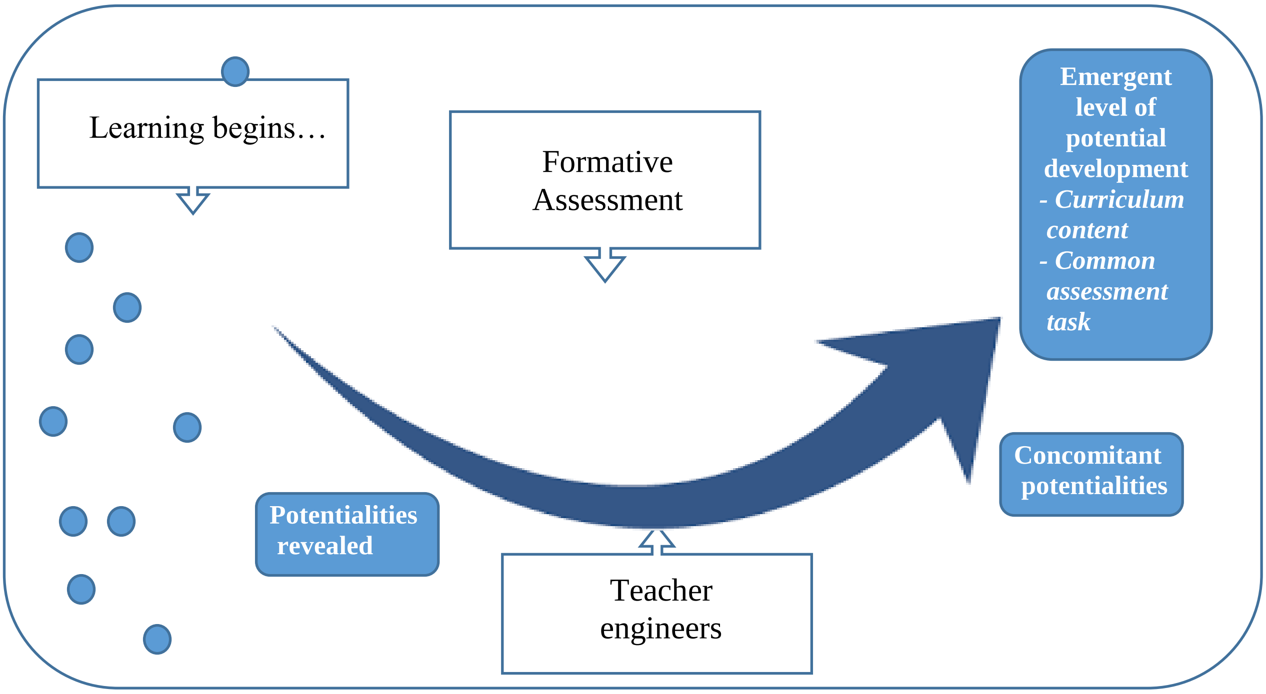


Figure 6-8 An impression of findings from Phase Two, incorporating Vygotsky's ZPD.

In this inquiry, the teacher artfully engineered opportunities to engage and reflect upon learning throughout the inquiry, to gain feedback of the students' ZPDs. Although relating developmental processes to learning, Vygotsky's ZPD seems to depict learning as a journey or process of reaching particular levels, only to begin again. In inquiry, this translates to a journey where learners traverse a complicated series of emergent and concomitant potentialities, engineered by the teacher in response to feedback gained through formative assessment (Figure 6-8). Rather than the idea of alignment (considered in Figures 4-1 and 5-7), learning in inquiry now portrayed a more interactive journey where the classroom elements of assessment, teaching and learning were more responsive to each other. The next iteration of study will consider the pathways students took as they learned mathematics through inquiry, to clarify learning and how it might differ to learning in a more traditional school mathematics classroom.

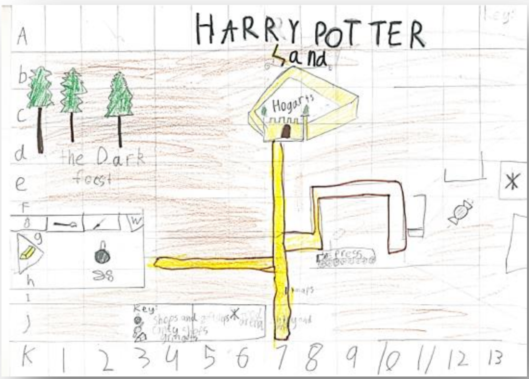
It has been acknowledged that learning is rarely a smooth advancement of taught concepts and skills yet research continues to define learning trajectories that can aid in sequencing tasks that allow students to develop and learn mathematics (Blair, 2008; Gravemeijer, 1999; Harel & Koichu, 2010; Simon, 1995; 2013). Some models have even attended to the creative tensions created for teaching based on constructivist principles (Simon, 1995). Constructivist notions of teaching and learning that inform inquiry place emphasis on students constructing meaning through mathematical experiences in personal ways (Cobb, Yackel & Wood, 1992; Confrey, 1991; Yackel & Cobb, 1996). In inquiry, the teacher is open to and willing to consider learning in broad ways (Fry & Makar, 2012). Feedback is evaluated by the teacher, although these evaluations are based on interpretations; interpretations of something that is meaningful to each student in very personal ways. Although learning trajectories continue to inform teaching sequences, it will be useful to articulate learning, as possible *conceptual corridors* (Confrey, 2006), in inquiry settings. Not with an aim to define a learning trajectory which can translate to other mathematics classrooms, but to consider the complex connections students make when learning mathematics in an inquiry context.

Difficulty lies in describing how a teacher engineered learning experiences based on feedback, without considering student learning. The classroom element of teaching in an inquiry classroom cannot be separated from the element of assessment, in particular when principles of formative assessment generate feedback about learning for the teacher to consider. Similarly, it becomes impossible to consider the classroom element of teaching in inquiry without consideration of the element of learning. In the next iteration of study, aspects of the classroom element of learning will be explored. A focus on formative assessment will continue to be incorporated into the next phase of study, to make student learning visible in this context for research purposes, and to inform teaching.

The next phase of data collection will take place in the same classroom context described here, later in the school year. I aim to openly consider student thinking, and describe it in ways that are particular to the inquiry context. Formative assessment will offer a source of insight into student learning. Potentialities engineered by the inquiry teacher in the third and final iteration of study will present opportunities for analysis of student learning. Analysis in this chapter considered how the teacher responded to feedback gained through formative assessment that was personal, varied and continually changing direction. Yet the teacher was not the only one to benefit from feedback. The next chapter will consider feedback that was generated by peers through formative assessment and how the learner responded to this information to move forward in their own learning.

Chapter 7 Phase Three: Learning Mathematics in an Inquiry Classroom

Students are now working in the Develop phase as they consider the inquiry question, Are maps useful? Already, the class has together pondered the features on maps that make them useful. They have explored colourful maps in glossy brochures from local theme parks, and simple Government-issued brochures that included maps of nearby National parks; online maps of local zoos and Google maps; to explore a satellite map of their own school and the features that online users can select and view. Text-book style maps studied in geography lessons have also been used to identify features. The students are creating their own maps, including particular features to make it the best. First they have to consider the usefulness of different mapping conventions and make choices about what to include on the maps they are creating. In this phase, the students share their drafts with others; the students that they previously explored features of real-world maps with. They're now familiar with the process of sharing their mathematical ideas with students in the class, to justify thinking and consider the thoughts of others. Students also offer feedback to their friends about their work, so they know if it needs improvement. Children sit in groups of three, in comfortable places around the classroom and spread out the work they have already completed. A quick glance around the room reveals students are proudly sharing their efforts and talking about what they have done, while others look on, nod, or comment. It starts to get a little noisy as some children start to place their maps on top of other ones to illustrate how their map has included a particular feature or not. The teacher notes the noise level and tentatively steps forward to interrupt, but she considers that there has not been enough time for each child to share their map and to respond to others in their group. Instead of ending the conversations, she interrupts only briefly, to remind the class that everyone needs to hear the comments from their own group only. Shortly, some students will share their own maps with the whole class; maps that students have identified as having useful features.



Research Aims

The overarching aim for research in this third phase of study was to identify features of how students learned mathematics through inquiry, supported by formative assessment, and how learning through inquiry enhanced and deepened the mathematical understandings that students developed. Research questions guiding exploration and analysis in Phase Three included:

Research question one: What learning opportunities arise in an inquiry classroom to develop mathematical understandings?

Supposition: Rich mathematical learning takes place in an inquiry classroom. The classroom teacher can orchestrate opportunities for learners to develop mathematical understandings when principles of formative assessment are engineered into inquiry lessons. Aligned with constructivist notions of learning, teachers in inquiry see value in students constructing mathematical insights within a framework of that individual's experience. They also encourage students to express alternative perspectives to better understand the perspectives of others (Confrey, 1991). In inquiry, challenges to learning present perturbations to students and ways of understanding and thinking are developed in the process of working things out (Harel & Koichu, 2010). These challenges may take place in interactions with the teacher and/or peers. Classroom interactions in one inquiry classroom, that reflect rich mathematical learning including perturbations which students face, will be explored in ways that consider constructivist notions of learning mathematics.

Research question two: How can a student learn mathematics through inquiry?

Supposition: Inquiry presents a messy context in which a student can learn mathematics. Research which describes mathematical learning deriving from carefully sequenced mathematical tasks (learning trajectories that pass through a series of conceptual steps, Simon et al., 2010) has been critiqued as simplistic with little regard for the complexity of the learning process (Harel & Koichu, 2010). Learning in one inquiry classroom will present opportunities for students to overcome challenges, to assimilate and accommodate new ideas with old knowledge, a key notion of Piaget's (1964) developmental theory of learning, more recently adopted by Harel & Koichu (2010) to operationalise mathematical learning. A key feature of inquiry, common to all phases of this study, is learning through participation with others in a community of inquiry (Goos, 2004). Illustrations from one inquiry classroom will be used to characterise how students learn in this inquiry context with consideration of interactions with peers.

These questions guided analysis of data in this phase to illustrate learning in an inquiry classroom. Data collected in this classroom were analysed to firstly uncover points in time for students where they seemed to 'hold onto' moments of being unsure in solving inquiry problems. This can be defined as the transitional moments where a learner fits certain experiences into certain thoughts to step forward (Duckworth, 2006). Characteristics of learning already offered by literature in this chapter so far are grounded in the Piagetian premise of knowing as reaching (does not imply finality; is temporary) equilibrium (Harel & Koichu, 2010; Piaget, 1952; 1964). When faced with doubt or disequilibrium, inquiry is part of the thinking process on the way to certainty (Dewey, 1938b). A conceptual definition of learning offered by Harel and Koichu (2010) was used to consider such moments in one inquiry, where students reached and overcame doubt. Harel and Koichu's (2010) framework for the learning and teaching of mathematics, based on earlier work by Harel (2008a; 2008b; 2008c), was used to characterise student learning in the inquiry classroom presented in this chapter, to analyse moments when students overcame perturbations to move ahead in their mathematical learning through inquiry.

Research in this phase of study built on the findings from previous chapters. In the first phase, learning was made visible through the implementation of formative assessment practices and considered broadly to reflect learning beyond the intent of the curriculum. In the second phase of study, the focus was on how the classroom teacher used feedback gained through formative assessment to influence teaching choices in inquiry, and to identify characteristics of an inquiry teacher. This presented the teacher as an artful engineer, responsive to the needs of her learners while being aware of having to guide students towards an *emergent LPD*. An *emergent LPD* was outlined as a way for the teacher to cater for objective notions of learning knowledge to do with the world, while incorporating the potentialities that students presented. This catered for constructivist notions that learning or knowledge is constructed in personal ways, and sociocultural beliefs that this process occurs through participation in a community of learners. Principles of formative assessment that valued learning as participation and beyond narrowly stated content, were again integrated into the design of this third phase of the study, to further consider how characteristics of formative assessment might align with teaching and learning using this approach.

Analysis in this chapter sought to characterise learning in one Year 3 inquiry mathematics classroom, in relation to the already highlighted characteristics of assessment and teaching in inquiry. After all, how can a teacher assess learning in inquiry without understanding how learning takes place? How will they know what to look for?

Chapter Outline

This chapter will consider mathematical learning for students in a Year 3 inquiry classroom. This was the same class of students presented in the previous chapter yet this inquiry took place later in the same year of their learning. Participants and the design of the inquiry for this phase of study will be outlined initially, to frame the learning experiences to be explored. Planned formative assessment opportunities were used to generate data to explore learning in this classroom. Analysis applied a framework based on an operationalised definition of learning (Harel, 2008a; 2008b; 2008c; Harel & Koichu, 2010) to characterise learning for one student as he interacted with others in a small group, to create useful maps. Characteristics of learning which Harel & Koichu applied in their framework were shown to align with characteristics of learning in inquiry and are referred to often in this chapter:

> "Learning" in DNR is operationally defined as a continuum of disequilibrium-equilibrium phases manifested by (a) *intellectual needs* and *psychological needs* that instigate or result from these phases and (b) *ways of understanding* or *ways of thinking* that are utilised and newly constructed during these phases. (Harel & Koichu, 2010, p. 116)

In this phase I particularly made use of the model for learning that Harel and Koichu developed to characterise the sequence of disequilibrium-equilibrium phases that a student experiences when learning through inquiry. Retrospective analysis considered how findings from this phase interrelated with findings from previous phases of study, to contribute to a greater understanding of the theoretical implications of assessment, teaching and learning in this context.

Preparation and Design: Are Maps Useful?

This phase of study continued to take place in the Year 3 classroom presented in Chapter 6. In between these phases of study, the students had participated in a small number of inquiry learning experiences and classroom norms of inquiry were becoming more established. This inquiry focused on an exploration of the features maps use that are effective. Mathematically, this content was situated in the Measurement and Geometry strand of the Australian Mathematics Curriculum, in the sub-strand of Location and transformation. Content described using simple grids to show positions and pathways. Inquiry pedagogy was slowly being incorporated into statewide curriculum resources and it was surprising to now find that inquiry units of work existed for teachers to implement in their own classrooms. The C2C unit of work (State level designed curriculum referred to in the previous chapter (The State of Queensland, 2012)) that generally guided teaching and learning in the other Year 3 classes at this school provided five lesson for teachers to implement a

Mathematical guided inquiry (MGI) exploring maps.[3] This would allow space (and time) in the curriculum for the inquiry experience designed for this study to span one week also. The inquiry stages are outlined in Chapter 4 of this study (Table 4-1). Students worked collaboratively in small groups of two or three for previously identified teaching and learning needs, not to suit the requirements of this study. Groups included students of mixed achievement levels to make sure that wherever a group included a student with low school grades, there would also be a peer with higher grades.

The *emergent LPD* was also informed by the C2C (The State of Queensland, 2012) common assessment task to be used across all Year 3 classes. Teaching and learning experiences needed to support students with successful completion of the common assessment task; teaching a game involving position and movement to a partner. Features of this task included a written plan of how to play a game designed by students, and accompanying digital explanations. A monitoring task checklist was included to be completed by the teacher for each student in the class (Figure 7-1). It was expected that recordings would be transcribed by the teacher and used to determine what students could do at the end of the unit of study. This monitoring task checklist was not included in the teaching experiences designed for study in this phase but has been included here to highlight the time-consuming measures already expected of teachers in classrooms to identify learning. I would also like to consider how the checklist was inadequate in being able to capture student thinking about the topic, only what students could or could not do.

The checklist required teachers to make judgments on how well their students could conduct particular tasks, for example if the student could interpret simple maps independently. Although it can be said that the checklist could help teachers to identify gaps in their students' knowledge of mapping features, it did not include a space, or questioning, for a teacher to record perturbations or potentialities that students might have. The feedback generated by the teacher related to the mastery of particular skills or convergent-type assessment (Torrance & Pryor, 2001). It was intended to be used to inform future teaching yet it was based only on the teacher's interpretations rather than listening to what students know. For an inquiry setting, such a checklist would reflect a narrow description of learning that considered curriculum content only as the learning goal. Yet requirements of the common task (checklist in Figure 7-1) contributed to the *emergent LPD* for this inquiry, the idea of following and giving directions on a grid map.

[3] *Mathematical guided inquiries are teacher-supported, student-centred approaches to learning mathematics where students address open-ended problems, and require mathematical evidence to support their answers (The State of Queensland, 2012).

Monitoring task checklist: Exploring position and pathways using simple grid maps

	No assistance required	Some assistance	A lot of assistance	Not able to do this task
Mathematical guided inquiry — How can you use positions and pathways in a game?				
Can the student: interpret simple grid maps?	☐	☐	☐	☐
locate positions on a map?	☐	☐	☐	☐
follow and give directions to show pathways on a grid map?	☐	☐	☐	☐
explain how position and pathways are used in real world situations, for example: games?	☐	☐	☐	☐
Does your student require extra assistance from the teacher? Comment if necessary:				

For your teacher

	Returned	Date	How returned
Photograph of positional and movement word chart (Lesson 6)	☐		
Photograph of positional and movement word chart (Lesson 8)	☐		
Sheet 9 — My plan: How to play (Lesson 9)	☐		
Recording or transcript of students' explanation of game (Lesson 9)	☐		

Figure 7-1 Monitoring checklist designed by C2C (The State of Queensland, 2012).

I would like to make note here, that as teacher and researcher, I planned the inquiry teaching and learning experiences in the previous two iterations of study. In this phase, planning took place in collaboration with my teaching partner. She was also familiar with the inquiry pedagogy and my own research interests, and together we agreed that we could plan an inquiry for our class where students could develop understandings more broadly than the content described by the curriculum. The openness of our inquiry question *'Are maps useful?'* would generate a need for students to define ambiguities (Makar, 2012). For example, what are maps and how are they used; and what features do maps include and how can we judge their usefulness. Students would apply their criteria to maps they created as a way of defending their own solutions and processes to their peers.

As part of the design research methodology (Cobb et al., 2003), principles of formative assessment from the first two phases of study were incorporated into the design of this inquiry to generate data about learning. Making learning visible through formative assessment provided opportunity to glimpse into student thinking that was personally relevant to learners' experiences. Elements of teaching inquiry, identified in the previous phase, also contributed to the design of this third iteration. They included notions of ZPD (Vygotsky, 1978) as a way to identify learning progression, to highlight concomitant potentialities that the teacher artfully engineered into her teaching during inquiry, to move students forward in their mathematical development. The idea of learning in inquiry as a journey will be expanded upon in this chapter to analyse moments when learning took place, through the process of reaching equilibrium between assimilation and accommodation of new mathematical ideas (Inhelder, Chipman & Zwingman, 1976; Piaget, 1964). Analysis considered the characteristics of learning mathematics through inquiry. This will have ramifications for how to assess learning in ways that align with this pedagogy, and to understand how teaching can support and deepen the learning that takes place.

Data and Analysis

Data in this iteration of study was considered in relation to the research questions outlined above and so artefacts collected as data focused, therefore, on student learning. These included: products of students' work generated through formative assessment opportunities, classroom discussions and video observations. My research focus included wanting to understand opportunities that supported learning in inquiry, and a more intimate understanding of how student learning took place. Students' mathematical understandings were conveyed through their own words or illustrations. Constructivist researchers Steffe and Thompson (2000) described the importance of attributing mathematical realities to students that were independent of the researcher's mathematical realities. I was interested in the perturbations that presented to students throughout the inquiry as part of the learning process, and was aware that these may not always be recorded in writing by the students. Discussions also provided opportunities to elicit learning (Wiliam, 2011a), to find out where students were in their learning. Whereas formative assessment literature mostly described the responsibility of facilitating discussions as the teacher's, I have included discussions between students working in small groups, as an opportunity for formative assessment. Sometimes I was included in these conversations as the classroom teacher and sometimes the students lead their own conversations without a teacher present. These recordings offered a chance to listen to students as they articulated their own ideas about the mathematics they used.

Products of students' work were also collected from each phase of this inquiry, planned for in the design of the teaching and learning experiences. The data collected captured moments when students pondered the inquiry. These are articulated in the next section of this chapter. These artefacts were collected to compare the students' mathematical considerations at different stages throughout the inquiry. Video observations also assisted in capturing learning during classroom discussions, as scaffolded or guided by the teacher and by the students themselves. In a more traditional school mathematics classroom where the teacher focuses on evidence of applying taught knowledge and/or procedures correctly, much student pondering might go unnoticed.

Data analysis is presented in the order in which analysis took place, to build on properties identified and to illustrate how categories regarding the classroom element of learning were formed. Firstly, an overview of the formative assessment practices engineered in this inquiry is outlined. These practices primarily considered learning as participation in this community of inquiry. Formative assessment practices used in this classroom are outlined to frame the particular events that were used in this chapter as data to illustrate learning. Analysis will highlight dimensions of an active learner in this inquiry classroom. Next, learning for one student will be illustrated using the DNR framework (Harel & Koichu, 2010) and analysis focused on the roadblocks he faced in collaboration with peers. Finally, analysis of student learning generally explores dimensions of learning through participation with peers. This is articulated to reflect duality between ways of thinking (*WoThs*) and ways of understanding (*WoUs*), and the resulting intellectual and psychological needs (Harel & Koichu, 2010), sufficient in itself to model someone's learning within the DNR conceptual framework. Comparative analysis of these events assisted in refining the properties and dimensions of learning in this inquiry classroom.

This chapter explores characteristics of the inquiry classroom from the student's perspective, as a learner of mathematics. Analysis aimed to consider learning opportunities in an inquiry classroom, to gain insight into how students learned mathematics and the characteristics of learning in this context. The Discussion Chapter will consider these findings in relation to the classroom elements of assessment and teaching.

Learning made visible through formative assessment

The use of all five of Wiliam's strategies for effective formative assessment (Figure 2-1) were evident in this inquiry (Table 7-1). The left-hand column shows in which phase particular activities or tasks took place in this inquiry. These activities were generally norms and practices already established in this classroom community of inquiry and tasks that were familiar to the students. The right-hand column highlights the key formative assessment strategy used, to show how teaching was adaptive to the learner's needs (Wiliam, 2011a; b). Evidence of the use of these strategies showed support for the inquiry pedagogy to provide opportunities for formative assessment. Although each of Wiliam's five strategies were not evident in each phase, highlighted was the notion of 'activating' learners. An initial property of learners was forming which characterised the principle of being active, in each phase of inquiry.

Table 7-1 Relating key activities in the inquiry to key formative assessment strategies (Leahy et al., 2005; Wiliam, 2011a; b).

Key activities and related phase of Inquiry (Allmond et al., 2010)	Formative assessment strategy (Wiliam, 2011a&b)
Discover: **Concept maps completed in small groups, articulating each students' prior knowledge of maps** (Figure 7-2)**.** **In the centre of each concept map, students collaborated to generate a list of features that maps have, that they already knew of.**	Engineering effective classroom discussion that elicit **evidence of learning** **Activating** learners as the owners of their own learning, and **Activating** learners as instructional resources for one another.
Devise: **Sets of written instructions, directing peers to a position on a map. The focus was on thinking about features of maps and using directional language. Students provided feedback about the usefulness of directions.**	Clarifying, sharing and understanding learning intentions and **criteria for success**, Engineering effective classroom activities that elicit **evidence of learning**, **Providing feedback** that moves learning forward, and **Activating** learners as instructional resources for one another.
Develop: **Written lists of features of two different maps recorded in small groups. These lists were compared by students to identify and develop opinions on the usefulness of mapping features.**	Clarifying, sharing and understanding learning intentions and **criteria for success**, **Activating** learners as the owners of their own learning, **Providing feedback** that moves learning forward, and **Activating** learners as instructional resources for one another.
Maps completed by students as evidence of a useful map. These were shared in small groups for refining based on feedback from peers and the teacher.	Engineering effective classroom activities and learning tasks that elicit **evidence of learning**, **Providing feedback** that moves learning forward, **Activating** learners as instructional resources for one another, and **Activating** learners as the owners of their own learning.
Defend: **Written reflections[4], completed independently. Students completed 5 statements:** 1. **I used these features** 2. **I didn't use these features** 3. **My map is very useful for** 4. **My map is the best possible map because** 5. ***I learnt***	Engineering effective classroom activities and learning tasks that **elicit evidence of learning**, and **Activating** learners as the owners of their own learning.

Each task included the element of *activating* learners, in an attempt to make learning visible. I drew on Piaget's (1977) work on cognitive development to analyse this property further. He considered how the stimulus-response scheme, as a classical model of behaviorism, was relevant to his work on equilibration. He was critical of this process though, and questioned how one might respond to a stimulus without having some sort of scheme already in place to consider it. Rather than a simple

[4] I would like to attribute the design of this reflection sheet to my teaching partner. We were both responsible for classroom assessment.

stimulus-response mechanism, he acknowledged this relationship as circular where one sought equilibrium between a stimulus and the subject's already existing schemes (Piaget, 1977). In this inquiry classroom, feedback gained through formative assessment aimed to activate schemes already held by the learners, concomitant to or associated within their developing ZPDs; along their journey towards an emerging *LPD*. Piaget would consider this activation only taking place when it presented a perturbation to existing frameworks.

The open approach to problem solving in inquiry fosters opportunities for students to firstly make connections to context and purpose (Makar, 2012). Students also access the problem using pre-existing mathematical knowledge. Very quickly this becomes modifiable as students gain further information in collaboration with their peers. A learner in inquiry would need to be willing to take 'on board' the opportunities for feedback presented in learning experiences, when perturbations presented. Learners needed to be active and responsive to feedback if equilibration between processes of assimilation and accommodation could be achieved (Piaget, 1977). Although this considers dimensions of learners in inquiry as active and willing to take feedback 'on board', this dimension cannot be considered in isolation to the classroom elements of assessment and teaching.

Artful engineering of opportunities to generate feedback, needed to offer feedback that was close to each learner's *ADL* (Vygotsky, 1978), to perturb already existing understandings for each student. A difficult task for any teacher, shifting responsibility to peers allowed each student to gain feedback with a much shorter feedback loop (Furtak & Ruiz-Primo, 2008), that was more responsive to the thoughts and ideas being articulated. A social perspective of inquiry considers how this feedback is generated through participation with peers. In the Discover phase, students were activated to firstly consider their own ideas about maps before collaborating with peers to generate a shared list of mapping features. In the Devise phase, students wrote instructions to direct a peer towards a position on a map. Again, this opportunity intended to generate feedback within each student's ZPD, where willing students could act on feedback from their peers in real-time. In the Develop phase, students collaborated with peers to compare different maps. A responsive and active student could incorporate feedback from this interaction, into the map that they were creating. Finally, students shared with others the maps they created to seek feedback that would help them refine their creation. In this inquiry, feedback gained through formative assessment strategies also considered the interactions between students. Learners needed to be open and responsive to this feedback, willing to take it 'on board' to move forward in their own learning.

To sum up initial insights into student learning made visible through the use of formative assessment, coding of data highlighted properties of the classroom element of learning. Through intimate consideration of the data, dimensions were extracted (Corbin & Strauss, 2008) which illustrated the learner in a mathematics inquiry classroom, in ways that were familiar to the properties of teaching in the previous chapter (Table 7-2).

Table 7-2 Properties of learning in Phase Three, in relation to dimensions of teaching highlighted in Phase Two.

Phase Two: A teaching focus Four categories or higher level concepts underlying the classroom element of teaching in this inquiry:	**Phase Three: Learning mathematics in an inquiry classroom** Three main categories of learning characterised by initial coding:
1. Encompassing and comprehensive 2. Respectful	Learning-as-participation (Sfard, 1998a; Goos, 2004): • Collaborating in groups; teacher and peer-led. • Adding ideas to group discussions where learning was guided towards an emergent level of potential development. • Comparing or contrasting one's own thinking to that of others. • Willing to defend
3. A thirst for knowing	Expects roadblocks in learning (considered by Piaget (1952) as "bringing the new into the known", (p. 6); Makar, 2012): • Sees perturbations as challenges to overcome • Asks questions • Challenges
4. Aware and responsive	Active learners (Jaworski, 2006; Wiliam, 2011a): • Constructs ideas • Articulates thinking • Responds to feedback • Thinks as one thinks, connecting to already known knowledge

These three main categories are properties already highlighted in sociocultural literature related to the inquiry approach to teaching mathematics. New characteristics placed emphasis on the learner's responsibility to learn. These included pondering shared knowledge and accepting and acting on the knowledge of others (as feedback), a willingness to defend, challenge and contribute to others' learning, an expectancy to overcome perturbations and to confidently make connections to already known understandings. Key events in this inquiry are illustrated to explore these properties in relation to the higher level concepts of formative assessment and teaching in an inquiry classroom, from the previous two analysis chapters.

Relevant to each phase of this inquiry, yet not elaborated upon in Table 7-1 as a specific formative assessment activity, were the classroom conversations that frequently took place. Important checkpoints throughout inquiry, these conversations took place at the beginning and end of each inquiry lesson, yet frequently occurred throughout lessons. The occurrence was often spontaneous which is why conversations were not included as a key activity in Table 7-1. Students were familiar with sharing ideas in classroom discussions or with their peers in small groups. Discussions in the Devise phase highlighted developing classroom community understandings about maps, with the teacher's role in these discussions to scaffold the process of inquiry and make explicit references to more abstract mathematical ideas, including making links between commonsense meanings and new terms and concepts (Goos, 2004). Although classroom discussion was guided by the teacher, often students responded to each other directly. In smaller groups, the teacher would not be involved in some discussions at all. Video observations reflected children discussing their learning even when the teacher was not in close proximity, or worked with another group. This offered further evidence of the dimension of learners as active and willing to act on feedback.

In inquiry, students work together to solve problems that are designed to be ill-structured and include ambiguities (Fielding-Wells & Makar, 2012; Makar, 2007; 2012). As students worked through the process of solving the inquiry question, students often faced roadblocks that challenged their strategies, solutions, and the criteria students generated to refine ambiguous terms. A norm for learners in this inquiry classroom was now more established where roadblocks were seen as essential opportunities for learning (Makar, 2012; Steffe & Thompson, 2000). Conflicts with roadblocks resulted in student frustration, and/or opportunities to reconsider the mathematics they had chosen to use to move forward in their learning. Analysis of discussions between three particular students focused on the transitional moments for Dale in particular, of moving forward (Duckworth, 2006); when students hold onto the moment of being unsure about how to continue. Learners in this class were responsible for pondering roadblocks, and for challenging others in their work if they noticed conflicts between ideas. Constructivists devote considerable time to listening to students (Confrey, 1991; Duckworth, 2006) and analysis of this data included 'listening' to student learning through exploration and pondering how students overcame roadblocks to their learning in this inquiry classroom.

Illustrating Dale's learning

A supposition to the second research question above was that learning mathematics in inquiry considers the moments when students overcome challenges, reaching a sense of equilibrium as learners assimilate new ideas with old knowledge (Harel & Koichu, 2010; Piaget, 1952). Data were explored for opportunities that showed multiple references to learning for individual students. Rather than illustrating *conceptual trajectories*, or focusing on characterising a trajectory of teaching and learning interactions within conceptual boundaries, or to characterise possible learning environments, or to generate a list of understanding and thinking (Confrey, 2006; Harel & Koichu, 2010; Steffe, 2003; Steffe & Thompson, 2000), this research aimed to consider the properties of learning illustrated in the data; data which presented learning in inquiry as a journey of reaching equilibrium by overcoming roadblocks to move learning forward. An opportunity to do this is presented for one particular student, Dale, who worked collaboratively with Brianna and Georgia.

Specifically, learning for Dale is illustrated, as data revealed his emerging and developing understandings of mathematical features in maps. Perturbations or moments where Dale 'held' onto the moment of not knowing what to do were generated by interactions with his peers and are recorded as *Ways of Thinking (WoThs)* and *Ways of Understanding (WoU)s* about mapping. These are key features of the DNR framework presented by Harel & Koichu (2010). Elaborated on in the Literature Chapter of this thesis, I include here a short reminder of learning based on the DNR framework:

> …the dual relationship between perturbations and knowledge construction (duality – D) arise from the intellectual and psychological necessity (N) to develop knowledge or understanding. Repeated-reasoning (R) of the new ways of thinking and understanding developed assist the learner to internalize, organise and retain the new knowledge. (Fry, 2013, p. 308).

Each characteristic of the DNR framework was articulated upon in illustrating learning for Dale (Table 7-3). A summary of the events on his learning journey are included first to consider the context in which his learning takes place. These included activating what he already knew about the topic (activating existing schemes [Piaget, 1977]), taking feedback 'on board' that was relevant to his *ADL* (Vygotsky, 1978) by pondering his own ideas and the questions his group considered, and finally the evidence of learning shared in the map he created.

Embarking on his journey

Accepted practice in this inquiry classroom was activating and sharing prior knowledge at the beginning of an inquiry, through collaboration with peers. As part of formative assessment, a concept map (Figure 7-2) was completed in small groups during the Discover phase. Structuring cooperative learning tasks can support student oral discussion and this strategy originated in studies of cooperative learning (Gillies & Ashman, 1998). The students were encouraged to record their own ideas about maps on one sheet of A3 paper per group. Initially, there was no talk or collaboration of ideas and each student had their own space in which to write. Dale's contribution (Figure 7-2) begins with the idea of "x's on a map" (upside-down in relation to the centre of the circle). He referred to the convention of coordinates in mapping, by recalling maps in shopping centres that he knew, "2A is like DFO" (A local outlet mall). He did not yet use the mathematical language of ‘coordinates’ although considered the purpose of maps (How to get there? and How many of something?). One final question he posed was, “How to do it?” Dale seemed unsure at this moment of how to create the best map. In this activity, once students felt that they had written all that they already know about a topic, they were encouraged to share their ideas in the group. The centre of the page is the space where students recorded their collaborations; in this instance, features of maps that they agreed upon. Here we find the group now uses the word ‘coordinates’ (Figure 7-2).

Figure 7-2 Collaborative concept map completed by Dale, Brianna and Georgia.

Conversations

As the students compared the maps they created in the Develop phase of the inquiry (Table 4-1), I became interested in the feedback these conversations generated. During one of these conversations I participated with Dale’s group as their classroom teacher, to hear about their progress. Excerpt 1 is a transcript of part of their conversation, recorded using an iPad. Students were familiar with having an iPad in the classroom to record audio in conversations, or videos of interactions, or to take photos of students' efforts.

The students in this group were all looking at and discussing a draft version of Dale's map (Figure 7-3). I noted he had used letters and numbers on his map and began to ask about this feature. He noted in his response a perturbation that he had been considering (Excerpt 1, lines 3 & 4): if someone wanted to know where to go but they didn't have a map in front of them, how useful would the coordinates be? Were coordinates a useful feature on maps after all? Brianna considered his idea and saw the difficulty in using coordinates if both users did not have the same map in front of them to refer to (Lines 5 & 6). On line 7, the teacher reiterated the idea; the teacher didn't have a map in front of her and wondered where in the classroom the coordinates might lie. Later in the conversation, the teacher (Lines 14 & 15) interpreted this idea further and presented the idea that a map would need to be oriented a particular way to use it, yet Dale returned to his concern or perturbation (Lines 16 & 17). This is, that if the audience does not have a copy of the map in front of them, coordinates will not be helpful in offering direction. The perturbation that Dale introduced to the conversation may have manifested a need for this group to consider if maps always included coordinates as a feature, adding disequilibrium to their learning journeys as described in the DNR-oriented definition of learning.

Excerpt 1: First conversation

I considered Dale's perturbation using the DNR-oriented framework (Harel & Koichu, 2010). With respect to the problem posing act of making the most useful map, Dale's learning journey started with the collaborative brainstorming between Dale, Brianna and Georgia (Figure 7-2). Here they agreed that coordinates were a useful mapping feature or convention. In the first conversation (Excerpt 1), Dale's perturbation or understanding (recorded as his *WoUs* on Table 7-3) started to change as he considered coordinates only being useful when all users had the same map to read from (Lines 3 and 4).

Dale: I put that on there. It's for coordinates so that you could say go to 3, D and you'll know where you are.

Teacher: Perfect, that's where Ian sits

Dale: But when you're in the classroom I was thinking you can't actually look if you don't have the map in front of you, so I was thinking...

Brianna interrupts: Because you wouldn't have exactly the same (map)... it says 4, D and you wouldn't know exactly where that is.

Teacher: Where's the 4 and where's the D? Is that what you mean?

Georgia: Yeah

Teacher: That's a good point. So did you do the grid after you had drawn the classroom or did you do the grid and then draw the desks?

Dale: I did it after

Teacher: So you drew the desks and then you put the grid over the top

Dale: Yeah, and I've put some keys over there like the computer, computer desk, chair, teacher's desk

Teacher: So the problem is if I came into the classroom, can I borrow that (map)? And you said I want you to go to D4 that I might come in and would not know which way to put the map, is that what you are saying?

Dale (takes the map in his hands to illustrate): If you don't even have the map and you say go to D4. Is D4 over there or? Or is it? Where's D4?

Teacher: So you'd actually have different instructions. You wouldn't say to someone… like that.

Dale: You need to say, a book to show you where

Teacher: you need a book to show– so you need a map?

Dale: If you don't have a map, you can still direct them by saying one quarter-turn that way to the right, one quarter-turn to the left, and a half-turn

Teacher: So those things would still help you, with the map

Dale: Yep, no without the map

Teacher: So that's the language you'd use without the map? But with the map you would use coordinates?

All agree

Teacher: Cool

Georgia: You would need a copy of that map for yourself and the person...

Dale (interrupts): No, you'd just give 'em that one

All talk at once

Dale started to think (*WoThs* on Table 7-3) that when relying on oral instructions, coordinates were not useful after all. The duality principle of the DNR framework could explain how Dale would have a need to reconcile, or accommodate this new idea or perturbation. The necessity principle (Harel & Koichu, 2010) caters for a student's intellectual need to learn new knowledge and in this case, this was the need to know if coordinates were useful. Further practical investigation of this concept would be required for Dale to eliminate the perturbation, or to identify the missing components of Dale's state of equilibrium. Opportunities for Dale to further develop his reasoning about coordinates as a mapping feature could assist Dale in internalisation, organisation and retention of this knowledge, as part of the repeated-reasoning principle of the DNR framework.

Later in the Develop phase, the students explored the similarities and differences between the maps that each had designed in their group. As the teacher, I was there to assist all students in the class and so I left the iPad with Dale, Brianna and Georgia on record while I worked with groups close by. This second conversation quickly turned to disagreement about their use of coordinates.

Excerpt 2: Second conversation

The second conversation (Excerpt 2) began with the three students looking for the use of similar and different mapping features in the maps they produced, and their conversation again considered the mapping feature of coordinates.

Brianna: I haven't got coordinates. Yes I have. I've got this one

Georgia (ignoring): and differences would be... coordinates. Coordinates.

Dale: No we all have coordinates. (Points) Coordinates, coordinates, coordinates.

Georgia: She doesn't have coordinates! (Points to Brianna's map)

Dale: Yeah, look, she's using the grid lines

Brianna: I did that on purpose. I did that on purpose.

Georgia (Picks up the book to illustrate): No, but she's got no letters down the side. That's a grid, we've all got grids. She's only got lines.

Brianna (interrupts): I haven't finished my map yet.

Dale: You haven't got letters

Georgia: I've got letters

Dale: Ok, differences: coordinates. How do you spell coordinates?

For analysis and using the DNR framework, I considered Dale's *WoUs* and *WoThs* with respect to the problem of identifying similarities and differences between the students' three maps. Although Georgia was beginning to record the use of coordinates in the "*Different"* column in their books (Line 32), Dale quickly noted how all three maps did include the use of coordinates (Line 33). His response (Line 35) reflected Dale's understanding and thinking about grid lines as coordinates (*WoU* and *WoTh*). Next, Georgia argued that this was not the case (Line 37) which conflicted with Dale's *WoUs* and *WoThs*. He considered Georgia's argument as part of his need for certainty and looked closer at Brianna's map to find that she hadn't used letters. To the students, letters were a key element of coordinates. This opportunity to collaborate and compare maps assisted Dale in making a clearer definition of the mapping feature of coordinates, contributing to his repeated-reasoning and internalisation of this mathematical concept.

Dale's learning journey continues

The final point in time selected to reflect Dale's learning journey is in the Defend phase where students defended the mapping features they included in their own maps. The final map that Dale created (Figure 7-3) was the product of student's work referred to in the illustration of learning (Table 7-3). I included Dale's reflective response to the teacher's statements (activity outlined in Table 7-1) to support the categories of understanding and thinking I identified for Dale in this part of his learning journey. Each student was asked to create the best map: not in competitive terms but in regards to the context in which it would be useful.

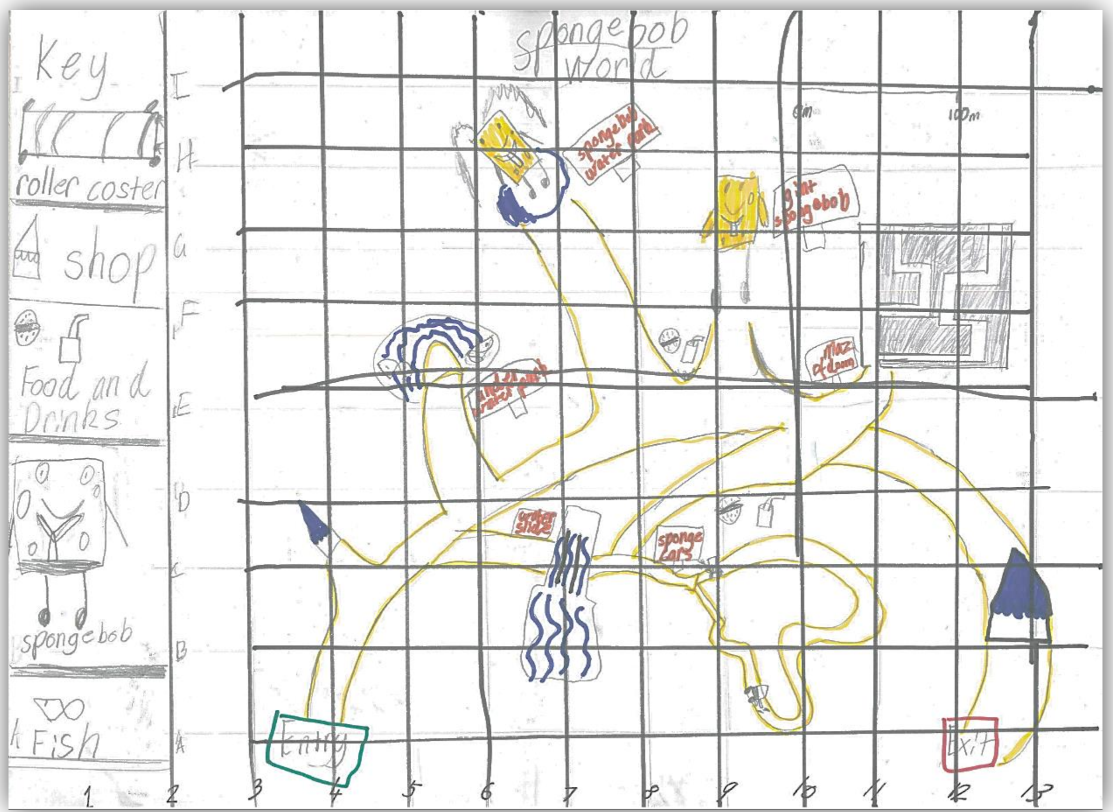
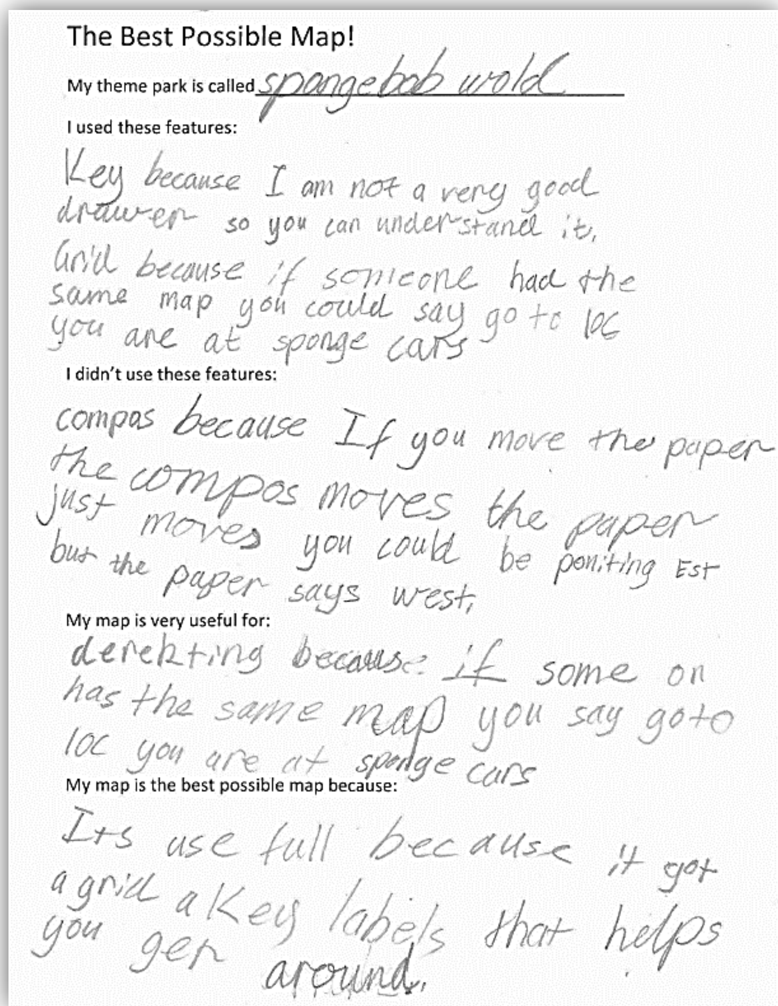


Figure 7-3 The final map that Dale created (Fry, 2013).

Features included on Dale's map included: a key, a graphic scale, and gridlines marked with letters and numbers so that coordinates could be used to describe positions and pathways (Figure 7-3). Dale defended his use of particular features in his reflection (Figure 7-4). Regarding his use of grid lines (I used these features): "… because if someone had the same map you could say go to 10C. You are at sponge cars." Dale's reference to both users having the same map may have related to previous conversations with peers (Excerpts 1 & 2). He reflected on the usefulness of his map by explaining how (Figure 7-4, My map is very useful for): "Directing because if someone has the same map, you say go to 10C. You are at sponge cars." Please note that although Sponge Cars is situated on the C line coordinate, it does not quite match the 10 coordinate reference Dale makes. I am unsure why he made this error; whether it reflected a flippant comment he made, or incorrect reading, is unknown



The Best Possible Map!

My theme park is called spongebob wrold

I used these features:

Key because I am not a very good drawer so you can understand it, Grid because if someone had the same map you could say go to 10C you are at sponge cars

I didn't use these features:

compos because If you move the paper the compos moves the paper just moves you could be poniting Est but the paper says west,

My map is very useful for:

derekting because if some on has the same map you say goto 10C you are at sponge cars

My map is the best possible map because:

Its use full because it got a grid a Key labels that helps you get around.

The Best Possible Map!
My theme park is called *Spongebob (world)*

I used these features: *Key because I am not a very good drawer so you can understand it. Grid because if someone had the same map you could say go to 10C you are at sponge cars*

I didn't use these features: *(Compass) because If you move the paper the (compass) moves the paper just moves you could be (pointing East) but the paper says west.*

My map is very useful for: *(directing) because: if (someone) has the same map you can say go to 10C you are at sponge cars*

My map is the best possible map because: *Its (useful) because it got a grid a key labels that helps you get around.*

Figure 7-4 Dale's written reflection.

Four stages or events in Dale's learning journey were presented in this chapter. These began with Dale brainstorming what he already knew about maps and collaborating with Brianna and Georgia to generate a list of useful mapping features (Referred to as Collaborative Brainstorming in Table 7-3). Two excerpts were presented from separate conversations about draft maps that the students created (First and second conversation). These excerpts illustrated how perturbations can be generated amongst peers, and how students can reach consensus during the feedback process in small groups of students. Finally, the map that Dale created and his reflections on learning were included in this journey (Products of student's work). These four stages or events are summarised in Table 7-3 using the DNR framework (Harel & Koichu, 2010).

In reference to the language used in analysis of Dale's learning experiences, key terms were derived from the mathematical content related to content descriptions identified by The Australian Curriculum (Table 4-1) and the common assessment task (Figure 7-1). The intellectual and psychological needs of Dale were described practically or using language presented in the work by Harel and Koichu (2010). It is also worth noting that in their paper the perturbations were generated by the researchers, whereas in this study, perturbations were generated through interactions with peers throughout different stages of the inquiry.

I would now like to offer specific explanations of the features of the DNR framework in relation to analysis in this phase of study (Table 7-3). I return to analysis of the collaborative brainstorming Dale undertook with Brianna and Georgia (Figure 7-2). Dale's *WoUs* were considered with respect to the act of problem posing and the act of interpreting, in this instance, coordinates. With respect to problem posing, at the brainstorming stage the problem being considered by Dale had been posed by the teacher: what were the features of maps. By himself he had not recorded the word 'coordinates' although after collaboration with his peers, it was agreed that coordinates were a feature of maps. The students had shared the idea that coordinates were numbers and letters "down the side" (Figure 7-2). Dale's *WoThs* at this early stage of the inquiry included the conjecture that maps contained many features, including coordinates. It is the interdependency between these two categories of Dale's knowledge (*WoUs* and *WoThs*) that is a foundational principle of the DNR framework: Duality as the reciprocity between the two categories where change in one cannot occur without related change in the other, in this instance, coordinates were one of many features of maps that includes combinations of numbers and letters.

The Necessity principle is the second of Harel and Koichu's (2010) principles to deal with a student having the need for learning new knowledge. Expressed as psychological and intellectual needs (Table 7-3), I refer to the inquiry question of 'What is the best map?' This question established the context for finding out what features to include on the best map. In terms of Dale's need to solve the problem of creating the best map, I considered perturbations that Dale may have come across. In this phase of his learning, Dale had not considered coordinates as a feature, yet his peers had. He had the chance to eliminate this perturbation through collaboration with his peers. Disequilibrium-equilibrium manifested as Dale was still unsure of what the best map is.

In the paper by Harel and Koichu (2010), which used the DNR framework to operationalise learning for their participant (Burt), the illustration they included did not make explicit reference to the principle of Repeated-reasoning: the third foundational principle of the DNR-oriented definition of learning. Although not explicitly stated in either illustration, the notion of repeated-reasoning for Dale is reflected in the repeated opportunities to consider the feature of coordinates that inquiry presented. Dale constructed his knowledge of coordinates during the disequilibrium-equilibrium phases in each interaction or event. His knowledge of coordinates was built on and included consideration of the knowledge of his peers, incorporating his need to answer the inquiry question to produce the best map.

Table 7-3 summarises the learning journey for Dale at each of these stages or events as a continuum of disequilibrium-equilibrium phases. According to Piaget, these perturbations existed because he already had in place some organisational scheme regarding maps, evident in Figure 7-2 at the beginning of his learning journey. Actively, he considered features of maps that included letters and numbers; with peers, he pondered the usefulness of coordinates when users did not have a copy of the map in front of them (Excerpt 1); he became unsure of his understanding of coordinates as a feature of maps (Excerpt 2); and successfully created a map using this feature (Figure 7-3), noting how all users would need a copy of the same map (Figure 7-4).

Table 7-3 An illustration of Dale's learning using Harel and Koichu's (2010) DNR-oriented framework, modified from Fry (2013).

		Collaborative Brainstorming	**First conversation**	**Second conversation**	**Products of Student's work**
Ways of understanding	**With respect to problem posing act:**	Considers features of maps	If you don't have a map in front of you, what use are coordinates?	Similarities and differences between map features	Can successfully create a map
	With respect to the act of interpreting coordinates:	Combinations of letters and numbers are places on a map	Coordinates are only useful if all users have the same map	Grid lines are coordinates	Uses coordinates to show location on a map
	Ways of thinking:	Maps contain many features	Not all features on maps are useful	Grid lines show coordinates	Coordinates are useful on maps
		Coordinates are a useful feature	Oral instructions do not rely on coordinates	All three maps use coordinates – this is a useful feature	My map is the best
	Psychological and intellectual needs:	The need to solve the problem *What is the best map?*	The need to know if you always need coordinates	The need for certainty	The need to justify choices made
	Ways of eliminating perturbations:	Collaboration with peers	Further investigation through discussion and use of maps made	Collaboration of ideas	Completing the task
	Missing components of the state of equilibrium:	Uncertainty about what is 'best'	Further practical investigation using maps	Clear definition of coordinates	

The DNR framework was useful for illustrating the complexity of student learning in an inquiry context. The framework illustrated Dale's learning as multi-dimensional, developing through peer interactions in a community of inquiry, where feedback influenced decisions he made. Dale actively considered feedback in situations artfully engineered by the teacher, by connecting feedback to ideas he already knew about the topic. He challenged his peers, providing further feedback to them and even questioned his own ideas. Dale had to consider their ideas also, adding a dimension of pondering to the classroom element of learning; pondering one's own thoughts (being active) and the thoughts of others, especially when they might conflict with one's own thoughts. The process of equilibration can take place in these moments as students develop thinking through the construction of understanding, and have multiple opportunities to do so in an inquiry context. Richness in learning can be found in the moments that students 'hold' onto when they don't know the answer to a problem, when they ponder the perturbations of others or challenge others with their own ideas.

Classroom reflections: "I learnt…"

Although a range of activities were planned for to make learning visible, I would like to pay particular attention to the written reflections produced by students in the Defend phase of this inquiry. In this phase of study I was most interested in understanding the link between assessment and learning and sought to know what the students had learned in the inquiry, from their own perspectives and in their own language. Students were required to independently complete five statements in relation to their own learning as part of reflection in the Defend phase of the inquiry (See Figure 7-4 as an example). Intended only for the teacher as the audience, these reflections were not intended to display or to share with peers. The reverse side of the reflection sheet was intentionally left blank. Here students were asked to openly respond to the sentence-starter 'I learnt'. I wondered how, in these statements made by learners, they would describe their own learning. Through comparative analysis, I categorised the statements students made as five general *WoUs* about maps (a-e), and the students' related *WoThs* (Table 7-4).

Table 7-4 A synthesis of learning statements written by students in the Defend phase of the inquiry.

Ways of Understanding	Ways of Thinking
a) Maps have different purposes	- There are different maps - To draw attention and persuade
b) Maps are/need to be useful	- Some maps aren't useful - To find your way around, where to go and how to get there - To find things, destinations, where you are - To show directions if you are lost
c) Maps are/need to be helpful	Characteristics: - Simple and clear Features include: - Places/rides, pictures, signs, labels, key, alphanumeric grid, coordinates, pathway/paths, titles, compass, symbols
d) All users need the same map	- Or they won't have a shared reference point (based on discussions between Dale and his peers)
e) Difficult to create	- Hard to make

Each response to the statement 'I learnt' was related to each student's overall learning about mapping through the inquiry. The students were reminded to consider what the 'big' ideas were that they had learned, compared to what they knew about mapping previously. Through open coding, I firstly identified concepts related to *WofUs* about maps that the students shared. Categories were identified and refined to describe the cognitive products associated with the mental acts carried out in all interactions in this inquiry; or *WoU*s. Approaches that the students used to come to these understandings included looking at the problem in different ways. These approaches, or *WoTh*, depended on how learners represented, came to, or interpreted their understanding about the related *WoU*. For example, some statements written by students elaborated on the understanding that maps are, or need to be, helpful (Table 7-4, *WofU*: c). Thinking about being helpful, the students described how maps could be helpful: they needed to be simple and clear, and include particular features (Table 7-4, *WoThs* about c). The statements each student wrote about their own learning in this inquiry show the various *WoUs* developed, and the *WoThs* articulated by the students.

A key feature of the DNR perspective of learning presented by Harel & Koichu (2010) is the duality principle that states: "Students develop ways of thinking through the production of ways of understanding, and, conversely, the ways of understanding they produce are impacted by the ways of thinking they possess" (p. 899). In this study, as students pondered which features maps included (*WoThs*), we see that their *WoUs* that maps are helpful, helped them to consider which features to include. This duality principle summarises the process of developing *WoThs* and, conversely, *WoUs* (Harel & Koichu, 2010). The duality principle is applied to individual learners by its authors in their 2010 paper whereas I have also considered interactions with peers in a classroom community of inquiry.

I would like to relate the synthesised *WoUs* and *WoThs* (Table 7-4) to the notion of concomitant potentialities from the previous phase of study (Phase Two: A teaching focus). The DNR-oriented definition of learning (Harel & Koichu, 2010) considered that the reciprocity of the collective *WoUs* and *WoThs* of students in this class were developed because there was a need to learn this knowledge (the Necessity principle) which was developed during the inquiry, *Are maps useful?* The *emergent LPD* was articulated in the Preparation and Design section of this chapter, as a focus on the features maps use that are effective. The *WoUs* and *WoThs* identified, as written by the students in their reflections, reflected concomitant potentialities presented by the students. Although the concomitant potentialities were not explicitly stated by the curriculum, the potentiality of a need for maps to be useful for example, was fostered by the ambiguity of the inquiry question, *Are maps useful?* The multiple iterations for students to consider the usefulness of maps through the inquiry approach supported students to reason repeatedly on the concomitant potentialities that arose.

Retrospective Analysis

This chapter presented analysis from the third phase of study exploring the classroom element of learning in inquiry. An aim was to illustrate learning in ways grounded already in theory and empirical research (Harel & Koichu, 2010; Makar, 2012). Recently, Simon (2013) presented a complementary paper to overlap with and highlight the slight differences between his own research and the 2010 paper by Harel and Koichu. He argued that mathematics instruction that presented problems for students to solve, where discussion brought about learning, was "theoretically weak" (Simon, 2013, p. 287). He added how promoting disequilibrium would not in itself directly foster the learning of mathematics intended, or the need for something that the learner could not yet conceptualise. How can students learn what they do not already know? Promoting disequilibrium through inquiry may foster a need to learn something. The previous phases of study in this thesis have considered a notion of developing a sense of the need for (mathematics), as a belief to characterise learning in inquiry. Disequilibrium in inquiry only occurred when the learner already had in place some understanding or scheme about the topic. The learner in inquiry must be willing to embark on a learning journey and feedback can assist them in gaining confidence to move forward, when they act upon this information.

In reference to illustrating an example of a productive way to analyse a learning process, I turn again to the paper that Simon presented (2013), where he critiqued the analysis presented to illustrate Burt's learning (Harel & Koichu, 2010). Simon suggested that four factors must be in place for productive engagement in analysing a learning process. I would like to take this opportunity to respond to each of these:

1. **Evidence of learning:** Dale’s mathematical learning about mapping was illustrated through four different artefacts: brainstorming independently and collaboratively with peers (Figure 7-2), two separate conversations with peers (Excerpts 1 & 2), and in the production of what Dale considered to be the best map (Figure 7-3). This data reflected aspects of ‘pre and post data’ as Simon (2013) suggested. This was to show learning as progression over time, to consider the transitions Dale made from one conceptual state (in this instance about coordinates) to another. This would not be possible to be captured in quantitative evidence of learning.
2. **Span of the data within which the learning occurred:** Simon (2013) particularly stressed the importance of articulating the span of the data within which the learning occurred. In contrast to Harel and Koichu’s work (2010), he added that this should be timely. Illustrations of learning for Burt (Harel and Koichu’s participant – a mathematics teacher) reflected three interview protocols that took place over 20 months. Although not articulated clearly, learning for Erin (Simon’s participant – a prospective elementary teacher in their final year of study) reflected five sessions which may have taken place over 11 days/2 weeks. In this study, the lessons within which Dale’s learning was illustrated took place over one week. This is clearly articulated in the description of the preparation and design of this phase.
3. **Learning occurs over a short period of time:** Simon (2013) stressed that “concepts that can be developed in a short focused study are optimal” (p. 288). In this study, learning about the usefulness of features on maps was considered over the course of five-lessons, a short mathematics inquiry. Dale’s mathematical learning was illustrated in all phases of the inquiry in a variety of ways: written work (both independently completed and in collaboration with peers), in conversations (between the teacher and the particular groups of students described here), and in the artefact generated by the student; his map.
4. **Data must provide a continuous trace of the unit of analysis:** Learning for a single student, Dale, was presented as a unit of analysis in this chapter. Simon (2013) claimed the difficulty in making claims about the process of learning when part of this process can take place during conversations with peers or in solving homework problems for example. In this phase of study, I have not tried to claim the evolution of Dale’s thought process as an individual construction. Instead I have particularly valued contributions from peer interactions as part of

> the classroom community of inquiry. It might prove extremely hard to reduce gaps in data on the learning process for any one student as Simon suggested: in a clinical study a student may look out the window and see a sign showing direction that might contribute to the process, or they may see an ant trekking across the floor beneath them that impacts on their understanding of position and pathways.

In Simon’s study (2013), he conducted analysis using one-on-one teaching experiments, addressing each of these issues. I have responded to each of these factors to consider the mathematical learning for a student in an inquiry classroom, to illustrate a complex and highly interactive view of learning. I used formative assessment strategies to make learning visible in this context and with the rich amount of data collected, illustrated learning for one student. As foregrounded, my intent was not to articulate a particular learning trajectory, or *conceptual trajectory* (Confrey, 2006). Such is the nature of inquiry; learning is broad and varied for each student. It does not always match the *emergent LPD* as intended, and can be very difficult to describe for any one student, especially in relation to assessing learning as a pursuit of education.

For one student, Dale, mathematical learning through inquiry was illustrated using a framework that valued disequilibrium as a way to foster mathematical learning. I have noted the contrasts in this study to the one presented by Harel and Koichu (2010): participants presented in this chapter were young children (8 and 9 year olds), learning mathematics through the pedagogy of inquiry. Burt (in Harel & Koichu, 2010) was a mathematics teacher who was presented with problems designed by researchers, designed to present him with perturbations to overcome. Dale, on the other hand, was presented with perturbations generated by his peers. The idea of using coordinates as a feature of maps was not a learning intention stated by the curriculum. Neither was it a curriculum intention to generate criteria about the best features to include on a map. Yet this concomitant potentiality was reflected in the data for one student, Dale.

Research question one: What learning opportunities arise in an inquiry classroom to develop mathematical understandings?

Formative assessment opportunities had been artfully engineered to make learning visible in this inquiry classroom. Yet little was known initially about how these opportunities specifically moved learners forward (in their own, personally constructed learning) in an inquiry context. Analysis of the classroom element of teaching in the previous phase had considered how the classroom teacher integrated curriculum intentions into the students’ ZPDs. The *emergent LPD* was artfully engineered by the classroom teacher as she considered the potentialities of students’ learning progressions to solve the inquiry problem in personal ways. Analysis in this phase of study explored how feedback stimulated a response in the learner when it built on already established schemes (Inhelder et al., 1976; Piaget, 1964). These schemes represent the personal approaches to problem solving that students draw on. The feedback in inquiry becomes close and personal when it takes place in collaborative teaching and learning experiences, as is the nature of learning in a classroom community of inquiry (Goos, 2004). Rather than rely on behavioristic models of stimulus-response schemes, Piagetian notions would acknowledge how in inquiry, schemes are activated when perturbations to learning, that are close and personal, present.

Learning opportunities analysed in this chapter focused on the learning journey of one particular student, Dale, as he collaborated with his peers (Brianna and Georgia) and the classroom teacher in the mathematical inquiry, *Are maps useful?* This presented a learning journey where perturbations meant having to ponder challenges, and to eventually overcome them. Although analysed using a framework based on developmental theory, the journey that presented was not a straightforward learning trajectory that I could propose to apply to any learner learning about maps (Simon et al., 2010; Steffe, 2003). Instead, the learning opportunities were artfully engineered by the teacher to incorporate concomitant potentialities that enabled students to approach solving the inquiry question personally, in the social context of inquiry. The illustrated *WoUs* and *WoThs* that Dale developed were influenced by feedback that was within his ZPD, and received close in time of being unsure (Excerpts 1 & 2). These learning opportunities allowed him to act on feedback and to move his learning forward towards the *emergent LPD*. Although the process of equilibration for Dale has not been expanded upon in this chapter, analysis of the data instead illustrated the *WoUs* and *WoThs* developed throughout the process of equilibration (Piaget, 1977).

Research question two: How can a student learn mathematics through inquiry?

In an inquiry classroom the learning journeys students embark upon are personal. In this phase, learners connected to pre-existing knowledge/schemes when they discussed what was personal and meaningful to them, striving to make meaning to their peers (Collaborative brainstorming, Figure 7-2). Learners themselves seemed to access the ZPDs of their peers and conversations offered feedback that was personal and meaningful (Excerpts 1 & 2). Students learned as they pondered the ideas of others and confidently challenged peers to consider their own thinking about the problem. Students also needed to show willingness to embark on this learning journey and be accepting of feedback they received, even when that feedback presented a roadblock or perturbation. Opportunities to articulate thinking (in writing or through discussion as formative assessment) assisted students in their processes of equilibration, although possibly a temporary state rather than a final destination (including reflections on learning by students in Table 7-4).

The 4D inquiry framework (Allmond et al., 2010) offered learners multiple opportunities to consider an inquiry question, in multiple phases: Discover, Devise, Develop and Defend phases. Frequent formative assessment offered students a chance to consider the mathematics they are learning about in a variety of ways. The theoretical framework offered by Harel and Koichu (2010) provided a way to illustrate learning for one student in this inquiry classroom (Dale), showing that the foundational principles of Duality, Necessity and Repeated-reasoning can be supported using an inquiry approach. I would like to add to this framework the consideration that perturbations that provoke learning can present through interactions with peers and that these perturbations are valid in the students' minds: for Dale, we can see evidence of his consideration of coordinates throughout his learning journey. Perhaps it is only that Dale is open to the feedback he received, or actively considered feedback offered to him, that is part of the culture established through inquiry.

Potential Innovations to Consider

There are many opportunities for peers to collaborate in an inquiry classroom environment, both in small groups and with the classroom community in general. When principles of formative assessment are artfully engineered into teaching and learning experiences, active learners know that they are about to embark on a learning journey no matter where they are in their development. In inquiry, responsibility shifts to the students to activate what they already know about the inquiry question (existing schemes) and to ponder perturbations that present. The journey is unique and students need to be open to feedback they receive in order to overcome challenges they face. There is little chance that specific learning trajectories could articulate each and every student's learning journey in an inquiry setting. However, students need to be active in contributing feedback in collaborative situations. Asking questions and challenging peers can offer feedback that can be acted upon almost immediately. This is opposed to students receiving feedback from their teacher in a school mathematics classroom that might have a longer feedback loop; even if feedback is returned the following day after marking workbooks, for example.

There is much research on how students learn that attempts to theorise this process in deep ways, or offer explanations of how further research may be implicated (Black & Wiliam, 2006; Greeno et al., 1996; Harel & Koichu, 2010; Hofer & Pintrich, 1997; Simon et al., 2010). I have described the ability of peers in an inquiry classroom to be able to quickly understand the ZPDs of others, to offer feedback to other learners that is meaningful. I considered this ability as ingenious, where a goal of teaching might be to find out what is in the head of our students. Research by Goos (2004) already explores the learning potential of peer groups of comparable expertise in a classroom community of inquiry. Analysis in this phase considered that when some of this responsibility shifts to students they are quite capable of supporting their peers in their process of learning. Developing in learners a sense of responsibility for learning, is a skill which is developed over time as classroom norms become accepted and refined, until the teacher insists that students take responsibility for habits of reflection and self-monitoring, for example (Goos, 2004). Makar (2012) acknowledged how for teachers, striving to balance teacher control with giving students opportunities to take responsibility can be difficult. The inquiry community of practice fosters the importance of making thinking visible, to peers in particular as they collaborate to solve the inquiry question. Students expected to be listened to by the classroom teacher as she considered how students may have approached the inquiry question. Students expected their peers to listen as they tried to make meaning themselves. Peer interactions offered students more time to consider the ponderings of others when the classroom teacher was only able to offer feedback to one group or learner at a time.

I return to the notion of learning being a journey. Students choose to embark on their learning journeys with a goal in mind, yet this is not to determine the end point. Summative assessment may consider learning as a final destination where reaching the *emergent LPD* determines success. Further perturbations in Dale's learning may present when he notices other maps in the social environment around him, or when he sees the maps that other students in the class share and defend. In inquiry, potentialities are the beneficial connections to pre-existing schemes of knowledge that move learning forward. These are beneficial as they allow the learner to be active in response to further challenges to learning. Students gained feedback through formative assessment, which lead to potentialities to explore. This may be challenged through further consideration of potentialities through feedback, as a continued process of formative assessment. This presents learning in inquiry as a complex process, with an end-point that is unclear, and that is part of a complex web of interactions with peers, as engineered by the classroom teacher.

I have attempted to illustrate the directions of learning that a student may travel in Figure 7-5. The student begins with pre-existing notions about the topic (1) and through multiple iterations of formative assessment and responding to feedback, potentialities are encountered which can be guided towards the *emergent LPD*. This is not intended to show that the student is moving away from the *emergent LPD*, rather that there are many pathways or directions a student may take in solving the inquiry question.

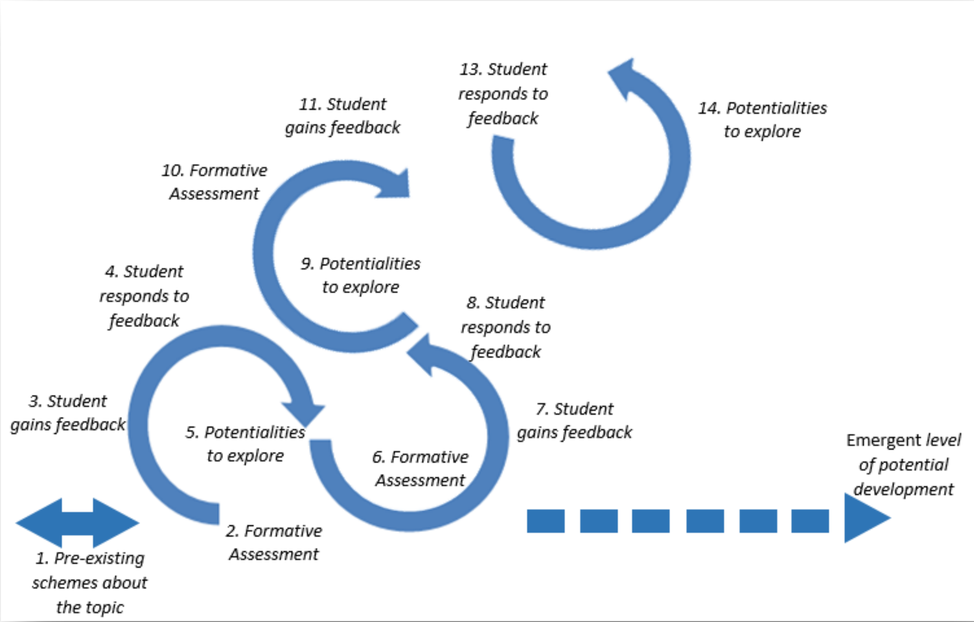


Figure 7-5 An impression of findings: Complex and frequent interactions create a cycle of formative assessment and feedback where learning progresses on personal pathways.

Chapter 8 Discussion

I begin this discussion with a reminder of my initial concern with the mismatch of descriptions of learning gained through assessment, compared to my own views of student learning in an inquiry classroom. Although this began with my own teaching experience in primary mathematics classrooms, assessment data that reported on student learning limited descriptions to curriculum content and only in terms of right or wrong. What seemed to be missing was any reflection of the openness of the approach to learning mathematics inquiry supported. Assessment described student learning narrowly yet inquiry encouraged students to explore multiple methods for solving mathematical problems, related to the inquiry context being explored.

An overall aim of this study was to gain insight into the complex, educational interactions in a mathematics inquiry classroom. The study sought to research and document how and what students learned through inquiry settings, to inform practical ways to consider teaching and assessment. As indicated in the Introduction Chapter of this thesis, the particular research focus centred on three particular elements in an inquiry classroom:

1. Through assessment, identify and describe learning in ways that are well-informed and respectful of students as learners of mathematics in inquiry (Assessment).
2. Consider how information gained through assessment informs my teaching about how to extend children's thinking further through inquiry (Teaching).
3. Elaborate on the process of learning (understandings, skills and procedures) in an inquiry context (Learning).

The three classroom elements of assessment, teaching and learning were used to structure the thesis and the three phases of study. In this chapter, I will first summarise the findings of the analyses from the three iterations of study. Interpretations and theoretical analyses have already been considered in each of the three analysis chapters, in response to the aims in each iteration. As part of design research (Cobb et al., 2003), iterations of study have incorporated previous findings into the design of the next phase of study. In each chapter these were outlined as *Potential innovations to consider.* This chapter considers the implications of these findings in an interrelated fashion, drawing on theoretical notions of beliefs presented by Duckworth (2006). Further potential innovations are considered to provoke new research into the inquiry pedagogy in primary mathematics classrooms. Findings from other studies will provide an opportunity to compare and contrast innovations established in this study, to integrate these innovations with educational research on assessment, teaching and learning in the mathematics classroom. A final section summarises the main findings and the lessons to be drawn from this study, and elaborates on how this work has already contributed to research in the mathematics education community.

Summary of Findings

Phase One: Assessment

The research aims of this phase of study centred on the classroom element of assessment and endeavored to understand: if current assessment practices aligned with the pedagogy of inquiry, to support teaching and learning in this context; and what else assessment revealed about teaching and learning. Research was undertaken in a Year 6 classroom where students were involved in the mathematical inquiry, *How much is a cubic metre?* This inquiry was designed to follow on from a commercially-designed unit which reflected styles of teaching and learning similar to a traditional school mathematics classroom. The common assessment task that accompanied the unit was used to evaluate student learning prior to the inquiry.

Analysis focused on the opportunities for assessment undertaken in this classroom and was informed by the PISA Assessment Framework (OECD, 2009) to characterise assessment generally. It revealed how the common assessment task, used to illustrate mathematical learning, fostered thinking at a reproduction level; one that did not consider any personal connections students might make in learning about this concept, nor learning in collaboration with peers, nor any intimate connection with mathematical content. These negative qualities seemed to be in contrast to student learning during inquiry. Analysis that focused on the information gained through formative assessment opportunities during the inquiry, revealed a window into learning that included connections to personal experiences and for some students, a relational view of learning in their responses (Dewey, 1891; 1938a). Analysis highlighted student thinking, fostered by the inquiry approach to teaching and learning, in the connections and reflection clusters (OECD, 2009) which were higher levels than what was valued by the common assessment task. The frequent occurrence of formative assessment practices in the inquiry setting reflected this classroom as highly interactive with a culture of 'seeking to know' about learning.

Theoretical analysis likened learning in these clusters (OECD, 2009) to the process of conception; an aspect of thinking presented by Dewey (1891). He summarised the process of conception for learners as a developmental process including abstraction, comparison and synthesis. In this classroom, the process of conception of ideas was supported concurrently by the fostering of student thinking in the connections and reflection clusters (OECD, 2009). Formative assessment supported mathematical learning at different stages throughout the learning process, by offering students opportunities to connect, reflect upon and synthesise their thinking about the mathematics being explored: thinking as the process of conception. The inquiry, as a means of assessing students' understandings of cubic metres, revealed deep thinking about mathematics. Consideration for the use of formative assessment continued in the next phase of study where attention turned to the classroom teacher as the person who would generally make decisions about the methods and kinds of assessment to use. How would an inquiry classroom teacher use information about learning gained through formative assessment, and how might they adjust their teaching based on this feedback? Design research would provide me with the opportunity to incorporate findings from this phase of study, to explore the classroom element of teaching.

Phase Two: A teaching focus

In the second phase of study, the research aim considered how a teacher in an inquiry classroom was responsive to feedback gained through formative assessment, to guide students towards particular learning goals. A Year 3 classroom provided the context as students participated in the mathematical inquiry *How many steps do you walk in a day?* Findings from the first phase, which characterised assessment as useful when it was frequent and formative, were integrated into the design of this iteration of study. Formative assessment generated feedback to make visible the development of learning for students (conception of thinking). The research question guiding analysis in this phase speculated how the teacher might adjust their teaching to support learning. This proved to be a complex process for the teacher: knowing 'where students were' in their learning involved a collective notion of all learners in the classroom, so teaching considered multiple perspectives in moving all students towards particular learning goals.

Theoretical analysis was informed by Vygotsky's notion of ZPD (1978), which highlighted how an awareness of a student's ZPDs can inform the teacher about how to scaffold learning for that student. Pre-assessment was included into the design of this inquiry to establish collectively, the *actual developmental level* (*ADL*) of the class. For the teacher, and in analysis of this data, this presented a messy picture of where students were beginning their learning journeys; just as if students in a running race started at various places on the track (and some not on the track at all!). The classroom teacher also had to consider the *level of potential development* (*LPD*) as a goal for learning. In the case of this classroom, the *LPD* was determined by curriculum and assessment structures yet proved only emergent in the design of the inquiry. As students collaborated to mathematically solve the inquiry question, they considered other mathematical concepts, processes and connections not articulated in the *LPD.* Rather than disregard these potentialities, the teacher engineered them into the design of the inquiry, as concomitant to the *emergent LPD.*

To apply Vygotsky's notion of *LPD* (or learning goals) in an inquiry context assumed that learning had an end-point in time. Rather than an end-point, Duckworth's (2006) belief framework was also used to consider further the *emergent LPD* in an inquiry learning journey. This framework characterised teaching and learning goals stated as objectives to be learnt, as '*the-way-things-are'* kinds of beliefs; traditionally measured through testing (Duckworth, 2006). Rather than rely on assessment alone to elaborate on learning in this inquiry, Duckworth's framework was used as the analytical tool to identify other kinds of beliefs fostered by the teacher as part of the pedagogy of inquiry. This proved useful not as a further element to add to assessment in an inquiry classroom generally, but as a way of openly considering that many other kinds of beliefs are fostered when an inquiry approach is chosen by the teacher to teach mathematics, beyond the scope of assessment generally. I will return to this framework to consider findings from all three phases in an interrelated fashion.

Upon reflection, through the use of formative assessment I had uncovered somewhat intangible information about personal and connected student learning in an inquiry classroom, beyond the constraints of classroom assessment (not designed for an inquiry approach to teaching mathematics). I was also aware of the difficulty in trying to engineer into an inquiry, the multiple directions in mathematics, or potentialities, that students presented in the process of inquiry. I wondered how these findings would contribute to further research exploring students' mathematical learning in an inquiry setting.

Phase Three: Learning mathematics in an inquiry context

Research aims in this final phase pondered the characteristics of learning opportunities in this classroom, and considered how students learned in inquiry. This iteration of study took place in the same Year 3 classroom explored in the previous chapter. *Are maps useful?* provided the inquiry context in which to provide a window into student learning. Principles of formative assessment from the first phases of study were implemented into the design of this inquiry to make learning visible and for research purposes, to generate data for comparison and to refine dimensions and properties to do with the classroom element of learning. The concept of an *emergent LPD* was applied in the planning for this phase, as well as the engineering of concomitant potentialities by the teacher into the inquiry. A common idea about learning presented by Dewey, Vygotsky and Piaget, was that learning is a process. Returning to the theoretical framework and put simply here, all three theorists also considered an element of overcoming challenges to learning as part of this process. This informed theoretical analysis of learning in this third phase of study; to consider classroom challenges or perturbations as part of the learning journey.

Just as in the previous cycle of study the classroom teacher was responsive to feedback, being a responsive learner could assist in the equilibration process of assimilation and accommodation described by Piaget first in 1952, and incorporated into the DNR- based definition of learning (Harel & Koichu, 2010). The DNR framework provided the analytical tool for theoretical analysis and was used to illustrate learning for one student, Dale. Learning for Dale included consideration of his own perturbations; being unsure of how to continue towards answering the inquiry question. These perturbations presented in collaboration with his peers through inquiry. Formative assessment opportunities that supported working collaboratively presented Dale with opportunities to learn that included challenging his own thinking and eliminating perturbations through discussion with peers, to complete what he thought was the best map. The illustration of Dale's learning reflected a complex and highly interactive view of learning in inquiry. It included the process of scaffolding taking place in interactions with peers rather than the teacher being alone in this process.

Also in this third phase of study, I considered learning for the class generally. Analysis of reflections made by students about their learning resulted in identifying a suite of *Ways of Understanding* (*WofUs*) and related *Ways of Thinking* (*WofThs*) about maps. These I related to the notion of concomitant potentialities presented above. Teacher engineering of these potentialities into teaching and learning experiences assisted in guiding students towards the *emergent LPD* of creating the most useful map. The teacher saw these potentialities as a way to support a sense of the need for mathematics to create a useful map, and as a way to incorporate students' personal solutions, brought into context through the inquiry. Opportunities for students to articulate their responses to reflective prompts may have assisted students in their own processes of equilibration or learning.

A key conclusion I made in this phase was concerned with how some of the responsibility to learn in an inquiry classroom shifts to the students. In this inquiry, learning would require the learner to be active in response to, and in the generation of, feedback. The inquiry context explored in this phase of study supported opportunities to be active through principles of formative assessment that valued collaborative sense-making.

An interrelated view of the classroom elements of assessment, teaching and learning, in an inquiry classroom

The theoretical implications from each iteration of study will now be considered in relation to each other. This will contribute to articulation of the *conceptual corridor* of inquiry mathematics: the possible space to be navigated successfully to learn mathematics (Confrey, 2006). Just as changes to one classroom component cannot take place without affecting the other components (Ramaprasad, 1983), this study seeks to consider all three classroom components in relation to the others. Ignorance of the network of relationships (in this sense, classroom components of assessment, teaching and learning) in a classroom could result in unintended consequences of organisational change (Ramaprasad, 1983). In fact, it could be argued that schools have responded to research in the field of assessment and data in educational contexts, without giving thought to how this may affect the broader sense of implementation. Making clear and definite changes to classroom pedagogy without consideration of related classroom elements may be a cause of the lack in teacher confidence with qualitative data, about learning that that does not seem to fit into curriculum expectations.

Duckworth's (2006) belief framework is used as the framework to synthesise the theoretical findings explored in each iteration of study. Elaboration on this framework can be found in the Literature Review of this thesis, in response to learning in a mathematics classroom. Rather than try to describe all the concepts there are for students to learn, Duckworth (2006) conceptualised four kinds of beliefs to organise the many things children might learn: those to do with world knowledge ('*the-way-things-are*' beliefs), self-interest ('*it's-fun*' beliefs), self-confidence ('*I-can*' beliefs) and shared ('*people-can-help*' beliefs). This framework is used to consider further the findings from all three phases of study. This analysis is integrated with the theoretical contributions from Dewey, Piaget and Vygotsky explored in each phase. Knowing if or how particular characteristics of assessment, teaching and learning in an inquiry classroom align to support each other can inform the design of these related classroom elements, to foster in students a deep, connected learning of mathematics.

The first kind of belief articulated by Duckworth (2006) regarded knowledge that had to do with the world, in the sense of a child's view of knowing that knowledge already exists and can be passed on to others. These kinds of beliefs to do with the world were promoted by the common assessment task that students completed in the first inquiry classroom. In her own research, Duckworth (2006) identified other learning that was important, yet not specified as curriculum concepts. The assessment task in Phase One (Figure 5-3) supported the notion that mathematical knowledge to do with the concept of volume already existed in the world and one had only to *learn* it (if one valued success as getting 100% of their answers correct). Torrance and Pryor (1998) described assessment which focused on contrasting errors with correct responses as convergent, or assessment *of* the learner *by* the teacher. Informed by the belief framework designed by Duckworth (2006), this assessment task only revealed a small picture of learning for students if this was all to be considered (information about learning captured by the assessment task reflects World knowledge only, in Table 8-1). For success in completing the relevant assessment task, students would only need to learn mathematics (as World knowledge taught by the teacher) at the reproduction level (OECD, 2009): low-level thinking to reproduce taught mathematical skills and knowledge (Table 8-1). Any other types of beliefs about learning were not considered when using assessment practices not designed specifically for the inquiry approach to teaching and learning. Further consideration of this inquiry using Duckworth's belief framework would reveal a broader picture of student learning gained through formative assessment.

Table 8-1 Beliefs to do with the way of the world considered by the summative assessment task.

Study Phase	**Beliefs about learning fostered (Duckworth, 2006):**			
	World knowledge	***Self-interest***	***Self-confidence***	***Shared***
1: A focus on assessment in an inquiry mathematics classroom	Summative (conventional) assessment practice revealed: - Traditional notions of learning mathematical rules and procedures - Reproduction level of mathematical thinking (OECD, 2009)			

In the inquiry classroom, the use of formative assessment revealed higher levels of student thinking about mathematics than required by the common assessment task: at the connections and reflections clusters (OECD, 2009). Formative assessment provided opportunities for students to conceptualise their thinking beyond the scope of mathematical content (as knowledge that is to do with the world). I would argue that formative assessment offered the time and space for broader thinking to take place. For example, Quickwrites (as a formative assessment technique) encouraged students to connect to prior knowledge and knowledge shared, and to reflect on all areas of mathematics to solve inquiry problems. To satisfy assessment constraints in a classroom context, this information could still be considered through assessment in terms of *world knowledge* or content knowledge.

Just as curriculum and syllabus documents (that informed which kinds of beliefs about the world one had to learn) informed teaching and learning in mathematics classrooms generally, it was revealed in the second phase of study that this learning focus would be considered only as emergent in the inquiry classroom. Analysis in the second phase considered how the classroom teacher utilised the wealth of information about her students' learning that formative assessment provided, to adjust her teaching. Scaffolding learning involved engineering potentialities into the collective ZPDs of her students. This assisted in relating learning personally to the contexts students invented and that were relevant to them (Table 8-2). Things to be learnt in this inquiry classroom moved beyond world knowledge, to consider beliefs to do with self-interest and self-confidence.

Table 8-2 World knowledge remained an element of teaching in inquiry in the second phase of study, although other beliefs about learning (Self-interest and self-confidence) were considered by the teacher in her practice and through assessment.

Study Phase	Beliefs about learning fostered (Duckworth, 2006):			
	World knowledge	***Self-interest***	***Self-confidence***	***Shared***
1: A focus on assessment in an inquiry mathematics classroom	*Summative (conventional) assessment practices revealed:* - *Traditional notions of learning mathematical rules and procedures* - *Reproduction level mathematical thinking (OECD, 2009)*			
2: A teaching focus in an inquiry mathematics classroom	Summative (conventional) assessment practices and curriculum content informed the design of the *LPD* or learning goals: - Considered emergent only in the ZPD sense (Vygotsky, 1978) - Collective *ALD* for students presented a messy beginning to the learning journey - Integrated potentialities presented by students as concomitant to the *emergent LPD*			

In the second phase of study, information gained about student learning during formative assessment was used to characterise where students were in their learning journeys and informed the teacher about adjustments to be made to teaching. Table 8-2 reflects how other beliefs learned by students could be engineered by the classroom teacher into the inquiry. Vygotsky's ZPD theory (1978) was the analytical tool used to articulate this journey as a collective notion of the whole class, with teacher scaffolding a key element. For example, pre-assessment (as a formative assessment technique) was used to collectively identify the students' *ALDs*, revealing an eclectic mix of starting points in students' learning journeys as different potentialities to explore. It could be said that this placed value on student beliefs to do with self-interest which I relate to '*it's-fun*' beliefs (Duckworth, 2006). Although curriculum and common assessment tasks defined the *LPD*, this was considered by the teacher as emergent only, as it did not take into consideration the potentialities students presented. The teacher engineered these potentialities into her teaching, concomitant to the *emergent LPD*, in an attempt to relate learning personally to the contexts each student presented. Rather than dismiss student thinking that was different to the teacher's solution, integrating these ideas validated them so students could develop beliefs to do with self-confidence. The nature of inquiry is that learning is student-centred as students design their own pathways to answer the inquiry question (Makar, 2012). For the teacher, difficulty lies in engineering potentialities into inquiry teaching when a collective learning goal is in sight. Vygotsky's ZPD was modified to represent a complex process of teaching that supported learning in an inquiry setting. It was used to articulate the teacher as being an artful engineer (where the word artful has already been used to describe the process of formative assessment (Clark, 2010) and engineering is a term commonly used to describe teaching that elicits evidence about learning (Black & Wiliam, 2009; Leahy et al., 2005; Wiliam, 2011a; 2011b). Feedback about learning was sought-after by the teacher so she could consider potentialities students presented, in the contexts the students' invented, as concomitant to the *emergent LPD*.

The third phase of study considered the concept of scaffolding as fostered by peers in collaborative interactions in an inquiry classroom. Illustrations of learning for one student, Dale, depicted a complex and interactive journey incorporating potentialities that presented to the student, in contexts that were important and close to him. In this iteration, we saw a shift in the focus on learning from knowledge of the world, to a broader view of learning that encompassed beliefs to do with self-interest, self-confidence and shared understandings (Table 8-3). The DNR framework (Harel & Koichu, 2010) provided one way to illustrate this.

Table 8-3 Beliefs about learning fostered in the inquiry classroom in the third phase of study.

Study Phase	**Beliefs fostered (Duckworth, 2006):**			
	World knowledge	***Self-interest***	***Self-confidence***	***Shared***
1: A focus on assessment in an inquiry mathematics classroom	*Summative (conventional) assessment practices revealed:* - *Traditional notions of learning mathematical rules and procedures* - *Reproduction level mathematical thinking (OECD, 2009)*			
2: A teaching focus in an inquiry mathematics classroom	*Summative (conventional) assessment practices and curriculum content informed the design of the LPD or learning goals:* - *Considered emergent only in the ZPD sense (Vygotsky, 1978)* - *Collective ALD for students presented a messy beginning to the learning journey* - *Integrated potentialities presented by students as concomitant to the emergent LPD*			
3: Learning mathematics in an inquiry classroom	- Formative assessment considered learning that took place in collaboration with peers. - The teacher integrated potentialities, concomitant to the *emergent LPDs* as learning goals. - Learning journeys for students were not only considered as an end point in learning. They included overcoming challenges presented by peers and the teacher. Learning journeys included learning all kinds of beliefs about things to be learned (Duckworth, 2006). - Responsibility for learning shifted to each student as they received feedback from their peers and generated feedback to others.			
	World knowledge	***Self-interest***	***Self-confidence***	***Shared***

The third phase of study illustrated student learning in inquiry as a process, based on a DNR-based operationalised definition of learning (Harel & Koichu, 2010). For one student, Dale, this presented a complex process that included feedback gained through frequent formative assessment, in interactions with peers and engineered teaching and learning experiences. Learning as a journey in inquiry considered perturbations in the Piagetian sense (1964; 1977) of the processes of assimilation and accommodation. Overcoming challenges to learning mathematics, presented through learning in inquiry, were described as reaching equilibrium (although perhaps only in a temporary state if learning is considered as a continuing process). In this phase, learning not only depicted the mathematical content intended for students to learn as the *emergent LPD*; it also reflected challenges to learning as concomitant potentialities, generated through peer discussions. In a traditional sense, a student's understanding is described through assessment in isolation to any other beliefs about learning, including collaboration with peers. If an inquiry pedagogy fosters collaboration, assessment also needs to consider and value this contribution to the learning process.

The inadequacy of a common assessment task was evident in the first phase of study when analysis of the information revealed about learning in the assessment task only reflected the reproduction of taught skills and knowledge. This study unveiled how inquiry can foster higher-levels of thinking about mathematics through the complex interactions in inquiry settings. Findings considered how the teacher interweaved student thinking about inquiry problems into the planning of teaching and learning experiences. Analysis of learning for one student revealed how the conception of his ideas included challenges to his own thinking presented by peers, and opportunities to reach equilibrium between his own thoughts and the conflicting ideas of others. Formative assessment proved one way to consider what students know and can do in inquiry settings. Analysis revealed how the teacher then considers information gained through formative assessment beyond the scope of curriculum content. In inquiry, the teacher tries to understand the many ways children come to learn mathematics by watching and listening for those transitional moments when learners fit certain experiences into certain thoughts, to move forward in their learning (Duckworth, 2006).

Where to Next?

The discussion of findings in this chapter aimed to provide information to deepen the reader's (and researcher's) understandings of assessment, teaching and learning mathematics in an inquiry context. Theoretical perspectives from Dewey, Vygotsky and Piaget were used as lenses for analysis in each phase of study. This has allowed insight into the classroom processes in an inquiry setting to reveal a complex, interactive learning journey for students. Integrating formative assessment principles into mathematical inquiry can support students in the conception of mathematical thinking, in ways that can be independent of their peers and teacher (and curriculum) and valued as such, that moves away from a sense of only rewarding correct answers. Through inquiry, students built resilience as they overcame perturbations that were important to them. Although these characteristics of the inquiry learning journey are not traceable in a conventional assessment sense, this does not lessen the emphasis to be placed on supporting these characteristics through inquiry learning experiences.

Key to this study was investigating ways to align the particular classroom elements with the pedagogy of inquiry. In reflection, it seemed more sensible for teachers of inquiry to instead incorporate multiple aspects into the planning of assessment, teaching and learning. These aspects include: providing opportunities for students to conceptualise their own thinking about mathematical experiences personal to them, incorporating mathematical potentialities presented by students which are relevant to the problem they are solving into assessment practices, valuing moments when students overcome difficulties in solving an inquiry problem, and acknowledging that this can happen through interactions with peers as they work together collaboratively. Assessment practices in inquiry need to reflect learning as high-level, collaborative, rich and broad mathematically. Possibly, only the students themselves may be able to offer intimate descriptions of learning mathematics in an inquiry setting that reflects what students know and can do. Teachers have the responsibility to negotiate and establish with their students ways of doing this which are frequent and flexible in order to cope with the complex teaching and learning approach that inquiry supports.

Just as I have considered the use of Duckworth's belief framework (2006) to decipher ways of learning other than mathematical content (Tables 8-1, 8-2 & 8-3), I consider the impact that this may have at the classroom level. To me, it would prove disastrous to add further assessable elements at the classroom level, to be monitored through additional assessment processes and reported on by teachers to a variety of people who might consider and use this information, other than students. Instead, I hope that these findings will instead illustrate that deep mathematical thinking takes place in inquiry mathematics classrooms, and that a range of personal and important beliefs can also be fostered that are not only related to mathematical content. When assessment does not aim to uncover deep mathematical thinking that is connected and personal, then why should learning aim to do so. It is important for educators to consider that when information about student learning is gained through assessment that focuses on low-level, reproduction of taught knowledge, then it cannot broadly consider what students learn through inquiry. If educators aim to make mathematical learning meaningful to students, and for high-level thinking to take place in their mathematics classrooms, then inquiry can provide the classroom environment to do so. Inquiry can provide the means of assessing learning, evident through the frequent interactions that take place between students, and with their teacher.

Potential Innovations to Consider

Findings from this study strongly suggested value in students learning mathematics through inquiry and the importance of the inquiry learning journey as part of this process. If the feedback question guiding this study of *Where to next?* is relative to a goal, and inquiry is an ongoing process of inquiry, settling doubt and further doubt, then *Where to next?* will continue to be a question that drives my own inquiry learning journey to continue. This section outlines the potential innovations for further research as well as the contributions to literature on assessment, teaching and learning.

Assessment in an inquiry classroom

In the inquiry classrooms presented in this study, students were encouraged to solve problems in multiple ways and to develop efficient procedures to deepen mathematical understandings. To represent a cubic metre in the first phase of study, Laura's group connected the idea to describing the size of a dog kennel (Figure 5-4) whereas Paul's reflection referred to a pallet of soft drink cans and included thoughts about transport and finance (Figure 5-5). Common assessment tasks constrained how learning was measured or described in the first phase of this study, fostering lower level thinking skills. Similarly in Phase Two, students were encouraged to interpret data collected to consider the typical number of steps to and from a particular destination. Three different responses (Figures 6-5, 6-6 & 6-7) reflected how differently students thought about how to estimate a typical number/amount. Classroom reflections in the third phase of study reflected a range of different understandings about maps and students were able to include different features in the maps they created that were useful to include. Flexibility is a valuable skill to have in mathematics classrooms. It is represented by the proficiency of Fluency presented in the current Australian Curriculum (ACARA, 2012) and reflects creative and innovative ways of thinking as a 21st century skill in real-life situations (Binkley et al., 2012; Masters, 2015b). In inquiry settings, fluency is illustrated when students choose mathematical strategies that are most effective and useful (Makar, 2012). In all three phases in this study, fluency was reflected in the diversity of responses to the inquiry questions. The notion of an *emergent LPD* catered for students to diverge from curriculum objectives; to deviate in directions which students deemed useful. The nature of inquiry is that students think divergently about the mathematics they are using. Future research could explore the concomitant potentialities presented by learners in other inquiry classrooms with an aim to develop broad, flexible and practical assessment tools that are able to incorporate concomitant potentialities into descriptions of learning and development over time.

Formative assessment opportunities were designed in this study to make learning explicit and seemed to complement the inquiry approach to learning. Yet the formative assessment in each regard was focused on particular learning goals and successful completion of common assessment tasks. This is the notion of conformative assessment (Torrance, 2012) where students can only achieve set learning goals. Torrance (2012) critiqued how learners could be flexible when objectives were explicitly stated. A contribution to assessment literature that builds on the idea by Torrance (2012), might be that inquiry can foster divergent ways to describe learning and that these descriptions may lie within the realm of the students as the learners themselves, requiring as much worth and respect as a teacher's observations. In this study for example, classroom conversations revealed mathematical thinking as students participated in discussions (Phase Two) and conversations between peers focused on mathematical issues important to context (Phase Three). It would be difficult for the teacher to capture the kinds of information about learning gained during these moments if conformative assessment restricted descriptions to particular content.

In this study, evaluations of learning were made by peers, self and the teacher in ways that were considerate of context. Although it was my initial concern that assessment was unable to capture mathematical learning in inquiry, there may be an important lesson here that the teacher is unable to describe student learning, only student *doing*; the visible actions that students can do. Perhaps school summative assessment tasks are only capable of reporting *doing* in terms of compliance, to judge and grade, which would then seem to be a straightforward task (Masters, 2015b). If this is the case, educators should refrain from using terms such as assessment of learning, and in particular, refrain from using summative assessment tools in inquiry mathematics settings which are not designed for use in this context.

A recommendation for future research would be to conduct research that focuses on how students are able to manage and conduct the process of describing or measuring their own learning, through negotiation with their classroom teacher in an inquiry context. This kind of study would need to be longitudinal, to see how students progressed in the skill of doing so, and to consider how assessment could be refined. An aim would be to develop assessment processes capable of monitoring the long-term development of students' general capabilities and deep understandings of concepts and principles (Masters, 2015b). Such a study could be set in a variety of inquiry classrooms, of students at different ages, to consider developmentally, how this might progress. Researchers could work closely with classroom teachers to develop prompts and reflections for students to respond to, and opportunities to meet would allow for teachers to collaborate with others on what they are learning from the process. Aspects of refinement could be supported through a design research approach as this methodology which would allow for iterations of engineered phases to test particular conjectures (Cobb et al., 2003; Confrey, 2006), or in the case of this study, suppositions to consider. The design research approach would also allow the work to be conducted in classrooms, to consider the complexity of this context.

This study has shown how formative assessment can support the inquiry learning journey. The processes of formative assessment were intertwined with the learning experiences in each phase of study. The purpose of formative assessment in each case was to make learning visible yet it allowed opportunity for student thinking, or for learning to take place. Formative assessment was present in the interactions between students, in classroom conversations, in bookwork; it permeated throughout the learning experiences and the interactions between teachers and students. The inquiry episodes presented in this study reflected collaborative relationships between the teacher and her students in an attempt to balance content and assessment criteria with learning. For formative assessment to be transformative, Torrance (2012) recommended this balance and for teachers to "consider unintended outcomes as an indication of success rather than a lack of compliance" (p. 339). In inquiry, assessment intentions need to shift away from notions of success in terms of curriculum content, to incorporate unintended outcomes as valuable learner attributes. Future research into the use of formative assessment in inquiry mathematics classrooms could focus on unintended learning over a longer period of time, to see how these ideas or understandings contribute to a student's mathematical learning over time.

Teaching and learning in mathematical inquiry classrooms

There is opportunity for further research in inquiry classrooms to operationalise the process of conceptualising mathematical ideas through an inquiry approach. Conceptualisation of mathematical ideas was explored in this study in the first phase, as outlined by Dewey (1891) in the thinking process of conception. My own research brings to light the complexity teachers face when planning and implementing inquiry teaching episodes. In mathematical inquiry, learning journeys are personal and students can begin their journeys at different 'places' to their peers in the one classroom. Potential research from this study would not attempt to characterise hypothetical learning trajectories to further the continued research by Confrey (2006), and Simon et al., (2010). Instead, this research would present the perspective that learning journeys are personally constructed and situated in inquiry learning experiences. Due to the complexity of already established experiences and conceptions, and the context of the inquiry which students may invent, it would be impossible to expect student learning as neat, conceptual steps that students could expect to follow. Explicating inquiry learning trajectories could have the effect of narrowing learning or the potentialities students could present. Rather, future research into learning mathematics in inquiry classrooms could contribute to the earlier work of Tzur and Simon (2004); to build qualitative explanations of personal learning journeys that include the processes of conceptualising mathematical ideas, and that considers moments when students are stuck as valuable perturbations to overcome (Harel & Koichu, 2010; Piaget, 1952). Potential research would need to consider learning in relation to information gained through formative assessment opportunities, beyond the scope of mathematical content only, and in relation to particular interactions that provoked student learning. This information could highlight the range of potentialities that students might present and be used to inform teaching in inquiry settings.

I would like to consider that findings from this study could contribute to the field of work on DNR-oriented learning (Harel & Koichu, 2010). To illustrate learning for one student, Harel and Koichu articulated all aspects of the DNR-oriented framework over three interviews: duality between *WoUs* and gradually evolving *WoThs*, the psychological and intellectual necessity to pursue problematic situations in particular ways, repeated opportunities to reason about the problematic, and solving the particular problem. There are at least four differences however, in how Harel and Koichu illustrate learning for their participant (Burt), to the illustration of learning for Dale in this study. Firstly, in the 2010 paper, the perturbation that Harel and Koichu wished Burt to consider, had been carefully selected by the researchers and based on previous research on US teachers' difficulties in solving problems involving division of fractions. The research aim which Harel and Koichu designed for their participant, a high school mathematics teacher in the US, was for him to experience detectable perturbations for research purposes. In my study it was unknown which perturbations to expect although inquiries were designed to include ambiguities for students to address. The authentic perturbations that students faced were naturalistic in that they were important for the learner to solve as part of the process of solving the inquiry problem.

Secondly, although the nature of the DNR definition of learning operationalised learning for Burt, and this was useful in a research sense (Harel & Koichi, 2010), I applied the framework at the classroom level. Rather than describe learning taking place in an isolated instance, I included in the DNR framework aspects of collaboration between Dale and his peers. Perturbations arose for Dale in these experiences as part of the inquiry process. A collaborative or social aspect of learning seems to be missing from the DNR framework although for the purposes of Harel and Koichu's experiment, learning was considered as it took place during the conduct of the interview with the interviewer refraining from revealing anything evaluative about Burt's responses. Illustrating learning in a classroom community of inquiry using the DNR framework would need to include further consideration of learning as participation and the discourse that takes place in collaborative inquiry settings.

The Harel and Koichu (2010) paper thoroughly outlined the protocol, or methodology that they used in interviewing their participant, Burt. It is interesting to note that Burt is a US, high school mathematics teacher. He was prompted to think aloud in his interviews about fraction problems with "keep talking" or being asked "What are you doing right now?" The third difference between the study by Harel and Koichu (2010) and this one, is that Dale is a Year 3 student (7 or 8 years old) who came to realise that coordinates are not always a useful feature to include on a map (Excerpt 1, Chapter 7). Dale realised this through conversations and collaboration with his peers and his need to defend his own ideas arose in these situations. There was no need for outside prompting by researchers; the reasoning was a natural part of Dale's engagement with his peers in an inquiry classroom. It would be useful to conduct further research into mathematical inquiry using the DNR framework to refine illustrations that incorporate students' voices and their own mathematical language, especially when the use of this framework was modified to consider student-generated perturbations rather than artificially generated perturbations in interview scenarios.

The fourth difference between Harel and Koichu's (2010) study and my own, concerns the way in which learning was conducted. The DNR framework was not designed to illustrate learning in an inquiry setting yet it has been used to do so in this study. In Burt's case (Harel & Koichu, 2010), it would seem that no teaching about fractions took place before, during or after the interviews and Burt was left to ponder solving the problem relying on his own previous and current experiences. Over the three occasions Burt was interviewed, he revisited the same problem each time over a period of 20 months (Harel & Koichu, 2010). In this study, the framework was applied to an inquiry setting, with the time period in which learning was illustrated being only one week. The situation in which Dale considered the mathematical idea of including coordinates on maps was different each time. This would be difficult for a classroom teacher to capture of every student in her class although the intention in Phase Three was not to generate a tool for doing so. The purpose of using the DNR framework in this study was to illustrate mathematical learning in an inquiry setting as complex and multi-dimensional, that included phases of dis-equilibrium and equilibrium. In this sense, the DNR framework has proved useful.

Kinds of beliefs to consider

Findings from this study were not intended to define all the mathematical concepts to be learnt in these inquiry classrooms, for each particular student. Instead, learning was illustrated as a complex journey that included: doubt, a process of coming to know, building a sense of the need to know, and overcoming doubt only to lead to further doubt. In classrooms where educators strive to teach all the content there is to know, there is a risk that not all students will learn all of it. In research attempting to identify all the chief concepts learned by students, Duckworth (2006) preferred to use the term beliefs to encompass what students come to know, rather than concepts. The four kinds of beliefs she identified are referred to in different stages of this study, to provide a lens for describing learning beyond mathematical content. In each phase of this study the curriculum intent was made explicit, yet findings from this study suggested that there was more to consider when describing learning in inquiry.

It would be beneficial to design further design-research opportunities into inquiry classrooms on a larger scale, to assist in characterising more broadly what else is learned beyond mathematical content. It is an intention that curriculum content explored in classrooms is reflected in the information about learning gained through assessment. Duckworth (2006) considered how curriculum intent was mainly concerned with knowledge of the world. She considered these beliefs as '*the-way-things-are*' beliefs adding further comment that almost without exception, '*the-way-things-are*' beliefs are lesson-by-lesson objectives. Findings from my own research have highlighted how content from the curriculum was carefully considered by the teacher in each inquiry yet information gained through formative assessment not only referred to particular curriculum objectives. Phase Two explored how the classroom teacher engineered further potentialities into the scaffolding of learning; to value student ideas and to incorporate personal experiences. Research conducted in a larger number of inquiry classrooms could potentially consider the ways other teachers engineer curriculum content into the planning of inquiries, frequently and flexibly throughout lessons, to further characterise the important role the teacher has in an inquiry classroom to scaffold learning experiences.

Further research in inquiry classrooms could also provide opportunity to consider other kinds of beliefs that Duckworth (2006) articulated. A closer look at the kinds of '*It's-fun*' and '*I-can*' beliefs that students develop in inquiry classrooms in particular, could contribute to the work on growth mindset by prolific writer, Carol Dweck. Her work over the last 20 years has not been included in the literature in this study, but her notion of growth mindset is of high importance generally and in the field of mathematics education (Boaler, 2013; Dweck, 2008; 2009; 2010; Masters, 2013c; Suurtamm, Quigley & Lazarus, 2015; Tomlinson, 2014; Yeager & Dweck, 2012). In inquiry classrooms, a greater link to growth mindset could be explored to consider: how students persist to overcome challenges to learning in inquiry, how inquiry creates a culture of risk-taking, the pace or speed of learning through inquiry and the understandings that develop, and how the concept of building a growth mindset can be illustrated through inquiry. I exclude the notion of mathematics in each of these considerations to avoid offering evaluations in terms of mathematics curricula only.

Although Duckworth (2006) described four kinds of beliefs about learning, further beliefs could characterise teaching in an inquiry context. For example, in this study I considered the idea of teaching beliefs that had to do with '*gaining-a-sense-of-the-need-for'* mathematics; possibly mathematics that is not yet known by the learner. This would mean that the teacher in a classroom community of inquiry would need to have knowledge of their students' ZPDs (Vygotsky, 1978), to plan perturbations that guide learners towards a sense of needing a more efficient way of doing something that someone in the mathematics community may have already refined. In the third phase of this study for example, '*gaining-a-sense-of-the-need-for'* mathematics was characterised as a need for mathematical features on maps to assist with describing position and pathways. Children did not have to create their own directional language yet consistent language was required to describe direction such as cardinal terms including North, South, East and West. Many maps do not include all mapping conventions. For example, digital, interactive maps available in shopping centres may not include cardinal direction terms. Whereas a traditional pedagogy might teach students the belief that simple grid maps can show position and pathways ('*the-way-things-are*' belief described as curriculum content), research could consider how the classroom teacher guides students towards '*gaining-a-sense-of-the-need-for'* the mathematics being explored in other inquiry classrooms. Pedagogy that focuses on skill and knowledge acquisition may not provoke learners to gain a sense of the need for mathematics whereas in this study, perturbations (in discussions with peers and engineered by the teacher in classroom discussions) generated a need for mathematics that could make problem solving seem more effective to students.

Further opportunities for research into teaching and learning in mathematical inquiry classrooms have been presented through this study. An initial aim was to see how an inquiry classroom environment which aligns teaching, learning and assessment is supportive in deepening mathematical understandings yet this research has only uncovered how these aspects do not align. Qualitative assessment information that better illustrates learning should be considered when making school-based decisions around assessment, to create a broader view of assessment in data-driven reports. Information about learning should include descriptions of learning beyond curriculum content and in terms of the context in which the mathematics was useful. Student reflections and conversations should also be included placing value on personal connections students make.

Learning in an inquiry classroom presents opportunities for students to identify, explore and investigate mathematical concepts in context. Not officially measured are: the unintended mathematical concepts explored, transfer of students' prior and new knowledge, mathematical reasonings generated in rich conversations where students are required to argue and defend their mathematical reasoning, incorrect application by students of mathematical understandings and how this contributes to deeper understandings, the development of essential collaborative skills, how students creatively solve mathematical problems, nor the ability to pose and solve mathematical problems. This list is not exhaustive yet is indicative of the lack of understanding about methods to assess what is learnt in an inquiry classroom. Suggestions about how the classroom elements of assessment, teaching and learning can support each other in an inquiry setting need to be clear and helpful if they are to permeate into mathematics classrooms.

A Concluding Note

This research, as my own inquiry into teaching and learning mathematics through inquiry, had the intentions of articulating any benefits of the pedagogy to the teaching and learning of mathematics. Findings from this study were considered in relation to everyday classroom experiences; in real situations. Theoretical analysis supported findings to contribute to theory about assessment, teaching and learning (Dewey, Vygotsky and Piaget). Analysis built on the findings in each iteration in an attempt to align practices. Conclusions made highlighted the relationships between assessment, teaching and learning mathematics through inquiry, as complicated.

Findings from each phase of study have been presented at different mathematical conferences, to be shared with other researchers interested in the field of mathematics education. Early findings from Phase One were presented (Fry, 2011) at a conference jointly held by the Mathematics Education Research Group of Australasia (MERGA) and the Australian Association of Mathematics Teachers (AAMT). A revised version was later published in the Australian Primary Mathematics Classroom magazine, explaining to Australian teachers how formative assessment supported learning mathematics through inquiry (Fry, 2014). In 2013, I presented at a MERGA conference in Melbourne my initial findings from Phase Three, on the importance of students 'holding' the moment when they are unsure of learning through inquiry (Fry, 2013). More recently I was able to share the findings from Phase Two at the 2015 MERGA conference (Fry, 2105). I look forward to publishing my findings from this study to build on and contribute to the innovations of others.

The findings from this study revealed the complexity of the inquiry approach and invites further opportunities for research to be conducted into this field. To strengthen research in this area, it was also essential to elaborate on the theoretical underpinnings of the pedagogy to act as lenses for analysis for further theoretical contributions. My own end goal was to consider how classroom practice could change to value learning that is personally constructed, and for assessment information to more closely resemble this learning journey.

My thesis was organised around three key questions that are important principles of formative assessment (Hattie & Timperley, 2007): *Where am I going*, *How am I going* and *Where to next*. Answers to *Where am I going?* would clarify what effective classroom practices I should aim for through a review of related literature, to establish a learning goal. Rather, the review showed a dominance of a scientific emphasis in mathematics education that was making it difficult for more constructivist pedagogies to persist (Schoenfeld, 2007; Shepard, 2000). The literature highlighted such contrasting perspectives about the means or purpose of mathematics education, reflected over time as shifting yet somehow remaining constant to inform classroom practices today. The inquiry pedagogy was outlined which aligned closely with claims that mathematical teaching and learning should be relevant and meaningful to learners. Literature regarding assessment reflected ideals that aligned more with notions of skill and knowledge acquisition. Research in the field of assessment highlighted the importance of formative assessment as the key strategy to improve learning and principles of learning mathematics in an inquiry setting were outlined to make clear to the reader the means of mathematics education in this context; to learn and use mathematics that is deeply contextual, personally connected, and relevant. My research would attempt to align these different perspectives regarding teaching, learning and assessment to inform and contribute to research on effective, mathematics classroom practice.

Three classroom episodes were designed to explore the pedagogy in practice, to provide information about *How am I going?* Rather than self-evaluation, *how am I going* would refer to three classrooms implementing the pedagogy of inquiry to teach mathematics. The summary of findings provided in this chapter have articulated how the classroom elements of assessment, teaching and learning are complex in relation to each other, while highlighting particular benefits of the approach such as higher-level thinking about mathematics. In this final chapter, potential innovations to be considered provided information to the question *where to next* and it is hoped that these ideas for future research will eventuate.

"Look at all the different maps you created! They are all so different!" The students walk around the room to see all the maps that they made. Each student left their map on their desk for others to have a chance to look at closely. The teacher also walks around adding positive comments about features she thinks are effective and this encourages other students to add their own remarks. The students are clearly proud of their efforts and some hover for longer over particular maps, either because their friend made it or because it somehow caught their interest. The classroom teacher has already viewed the students' maps as she needed to compare their efforts with learning in the other Year three classes. The teacher has seen the map for: Portal world, Extreme Wild Wet World, The land of dinosaurs, Fairy land, Sock world and Freaky land, and many more. Having her students create their own maps seemed a successful a way to have her class learn about mapping conventions. When the teacher had to allocate 'marks' to her students' work it seemed that no-one had failed: all students had shown that they could interpret simple grid maps and locate positions on a map. The students had used their own maps to direct peers (to different locations on the map) and feedback had already been provided to improve efforts so that everyone could follow and give directions. It had been very difficult to justify to the other Year three teachers the marks she had given her students. All the students had achieved the assessment requirements through the inquiry;

there were no failed attempts. Now there weren't enough 'D's in her class to show the grade balance required for the task.

"I like Harry Potter land," Brianna quietly suggests to her teacher. "Imagine if you could really go there!"

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