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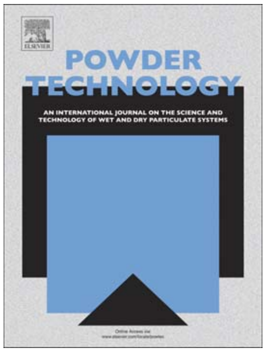

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Determining the significance of flotation variables on froth rheology using a central composite rotatable design

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Abstract

Froth performance in a flotation cell is expected to be affected by froth rheology due to change in the froth transportation rate. However, very little study has been performed to investigate how froth rheology responds to flotation variables. This paper presents an experimental program performed to study the effects of flotation variables (i.e. feed grade, feed particle size, froth height, superficial gas velocity and impeller speed) on the froth rheology. These conditions were varied using a central composite rotatable design (CCRD).

Froth rheology was found to change significantly with a variation in flotation conditions and exhibited shear-thinning behaviour. Assuming the froth moving towards the flotation lip is an open channel flow, the shear rate in the froth was calculated to be less than 4 s^{-1}. Results of the CCRD experiments showed that the flotation variables have different effects on the froth rheology. The interactions between these flotation variables in determining froth rheology were also analysed. A shear rate specific empirical model was developed to relate froth rheology to the flotation variables and their interactions.

Keywords: CCRD; shear rate; froth rheology; flotation

1. Introduction

Flotation is a process to separate valuable mineral from gangue to produce a concentrated mineral product. Flotation consists of pulp and froth phases. The hydrophobic particles are attached to the bubble surface in the pulp phase and then transported to the pulp surface. The mineralized air bubbles are accumulated at the pulp surface and form the froth phase. The froth phase is a complex system in which valuable minerals are further concentrated. Upgrading occurs as hydrophilic and weakly hydrophobic particles which were entrained into the froth drop back with the water into the pulp resulting in an increase in mineral grade. As the froth moves upwards and is transported towards the concentrate launder, bubbles coalesce and burst and valuable minerals are also lost back to the pulp phase. The froth transportation characteristics (and thus froth rheology) will affect the time the froth takes to reach the launder and thus the degree of drainage and bubble bursting that occurs. This affects the amount of valuable mineral particles that are recovered and the resulting concentrate grade.

Several methods have been proposed to model froth transportation [1]. For example, Zheng et al. [2] and Contreras et al. [3] divided the froth into two zones – below the launder lip where the froth rises vertically and above the launder lip where the froth moves horizontally towards the lip. Zheng et al. [2] have mentioned that their model poorly predicts froth transportation time at deep froth depth and this was attributed to changes in froth viscosity which was not considered in their model. Harris [4] has recently proposed a different type of froth transportation model which predicts the height of the froth zone above the cell launder lip by minimising the energy of the system taking into account both froth stability and froth rheology effects. However, this model is yet to be validated in flotation systems.

In addition to the recognition that froth rheology is an important parameter that should be incorporated into a froth transportation model, the importance of froth rheology and its effects on flotation performance have been directly demonstrated by many researchers. Farrokhpay [5, 6] and Shi and Zheng [7] have highlighted that froth rheology can affect froth mobility, as well as froth stability, and ultimately influence the flotation performance. Shi and Zheng [7] reported a correlation between froth rheology and grade of hydrophobic and hydrophilic minerals (chalcopyrite and quartz, respectively). Moudgil [8] observed a direct correlation between froth viscosity and flotation recovery and, on the other hand, an inverse correlation between froth viscosity and the phosphate mineral grade. Moolman et al. [9] also observed a correlation between froth viscosity and mineral recovery in phosphate flotation. Neethling and Cilliers [10] have simulated the effect of froth washing on flotation performance by incorporating the viscosity of the fluid in the froth as a variable.

Rheology is a measure of the flow characteristics of a substance. It is usually represented by a rheogram which is a measure of the shear stress of a fluid when subject to different shear rates[11]. In general, the substance can either exhibit Newtonian or non-Newtonian behaviour, with the latter including dilatant, plastic, pseudo-plastic and Bingham behaviours. Various types of rheograms are illustrated in Figure 1[12]. Apparent viscosity is the ratio of shear stress to the shear rate, which is constant in a Newtonian flow, but it is shear rate dependent in a non-Newtonian flow. Shear rate is the rate at which a progressive shearing deformation is applied to a material. The shear rate for a fluid between two parallel plates, one moving at a constant speed and the other one stationary, is defined by the ratio of the relative velocity between the two plates to the distance between the two parallel plates. Shear stress is a measurement of the force of friction from a fluid acting on a body in the path of that flow. A flotation froth flow moving towards the launder lip in a flotation machine can be considered as similar to an open channel flow. Shear stress in this case is the force of the moving fluid against the bed of the channel.

Rheology of flotation pulps is usually measured by a cup and bob style rheometer (Figure 2a) but the authors [13] demonstrated that the rheology of flotation froth can be better measured using a vane head encapsulated in a tube (Figure 2b). The vane is rotated at a set speed, and the resulting torque that arises due to the drag force of the froth is measured. The speed and measured torque of the vane can be converted into shear and strain parameters using a series of equations. The surrounding tube is required to remove adverse effects on the measurement caused by turbulence arising from horizontal flow of the froth.

It should be noted that while two phase system (e.g. gas-liquid foam and solid-liquid suspension) has been the subject of many studies. Foam structure and rheology has been well reported by Bikerman in his milestone book [14]. The fluid behaviour of liquid foam has been reviewed by Höhler and Cohen-Addad [15]. Mewis and Wagner [12] introduced the effect of solid concentration and particle shape on the suspension rheology. However, there is very little information available about the rheological behaviour of three phase flotation froth. Researchers have reported Pseudo-plastic for two-phase foam systems [16, 17]. As a flotation froth is a three-phase system (liquid-gas-solid), its rheological behaviour may be different to that of foam. Shi and Zheng [7] concluded from their measurements performed using a vane rheometer that flotation froth has pseudo-plastic characteristics. The validity of their results, however, could be questioned because they did not perform their measurements with the vane surrounded by a tube and therefore the horizontal flow of the froth is likely to have affected their rheology measurements. In addition, they used a rheometer with relatively low sensitivity with a mechanical bearing which was suitable for measurement only in the relatively high shear rate range (above 2.5 s^{-1}). Therefore, it was difficult to perform measurements at low shear rate and investigate whether the froth has a yield stress or not. An improved approach to measure froth rheology in-situ using a more sensitive air-bearing rheometer has recently confirmed that froth has a pseudo-plastic nature with a minor yield stress [13]. Therefore, it is expected that viscosity of the froth will change as the froth velocity (and thus the shear rate imparted to the froth) varies.

The aim of the current work was to investigate how changes in flotation operating variables such as air rate, froth height and impeller speed as well as flotation feed properties such as feed grade and particle size can influence the froth rheology. The results will assist to develop more conclusive prediction for froth transportation models in order to optimise froth phase performance, and ultimately improving flotation performance.

2. Experimental Design, Set up and Materials

2.1 Central Composite Rotatable Design (CCRD) of the experiments

After careful consideration of which parameters could potentially affect the characteristics of a flotation froth (as well as practical limitations) five parameters were chosen for investigation. These included the froth height, gas rate, impeller speed, particle size, and feed grade.

To determine the extent to which these parameters affect froth rheology, it was decided to perform a series of continuous laboratory flotation tests in which the chosen parameters were tested at different levels. To reduce the number of experiments to be performed, a Central Composite Rotatable Design (CCRD) was employed.

CCRD is a method which not only allows the factors that have a significant effect on the measured result to be determined with statistical precision but also allows the development of a second order regression model which can be used to estimate the relationship between the factors and the response variable for use in process optimisation [18]. It is particularly useful in laboratory or pilot plant trials in which the level of the factors can be controlled. It has also occasionally been used in full scale plants where close control of factor levels is possible. Although it should be mentioned that CCRD design can be only used with quantitative, not qualitative, factors [18, 19]. CCRD is based on the standard 2-level factorial design but with additional points added axially at a fixed distance from the centre to provide the quadratic terms in the response surface model. A fixed number of centre point runs are also prescribed to estimate experimental error. A CCRD provides almost as much information as a full 3-level factorial but requires significantly less runs and has other statistical advantages. The number of experiments can be further reduced if a fractional composite design is employed rather than a full CCRD. In this study, the CCRD design employed was based on a half 2-level factorial and involved seven repeats of the centre point condition to determine the repeatability of the experimental procedure. This involved performing 33 tests (rather than the 125 tests required for a full three level factorial design). The levels at which each factor was tested was chosen based on those typically observed in industrial applications as well as practical limitations associated with doing test work in the laboratory at pilot scale. The conditions used in each of the 33 tests (and the randomised run order) were determined using commercial statistics analysis software (Minitab 17) and are shown in Table 1.

2.2 Experimental set up and materials

The tests described in this paper were performed in a bottom driven 20 L pilot scale flotation cell with cross sectional dimensions of 30x30 cm. The cell was operated continuously in closed circuit with a conditioning tank. In order to achieve continuous steady state operation, the concentrate and tailing produced in the flotation experiment were recycled back to the conditioning tank, mixed and then fed again to the flotation cell. Concentrate flowrate was monitored to ensure a steady-state condition was achieved prior to froth measurement and metallurgical sampling of the feed, concentrate and tailing.

The concentrate lip height of the flotation cell can be modified to change the distance between the launder lip and the pulp-froth interface (i.e. froth depth) without affecting the pulp volume. Air was injected at the base of the cell, below the impeller mechanism, and the flow rate was controlled by an ABB air flowmeter. Impeller speed was measured by a tachometer and was varied by changing the impeller motor speed.

The froth rheology measurement was conducted using a 6-blade vane (22 mm in diameter and 16 mm height) attached to an air-bearing rheometer (Anton Paar DSR301). A tube (74 mm in diameter and 150 mm height) was used to encircle the vane to eliminate the effect of the horizontal froth flow on the rheology measurement as previously discussed by the authors [13]. The vane was positioned in the middle of the cell with its upper edge immersed 2 cm into the froth. During the froth rheology measurement, the torque values were measured by increasing the vane speed from 1rpm to 15 rpm in equal increments. A total of 5 torque values were measured in each test with a 5 second interval between each measurement. Each series of torque measurements were replicated five times for each test. The vane was only immersed in the froth for the period of the rheology measurements and then moved away to not impede the flotation froth movement.

A digital video camera (Sony ACC-FV50B) was mounted above the flotation cell to record froth movement and the videos were analysed by contracted software [20] to determine the froth velocity profile towards the cell launder lip. A single light source was mounted above the froth surface as this results in a single bright light on each bubble–a requirement of the froth analysis algorithm used in the analysis software.

A diagram of the experimental set up is shown in Figure 3. The flotation feed was prepared by grinding pure chalcopyrite and mixing it with silica (P_{80}=73 μm) in order to achieve a feed percent solids of 40 wt%. In order to study how froth rheology responds to variations in ore properties, feed preparation was varied to change the feed grade and particle size, variables that often change significantly in the feed to the different flotation cells within a circuit. The change of the feed grade was achieved by varying the amount of chalcopyrite mixed with the silica. The particle size of chalcopyrite in the feed (and therefore the froth) was varied by changing the chalcopyrite grind time.

Sodium ethyl xanthate and Dowfroth 250 were used as the collector and the frother at dosages of 2.0 g/t and 14.7 ppm, respectively. These reagents were kept constant in this study. It is however acknowledged that flotation reagents can play a significant role in influencing flotation behaviour and there are plans to investigate this in future work.

3 Rheological results

3.1 Calculation of shear rate in the froth phase

It will be demonstrated later in this paper that the froth has non-Newtonian characteristics, which means that the viscosity of the froth is shear rate dependent. As the shear rate is expected to be related to froth velocity which is not constant throughout the froth phase, it is necessary to estimate the shear rate in the froth to enable an evaluation of the froth viscosity profile.

As previously discussed, the froth phase is usually divided into two zones: below and above the level of the flotation launder lip. The froth below the launder lip consists of only vertical flow, the velocity of which is relatively constant and mainly driven by the rate of the rising air. The froth zone above the launder lip level is usually called the transport zone and exhibits both vertical flow and horizontal flow towards the cell lip (i.e. ABDC in Figure 4a) [2, 3]. At any vertical height within the transport zone, the horizontal froth velocity increases from the back wall of the cell to the launder lip. Figure 4b is an example of a typical horizontal froth velocity profile on the top of the froth measured in this study. The horizontal flow is affected by gas rate which raises the froth level and, as a consequence of gravity, provides a driving force towards the lip. Horizontal flow, however, will also be affected by both the froth stability and froth rheology. Poor froth stability means that more gas escapes to the atmosphere resulting in less of a driving force to the lip. Froth rheology affects the resistance to the horizontal flow and comes about largely due to ‘the friction’ between the froth layer below the level of the launder lip which is stagnant and the froth layers above the launder lip which are moving. Given that the contact area between the froth and the cell wall is relatively small, any drag force as a consequence of the wall resisting froth flow can be neglected.

The froth in this study was found to be shear-thinning in nature, which will be explained in section 3.3. Therefore, the froth viscosity will not be uniform but a function of the shear rate. Hence it is necessary to estimate froth shear rate to determine froth apparent viscosity. Shear rate in the froth phase is related to the horizontal velocity difference between froth layers, which can be calculated by evaluating the vertical velocity gradient. The horizontal flow of the transport zone can be regarded as being similar to flow in an open channel with an unchanging geometry if it is assumed that the froth height above the launder lip is constant across the cell cross section. For flow in an open channel, the velocity at the bottom must be zero (no-slip condition) and the velocity must be at its maximum at the top surface [21]. Imagining any cross section of the froth, the driving force applied is a 'body force' that acts throughout the flow, whereas the resisting drag force acts only at the level of the launder lip. At any position x across the transport zone (Figure 4a), the flow velocity must therefore increase monotonically as one moves vertically upward through the froth To simplify the analysis of the vertical velocity profile, it is assumed that the horizontal froth velocity increases linearly from zero at the level of the launder lip to the maximum at the surface at each point as shown in Figure 5.

Shear rate $\dot{\gamma}$, defined as the velocity gradient [22], refers to the velocity difference between the horizontal layers in this study. It has been assumed that the horizontal froth velocity increases linearly from zero at the level of the launder lip to the maximum at the surface at each point. Thus the velocity gradient between froth layers is a constant which can be calculated by the ratio of the top froth velocity, u_t to the froth height above the launder lip, h_f (Equation 1). Shear rate is therefore linearly related to the velocity of the froth at the surface. It will change with position x and will follow the same trend as shown in Figure 4b increasing from the cell back wall to the launder lip.

$$\dot{\gamma} = \frac{du}{dy} = \frac{u_t}{h_f} \quad (1)$$

The froth movement was recorded for all conducted tests using a video camera, and images were analysed to get the surface froth velocity profiles. The froth height above the launder lip, h_f, was measured using a ruler. The froth shear rates for each test were then calculated using Equation 1. In order to get an idea of the range of shear rates experienced in a froth, the maximum froth shear rates calculated in all conducted tests have been plotted in Figure 6. It can be seen that the maximum froth shear rate is less than 4 s^{-1} in all 33 tests, which is much smaller than the mean shear rate in the pulp phase that can be up to 90 s^{-1} [23].

3.2 Repeatability of froth rheological measurements

As outlined earlier, a total of 33 tests were performed according to the CCRD. A total of 7 out of these 33 tests (Run order 1, 9, 10, 12, 18, 19, 26) were performed under identical flotation conditions to enable an evaluation of experiment repeatability. Figure 7 shows the torque measurements obtained for these 7 repeated flotation tests. Table 2 summarises the coefficient of variation (CoV) which is the ratio of standard deviation to the mean torque value calculated using the readings at each vane speed. A lower CoV% value indicates better repeatability. In general, the CoV% values decrease upon increasing the vane speed. Except for the lowest vane speed significantly influenced by turbulence, the CoV% is considered to be at a relatively low level. It is therefore concluded that the same froth was generated under the same condition at different times. It is also concluded that the froth rheological property as represented by torque values in this section was accurately measured.

3.3 Froth rheograms

To obtain the froth rheograms for each experimental condition, the vane speeds and torque values were converted to shear rate and shear stress using a methodology recently developed by the authors [13]. During froth rheology measurement, a tube was employed around the vane to eliminate any adverse effects caused by the horizontal froth flow that could mask the real froth rheological characteristics. The froth can be either fully or partially sheared in this tube. If the froth is fully sheared, wall effects could occur and affect the froth rheology calculation. The authors have reported that two different equations for converting vane speed to shear rate need to be used depending on the existing scenarios [13]. Stickland and co-workers at the University of Melbourne [24] concluded that a diameter ratio of three of the tube to the vane was sufficient in practice to eliminate the wall effect. Nguyen and Boger [25] have also mentioned the diameter ratio of two was enough to minimize any effects caused by the rigid boundaries. As the diameter ratio of the tube to the vane was 3.36 in this work, the conversion of vane speed to shear rate was treated as in a partially sheared froth.

Figure 8a shows shear stress versus shear rate values of Runs 1 and 6 as an example. It can be clearly seen that the froth studied in this work has shear-thinning behaviour. The value of shear stress approaches zero as shear rate is close to zero. This indicates that the froth studied in this work has very little yield stress. In order to better interpret how froth rheology varies with shear rate, the apparent viscosity (obtained from the ratio of shear stress to shear rate at each point in Fig 8a) was plotted versus shear rate. It was found that the experimental data can be fitted well with the power law model (Equation 2) as shown in Figure 8b.

$$\eta = \mu \cdot \dot{\gamma}^{n-1} \qquad (2)$$

where η is apparent viscosity, μ is consistency index, $\dot{\gamma}$ is shear rate and n is flow index (dimensionless) which n indicates the deviation of the fluid from Newtonian fluid:

$n>1$: shear-thickening fluid (i.e. dilatant fluid)

$n<1$: shear-thinning fluid (e.g. plastic or pseudo-plastic fluid)

$n=1$: constant viscosity (μ is the reference viscosity of a Newtonian flow when $n=1$) [11].

The apparent viscosity data were plotted against shear rate in a log-log scale (Figure 9). The resulting straight lines in the log-log scale could better show the difference between rheology behaviour of tested conditions. Figure 9 shows that froth apparent viscosity at local shear rate varies in different tests. It is hypothesised that the change in the flotation condition in the CCRD program can influence the froth rheology. Therefore, it is necessary to investigate the significance of flotation conditions on the froth rheology and the findings can be used as a guide to change flotation conditions when froth rheology needs to be adjusted. This is discussed in the next section.

4 Discussion of Results from Regression Analysis

4.1 Significance of flotation operating variables on froth rheology

One of the advantages of the CCRD is that it allows for the development of a statistically sound model that can be used for optimisation or to provide a more comprehensive understanding of the effect of different variables on the response being investigated.

It was shown that froth rheology varies with changes in flotation conditions. Using the results from the 33 CCRD flotation experiments and Minitab 17 software, a regression model was developed in which apparent froth viscosity was related to the flotation conditions. It should be noted that the relationship is not constant, with the coefficients of each term of the equation being dependent on shear rate. Equation 3 shows the equation developed for a nominal shear rate of 2 s^{-1}.

$$\eta = 0.426+0.057\cdot FH + 0.182\cdot J_g+0.000416\cdot IS-0.01188\cdot CS+1.382\cdot CG+0.4053\cdot J_g^2-0.000026\,CS^2-0.1924\cdot FH\cdot J_g+0.002804\cdot FH\cdot CS+0.0566\cdot FH\cdot CG-0.00894\cdot I_s\cdot CG-0.00961\cdot P_{80}\cdot CG \qquad R^2=0.97 \qquad (3)$$

η: froth apparent viscosity (Pa·s)

$J_{g:}$ superficial gas velocity (cm/s)

FH: froth height (cm)

IS: impeller speed (rpm)

CS and CG: chalcopyrite particle size (μm) and copper grade (%), respectively.

It is noticed from Equation 3 that froth apparent viscosity is correlated both linearly and non-linearly with the flotation operating conditions. The interactions between the flotation operating conditions are also exhibited in Equation 3. As froth exhibits non-Newtonian rheological characteristics, froth apparent viscosity varies with shear rate. Therefore, the response of froth apparent viscosity to the flotation variables cannot be evaluated at a single nominal shear rate. Thus, it is necessary to determine the significance of flotation variables on the froth apparent viscosity at different shear rates within the desired range of shear rate. Table 3 shows the significance of flotation variables and their interactions on froth apparent viscosity at different shear rates. Interestingly, Table 3 shows that these significant variables do not change with the shear rate (within the range). Statistically, a variable can be called significant to the response when its significance value is usually more 95%. It is, therefore, concluded that all these five flotation variables can significantly affect the froth apparent viscosity throughout the shear rate range (either in a linear or in a non-linear way).

4.2 Trends – effect of flotation operating variables on froth rheology

It was found that these five flotation variables influence the froth apparent viscosity in different ways (Figure 10). In general, flotation conditions, except superficial gas velocity and chalcopyrite size, exhibit a linear relationship with froth apparent viscosity. Froth height and copper grade have a positive correlation with froth apparent viscosity while impeller speed and chalcopyrite particle size have a negative correlation with froth apparent viscosity. The correlation between gas rate and froth apparent viscosity depends on the specific value of the superficial gas rate. Another conclusion that can be drawn is that chalcopyrite size results in a larger change in froth apparent viscosity than the other parameters (over the ranges tested). It is necessary to point out that Figure 10 is plotted at shear rate of 2 s^{-1}. However, the trend has been validated to be applicable to other shear rates between 0 and 4 s^{-1}. The speculated reasons for the results observed in Figure 10 are provided as follows:

1. There is no doubt that changing superficial gas velocity significantly changes both the froth composition and its structure. Given it is not clear yet how the froth properties change upon superficial gas velocity and what froth properties are the main drivers determining froth rheology, it is difficult to explain the reason why the froth apparent viscosity does not monotonically increase or decrease upon increasing superficial gas velocity. This will be studied in future work.
2. Increasing froth height increases the froth residence time and it is known that the grade of flotation product can improve with the froth height [26]. Intuitively it is logical to assume that the drainage of water within the froth will also increase [27, 28]. Hence increasing froth height decreases water hold-up causing a drier froth, and consequently enhancing froth viscosity.
3. Increasing copper grade of the feed will increase the volume concentration of solid particles in the froth. It is well known that the volume concentration of solids in a two-phase flow (i.e. liquid and solid) has a positive contribution to the suspension viscosity [29, 30]. It is expected that this relationship will also apply to a three-phase flow such as a flotation froth. Increasing the concentration of solid particles in the froth could increase bubble loading and make the froth more difficult to shear. In addition, hydrophobic particles coating air bubbles can reduce the bubble surface tension and reduce the probability of coalescence. Increasing feed grade is often observed to stabilise the froth and decrease bubble size in the froth phase. Bubble size has been shown to be inversely correlated with viscosity in foam rheology studies [17, 31], which could also be applicable to the froth in this study. Decreasing bubble size and increasing solid concentration

(expected to occur with an increase in feed grade) could both be the reason for the observed increase in the froth apparent viscosity. Froth properties will be investigated in future work to understand the mechanisms involved in this process.

4. Froth apparent viscosity decreases as chalcopyrite size in the feed increases. This is similar to what was previously observed by other researchers in two-phase suspensions [32, 33] and it is therefore likely that this relationship also applies to a three-phase flotation froth. In addition, it is well known that fine particles have a higher recovery than coarse particles through the froth. Fines are less likely to detach from bubbles and drain from the froth if they have reported to the froth by true flotation and they also do not drain as easily if reporting to the froth by entrainment [34-36]. It is therefore speculated that, for the same feed rate of solid flowing through the flotation system, a greater mass of particles will be contained within the froth phase when the feed particle size is finer, which means higher bubble loading. As discussed in the previous point, this may result in a higher froth viscosity. Another potential reason is that particles size can strongly affect bubble size. There is therefore the potential in this study that the reduction in chalcopyrite size decreases the bubble size but increases the froth apparent viscosity. There is no conclusive agreement in the literature regarding the effect of particle size on the bubble size and froth stability [37]. Tao et al. [38] showed that particles <150 μm decrease the froth stability at lower concentrations but on the other hand, enhances the froth stability at higher concentrations, while particles <30 μm always resulted in lower froth stability. The froth-destabilising effect of fine hydrophobic particles was attributed to the fact that fines adsorb greater amount of frother (as they have a higher surface area) resulting in a lower frother concentration in the solution. However, it is generally believed that, in a medium particle size range, decreasing particle size can enhance froth stability and decrease bubble size by slowing down the bubble coalescence.
5. Increasing impeller speed was found to decrease froth apparent viscosity. The mechanism of how impeller speed affects froth rheology is not clear and needs to be further investigated. However, it is likely due to the turbulence caused by the impeller in the flotation cell, and consequently, changing the composition within the froth, as well as the froth structure.

It has been speculated that the flotation variables affect froth rheology by changing the froth properties. Future work will investigate how the froth properties change in the CCRD experiments and whether correlations can be developed between froth properties and the apparent froth viscosity.

4.3 Effect of interaction between flotation variables

Flotation variables often interact with one another to produce the ultimate flotation results. So far in the paper, only the sole effect of the investigated variables on the froth viscosity was discussed. However, it cannot be neglected that interactions between these variables could contribute more significantly than the individual variables.

Figure 11 displays the interactions between the flotation variables in determining the froth rheology in the current study. Each interaction is titled at the top of the sub-figure. It shows that 5 interactions (out of all possible 10 interactions) are statistically significant. For example, there is an interaction between froth height and chalcopyrite size in determining froth rheology. In the presence of fine chalcopyrite (P_{80}=20 µm), froth apparent viscosity decreases upon increasing froth height, while the froth height does not significantly affect froth apparent viscosity when medium chalcopyrite (P_{80}=80 µm) was used. On the other hand, froth height is positively related to the froth apparent viscosity in the presence of coarse chalcopyrite size (P_{80}=140 µm) meaning that froth viscosity increases if froth height increases. Again, it needs to be highlighted that Figure 11 is plotted at a fixed shear rate of 2 s^{-1}. However, these trends were also validated at other shear rates of 0.5, 1, 3 and 4 s^{-1}.

4.4 Predicted versus measured froth apparent viscosity

The accuracy of the developed model in Equation 3 was investigated in this section. R^2, the square of the correlation coefficient of this relationship is 0.97. The predicted apparent froth viscosity for each of the 33 CCRD experiments is plotted against the measured result in Figure 12. The points in Figure 12 are randomly distributed close to both sides of the diagonal line of the plot, indicating this model can predict froth apparent viscosity well at the local shear rate of 2 s^{-1} using the measured flotation operational variables. This empirical model is only valid for the froth at the specified shear rate. It should be noted, however, that while the coefficients associated with each term in this model may change at different shear rates, it was found that the model structure remained similar.

From Equation 3 we can conclude that viscosity is not only affected by the variables changed in the system but there are also a number of terms that involve variable interactions. Presumably these are a consequence of how these variables interact to effect the composition and structure of the froth, something that will be investigated in detail in future work.

5 Conclusions

The effect of flotation variables on froth rheology was investigated by performing a CCRD test program in a 20 L pilot-scale continuous flotation cell. The shear rate in the froth was calculated by assuming the froth flow is similar to a flow in an open channel. The local shear rate was calculated as the ratio between the horizontal flow velocity towards the launder lip at a local point cross the flotation cell section to the froth height above the launder lip. It was revealed that the shear rate in the froth phase is less than 4 s^{-1}, which is much lower than the shear rate of pulp phase.

Froth apparent viscosity versus shear rate could be fitted well in the power-law model. The froths produced in these experiments exhibited shear-thinning characteristics with very little yield stress. It was concluded that the flotation froths in this work had pseudo-plastic behaviour.

Froth apparent viscosity was found to vary significantly with variation in flotation conditions. Feed grade was positively related to froth rheology while particle size was negatively correlated with froth rheology. Besides, increasing froth height linearly increased froth viscosity while impeller speed had a reverse effect. On the other hand, froth rheology was found to have a nonlinear correlation with the superficial gas rate. A shear specific empirical model was developed to relate froth rheology to the flotation variables and the interactions between these variables. The predicted froth apparent viscosities showed a good agreement with the measured results.

It should be highlighted that one should not expect the outcomes of this research, especially the established empirical model, to be applicable to other systems (e.g. different ore properties or different flotation cell designs). However, the trends observed in this study can be treated as a guide as to the effect of flotation variables on froth rheology. Future work will be conducted to investigate how the froth properties resulting from changing the flotation variables affect froth rheology. These types of relationships will be more generic and less system dependent. In addition, samples collected during each test will be assayed to determine the overall metallurgical performance achieved in each experiment. Results will then be analysed to investigate the extent to which froth rheology and its effect on froth behaviour affects the flotation metallurgical performance.

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Table 1 The CCRD test program

Standard Order	Run Order	Froth height (cm)	Superficial gas rate (cm/s)	Impeller speed (rpm)	Chalcopyrite particle size P_{80} (μm)	Copper grade (%)
18	1	7	1.4	900	80	1.0
1	2	6	1.0	750	50	1.4
7	3	6	1.8	1050	50	1.4
4	4	8	1.8	750	50	1.4
9	5	6	1.0	750	110	0.6
3	6	6	1.8	750	50	0.6
5	7	6	1.0	1050	50	0.6
15	8	6	1.8	1050	110	0.6
19	9	7	1.4	900	80	1.0
20	10	7	1.4	900	80	1.0
12	11	8	1.8	750	110	0.6
21	12	7	1.4	900	80	1.0
14	13	8	1.0	1050	110	0.6
16	14	8	1.8	1050	110	1.4
10	15	8	1.0	750	110	1.4
11	16	6	1.8	750	110	1.4
8	17	8	1.8	1050	50	0.6
22	18	7	1.4	900	80	1.0
17	19	7	1.4	900	80	1.0
2	20	8	1.0	750	50	0.6
6	21	8	1.0	1050	50	1.4
13	22	6	1.0	1050	110	1.4
30	23	7	1.4	900	140	1.0
23	24	5	1.4	900	80	1.0
25	25	7	0.6	900	80	1.0
33	26	7	1.4	900	80	1.0
28	27	7	1.4	1200	80	1.0
32	28	7	1.4	900	80	1.8
27	29	7	1.4	600	80	1.0
26	30	7	2.2	900	80	1.0
24	31	9	1.4	900	80	1.0
31	32	7	1.4	900	80	0.2
29	33	7	1.4	900	20	1.0

Table 2 Coefficient of Variance in the torque values measured at different vane speeds

Vane speed (rpm)	1.0	4.5	8.0	11.5	15.0
CoV (%)	18.27	6.34	5.30	6.74	5.74

Table 3 The significance of flotation variables on froth apparent viscosity at different shear rates

Shear rate (s^{-1}) / Flotation variables	1	2	3	4
	Significance (%)			
Froth height (FH)	98.9	99.2	98.9	98.4
Superficial gas velocity (J_g)	84.5	22.6	52	86.5
Impeller speed (IS)	100.0	100.0	100.0	100.0
Chalcopyrite size (CS)	100.0	100.0	100.0	100.0
Copper grade (CG)	99.8	100.0	100.0	100.0
$J_g \cdot J_g$	99.9	99.9	99.9	99.9

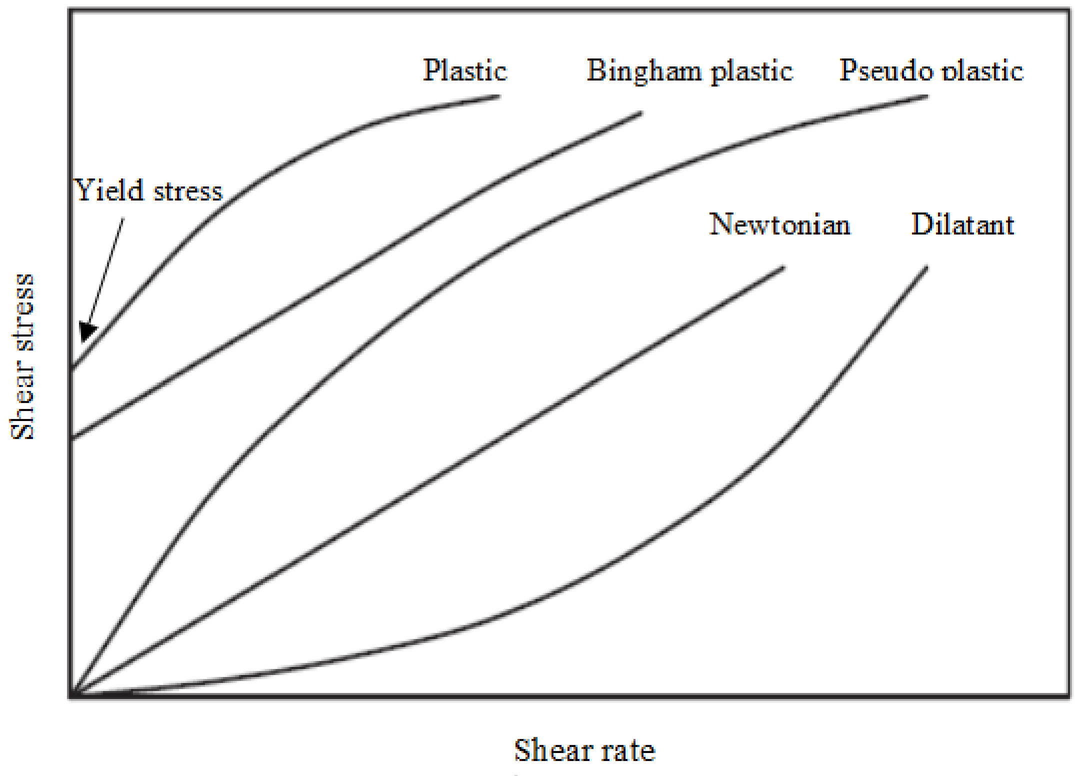


Figure 1 Schematic diagram of shear rate as a function of shear stress for different types of fluid (after Mewis and Wagner [12])

Figure 2 Example of (a) ‘bobbin and cup’ and (b) ‘vane’ rheology measuring heads [13]

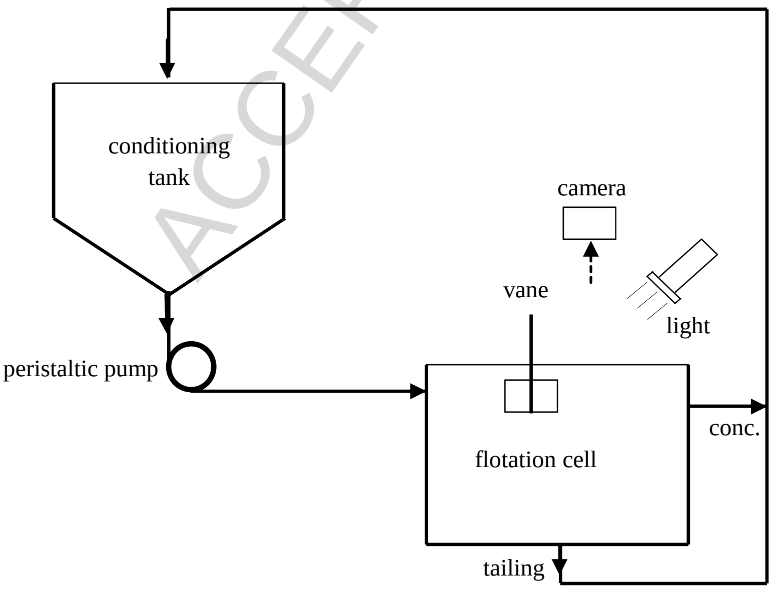


Figure 3 Diagram of the experimental set up

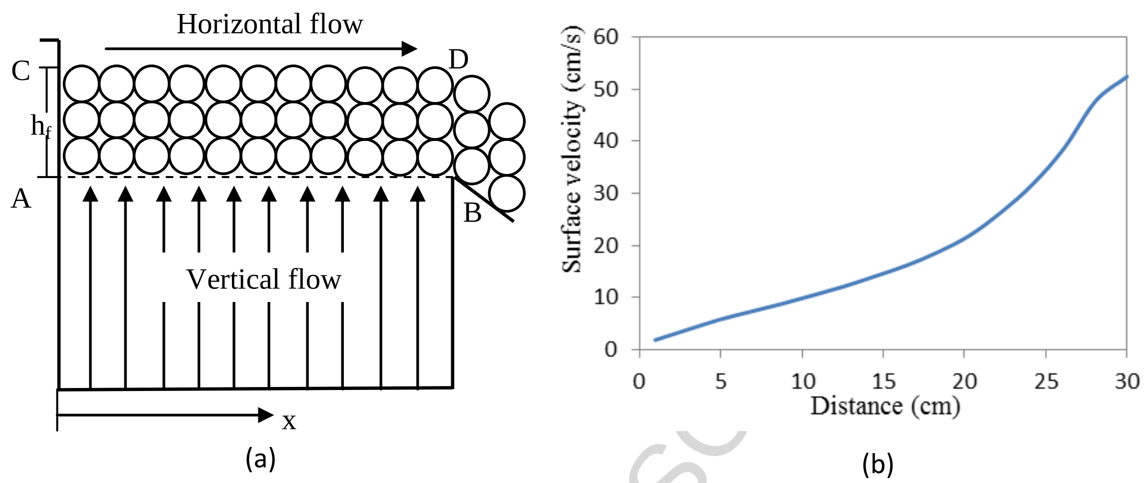


Figure 4 A schematic diagram of the horizontal froth velocity profiles

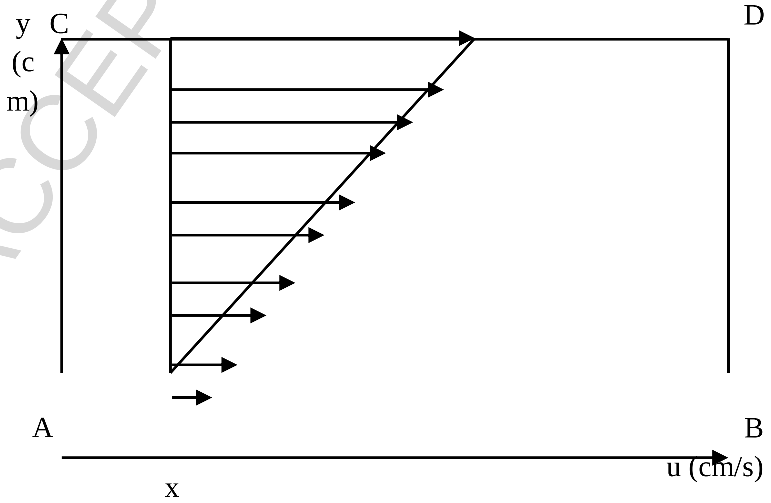


Figure 5 A schematic diagram of the horizontal froth velocity profile assumed at position x

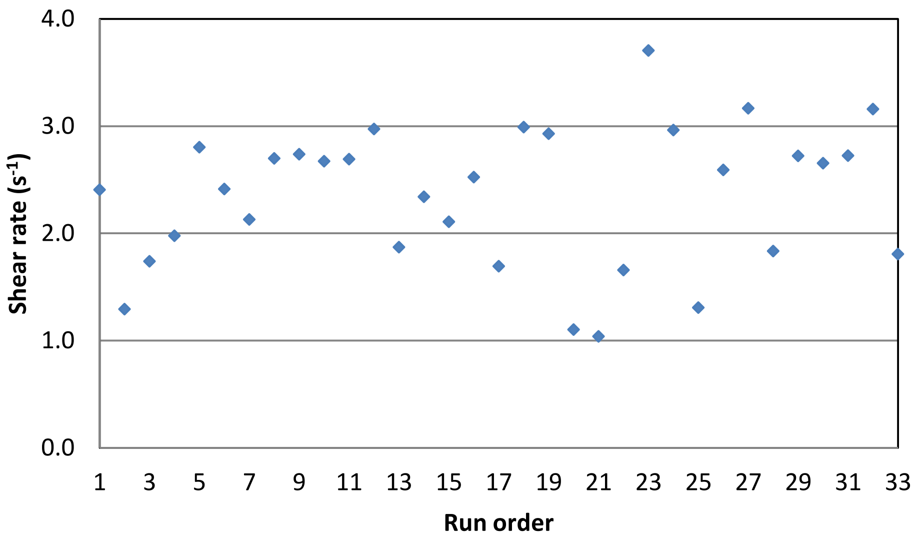


Figure 6 The maximum froth shear rate measured in each test conducted in the CCRD program

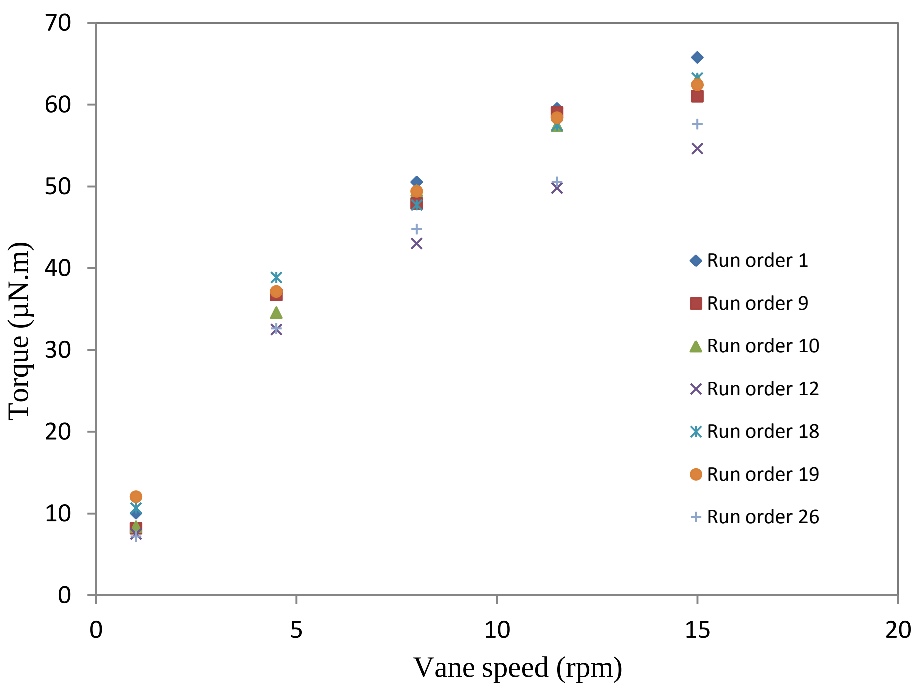
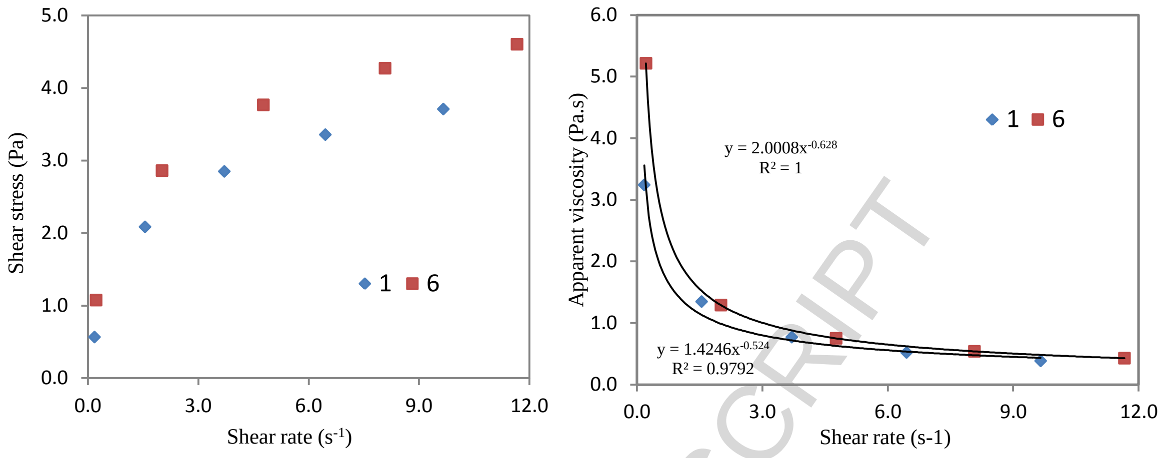


Figure 7 Torque values measured for each of the repeat tests of the CCRD design



(a) (b)

Figure 8 The shear-thinning behaviour of flotation froth

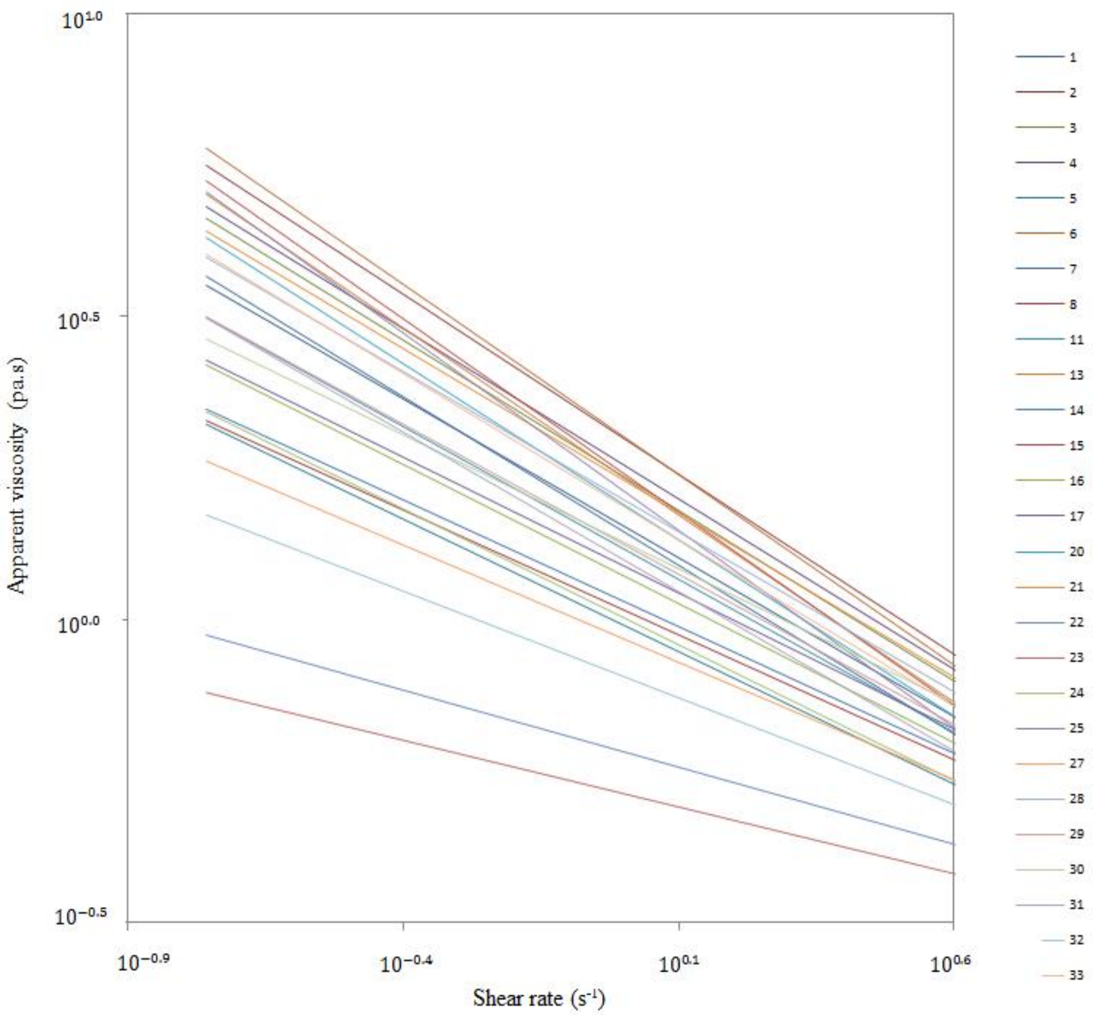


Figure 9 The dependence of froth apparent viscosity on shear rate

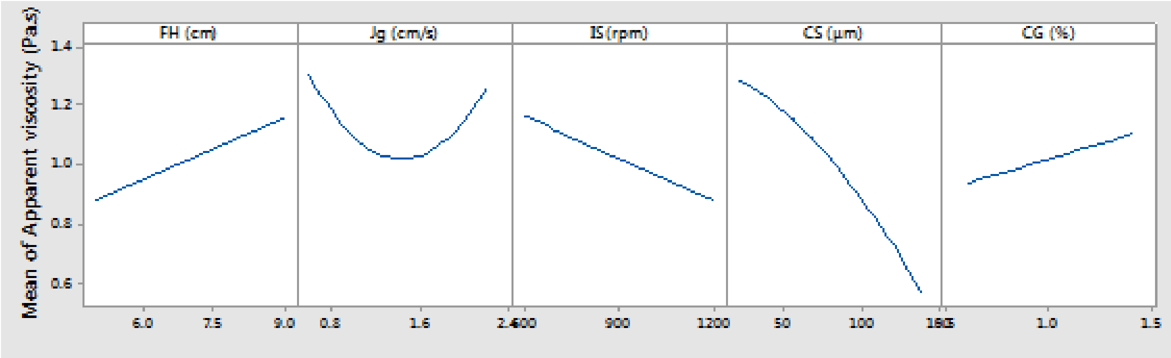


Figure 10 Correlations between flotation conditions and froth apparent viscosity (FH: froth height; J_g: superficial gas velocity; IS: impeller speed; CS: Chalcopyrite particle size; CG: copper grade)

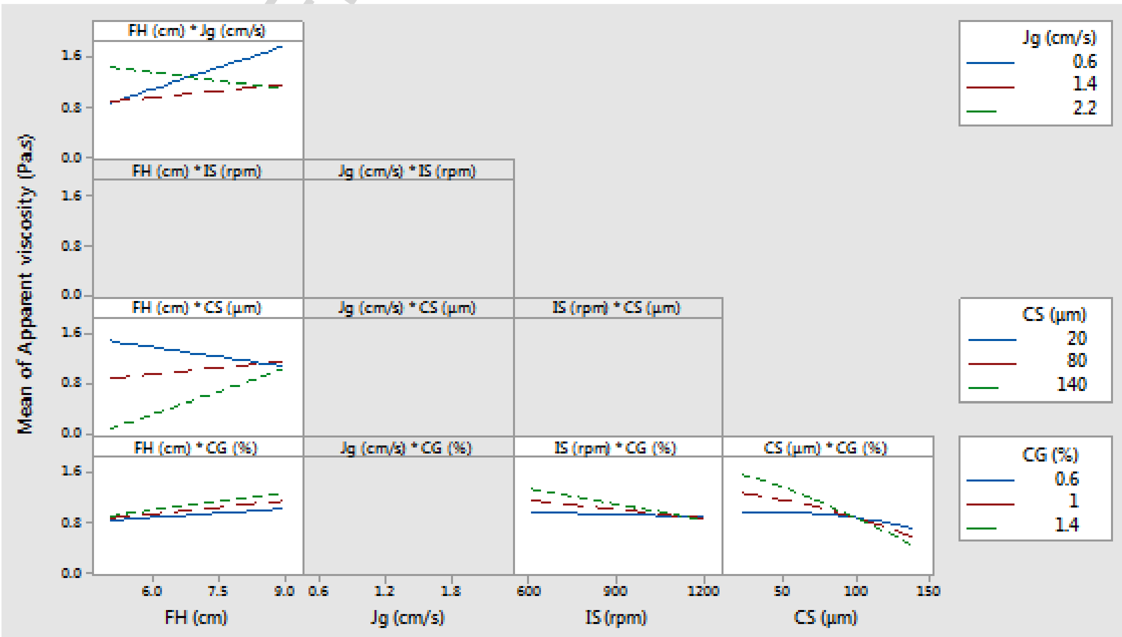


Figure 11 Interactions between flotation variables in determining froth rheology (FH: froth height; J_g: superficial gas velocity; IS: impeller speed; CS: Chalcopyrite particle size; CG: copper grade)

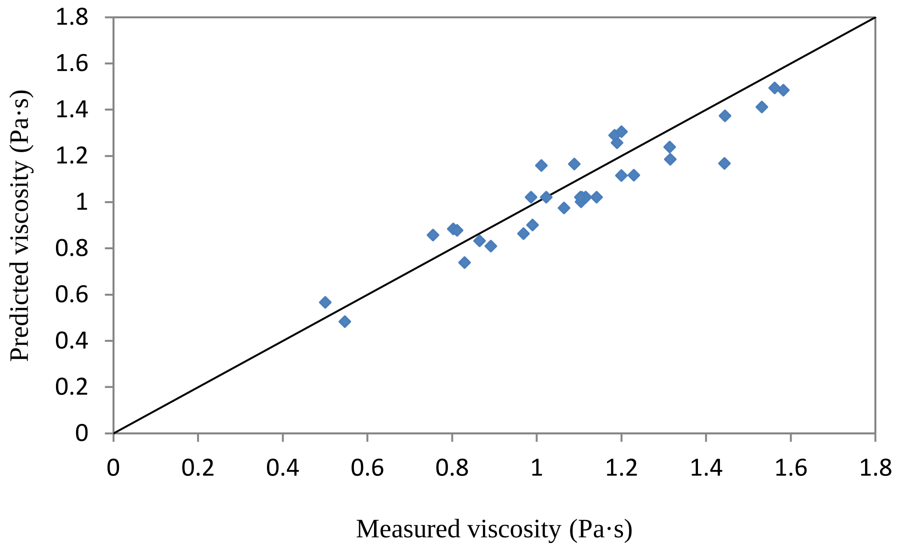
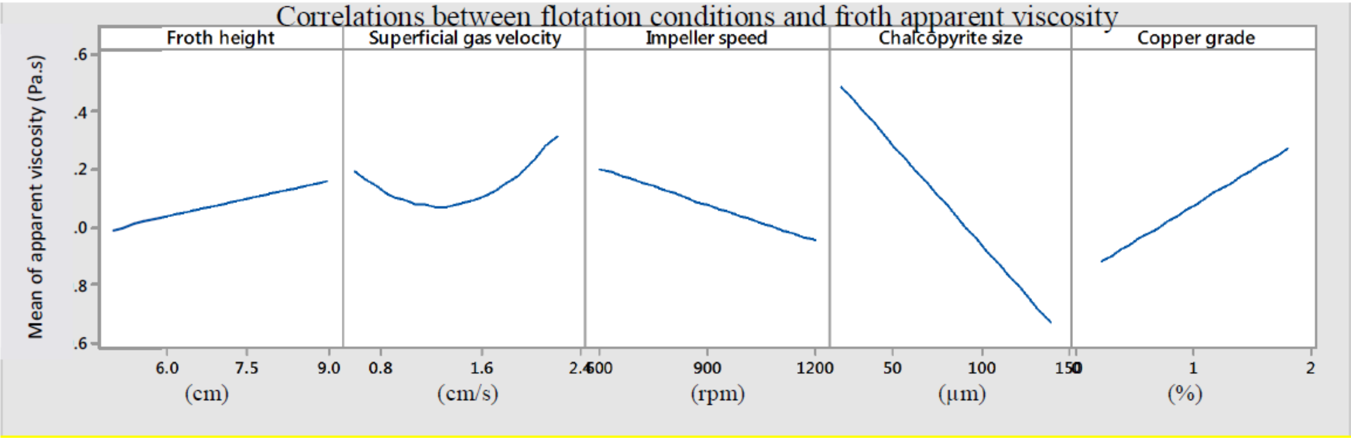


Figure 12 Comparison between predicted froth apparent viscosities and experimental results



Graphical abstract

Highlights

- Froth viscosity is not constant in the froth phase and it depends on the shear rate
- Froth phase shear rate is less than 4 s^{-1}, much lower than pulp phase shear rate
- Froth apparent viscosity varies significantly with variation in flotation condition