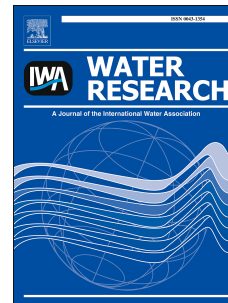


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Recent Advances in Mathematical Modeling of Nitrous Oxides Emissions from Wastewater Treatment Processes

Dr. Bing-Jie Ni, Zhiguo Yuan



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26 development of mitigation strategies for a wastewater treatment plant taking into the specific
27 design and operational conditions of the plant.

28

29 **Keywords:** AOB, model, nitrous oxide, hydroxylamine oxidation, AOB denitrification,
30 heterotrophic denitrification

31

32 **1. Introduction**

33 Nitrous oxide (N_2O) not only is a significant greenhouse gas, with an approximately
34 300-fold stronger warming effect than carbon dioxide (IPCC, 2007), but also reacts with
35 ozone in the stratosphere leading to ozone layer depletion (Portmann et al., 2012). It can be
36 produced and directly emitted from wastewater treatment systems (Foley et al, 2010, Ahn et
37 al., 2010a, 2010b, Ye et al., 2014). Although N_2O emission factors reported for full-scale
38 systems are relatively low, from 0.01% to 1.8% of influent total nitrogen (TN) (Ahn et al.,
39 2010a), N_2O emissions can contribute substantially to the carbon footprint of wastewater
40 treatment plants (WWTP). It should be noted that an emission factor of 1.0% would already
41 increase the carbon footprint of a WWTP by approximately 30% (de Haas and Hartley, 2004,
42 Law et al., 2012). Therefore, the development of reliable predictive tools for quantifying and
43 mitigating N_2O emission is important for achieving greenhouse gas neutral wastewater
44 treatment (Ni et al., 2013a, 2013b).

45 The N_2O emission data collected from wastewater treatment plants (WWTPs) to date
46 show a huge variation in the N_2O emission factor (the fraction of influent nitrogen load
47 emitted as N_2O), ranging between 0.01% and 1.8%, and in some cases even higher than 10%
48 (Kampschreur et al., 2009, Ahn et al., 2010a, 2010b, Foley et al., 2010, Wang et al., 2011). A
49 high degree of temporal variability in N_2O emission has also been observed within the same
50 WWTP (Ahn et al., 2010a, Ye et al., 2014). The observed variability is in clear contrast with

51 the fixed emission factors currently applied to estimating N₂O emissions from wastewater
52 treatment as recommended by the United Nation's Intergovernmental Panel on Climate
53 Change (IPCC) and various governments (IPCC, 2007, EPA, 2012). A major problem with
54 the use of fixed emission factors is that the link between emissions and process
55 configurations and operating characteristics is not considered. As such, the estimates do not
56 account for the variable process conditions in different plants and do not encourage
57 mitigation efforts (Ni et al., 2013a).

58 Mathematical models have been widely applied to the prediction of nitrogen removal in
59 wastewater treatment, and are gaining more attention for the prediction of N₂O accumulation
60 and emission during nitrification and denitrification processes (CH2MHill, 2008, Ni et al.,
61 2011, Corominas et al., 2012, Pocquet et al., 2013, Guo and Vanrolleghem, 2014; Harper et
62 al., 2015). The ability to predict N₂O production by modeling provides an opportunity to
63 include N₂O production as an important consideration in the design, operation and
64 optimization of biological nitrogen removal processes (Ni et al., 2011, 2013a). Furthermore,
65 mathematical modelling should be a more appropriate method for estimating site-specific
66 emissions of N₂O than the oversimplified model with fixed N₂O emission factors (Corominas
67 et al., 2012, Ni et al., 2011, 2013a, Mampaey et al., 2013, Pocquet et al., 2013, Guo and
68 Vanrolleghem, 2014). In addition, mathematical modeling provides a method for verifying
69 hypotheses related to the mechanisms for N₂O production, and thus serves as a tool to support
70 the development of mitigation strategies (Ni et al., 2013b).

71 N₂O modelling has evolved rapidly in the past few years, with models based on various
72 production pathways proposed. These models have been calibrated with data obtained from
73 laboratory reactors and full-scale wastewater treatment plants operated under various
74 conditions. Each of these models has its underlying assumptions and has been
75 calibrated/validated to various degrees based on the understanding of the processes of the

76 distinct model creators, which displayed various predictive abilities (usually good fit with own
77 data but fail with foreign data). Despite the obvious importance of N₂O modeling, and the
78 increasing number of publications, there has never been any attempt to summarize all the
79 modeling information in a comprehensive review. Therefore, this review aims to clarify, to
80 compare, and to provide guide for the use of these models. The existing mathematical models
81 describing all the known microbial pathways for N₂O production as well as their underlying
82 assumptions are reviewed, discussed and compared, including the single-pathway and
83 two-pathway models of AOB, the N₂O models of heterotrophic denitrifiers, and the integrated
84 N₂O models by both AOB and heterotrophic denitrifiers. An overview of the model
85 evaluations using lab-scale and full-scale experimental data is also presented to provide
86 insights into the applicability of these N₂O models under various conditions.

87

88 **2. N₂O Production Pathways in Wastewater Treatment**

89 N₂O is produced during biological nitrogen removal in wastewater treatment, typically
90 attributed to autotrophic AOB (Tallec et al., 2006, Kampschreur et al., 2009, Chandran et al.,
91 2011) and heterotrophic denitrifiers (Kampschreur et al., 2009, Lu and Chandran, 2010, Pan
92 et al., 2012). Although N₂O might be potentially produced through chemical pathway
93 (Schreiber et al., 2009; Harper et al., 2015), there are three main microbial pathways involved
94 in N₂O formation (Figure 1), namely the NH₂OH oxidation, nitrifier (AOB) denitrification,
95 and heterotrophic denitrification pathways (Wunderlin et al., 2012, 2013).

96 **2.1. N₂O production by AOB**

97 AOB are chemolithotrophs that oxidize ammonia (NH₃) to nitrite (NO₂⁻) via
98 hydroxylamine (NH₂OH) as their predominant energy-generating metabolism (Arp and Stein,
99 2003, Arp et al., 2007) (Figure 1A). The first step is catalyzed by ammonia monooxygenase
100 (AMO) where NH₃ is oxidized to NH₂OH with the reduction of molecular oxygen (O₂). In

101 the second step, NH_2OH is oxidized to NO_2^- by hydroxylamine oxidoreductase (HAO), with
102 O_2 as the primary electron acceptor. However, AOB contain a periplasmic copper-containing
103 nitrite reductase (NirK) and a nitric oxide reductase (Nor) (Hooper et al., 1997, Chandran et
104 al., 2011) (as shown in Figure 1A). NirK could speed up NH_2OH oxidation by channeling
105 electrons from the cytochrome pool to NO_2^- (to form NO) and thus play a facilitative role in
106 NH_3 oxidation itself (Hooper et al., 1997, Chandran et al., 2011). AOB also possess the
107 inventory to alternatively convert NO into N_2O , using a haem–copper nitric oxide reductase,
108 sNOR (Chandran et al., 2011).

109 Although N_2O is not an obligate intermediate in NH_3 oxidation, N_2O can be produced by
110 AOB through two major pathways according to the current understanding (Figure 1A): i)
111 N_2O as a byproduct of incomplete oxidation of NH_2OH to NO_2^- , typically referred to as the
112 NH_2OH oxidation pathway (Poughon et al., 2000, Chandran et al., 2011, Stein, 2011a, Law et
113 al., 2012), and ii) N_2O as the final product of AOB denitrification with NO_2^- as the terminal
114 electron acceptor and NO as an intermediate, the so-called nitrifier or AOB denitrification
115 pathway (Chandran et al., 2011, Ni et al., 2013b, Stein, 2011b).

116 It is generally accepted that NO_2^- and NO reduction for N_2O production is carried out by
117 AOB under oxygen limiting or completely anoxic conditions (Kampschreur et al., 2009, Law
118 et al., 2013). Increased N_2O production under high NO_2^- concentrations has been suggested to
119 be due to AOB denitrification (Yang et al., 2009, Yu et al., 2010). On the other hand, there is
120 also evidence supporting N_2O production from NH_2OH oxidation by AOB. The higher NH_3
121 oxidation rate could result in the accumulation of NH_2OH and other reaction intermediates
122 such as NO or NOH (Law et al., 2012), which in turn result in N_2O formation with detailed
123 reactions yet to be fully elucidated (Chandran et al., 2011, Stein, 2011a).

124 **2.2. N_2O production by heterotrophic denitrifiers**

125 N_2O is a known intermediate in heterotrophic denitrification (von Schulthess and Gujer,

126 1996, Pan et al., 2012, 2013a). Heterotrophic denitrification converts the nitrate and/or nitrite
127 generated from autotrophic nitrification to nitrogen gas (N_2) thus removes nitrogen from
128 wastewater. It consists of four consecutive steps, which produce three obligatory
129 intermediates, namely NO_2^- , NO and N_2O . These steps are individually catalyzed by four
130 different denitrification reductases, i.e., nitrate reductase (Nar), nitrite reductase (Nir), NO
131 reductase (NOR) and N_2O reductase (N_2OR). N_2O is produced by the sequential action of the
132 NO_3^- , NO_2^- and NO reductases (Figure 1B).

133 Many factors could affect the denitrification process and thus impacting N_2O emission,
134 such as chemical oxygen demand (COD) to N ratios, the substrate and biomass types, pH
135 levels, temperature, among others (Lu and Chandran, 2010, Pan et al., 2012, 2013a). On the
136 other hand, the four parallel denitrification steps could also exert influence on each other
137 through electron competition, which could result in accumulation of various intermediates
138 including N_2O . The four denitrification steps all require electrons from carbon oxidation, and
139 they could face competition for electrons when the electron supply rate from carbon
140 oxidation does not meet the demand for electrons by the four steps of denitrification
141 combined (Pan et al., 2013a).

142

143 **3. Modeling of N_2O Production by AOB**

144 As the fundamental metabolic pathways for N_2O production by AOB are now coming to
145 light (Kampschreur et al., 2007; Schreiber et al., 2009; Yu et al., 2010; Okabe et al., 2011;
146 Stein, 2011a; Perez-Garcia et al., 2014; Castro-Barros et al., 2015; Harris et al., 2015),
147 several mechanistic models have been proposed for N_2O production by AOB in mixed culture
148 based on one or two of the known N_2O production pathways of AOB, i.e., AOB
149 denitrification and NH_2OH oxidation pathways. To date, two categories of N_2O models by
150 AOB in mixed culture have been proposed, which are represented by single-pathway models

151 and two-pathway models. Tables S1 in the supplementary information (SI) lists the
152 definitions of the all the state variables used in the two categories of models.

153 **3.1. Single-pathway models**

154 Six different single-pathway model structures available in literature are presented in
155 Table S2 in SI, detailed with their kinetic and stoichiometric matrices. Table 1 presents the
156 key differences among the model structures of these single-pathway models by AOB.

157 Model A (Ni et al., 2011) and Model B (Mampaey et al., 2013) are based on the AOB
158 denitrification pathway. In Model A (Table 1, Ni et al., 2011), AOB denitrification with NO_2^-
159 as the terminal electron acceptor produces NO and subsequently N_2O by consuming NH_2OH
160 as the electron donor. Similarly, in Model B (Table 1, Mampaey et al., 2013), AOB
161 denitrification occurs in parallel with ammonium oxidation, reducing NO_2^- to NO and then to
162 N_2O with ammonium as the electron donor. The key difference between these two models is
163 that in Model A, dissolved oxygen (DO) is assumed to inhibit nitrite and NO reduction by
164 AOB, while in Model B, this inhibition is absent. A further minor difference is that ammonia
165 oxidation is modelled as a two-step (ammonia to hydroxylamine and then to nitrite) process
166 in Model A, but as a one-step process (ammonia to nitrite) in Model B.

167 Model A1 (Pocquet et al., 2013) and Model B1 (Guo and Vanrolleghem, 2014) are also
168 based on AOB denitrification pathway, which are the two modified versions from Models A
169 and B to describe N_2O production in several studies (Pocquet et al., 2013, Guo and
170 Vanrolleghem, 2014). In Model A1 (Table 1, Pocquet et al., 2013), the oxygen inhibition of
171 the AOB denitrification pathway was removed. In addition free ammonia (FA) and free
172 nitrous acid (FNA) were considered as the substrate for the AOB reactions, in order to
173 explicitly consider the effect of pH variation. In Model B1 (Table 1, Guo and Vanrolleghem,
174 2014), oxygen limitation and inhibition was added through a Haldane function in both the
175 kinetics of nitrite reduction and NO reduction processes (Guo and Vanrolleghem, 2014).

176 Inhibition by FA was also considered in Model A1 and both inhibition by FA and FNA were
177 included in Model B1.

178 Model C (Law et al., 2012) and Model D (Ni et al., 2013b) are based on the NH_2OH
179 oxidation pathway. Model C assumes that N_2O production is due to the chemical
180 decomposition of the unstable NOH , an intermediate of NH_2OH oxidation (Law et al., 2012).
181 In contrast, Model D assumes that the reduction of NO , produced from the oxidation of
182 NH_2OH , resulted in N_2O production by consuming NH_2OH as the electron donor. Model D
183 (Table 1, Ni et al., 2013b) assumes that DO has no inhibitory effect on NO reduction (Yu et
184 al., 2010), as in Model B.

185 **3.2. Two-pathway models**

186 A new approach has been employed to integrate the two N_2O production pathways of
187 AOB into a two-pathway model, i.e., decoupling approach based on electron balance. Two
188 different two-pathway N_2O model structures of AOB in mixed culture are presented in Table
189 S3 in SI, detailed with their kinetic and stoichiometric matrices. Table 1 compares the key
190 differences between these two two-pathway models by AOB.

191 In Model E (Table 1, Ni et al., 2014), the complex biochemical reactions and electron
192 transfer processes involved in AOB metabolism are lumped into three oxidation and three
193 reduction reactions (Figure 2A). Electron carriers are introduced as a new component in the
194 model to link electron transfer from oxidation to reduction. By decoupling the oxidation (E-1
195 to E-3 in Figure 2A) and reduction (E-4 to E-6 in Figure 2A) reactions through the use of
196 electron carriers, the electron distribution between O_2 , NO_2^- and NO as electron sinks is
197 modeled through assigning different kinetic values to Processes E-4, E-5 and E-6 with respect
198 to electron carriers, which are provided by Processes E-2 and E-3. In this way, the model can
199 predict the relative contribution of the two pathways to total N_2O production by AOB, as well
200 as the shifts of the dominating pathway at various DO and nitrite levels conditions.

201 Model F (Peng et al., 2015a) is based on decoupling approach with both electron and
202 energy (ATP) balance, which are proposed by extension of Model E to describe the
203 dependency of N₂O production by AOB on inorganic carbon (IC) concentration (Peng et al.,
204 2015a). In Model F (Table 1, Peng et al., 2015a), in addition to the electron carriers that link
205 electron transfer from oxidation to reduction, Adenosine triphosphate (ATP)/Adenosine
206 diphosphate (ADP) are also introduced as a component in the model (Table 1) to link energy
207 generation to IC fixation for biomass growth (Figure 2B). The energy distribution between
208 ammonia oxidation, NO₂⁻ reduction and oxygen reduction as energy source (ATP) is modeled
209 through assigning different kinetic values to Processes F-1, F-5 and F-6 with respect to ADP,
210 which are consumed by Processes F-7 with IC as substrate for AOB growth. In this way, the
211 possible effect of IC on AOB growth and subsequently the N₂O production from different
212 pathways by AOB can be explicitly described when the IC concentration in the bioreactor
213 varies temporarily or spatially, with N₂O production increasing with the increase of IC levels.

214 215 **4. Modeling of N₂O Production by Heterotrophic Denitrifiers**

216 To predict denitrification intermediates accumulation, denitrification needs to be
217 modeled as a multiple-step process (von Schulthess and Gujer, 1996). Four-step
218 denitrification models have been proposed and widely applied to predict the accumulation of
219 all denitrification intermediates including N₂O (Kampschreur et al., 2007, Hiatt and Grady,
220 2008, Ni et al., 2011, Pan et al., 2013b). To date, two distinct concepts have been proposed
221 (Table 1), which are represented by the Activated Sludge Model for Nitrogen (ASMN) (Hiatt
222 and Grady 2008) and the Activated Sludge Model with Indirect Coupling of Electrons
223 (ASM-ICE) (Pan et al., 2013b), respectively. Table S4 in SI lists the kinetic and
224 stoichiometric matrices for the two models, which are fundamentally different in describing
225 the electron allocation among different steps of heterotrophic denitrification (Table 1).

226 **4.1. Activated sludge model for nitrogen (ASMN)**

227 The “*direct coupling approach*”, represented by ASMN (Model G in Table 1, Hiatt and
228 Grady 2008), with which the carbon oxidation and nitrogen reduction processes are directly
229 coupled in the model. This type of model describes each of the four steps as a separate and
230 independent oxidation-reduction reaction (Table S4 in SI), with the kinetics of each step
231 modeled according to the nitrogen reduction reaction kinetics using a stoichiometric
232 relationship obtained through electron balance. Model G ignores the fact that the nitrogen
233 oxides reduction and carbon oxidation are carried out by different enzymes with their specific
234 kinetics, and consequently either of the two processes could limit the rate of denitrification.
235 In addition, this coupling approach describes each denitrification step independently with its
236 rate not being affected by other denitrification steps that draw electrons from the same
237 electron supply. Essentially, the carbon oxidation rate is modeled as the sum of the carbon
238 requirements by all denitrification steps, with the underlying assumption that electron supply
239 will always be able to meet the predicted total electron demand.

240 **4.2. Activated sludge model with indirect coupling of electrons (ASM-ICE)**

241 The “*indirect coupling approach*”, proposed by Pan et al. (2013b) and named as
242 ASM-ICE, with which the carbon oxidation and nitrogen reduction processes are decoupled.
243 Electron carriers are introduced as a new component in this model to link carbon oxidation to
244 nitrogen oxides reduction, with carbon oxidation reduces carriers and nitrogen oxides
245 reduction oxidizes carriers (Model H in Table 1, Pan et al., 2013b). In this way, each step of
246 heterotrophic denitrification can be regulated by both the nitrogen reduction and the carbon
247 oxidation processes. The possibility of the carbon oxidation or electron transfer being a
248 limiting step in denitrification is thus considered in the model. In heterotrophic denitrifiers,
249 competition for electrons may occur between the four reduction steps when the electron
250 supply rate from the oxidation process could not meet the demand for electrons by the four

251 reduction steps (Pan et al., 2013b), which plays an important role in the accumulation and
252 emission of N_2O (Pan et al., 2013a). The electron competition between the four denitrifying
253 steps can be modeled through assigning different values to the affinity constants responsible
254 for Processes H-2, H-3, H-4 and H-5 with respect to *Mred*, which are provided by Processes
255 H-1. Model H can be used as a practical tool for predicting N_2O accumulation during
256 denitrification, with the complex biochemical reactions and electron transfer processes
257 involved in biological denitrification by different microbial species being lumped into one
258 oxidation and four reduction reactions that are linked through electron carriers.

259

260 **5. Integrated N_2O Models Incorporating AOB and Heterotrophic Denitrifiers**

261 N_2O is generally produced/consumed by both AOB and heterotrophic denitrifiers in
262 WWTPs (Kampschreur et al., 2009, Law et al., 2012). Therefore, the integrated N_2O models
263 incorporating N_2O production/consumption by both AOB and heterotrophic denitrifiers
264 would contribute to more powerful models that predict the N_2O dynamics more accurately in
265 WWTPs, which could also be useful tool for the development of N_2O mitigation strategies.

266 Two approaches have been reported to integrate the N_2O production/consumption by
267 both AOB and heterotrophic denitrifiers into a comprehensive N_2O model: i) ASM-type
268 models that combine one of the single-pathway models of AOB (e.g., Models A-D, Table S2)
269 with ASMN of heterotrophic denitrifiers (Model G, Table S4) (Ni et al., 2011, Pocquet et al.,
270 2013, Guo and Vanrolleghem, 2014, Spérandio et al., 2014), and ii) Electron balance based
271 model that integrate the electron carrier based two-pathway model of AOB (Model E, Table
272 S3) and ASMN (Model G, Table S4) (Ni et al., 2015). Both modeling approaches have been
273 successfully applied to describe N_2O emissions from mixed culture
274 nitrification-denitrification systems and to identify the relative contributions between AOB
275 and heterotrophic denitrifiers to total N_2O production (Ni et al., 2011, 2013b, 2015,

276 Spérandio et al., 2014). A third potential approach to integrate the N₂O
277 production/consumption by both AOB and heterotrophic denitrifiers could be a full electron
278 balance based model integrating the electron carrier based two-pathway model of AOB
279 (Model E, Table S3) and electron carrier based model of heterotrophs (Model H, Table S4),
280 which though require future testing. It should be noted that the possible consumption of N₂O
281 by heterotrophic denitrification as a N₂O sink may occur and reduce overall N₂O production
282 in integrated model under the conditions of high COD to N ratio and/or low DO level.

283

284 **6. Model Calibration, Validation and Selection**

285 The N₂O models have to be tested to predict N₂O emission data from experiments in
286 order for the models to be developed into a useful tool for practical applications. During past
287 years, measurement campaigns have been performed by many studies. All the available N₂O
288 models have been evaluated with experimental data collected from different systems to reveal
289 the performance of these models under various process conditions and shed light on the
290 conditions under which each of the models would be suitable to facilitate their applications.

291 **6.1. Model Evaluation against Experimental Data**

292 The six single-pathway models of AOB (Models A-D, Table 1) was evaluated and
293 compared (Ni et al., 2011; Ni et al., 2013a; Spérandio et al., 2014) based on their ability to
294 capture the observed N₂O production results from different experiments (Yang et al., 2009;
295 Kim et al., 2010; Law et al., 2012; Spérandio et al., 2014). Model A could well predict the
296 observed trend of decrease in N₂O production at high DO concentrations (Yang et al., 2009),
297 whereas Model B was not able to predict such trend due to the absence of oxygen inhibition
298 on AOB denitrification in Model B (Ni et al., 2013a). Model B could not describe well the
299 N₂O peak that is likely related to the dynamics of NH₂OH (Ni et al., 2013a), which was not
300 included in Models B and B1. Models A, A1, B and B1 have been tested to be able to

301 reasonably describe N_2O production data with high nitrite accumulation (Spérandio et al.,
302 2014). In contrast, both Models C and D were not able to capture the observed dependency of
303 N_2O production on nitrite availability (Yang et al., 2009, Kim et al., 2010, Spérandio et al.,
304 2014) due to the fact that the two models are linked to incomplete NH_2OH oxidation.
305 However, Models C and D were able to reproduce the experimental observations that the N_2O
306 production increased/decreased with increasing/decreasing DO concentration (Law et al.,
307 2012). The kinetic structure of Model B also ensured that the N_2O production rate is
308 dependent on oxygen availability, resulting in a similar N_2O dynamic trend (increase in the
309 N_2O production rate with a increase in DO concentration). On the contrary, Model A
310 predicted an opposite to such observation (Law et al., 2012). These results suggested that DO
311 inhibition might be required to describe AOB denitrification pathway and NH_2OH need to be
312 included as a necessary intermediate. The use of FA and FNA in model structures would be
313 recommend for a better description of the pH effect and possible FNA inhibition. NOH would
314 be preferably used as N_2O precursor for describing NH_2OH pathway under extremely high
315 nitrite accumulation condition whereas NO could be generally applied as intermediate for
316 N_2O production from NH_2OH oxidation under common wastewater conditions.

317 With respect to the two-pathway models of AOB, Model E has satisfactorily described the
318 N_2O data from several different nitrifying cultures (partial nitrification culture or/and full
319 nitrification culture) and under various DO and NO_2^- concentration conditions (Ni et al., 2014,
320 Peng et al., 2014; Sabba et al., 2015). Model F has also well predicted these different
321 nitrifying cultures (partial nitrification and full nitrification culture) and under various IC
322 conditions (Peng et al., 2015a). These two-pathway models also successfully predicted shifts
323 of the dominating pathway at various DO, nitrite and/or IC levels (see Figure 3), consistent
324 with experimental observations that N_2O was produced from both nitrifier denitrification and
325 NH_2OH oxidation pathways by AOB (Ni et al., 2014; Peng et al., 2014). The model results

326 suggested that the contribution of AOB denitrification decreased as DO increased,
327 accompanied by a corresponding increase in the contribution by the NH_2OH oxidation
328 pathway, which were verified by the site preference (SP) isotopic measurements (Peng et al.,
329 2014). Although the electron based two-pathway models (Models E and F) have been
330 demonstrated to be effective, electron carriers may not necessarily be the only approach to the
331 integration of the two pathways into one model. The possible alternatives/simplifications
332 could be evaluated in the future.

333 For denitrifying N_2O models, Model G was generally able to reproduce the nitrate, nitrite
334 and N_2O profiles when only one nitrogen oxide species was added (Ni et al., 2011, Pan et al.,
335 2015), but Model G failed to reproduce the results when two or more nitrogen oxide species
336 were added together. In contrast, Model H was shown to be able to describe general COD
337 consumption, nitrate reduction and nitrite accumulation by enriched denitrifying culture (Pan
338 et al., 2015), the influence of nitrite and N_2O addition on nitrate reduction, as well as the
339 experimental results when one or more nitrogen oxide species were added (Pan et al., 2015).
340 Therefore, the decoupling approach of Model H (Table 1) might be essential to describe the
341 electron competition process among the four denitrifying steps.

342 **6.2. Selection of Models for N_2O Prediction**

343 The model evaluation results strongly suggest that appropriate selection of available N_2O
344 models is important for accurate N_2O prediction in different engineering nitrogen removal
345 systems under different operational conditions. Table 2 present a possible guideline for model
346 selection in their further applications.

347 For N_2O production by AOB, the single-pathway models (Models A-D) have simpler
348 structures (one single pathway involved) and fewer parameters, which bring convenience to
349 model calibration (Table 2), and could be used preferentially under certain conditions,
350 although they are not be able to reproduce all the N_2O data. The two-pathway models (Models

351 E-F) have the potential to describe all the N_2O data with different operational conditions, but
352 may require more efforts on model calibration because of more parameters. Specifically
353 (Table 2), Models A, A1, B and B1 might be used to describe the regulation of N_2O
354 production by nitrite (or FNA) concentration. Models C and D might be able to describe N_2O
355 emissions from the systems with the condition of relatively high DO levels and low nitrite
356 accumulation that likely favoring the NH_2OH oxidation pathway for N_2O production. In
357 addition, according to the analysis by Peng et al. (2015b) (Figure 4), it is critical that the DO
358 concentration in the system is well controlled at a constant level for the AOB denitrification
359 model to be used (e.g., Model A). The NH_2OH oxidation model (e.g., Model D) can be
360 applied under high DO conditions. Under other conditions, the two-pathway models (e.g.,
361 Model E) should be applied. Model E could be used under varying DO and NO_2^- but constant
362 IC conditions while Model F should be applied under highly dynamic IC condition.

363 For N_2O production by heterotrophic denitrifiers, Model G can be used to predict the
364 overall nitrogen and COD removal performance in a wastewater treatment plant as in most
365 cases the low level accumulation of denitrification intermediates do not significantly affect
366 the overall nitrogen removal rate. However, in the context of predicting the N_2O production
367 by heterotrophic denitrifiers, Model G is inadequate due to its structurally deficient in
368 describing the electron competition process in denitrification. Model H enhanced our ability
369 to predict N_2O production by heterotrophic denitrifiers and has the potential to describe all
370 the N_2O data under different conditions, but requires information on both the carbon
371 oxidation reaction kinetics and the nitrogen reduction kinetics.

372 **6.3. Key Kinetic and Stoichiometric Parameters**

373 Table S5 in SI summarizes the typical values of the model parameters that have been
374 reported in literature, which could serve as default values for the future applications of the
375 available N_2O models (Tables S2-S4). The continued testing against more experimental data

376 would delineate a range/pattern in parameter values. It should be noted that these parameters
377 were estimated under different conditions of temperature, sludge retention time and feeding
378 composition, and therefore correction factors must be adjusted by, for example, Arrhenius
379 equations (Snip et al., 2014). Furthermore, the parameter values estimated during batch
380 experiments may not be adequate for the continuous process and may not be compatible with
381 the values of other parameters (Ni et al., 2013a, Spérandio et al., 2014, Snip et al., 2014).

382 For the six single-pathway models of AOB (Models A-D in Table S2), the model
383 parameters were obtained after significant calibration efforts, and thus some of the parameters
384 showed high variation (more than 100%) among case studies during model evaluations (Ni et
385 al., 2011, Ni et al., 2013a, Spérandio et al., 2014). Among them, the half saturation constant
386 for nitrite or FNA ($K_{\text{NO}_2, \text{AOB}}$ or $K_{\text{HNO}_2, \text{AOB}}$ for Models A, A1, B, B1) and the reduction factor
387 for N_2O production (η_{AOB} , for all the six single-pathway models) were most highly variable
388 (see Table S5 in SI) and very influential on N_2O emissions (Spérandio et al., 2014).
389 Regarding the models based on AOB denitrification pathway (e.g., Models A, A1, B and B1)
390 the large variation of these two key parameters were related to the range of nitrite (or FNA)
391 concentration observed in each system (Spérandio et al., 2014), likely due to the adaptation of
392 enzymatic activity (NirK). Regarding the models based on NH_2OH oxidation pathway (e.g.,
393 Models C and D) the large variation of η_{AOB} might be dependent on the possible NO
394 accumulation in each system. High NO accumulation would lead to a low value for η_{AOB}
395 (Spérandio et al., 2014). Thus, calibration will be required for the application of the
396 single-pathway models regarding these key parameters (Table 2).

397 For the electron balance based two-pathway models of AOB (Models E and F in Table
398 S3), the affinity constants with respect to electrons (e.g., $K_{\text{Mred},3}$, and $K_{\text{Mred},4}$) are unique to
399 the two-pathway models and the key parameters governing the N_2O production via the two
400 pathways. The values represent the affinity of the corresponding reduction reaction to

401 electrons, with lower values indicating a higher affinity and thus a higher ability to compete
402 for electrons. For example, the estimated $K_{Mred,3}$ has a value that is about one magnitude
403 smaller than $K_{Mred,4}$ (Ni et al., 2014), indicating that O_2 reduction has a higher ability to
404 compete for electrons as the main electron acceptor during NH_2OH oxidation. Ni et al. (2014)
405 revealed that the absolute value of C_{tot} is not critical for model calibration and predictions,
406 and it is the ratios between parameters K_{Mox} , $K_{Mred,1}$, $K_{Mred,2}$, $K_{Mred,3}$, and $K_{Mred,4}$ and parameter
407 C_{tot} that affect the model output. Therefore, attention should be paid to these ratios for the
408 calibration and application of the two-pathway models (see Table 2).

409 Regarding the ASM-ICE of heterotrophic denitrifiers (Model H in Table S4), information
410 on both the carbon oxidation reaction kinetics and the nitrogen reduction kinetics was
411 required for its calibration and application (Table 2). Due to the lack of understanding of the
412 electron competition process in most of the previous studies, the respective reaction kinetics
413 of the carbon oxidation and nitrogen reduction processes were not well established. For
414 instance, the maximum carbon source oxidation rate ($r_{COD,max}$), which is the key parameter to
415 restrict the overall model predicted carbon oxidation (electron supply) rate, is not available in
416 literature and thus need to be measured or estimated (Pan et al., 2015). Similar to the
417 two-pathway models of AOB, the relative ratios between electron affinity constants ($K_{Mred,1}$,
418 $K_{Mred,2}$, $K_{Mred,3}$, and $K_{Mred,4}$) rather than their absolute values are important for the reaction rate.
419 Therefore, more efforts are needed to provide more information on these key parameters of
420 the ASM-ICE model for its further implementation (Table 2).

421

422 **7. Application of N_2O Models in Full-Scale WWTPs**

423 Mathematical modelling of N_2O emissions from full-scale WWTPs was firstly conducted
424 successfully by using ASM-type models that combine one of the single-pathway models of
425 AOB with ASMN of heterotrophic denitrifiers (Ni et al., 2013b). Ni et al. (2013b) applied a

426 model based on NH_2OH pathway model of AOB (Model D, Table 1) and ASMN (Model G,
427 Table 1) to describe the N_2O emissions from full-scale WWTPs. The model described well the
428 dynamic ammonium, nitrite, nitrate, DO and N_2O data collected from both an open oxidation
429 ditch (OD) system with surface aerators and a SBR system with bubbling aeration. Ni et al.
430 (2013b) also performed additional evaluations on the other three single-pathway N_2O models
431 of AOB (Model A, Model B and Model C in Table 1) to evaluate the experimentally observed
432 N_2O data from the two full-scale WWTPs. The results indicated that Model A could not
433 predict the N_2O data from either WWTP (Ni et al., 2013b, Spérandio et al., 2014). Models B
434 and C, on the contrary, obtained very similar fit between the model-predicted and
435 experimentally observed N_2O data (Ni et al., 2013b, Spérandio et al., 2014).

436 Dynamic simulations were also confronted to the data collected on the UCT process from
437 Eindhoven plant by using ASM-type models that combine one of the single-pathway models
438 of AOB with ASMN of heterotrophic denitrifiers (Guo and Vanrolleghem, 2014; Spérandio et
439 al., 2014). Model A1 + Model G, Model B1 + Model G and Model D + Model G were all
440 implemented for this plant and calibrated using data collected in a 1-month measurement
441 campaign. The conclusion was that all these models could be calibrated to the same level of
442 fit (Spérandio et al., 2014). They had similar performance and could follow the dynamic
443 variations in the measured N_2O data (see Figure 5). In addition, results showed that there was
444 less N_2O emission under wet-weather conditions compared to dry-weather conditions and all
445 the three models showed better simulation performance under dry-weather conditions than
446 wet-weather conditions (Spérandio et al., 2014).

447 Mathematical modelling of N_2O emissions from full-scale WWTPs was then conducted
448 successfully by using electron balance based model that integrate the two-pathway model of
449 AOB and ASMN of heterotrophic denitrifiers (Ni et al., 2015). Ni et al. (2015) applied an
450 integrated model incorporating the electron balance based two-pathway model of AOB

451 (Model E, Table 1) and ASMN of heterotrophic denitrifiers (Model G, Table 1) to describe
452 N_2O emissions from a step-feed full-scale WWTP. The model described well all the dynamic
453 ammonium, nitrite, nitrate, DO and N_2O emission data. Modeling results revealed that the
454 AOB denitrification pathway decreased and the NH_2OH oxidation pathway increased along
455 the path of the both Steps, with the Second Step of the full-scale WWTP having much higher
456 N_2O emission than the First Step. The integrated N_2O model captured all these trends
457 regarding the shifting/distribution between the different N_2O pathways in full-scale WWTP
458 (see Figure 6). A potential strategy to mitigate N_2O emission from this plant is also evaluated
459 using the model. The overall N_2O emission from the step-feed WWTP would be largely
460 mitigated if 30% of the returned activated sludge was returned to the Second Step with the
461 remained 70% returning to the First Step. The model could potentially serve as a powerful
462 tool for the prediction of N_2O emissions from full-scale WWTPs and development of
463 effective mitigation strategies, although it may require more efforts on model calibration.

464 It should be noted that there are still limited number of studies presented in literature
465 regarding the real application of N_2O models in full-scale WWTPs although many full-scale
466 measurement campaigns have been performed in different places during the past years. More
467 full-scale applications of the models using these full-scale N_2O data are still needed for the
468 models to be developed into a useful tool for practical applications. In addition, the
469 requirement of good fundamental knowledge on N_2O emission from modeller/engineer might
470 also hinder the N_2O model applications due to the complicated procedure for model selection
471 and calibration, which consequently limit the development of effective mitigation strategies.
472 Hopefully this review would facilitate the selection of suitable N_2O models, the estimation of
473 site-specific N_2O emissions and the development of mitigation strategies for a wastewater
474 treatment plant taking into the specific design and operational conditions of the plant.

475

476 **8. Conclusions and Perspectives**

477 In this work, the existing N₂O models available in literature based on the three major
478 N₂O production pathways were reviewed and compared to illuminate their structural
479 differences, their capabilities and inabilities describing experimental data and their potential
480 range of applications. The key conclusions are:

- 481 • The fundamental mechanism about N₂O production is still not fully understood, leading
482 to the structural differences of existing N₂O models and their capabilities/inabilities
483 describing experimental data under different conditions.
- 484 • For AOB, the two-pathway models have the potential to describe all the N₂O data, but
485 may require more efforts on model calibration. The single-pathway models could be used
486 under several particular conditions. For heterotrophic denitrifiers, the ASMN-type model
487 is preferred for predicting the overall nitrogen and COD removal performance with low
488 intermediates accumulation. The ASM-ICE type model has the potential to describe all
489 the N₂O data, but requires more information on reaction kinetics.
- 490 • The available lab- and full-scale data sets are not well consolidated with highly different
491 reactor set-ups, measurement methods, culture history, documentations, and/or
492 interpretations, which would possibly lead to the failure of model predictions.
- 493 • Although the good fundamental knowledge on N₂O emission from modeller is essential
494 for successful application, mathematical modeling of N₂O production has reached a
495 maturity that facilitates the estimation of site-specific N₂O emissions and the
496 development of mitigation strategies.

497 Although existing models still have limitations, their application will undoubtedly
498 increase in the near future. Work in the following areas is necessary in order to gain a better
499 modeling of N₂O emission:

- 500 • While the electron balance based model has been successfully applied to estimate

501 site-specific N₂O emissions and develop mitigation strategies for a specific WWTP, future
502 efforts should be devoted to comparing the selected models to real data from real WWTPs
503 to observe the key differences and to enhance their practical applications.

504 • The parameters obtained with different experiments and cultures should be compared and
505 synthesized, aiming to form a consistent pattern which could then be implemented in the
506 improvement/simplification of multiple-pathway model, and integrated with the models
507 describing other sections of the WWTPs to form a powerful plant wide model.

508 • Mathematical modeling of N₂O emission from biofilm systems should be conducted
509 using more monitoring data from such systems.

510 • The real application of N₂O models in full-scale WWTPs using more full-scale
511 measurement campaigns would still be required for the models to be developed into a
512 useful tool for practical applications. Model-based development of mitigation strategies
513 should be further conducted, with their validities being tested in real operations.

514

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519

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Table 1. Key differences among the single-pathway models by AOB, two-pathway models by AOB and N₂O models by heterotrophs

N ₂ O models		Model components	Stoichiometric	Kinetic
Single-pathway models by AOB	Model A - AOB denitrification	Using S _{NH4} and S _{NO2} ; With S _{NH2OH} .	Two-step NH ₄ ⁺ oxidation; Two-step NO ₂ ⁻ reduction; Cell growth during NH ₂ OH oxidation.	Two different oxygen affinity constants; Oxygen inhibition on NO ₂ ⁻ and NO reductions; Anoxic reduction factor.
	Model A1- AOB denitrification	Using S _{NH3} and S _{HNO2} ; With S _{NH2OH} .	Same as Model A.	Two different oxygen affinity constants; Without oxygen inhibition; NH ₃ inhibition on NH ₃ oxidation; Anoxic reduction factor.
	Model B - AOB denitrification	Using S _{NH3} and S _{HNO2} ; Without S _{NH2OH} .	One-step NH ₄ ⁺ oxidation; Two-step NO ₂ ⁻ reduction; Cell growth during all 3 processes.	Only one oxygen affinity constant; Without oxygen inhibition; Anoxic reduction factor.
	Model B1 - AOB denitrification	Same as Model B.	Same as Model B.	Only one oxygen affinity constant; NH ₃ and HNO ₂ inhibitions on NH ₃ oxidation; Haldane function for oxygen limitation; Anoxic reduction factor.
	Model C - NH ₂ OH pathway (via NOH)	Using S _{NH4} and S _{NO2} ; With S _{NOH} .	Three-step NH ₄ ⁺ oxidation via NOH; Cell growth during NH ₂ OH oxidation.	Two different oxygen affinity constants; NOH breakdown to produce N ₂ O.
	Model D - NH ₂ OH pathway (via NO)	Using S _{NH4} and S _{NO2} ; With S _{NO} .	Three-step NH ₄ ⁺ oxidation via NO; Cell growth during NH ₂ OH oxidation.	Two different oxygen affinity constants; NO reduction to produce N ₂ O; Without oxygen inhibition.
Two-pathway models by AOB	Model E	Using S _{NH3} and S _{NO2} ; With electron carriers.	Three-step NH ₃ oxidation; One-step NO ₂ ⁻ reduction; Without cell growth.	Applying electron competition concept; Without oxygen inhibition; Without anoxic reduction factor.
	Model F	Mostly same as Model E; With S _{CO2} ; With energy carriers.	Mostly same as Model E; With energy carriers involved; With cell growth considered.	Mostly same as Model E; With energy carriers involved; With effect of inorganic carbon considered.
N ₂ O models by heterotrophs	Model G	Without electron carriers.	Coupling carbon oxidation and nitrogen reduction (4 processes).	Without electron competition concept.
	Model H	With electron carriers.	Decoupling carbon oxidation and nitrogen reduction (5 processes).	With electron competition concept.

Table 2. Guideline for model selection for predicting N₂O production by AOB and heterotrophic denitrification

N ₂ O models	Single-pathway models by AOB	Two-pathway models by AOB	N ₂ O models by heterotrophs
Applicable conditions	<ul style="list-style-type: none"> ✓ Models A, A1, B and B1 to describe the regulation of N₂O production by nitrite (or FNA) ✓ Model A to predict possible DO inhibition on N₂O production at high DO levels ✓ Models A1, B and B1 to predict possible pH effect and FA/FNA inhibition on N₂O production ✓ Models C and D to describe N₂O emissions at high DO levels and low nitrite accumulation 	<ul style="list-style-type: none"> ✓ Model E to predict N₂O production at varying DO and NO₂⁻ with constant IC ✓ Model F to describe N₂O production under highly dynamic IC condition 	<ul style="list-style-type: none"> ✓ Model G to predict the overall nitrogen and COD removal performance with low level accumulation of denitrification intermediates ✓ Model H to describe N₂O production under different conditions
Inabilities of the models	<ul style="list-style-type: none"> ✓ Model A not to describe the increase of N₂O production with increasing DO ✓ Models B and B1 not to predict the N₂O production related to the dynamics of NH₂OH ✓ Models C and D not to predict the effect of nitrite accumulation on N₂O production 	<ul style="list-style-type: none"> ✓ Model E not to describe N₂O production with dynamic IC 	<ul style="list-style-type: none"> ✓ Model G not to describe N₂O production with electron competition
Key parameters for calibration	<ul style="list-style-type: none"> ✓ The half saturation constant for nitrite or FNA ($K_{NO_2, AOB}$ or $K_{HNO_2, AOB}$ for Models A, A1, B, B1) ✓ The reduction factor for N₂O production (η_{AOB}, for all the six single-pathway models) 	<ul style="list-style-type: none"> ✓ The affinity constants with respect to electrons (e.g., $K_{Mred,3}$, and $K_{Mred,4}$) ✓ The ratios among the affinity constants to electrons 	<ul style="list-style-type: none"> ✓ The N₂O production and reduction rates ✓ The relative ratios between electron affinity constants

Figure Legends

- Figure 1.** Simplified representation of the three N_2O production pathways by ammonia oxidizing bacteria (A) and heterotrophic denitrifiers (B): nitrifier denitrification, NH_2OH oxidation and heterotrophic denitrification pathways.
- Figure 2.** Simplified representation of the electron transfer and energy transform processes in the biochemical reactions (reaction numbers refer to Table S3 in SI) associated with N_2O production by AOB via the two production pathways: (A) Electron balance (Ni et al., 2014), and (B) Energy balance (Peng et al., 2015a).
- Figure 3.** The predicted contributions from the nitrifier dinitrification pathway and the NH_2OH pathway as well as their shifts using Model E (real data: symbols, model predictions: lines) for a partial nitrification (left panel adapted from Ni et al., 2014) and a full nitrification system (right panel adapted from Peng et al., 2014).
- Figure 4.** Summary of applicable regions for the AOB denitrification model, the NH_2OH oxidation model and the two-pathway model under various DO and NO_2^- concentrations. The applicable regions were insensitive to the variations of key parameters governing N_2O production by the two-pathway model (Peng et al., 2015b).
- Figure 5.** Model evaluation results for N_2O emissions using the measurement results at the beginning (BM) (upper panel), the middle (MM) (middle panel) and the end section (EM) (bottom panel) of the summer aeration package on the UCT process from Eindhoven plant by using ASM-type models that combine one of the single-pathway models of AOB (Models A1, B1 and C) with ASMN (Model G) of heterotrophic denitrifiers (Spérandio et al., 2014).
- Figure 6.** Model predicted percentage contributions from the three N_2O pathways to total N_2O productions at six different locations of the First Step (left panel) and the Second Step (right panel) in the step-feed full-scale WWTP, i.e., the nitrifier dinitrification pathway, the NH_2OH pathway and the heterotrophic denitrification pathway (Ni et al., 2015).

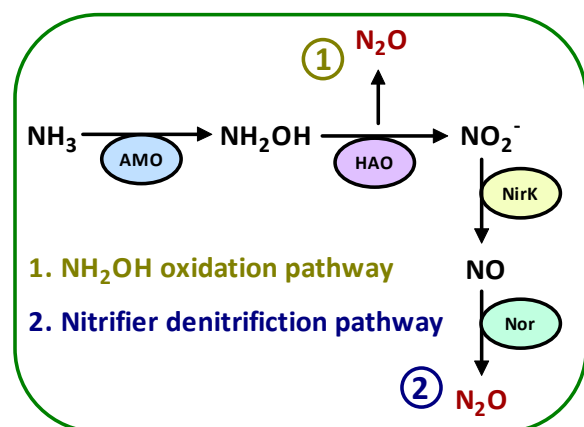
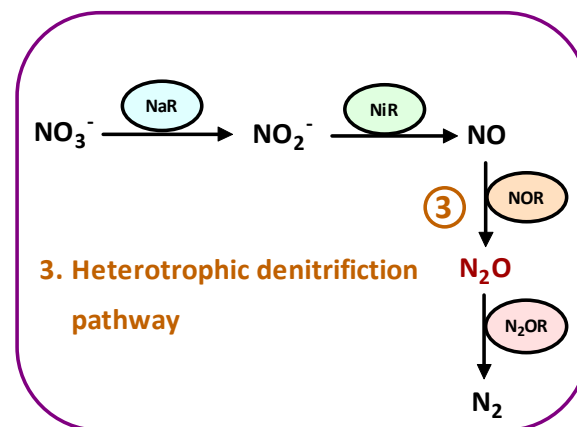
(A) Ammonia-oxidizing bacteria**(B) Heterotrophic denitrifiers**

Figure 1. Simplified representation of the three N_2O production pathways by ammonia oxidizing bacteria (A) and heterotrophic denitrifiers (B): nitrifier denitrification, NH_2OH oxidation and heterotrophic denitrification pathways.

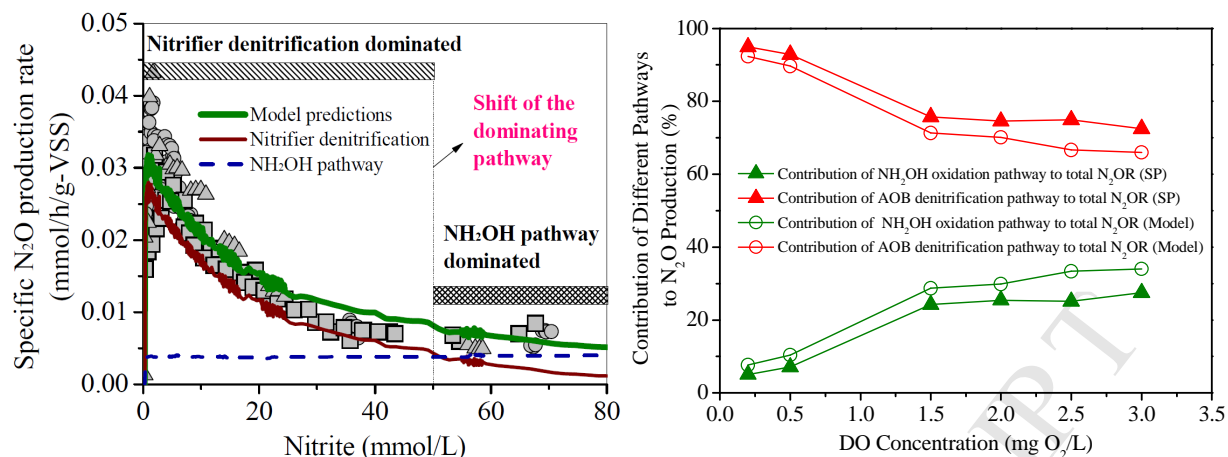


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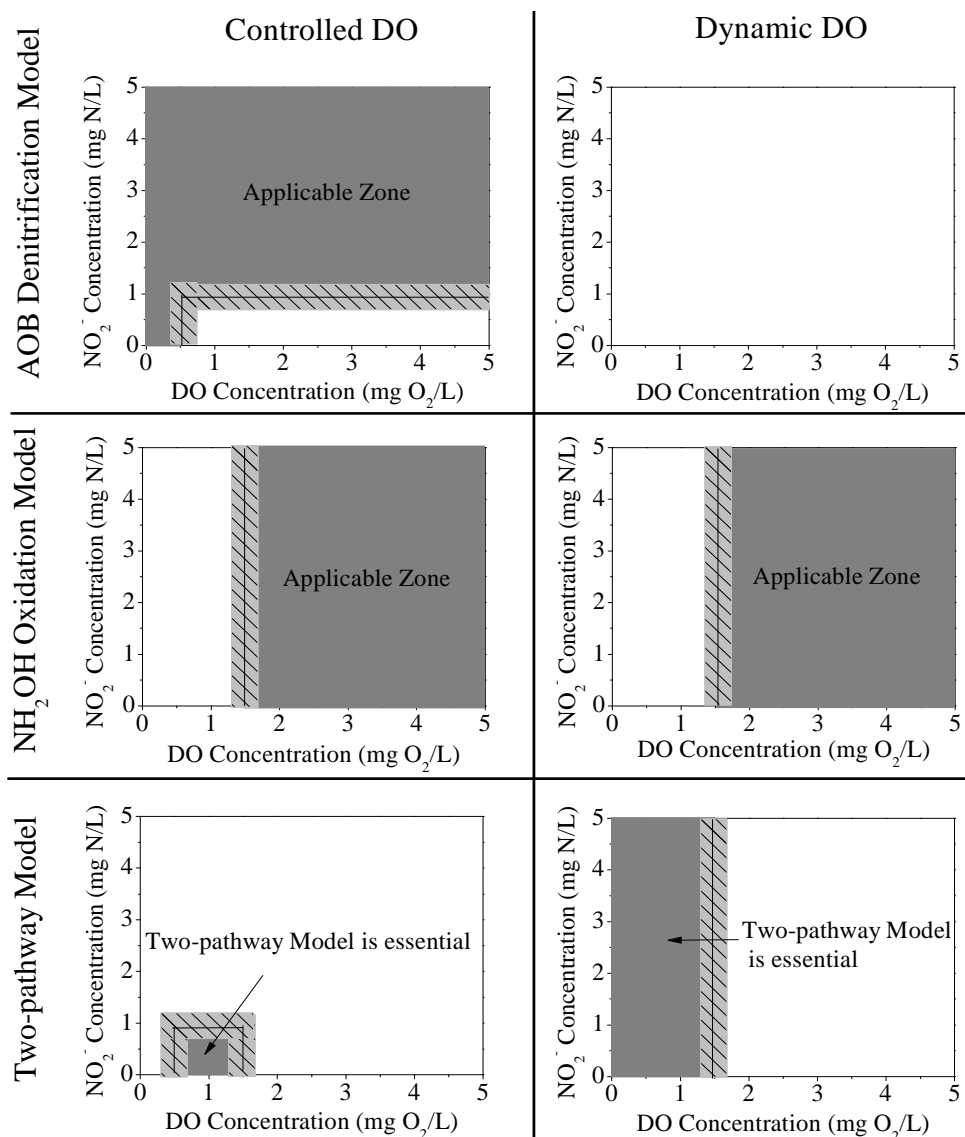


Figure 4. Summary of applicable regions for the AOB denitrification model, the NH₂OH oxidation model and the two-pathway model under various DO and NO₂⁻ concentrations. The applicable regions were insensitive to the variations of key parameters governing N₂O production by the two-pathway model (Peng et al., 2015b).

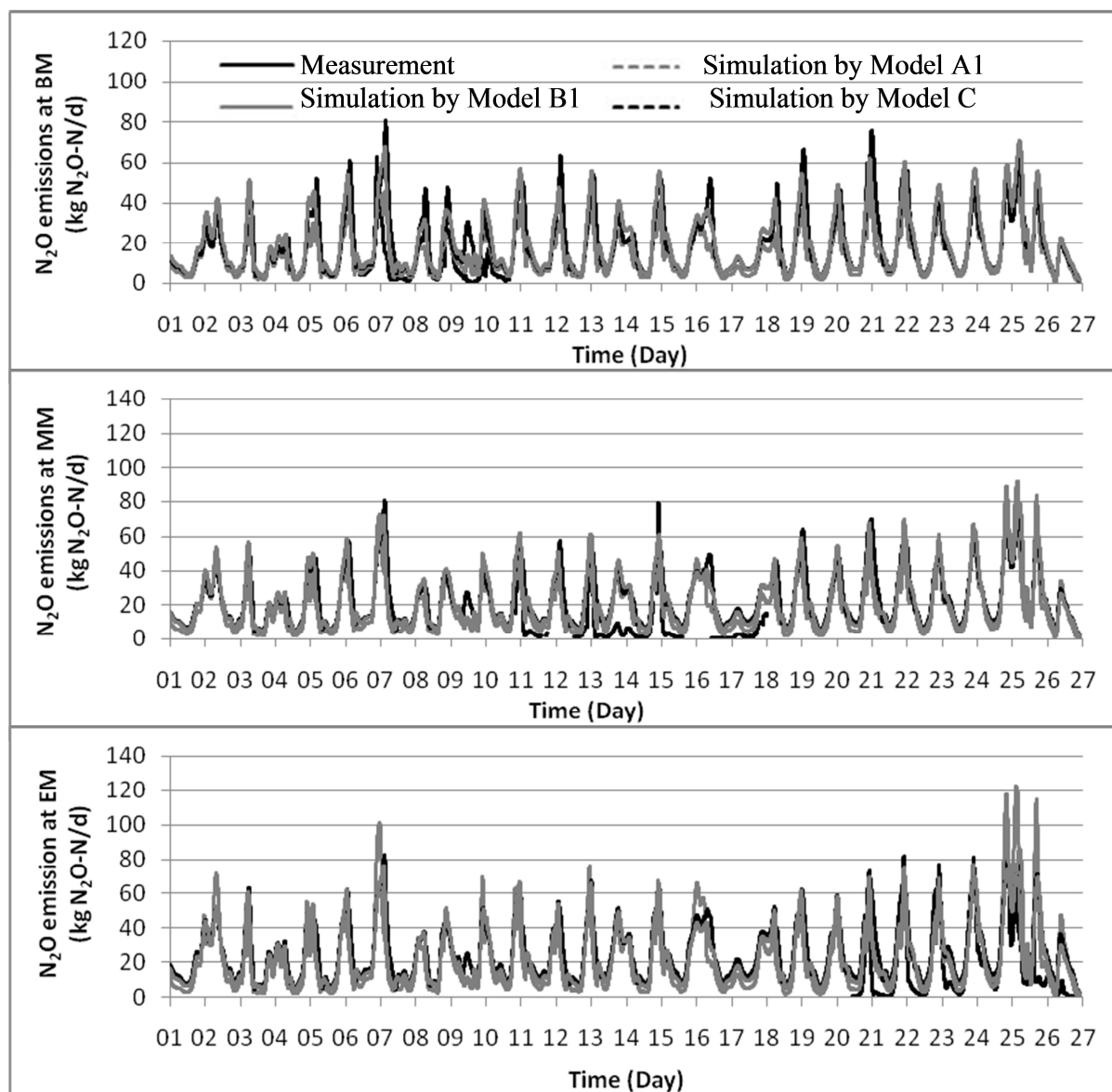


Figure 5. Model evaluation results for N_2O emissions using the measurement results at the beginning (BM) (upper panel), the middle (MM) (middle panel) and the end section (EM) (bottom panel) of the summer aeration package on the UCT process from Eindhoven plant by using ASM-type models that combine one of the single-pathway models of AOB (Models A1, B1 and C) with ASMN (Model G) of heterotrophic denitrifiers (Spérandio et al., 2014).

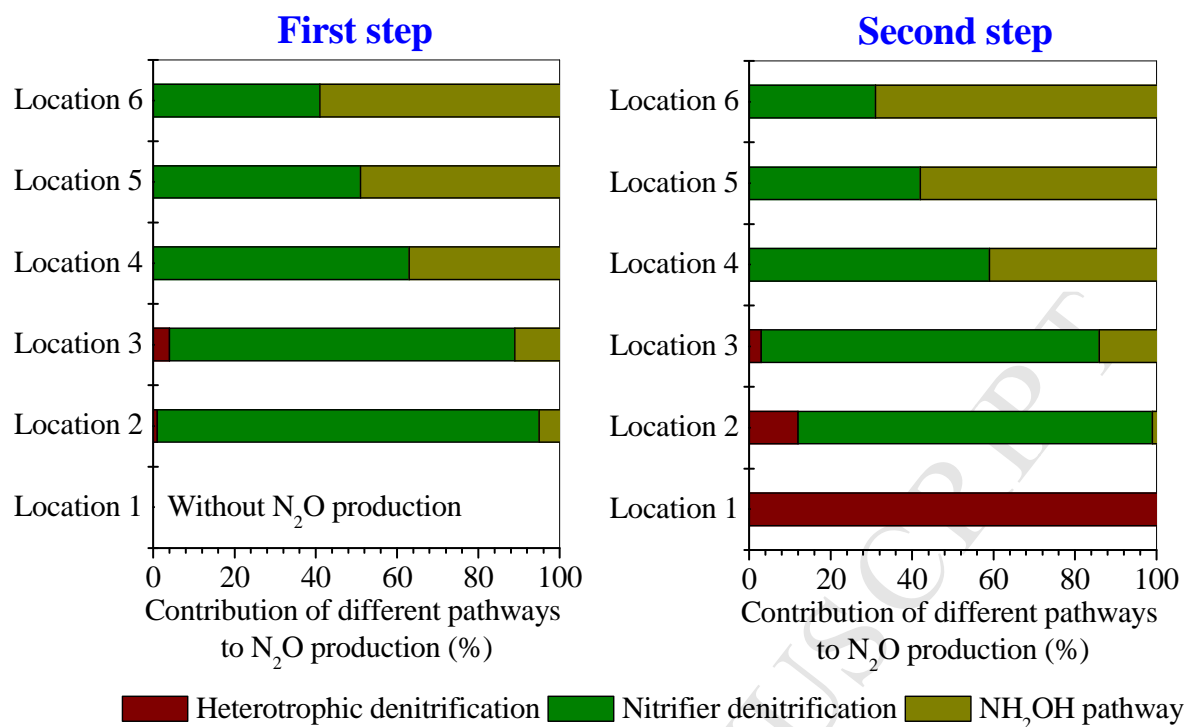


Figure 6. Model predicted percentage contributions from the three N₂O pathways to total N₂O productions at six different locations of the First Step (left panel) and the Second Step (right panel) in the step-feed full-scale WWTP, i.e., the nitrifier dinitrification pathway, the NH₂OH pathway and the heterotrophic denitrification pathway (Ni et al., 2015).

Highlights

- The models describing all the known microbial pathways for N₂O production are reviewed.
- The N₂O model structures as well as their underlying assumptions are compared.
- Model evaluations using lab-scale and full-scale experimental data are discussed.
- The key kinetic and stoichiometric parameters are summarized and analysed.
- The applicability of these N₂O models under various conditions is elucidated.

Supplementary Information

Recent Advances in Mathematical Modeling of Nitrous Oxides Emissions from Wastewater Treatment Processes

Bing-Jie Ni,* Zhiguo Yuan

Advanced Water Management Centre, The University of Queensland, St. Lucia,
Brisbane, Queensland 4072, Australia

***Corresponding author:**

Dr. Bing-Jie Ni

Advanced Water Management Centre

The University of Queensland

Australia

Phone: + 61 7 3346 3219

Fax: +61 7 3365 4726

E-mail: b.ni@uq.edu.au

The following is included as supplementary information for this paper:

Table S1. The definition of all model components

Variable	Description
S_{O_2}	Dissolved oxygen (DO) concentration
S_{NH_3}	Ammonia (NH ₃) concentration
S_{NH_4}	Ammonium (NH ₄ ⁺) concentration
S_{NO_2}	Nitrite (NO ₂ ⁻) concentration
S_{NO_3}	Nitrate (NO ₃ ⁻) concentration
S_{N_2}	Nitrogen gas (N ₂) concentration
S_{HNO_2}	Free nitrite acid (FNA) concentration
S_{NO}	Nitric oxide (NO) concentration
S_{N_2O}	Nitrous oxide (N ₂ O) concentration
S_{NH_2OH}	Hydroxylamine (NH ₂ OH) concentration
S_{NOH}	Nitrosyl radical (NOH) concentration
S_{Mred}	Reduced form of electron carrier (Mred) concentration
S_{Mox}	Oxidized form of electron carrier (Mox) concentration
S_{ADP}	Released form of energy carrier (ADP) concentration
S_{ATP}	Reserved form of energy carrier (ATP) concentration
S_{CO_2}	Inorganic carbon (IC) concentration
S_S	Readily biodegradable COD concentration
X_H	Heterotrophic denitrifiers (HD) concentration
X_{AOB}	Ammonia-oxidizing bacteria (AOB) concentration

Table S2. Process matrices for the six single-pathway N₂O models of AOB in literature

Process	Model Components							Kinetic rate expressions	
	S_{O_2}	S_{NH_4} (S_{NH_3})	S_{NH_2OH}	S_{NOH}	S_{NO_2} (S_{HNO_2})	S_{NO}	S_{N_2O}		X_{AOB}
Model A – AOB denitrification pathway (Ni et al., 2011)									
A-1	-1.14	-1	1					$\mu_{AOB,AMO} \frac{S_{O_2}}{K_{O_2,AOB,1} + S_{O_2}} \frac{S_{NH_4}}{K_{NH_4,AOB} + S_{NH_4}} X_{AOB}$	
A-2	$-\frac{2.29 - Y_{AOB}}{Y_{AOB}}$	$-i_{N,AOB}$	$-\frac{1}{Y_{AOB}}$		$\frac{1}{Y_{AOB}}$		1	$\mu_{AOB,HAO} \frac{S_{O_2}}{K_{O_2,AOB,2} + S_{O_2}} \frac{S_{NH_2OH}}{K_{NH_2OH,AOB} + S_{NH_2OH}} X_{AOB}$	
A-3			-1		-3	4		$\eta_{AOB} \mu_{AOB,HAO} \frac{K_{1,O_2,AOB}}{S_{O_2} + K_{1,O_2,AOB}} \frac{S_{NO_2}}{K_{NO_2,AOB} + S_{NO_2}} \frac{S_{NH_2OH}}{K_{NH_2OH,AOB} + S_{NH_2OH}} X_{AOB}$	
A-4			-1		1	-4	4	$\eta_{AOB} \mu_{AOB,HAO} \frac{K_{1,O_2,AOB}}{S_{O_2} + K_{1,O_2,AOB}} \frac{S_{NO}}{K_{NO,AOB} + S_{NO}} \frac{S_{NH_2OH}}{K_{NH_2OH,AOB} + S_{NH_2OH}} X_{AOB}$	
Model A1 – AOB denitrification pathway (Pocquet et al., 2013)									
A1-1	-1.14	-1	1					$\mu_{AOB,AMO} \frac{S_{O_2}}{K_{O_2,AOB,1} + S_{O_2}} \frac{S_{NH_3}}{K_{NH_3,AOB} + S_{NH_3} + (S_{NH_3})^2 / K_{1,NH_3,AOB}} X_{AOB}$	
A1-2	$-\frac{2.29 - Y_{AOB}}{Y_{AOB}}$	$-i_{N,AOB}$	$-\frac{1}{Y_{AOB}}$		$\frac{1}{Y_{AOB}}$		1	$\mu_{AOB,HAO} \frac{S_{O_2}}{K_{O_2,AOB,2} + S_{O_2}} \frac{S_{NH_2OH}}{K_{NH_2OH,AOB} + S_{NH_2OH}} X_{AOB}$	
A1-3			-1		-3	4		$\eta_{AOB} \mu_{AOB,HAO} \frac{S_{HNO_2}}{K_{HNO_2,AOB} + S_{HNO_2}} \frac{S_{NH_2OH}}{K_{NH_2OH,AOB} + S_{NH_2OH}} X_{AOB}$	
A1-4			-1		1	-4	4	$\eta_{AOB} \mu_{AOB,HAO} \frac{S_{NO}}{K_{NO,AOB} + S_{NO}} \frac{S_{NH_2OH}}{K_{NH_2OH,AOB} + S_{NH_2OH}} X_{AOB}$	
Model B – AOB denitrification pathway (Mampaey et al., 2013)									
B-1	$-\frac{3.43 - Y_{AOB}}{Y_{AOB}}$	$-\frac{1}{Y_{AOB}} - i_{N,AOB}$			$\frac{1}{Y_{AOB}}$		1	$\mu_{AOB} \frac{S_{O_2}}{K_{O_2,AOB} + S_{O_2}} \frac{S_{NH_3}}{K_{NH_3,AOB} + S_{NH_3}} X_{AOB}$	
B-2	$-\frac{2.29 - Y_{AOB,den}}{Y_{AOB,den}}$	$-\frac{1}{Y_{AOB,den}} - i_{N,AOB}$			$-\frac{1}{Y_{AOB,den}}$	$\frac{2}{Y_{AOB,den}}$	1	$\eta_{AOB} \mu_{AOB} \frac{S_{O_2}}{K_{O_2,AOB} + S_{O_2}} \frac{S_{NH_3}}{K_{NH_3,AOB} + S_{NH_3}} \frac{S_{HNO_2}}{K_{HNO_2,AOB} + S_{HNO_2}} X_{AOB}$	
B-3	$-\frac{2.29 - Y_{AOB,den}}{Y_{AOB,den}}$	$-\frac{1}{Y_{AOB,den}} - i_{N,AOB}$			$\frac{1}{Y_{AOB,den}}$	$-\frac{2}{Y_{AOB,den}}$	$\frac{2}{Y_{AOB,den}}$	1	$\eta_{AOB} \mu_{AOB} \frac{S_{O_2}}{K_{O_2,AOB} + S_{O_2}} \frac{S_{NH_3}}{K_{NH_3,AOB} + S_{NH_3}} \frac{S_{NO}}{K_{NO,AOB} + S_{NO}} X_{AOB}$

Model B1 – AOB denitrification pathway (Guo and Vanrolleghem, 2014)

B1-1	$-\frac{3.43 - Y_{AOB}}{Y_{AOB}}$	$-\frac{1}{Y_{AOB}} - i_{N,AOB}$	$\frac{1}{Y_{AOB}}$		1	$\mu_{AOB} \frac{S_{O_2}}{K_{O_2,AOB} + S_{O_2}} \frac{S_{NH_3}}{K_{NH_3,AOB} + S_{NH_3} + (S_{NH_3})^2 / K_{I,NH_3,AOB}} \frac{K_{I,HNO_2,AOB}}{K_{I,HNO_2,AOB} + S_{HNO_2}} X_{AOB}$
B1-2	$-\frac{2.29 - Y_{AOB,den}}{Y_{AOB,den}}$	$-\frac{1}{Y_{AOB,den}} - i_{N,AOB}$	$-\frac{1}{Y_{AOB,den}}$	$\frac{2}{Y_{AOB,den}}$	1	$\eta_{AOB} \mu_{AOB} \frac{S_{NH_3}}{K_{NH_3,AOB,den} + S_{NH_3}} \frac{S_{HNO_2}}{K_{HNO_2,AOB} + S_{HNO_2}} X_{AOB} DO_{Haldane}$
B1-3	$-\frac{2.29 - Y_{AOB,den}}{Y_{AOB,den}}$	$-\frac{1}{Y_{AOB,den}} - i_{N,AOB}$	$\frac{1}{Y_{AOB,den}}$	$-\frac{2}{Y_{AOB,den}}$	$\frac{2}{Y_{AOB,den}}$	$\eta_{AOB} \mu_{AOB} \frac{S_{NH_3}}{K_{NH_3,AOB,den} + S_{NH_3}} \frac{S_{NO}}{K_{NO,AOB} + S_{NO}} X_{AOB} DO_{Haldane}$
						$DO_{Haldane} = \frac{S_{O_2}}{K_{O_2,AOB,den} + (1 - 2\sqrt{K_{O_2,AOB,den} / K_{I,O_2,AOB}}) S_{O_2} + (S_{O_2})^2 / K_{I,O_2,AOB}}$

Model C – NH₂OH/NOH pathway (Law et al., 2012)

C-1	-1.14	-1	1			$\mu_{AOB,AMO} \frac{S_{O_2}}{K_{O_2,AOB,1} + S_{O_2}} \frac{S_{NH_4}}{K_{NH_4,AOB} + S_{NH_4}} X_{AOB}$
C-2	$-\frac{1.14 - Y_{AOB}}{Y_{AOB}}$	$-i_{N,AOB}$	$-\frac{1}{Y_{AOB}}$	$\frac{1}{Y_{AOB}}$	1	$\mu_{AOB,HAO,1} \frac{S_{O_2}}{K_{O_2,AOB,2} + S_{O_2}} \frac{S_{NH_2OH}}{K_{NH_2OH,AOB} + S_{NH_2OH}} X_{AOB}$
C-3	-1.14			-1	1	$\mu_{AOB,HAO,2} \frac{S_{O_2}}{K_{O_2,AOB,2} + S_{O_2}} \frac{S_{NOH}}{K_{NOH,AOB} + S_{NOH}} X_{AOB}$
C-4				-1	1	$k_{NOH} S_{NOH}$

Model D – NH₂OH/NO pathway (Ni et al., 2013b)

D-1	-1.14	-1	1			$\mu_{AOB,AMO} \frac{S_{O_2}}{K_{O_2,AOB,1} + S_{O_2}} \frac{S_{NH_4}}{K_{NH_4,AOB} + S_{NH_4}} X_{AOB}$
D-2	$-\frac{1.71 - Y_{AOB}}{Y_{AOB}}$	$-i_{N,AOB}$	$-\frac{1}{Y_{AOB}}$	$\frac{1}{Y_{AOB}}$	1	$\mu_{AOB,HAO,1} \frac{S_{O_2}}{K_{O_2,AOB,2} + S_{O_2}} \frac{S_{NH_2OH}}{K_{NH_2OH,AOB} + S_{NH_2OH}} X_{AOB}$
D-3	-0.57			1	-1	$\mu_{AOB,HAO,2} \frac{S_{O_2}}{K_{O_2,AOB,2} + S_{O_2}} \frac{S_{NO}}{K_{NO,AOB} + S_{NO}} X_{AOB}$
D-4			-1	1	-4	$\eta_{AOB} \mu_{AOB,HAO,1} \frac{S_{NO}}{K_{NO,AOB} + S_{NO}} \frac{S_{NH_2OH}}{K_{NH_2OH,AOB} + S_{NH_2OH}} X_{AOB}$

Table S3. Process matrices for the two two-pathway N₂O models of AOB in literature

Process	Model Components											Kinetic rate expressions	
	S_{O_2}	S_{NH_3}	S_{NH_2OH}	S_{NO_2}	S_{NO}	S_{N_2O}	S_{Mox}	S_{Mred}	S_{ADP}	S_{ATP}	S_{CO_2}		X_{AOB}
Model E – Decoupling approach, electron balance based model (Ni et al., 2014)													
E-1	-1	-1	1				1	-1					$r_{NH_3,ox} \frac{S_{O_2}}{K_{O_2,NH_3} + S_{O_2}} \frac{S_{NH_3}}{K_{NH_3} + S_{NH_3}} \frac{S_{Mred}}{K_{Mred,1} + S_{Mred}} X_{AOB}$
E-2			-1		1		-3/2	3/2					$r_{NH_2OH,ox} \frac{S_{NH_2OH}}{K_{NH_2OH} + S_{NH_2OH}} \frac{S_{Mox}}{K_{Mox} + S_{Mox}} X_{AOB}$
E-3				1	-1		-1/2	1/2					$r_{NO,ox} \frac{S_{NO}}{K_{NO,ox} + S_{NO}} \frac{S_{Mox}}{K_{Mox} + S_{Mox}} X_{AOB}$
E-4					-1	1/2	1/2	-1/2					$r_{NO,red} \frac{S_{NO}}{K_{NO,red} + S_{NO}} \frac{S_{Mred}}{K_{Mred,2} + S_{Mred}} X_{AOB}$
E-5	-1/2						1	-1					$r_{O_2,red} \frac{S_{O_2}}{K_{O_2,red} + S_{O_2}} \frac{S_{Mred}}{K_{Mred,3} + S_{Mred}} X_{AOB}$
E-6				-1		1/2	1	-1					$r_{NO_2,red} \frac{S_{NO_2}}{K_{NO_2} + S_{NO_2}} \frac{S_{Mred}}{K_{Mred,4} + S_{Mred}} X_{AOB}$
$S_{Mred} + S_{Mox} = C_{tot}$													
Model F – Decoupling approach, electron and ATP balance based model (Peng et al., 2015a)													
F-1	-1	-1	1				1	-1	-2/3	2/3			$r_{NH_3,ox} \frac{S_{O_2}}{K_{O_2,NH_3} + S_{O_2}} \frac{S_{NH_3}}{K_{NH_3} + S_{NH_3}} \frac{S_{Mred}}{K_{Mred,1} + S_{Mred}} \frac{S_{ADP}}{K_{ADP} + S_{ADP}} X_{AOB}$
F-2			-1		1		-1	1					$r_{NH_2OH,ox} \frac{S_{NH_2OH}}{K_{NH_2OH} + S_{NH_2OH}} \frac{S_{Mox}}{K_{Mox} + S_{Mox}} X_{AOB}$
F-3				1	-1		-1	1					$r_{NO,ox} \frac{S_{NO}}{K_{NO,ox} + S_{NO}} \frac{S_{Mox}}{K_{Mox} + S_{Mox}} X_{AOB}$
F-4					-1	1/2							$r_{NO,red} S_{NO}$

F-5	-1/2			1	-1	-1/3	1/3			$r_{O_2,red} \frac{S_{O_2}}{K_{O_2,red} + S_{O_2}} \frac{S_{Mred}}{K_{Mred,2} + S_{Mred}} \frac{S_{ADP}}{K_{ADP} + S_{ADP}} X_{AOB}$
F-6		-1	1/2	1	-1	-1/3	1/3			$r_{NO_2,red} \frac{S_{NO_2}}{K_{NO_2} + S_{NO_2}} \frac{S_{Mred}}{K_{Mred,3} + S_{Mred}} \frac{S_{ADP}}{K_{ADP} + S_{ADP}} X_{AOB}$
F-7				2	-2	15	-15	-1	1	$r_{AOB} \frac{S_{CO_2}}{K_{CO_2} + S_{CO_2}} \frac{S_{Mred}}{K_{Mred,4} + S_{Mred}} \frac{S_{ATP}}{K_{ATP} + S_{ATP}} X_{AOB}$
										$S_{Mred} + S_{Mox} = C_{tot,1}$
										$S_{ADP} + S_{ATP} = C_{tot,2}$

Table S4. Process matrices for the two types of four-step denitrification models describing N₂O production by heterotrophic denitrifiers

Process	Model Components								Kinetic rate expressions		
	S_{NO3}	S_{NO2}	S_{NO}	S_{N2O}	S_{N2}	S_S	S_{Mox}	S_{Mred}		X_H	
Model G – ASMN, the “direct coupling approach” adapted from Hiatt and Grady (2008)											
G-1	$-\frac{1-Y_H \cdot \eta_Y}{1.143 \cdot Y_H \cdot \eta_Y}$	$\frac{1-Y_H \cdot \eta_Y}{1.143 \cdot Y_H \cdot \eta_Y}$				$-1/(Y_H \cdot \eta_Y)$			1	$\mu_H \eta_{g1} \left(\frac{S_S}{K_{S1} + S_S} \right) \left(\frac{S_{NO3}}{K_{NO3}^{HB} + S_{NO3}} \right) X_H$	
G-2		$-\frac{1-Y_H \cdot \eta_Y}{0.571 \cdot Y_H \cdot \eta_Y}$	$\frac{1-Y_H \cdot \eta_Y}{0.571 \cdot Y_H \cdot \eta_Y}$			$-1/(Y_H \cdot \eta_Y)$			1	$\mu_H \eta_{g2} \left(\frac{S_S}{K_{S2} + S_S} \right) \left(\frac{S_{NO2}}{K_{NO2}^{HB} + S_{NO2}} \right) \left(\frac{K_{NO,2}}{K_{NO,2} + S_{NO}} \right) X_H$	
G-3			$-\frac{1-Y_H \cdot \eta_Y}{0.571 \cdot Y_H \cdot \eta_Y}$	$\frac{1-Y_H \cdot \eta_Y}{0.571 \cdot Y_H \cdot \eta_Y}$		$-1/(Y_H \cdot \eta_Y)$			1	$\mu_H \eta_{g3} \left(\frac{S_S}{K_{S3} + S_S} \right) \left(\frac{S_{NO}}{K_{NO}^{HB} + S_{NO} + S_{NO}^2 / K_{NO,3}} \right) X_H$	
G-4				$-\frac{1-Y_H \cdot \eta_Y}{0.571 \cdot Y_H \cdot \eta_Y}$	$\frac{1-Y_H \cdot \eta_Y}{0.571 \cdot Y_H \cdot \eta_Y}$	$-1/(Y_H \cdot \eta_Y)$			1	$\mu_H \eta_{g4} \left(\frac{S_S}{K_{S4} + S_S} \right) \left(\frac{S_{N2O}}{K_{N2O}^{HB} + S_{N2O}} \right) \left(\frac{K_{NO,4}}{K_{NO,4} + S_{NO}} \right) X_H$	
Model H – ASM-ICE, the “indirect coupling approach” adapted from Pan et al. (2013b)											
H-1						-1	$-(1-Y_H)$	$1-Y_H$	Y_H	$r_{COD,max} \left(\frac{S_S}{K_S + S_S} \right) \left(\frac{S_{Mox}}{K_{Mox} + S_{Mox}} \right) X_H$	
H-2	-1	1							1	-1	$r_{NO3,max} \left(\frac{S_{NO3}}{K_{NO3}^{HB} + S_{NO3}} \right) \left(\frac{S_{Mred}}{K_{Mred,1} + S_{Mred}} \right) X_H$
H-3		-1	1						$\frac{1}{2}$	$-\frac{1}{2}$	$r_{NO2,max} \left(\frac{S_{NO2}}{K_{NO2}^{HB} + S_{NO2}} \right) \left(\frac{S_{Mred}}{K_{Mred,2} + S_{Mred}} \right) X_H$
H-4			-1	$\frac{1}{2}$					$\frac{1}{2}$	$-\frac{1}{2}$	$r_{NO,max} \left(\frac{S_{NO}}{K_{NO}^{HB} + S_{NO}} \right) \left(\frac{S_{Mred}}{K_{Mred,3} + S_{Mred}} \right) X_H$
H-5				-1	1				1	-1	$r_{N2O,max} \left(\frac{S_{N2O}}{K_{N2O}^{HB} + S_{N2O}} \right) \left(\frac{S_{Mred}}{K_{Mred,4} + S_{Mred}} \right) X_H$
$S_{Mred} + S_{Mox} = C_{tot}$											

Table S5. Kinetic and stoichiometric parameters of all the N₂O models reviewed

Parameter	Definition	Typical values	Source
Model A – AOB denitrification pathway			
Y_{AOB}	Yield coefficient for AOB, g COD g ⁻¹ N	0.150	Ni et al. (2011)
$i_{N,AOB}$	Nitrogen content of biomass, g N g ⁻¹ COD	0.07	Ni et al. (2011)
$\mu_{AOB,AMO}$	Maximum AMO-mediated reaction rate, h ⁻¹	0.122	Ni et al. (2011)
$\mu_{AOB,HAO}$	Maximum HAO-mediated reaction rate, h ⁻¹	0.092	Ni et al. (2011)
$K_{O_2,AOB,1}$	S_{O_2} affinity constant for S_{NH_4} oxidation, g DO m ⁻³	0.043	Ni et al. (2011)
$K_{O_2,AOB,2}$	S_{O_2} affinity constant for S_{NH_2OH} oxidation, g DO m ⁻³	0.6	Ni et al. (2011)
$K_{I,O_2,AOB}$	S_{O_2} substrate inhibition parameter, g DO m ⁻³	0.112	Ni et al. (2011)
η_{AOB}	Anoxic reduction factor	0.074	Ni et al. (2011)
$K_{NH_4,AOB}$	S_{NH_4} affinity constant for AOB, g N m ⁻³	2.4	Ni et al. (2011)
$K_{NH_2OH,AOB}$	S_{NH_2OH} affinity constant for AOB, g N m ⁻³	2.4	Ni et al. (2011)
$K_{NO_2,AOB}$	S_{NO_2} affinity constant for AOB, g N m ⁻³	0.14	Ni et al. (2011)
$K_{NO,AOB}$	S_{NO} affinity constant for AOB, g N m ⁻³	0.0084	Ni et al. (2011)
Model A1 – AOB denitrification pathway			
Y_{AOB}	Yield coefficient for AOB, g COD g ⁻¹ N	0.150	Pocquet et al. (2013)
$i_{N,AOB}$	Nitrogen content of biomass, g N g ⁻¹ COD	0.07	Pocquet et al. (2013)
$\mu_{AOB,AMO}$	Maximum AMO-mediated reaction rate, h ⁻¹	0.216	Pocquet et al. (2013)
$\mu_{AOB,HAO}$	Maximum HAO-mediated reaction rate, h ⁻¹	0.062	Pocquet et al. (2013)
$K_{O_2,AOB,1}$	S_{O_2} affinity constant for S_{NH_4} oxidation, g DO m ⁻³	0.043	Pocquet et al. (2013)
$K_{O_2,AOB,2}$	S_{O_2} affinity constant for S_{NH_2OH} oxidation, g DO m ⁻³	0.6	Pocquet et al. (2013)
η_{AOB}	Anoxic reduction factor	0.20	Pocquet et al. (2013)
$K_{NH_3,AOB}$	S_{NH_3} affinity constant for AOB, g N m ⁻³	0.4575	Pocquet et al. (2013)
$K_{I,NH_3,AOB}$	S_{NH_3} substrate inhibition constant for AOB, g N m ⁻³	16	Pocquet et al. (2013)

$K_{NH_2OH,AOB}$	S_{NH_2OH} affinity constant for AOB, g N m ⁻³	2.4	Pocquet et al. (2013)
$K_{HNO_2,AOB}$	S_{HNO_2} affinity constant for AOB, g N m ⁻³	0.002	Pocquet et al. (2013)
$K_{NO,AOB}$	S_{NO} affinity constant for AOB, g N m ⁻³	0.004	Pocquet et al. (2013)
Model B – AOB denitrification pathway			
Y_{AOB}	Yield coefficient for AOB, g COD g ⁻¹ N	0.150	Mampaey et al. (2013)
$i_{N,AOB}$	Nitrogen content of biomass, g N g ⁻¹ COD	0.07	Mampaey et al. (2013)
μ_{AOB}	Maximum AOB growth rate, h ⁻¹	0.045	Mampaey et al. (2013)
η_{AOB}	Anoxic reduction factor	0.03	Mampaey et al. (2013)
$K_{NH_3,AOB}$	S_{NH_3} affinity constant for AOB, g N m ⁻³	1.0	Mampaey et al. (2013)
$K_{NO,AOB}$	S_{NO} affinity constant for AOB, g DO m ⁻³	1.0	Mampaey et al. (2013)
$K_{HNO_2,AOB}$	S_{HNO_2} affinity constant for AOB, g N m ⁻³	0.002	Mampaey et al. (2013)
$K_{O_2,AOB}$	S_{O_2} affinity constant for AOB, g DO m ⁻³	0.5	Mampaey et al. (2013)
Model B1 – AOB denitrification pathway			
Y_{AOB}	Yield coefficient for AOB, g COD g ⁻¹ N	0.180	Guo and Vanrolleghem (2014)
$Y_{AOB,den}$	Yield coefficient for AOB denitrification, g COD g ⁻¹ N	0.150	Guo and Vanrolleghem (2014)
$i_{N,AOB}$	Nitrogen content of biomass, g N g ⁻¹ COD	0.07	Guo and Vanrolleghem (2014)
μ_{AOB}	Maximum AOB growth rate, h ⁻¹	0.032	Guo and Vanrolleghem (2014)
η_{AOB}	Anoxic reduction factor	0.3	Guo and Vanrolleghem (2014)
$K_{NH_3,AOB}$	S_{NH_3} affinity constant for AOB, g N m ⁻³	0.007	Guo and Vanrolleghem (2014)
$K_{NH_3,AOB,den}$	S_{NH_3} affinity constant for AOB denitrification, g N m ⁻³	0.0041	Guo and Vanrolleghem (2014)
$K_{I,NH_3,AOB}$	S_{NH_3} substrate inhibition constant for AOB, g N m ⁻³	0.1	Guo and Vanrolleghem (2014)
$K_{NO,AOB}$	S_{NO} affinity constant for AOB, g DO m ⁻³	0.1	Guo and Vanrolleghem (2014)
$K_{HNO_2,AOB}$	S_{HNO_2} affinity constant for AOB, g N m ⁻³	0.00001	Guo and Vanrolleghem (2014)
$K_{I,HNO_2,AOB}$	S_{HNO_2} inhibition constant for AOB, g N m ⁻³	0.001	Guo and Vanrolleghem (2014)

$K_{O_2,AOB}$	S_{O_2} affinity constant for AOB, g DO m ⁻³	0.6	Guo and Vanrolleghem (2014)
$K_{O_2,AOB,den}$	S_{O_2} affinity constant for AOB denitrification, g m ⁻³	2.14	Guo and Vanrolleghem (2014)
$K_{I,O_2,AOB}$	S_{O_2} substrate inhibition parameter, g DO m ⁻³	4.68	Guo and Vanrolleghem (2014)
Model C – AOB NH₂OH/NOH pathway			
Y_{AOB}	Yield coefficient for AOB, g COD g ⁻¹ N	0.150	Law et al. (2012)
$i_{N,AOB}$	Nitrogen content of biomass, g N g ⁻¹ COD	0.07	Law et al. (2012)
$\mu_{AOB,AMO}$	Maximum AMO-mediated reaction rate, h ⁻¹	0.205	Law et al. (2012)
$\mu_{AOB,HAO,1}$	Maximum HAO-mediated reaction rate for NH ₂ OH oxidation, h ⁻¹	0.065	Law et al. (2012)
$\mu_{AOB,HAO,2}$	Maximum HAO-mediated reaction rate for NOH oxidation, h ⁻¹	0.43	Law et al. (2012)
K_{S1,O_2_AOB}	S_{O_2} affinity constant for S_{NH_4} oxidation, g DO m ⁻³	0.4	Law et al. (2012)
K_{S2,O_2_AOB}	S_{O_2} affinity constant for S_{NH_2OH} oxidation, g DO m ⁻³	0.056	Law et al. (2012)
k_{NOH}	Maximum reaction rate for NOH decomposition, h ⁻¹	0.79	Law et al. (2012)
$K_{NH_4,AOB}$	S_{NH_4} affinity constant for AOB, g N m ⁻³	2.4	Law et al. (2012)
$K_{NH_2OH,AOB}$	S_{NH_2OH} affinity constant for AOB, g N m ⁻³	0.7	Law et al. (2012)
$K_{NOH,AOB}$	S_{NOH} affinity constant for AOB, g N m ⁻³	0.7	Law et al. (2012)
Model D – AOB NH₂OH/NO pathway			
Y_{AOB}	Yield coefficient for AOB, g COD g ⁻¹ N	0.150	Ni et al. (2013b)
$i_{N,AOB}$	Nitrogen content of biomass, g N g ⁻¹ COD	0.07	Ni et al. (2013b)
$\mu_{AOB,AMO}$	Maximum AMO-mediated reaction rate, h ⁻¹	0.205	Ni et al. (2013b)
$\mu_{AOB,HAO,1}$	Maximum HAO-mediated reaction rate for NH ₂ OH oxidation, h ⁻¹	0.085	Ni et al. (2013b)
$\mu_{AOB,HAO,2}$	Maximum HAO-mediated reaction rate for NOH oxidation, h ⁻¹	0.567	Ni et al. (2013b)
K_{S1,O_2_AOB}	S_{O_2} affinity constant for S_{NH_4} oxidation, g DO m ⁻³	0.4	Ni et al. (2013b)
K_{S2,O_2_AOB}	S_{O_2} affinity constant for S_{NH_2OH} oxidation, g DO m ⁻³	0.073	Ni et al. (2013b)
η_{AOB}	Anoxic reduction factor	0.285	Ni et al. (2013b)

$K_{NH_4,AOB}$	S_{NH_4} affinity constant for AOB, g N m ⁻³	2.4	Ni et al. (2013b)
$K_{NH_2OH,AOB}$	S_{NH_2OH} affinity constant for AOB, g N m ⁻³	2.4	Ni et al. (2013b)
$K_{NO,AOB}$	S_{NO} affinity constant for AOB, g N m ⁻³	0.0084	Ni et al. (2013b)

Model E – Two-pathway model of AOB

$r_{NH_3,ox}$	Specific maximum ammonia oxidation rate, mmol/(g-VSS*h)	14.75	Ni et al. (2014)
$r_{NH_2OH,ox}$	Specific maximum NH ₂ OH oxidation rate, mmol/(g-VSS*h)	22.86	Ni et al. (2014)
$r_{NO,ox}$	Specific maximum NO oxidation rate, mmol/(g-VSS*h)	22.86	Ni et al. (2014)
$r_{O_2,red}$	Specific maximum oxygen reduction rate, mmol/(g-VSS*h)	48.02	Ni et al. (2014)
$r_{NO_2,red}$	Specific maximum nitrite reduction rate, mmol/(g-VSS*h)	3.06	Ni et al. (2014)
$r_{NO,red}$	Specific maximum NO reduction rate, mmol/(g-VSS*h)	1.6×10 ⁻²	Ni et al. (2014)
K_{O_2,NH_3}	Oxygen affinity constant for ammonia oxidation, mmol-O ₂ /L	1.9×10 ⁻²	Ni et al. (2014)
K_{NH_3}	Ammonia affinity constant for ammonia oxidation, mmol-N/L	1.7×10 ⁻¹	Ni et al. (2014)
K_{NH_2OH}	NH ₂ OH affinity constant for NH ₂ OH oxidation, mmol-N/L	5×10 ⁻²	Ni et al. (2014)
$K_{NO,ox}$	NO affinity constant for NO oxidation, mmol-N/L	6×10 ⁻⁴	Ni et al. (2014)
$K_{O_2,red}$	Oxygen affinity constant for oxygen reduction, mmol-O ₂ /L	1.9×10 ⁻³	Ni et al. (2014)
K_{NO_2}	Nitrite affinity constant for nitrite reduction, mmol-N/L	1×10 ⁻²	Ni et al. (2014)
$K_{NO,red}$	NO affinity constant for NO reduction, mmol-N/L	6×10 ⁻⁴	Ni et al. (2014)
K_{Mox}	S_{Mox} affinity constant for NH ₂ OH or NO oxidation, mmol/g-VSS	1×10 ⁻² ×C _{tot}	Ni et al. (2014)
$K_{Mred,1}$	S_{Mred} affinity constant for ammonia oxidation, mmol/g-VSS	1×10 ⁻³ ×C _{tot}	Ni et al. (2014)
$K_{Mred,2}$	S_{Mred} affinity constant for NO reduction, mmol/g-VSS	1×10 ⁻³ ×C _{tot}	Ni et al. (2014)
$K_{Mred,3}$	S_{Mred} affinity constant for oxygen reduction, mmol/g-VSS	6.9×10 ⁻²	Ni et al. (2014)
$K_{Mred,4}$	S_{Mred} affinity constant for nitrite reduction, mmol/g-VSS	1.9×10 ⁻¹	Ni et al. (2014)
C_{tot}	The sum of S_{Mred} and S_{Mox} , an assumed constant, mmol/g-VSS	1×10 ⁻²	Ni et al. (2014)

Model F – Two-pathway model of AOB

$r_{NH_3,ox}$	Maximum ammonia oxidation rate, mmol/(g-VSS*h)	14.75	Peng et al. (2015a)
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$r_{NH_2OH,ox}$	Maximum NH_2OH oxidation rate, mmol/(g-VSS*h)	22.86	Peng et al. (2015a)
$r_{NOH,ox}$	Maximum NOH oxidation rate, mmol/(g-VSS*h)	13.42	Peng et al. (2015a)
$r_{O_2,red}$	Maximum oxygen reduction rate, mmol/(g-VSS*h)	48.02	Peng et al. (2015a)
$r_{NO_2,red}$	Maximum nitrite reduction rate, mmol/(g-VSS*h)	3.06	Peng et al. (2015a)
$r_{NO,red}$	Maximum NOH decomposition rate, mmol/(mmol*h)	6×10^{-2}	Peng et al. (2015a)
r_{AOB}	Maximum AOB growth rate with CO_2 fixation, mmol/(g-VSS*h)	1.55	Peng et al. (2015a)
K_{O_2,NH_3}	Oxygen affinity constant for ammonia oxidation, mmol- O_2 /L	1.9×10^{-2}	Peng et al. (2015a)
K_{NH_3}	Ammonia affinity constant for ammonia oxidation, mmol-N/L	1.7×10^{-1}	Peng et al. (2015a)
K_{NH_2OH}	NH_2OH affinity constant for NH_2OH oxidation, mmol-N/L	5×10^{-2}	Peng et al. (2015a)
$K_{NOH,ox}$	NOH affinity constant for NOH oxidation, mmol-N/L	5×10^{-2}	Peng et al. (2015a)
$K_{O_2,red}$	Oxygen affinity constant for oxygen reduction, mmol- O_2 /L	1.9×10^{-3}	Peng et al. (2015a)
K_{NO_2}	Nitrite affinity constant for nitrite reduction, mmol-N/L	1×10^{-2}	Peng et al. (2015a)
K_{CO_2}	CO_2 affinity constant for carbon fixation, mmol-C/L	2.35	Peng et al. (2015a)
K_{Mox}	S_{Mox} affinity constant for NH_2OH and NOH oxidation, mmol/g-VSS	2.1×10^{-2}	Peng et al. (2015a)
$K_{Mred,1}$	S_{Mred} affinity constant for ammonia oxidation, mmol/g-VSS	$1 \times 10^{-3} \times C_{tot}$	Peng et al. (2015a)
$K_{Mred,2}$	S_{Mred} affinity constant for oxygen reduction, mmol/g-VSS	6.9×10^{-3}	Peng et al. (2015a)
$K_{Mred,3}$	S_{Mred} affinity constant for nitrite reduction, mmol/g-VSS	8.2×10^{-3}	Peng et al. (2015a)
$K_{Mred,4}$	S_{Mred} affinity constant for cell growth, mmol/g-VSS	$1 \times 10^{-3} \times C_{tot}$	Peng et al. (2015a)
K_{ATP}	S_{ATP} affinity constant for cell growth, mmol/g-VSS	4.4×10^{-3}	Peng et al. (2015a)
K_{ADP}	S_{ADP} affinity constant for ammonia oxidation, mmol/g-VSS	1.44×10^{-2}	Peng et al. (2015a)
$C_{tot,1}$	The sum of S_{Mred} and S_{Mox} , which is a constant, mmol/g-VSS	1×10^{-2}	Peng et al. (2015a)
$C_{tot,2}$	The sum of S_{ADP} and S_{ATP} , which is a constant, mmol/g-VSS	3×10^{-2}	Peng et al. (2015a)
Model G – ASMN of heterotrophic denitrifiers			
μ_H	Maximum specific growth rate, h^{-1}	0.26	Hiatt and Grady (2008)
Y_H	Yield coefficient for heterotrophs, g COD g^{-1} COD	0.6	Hiatt and Grady (2008)

η_Y	Anoxic yield factor, dimensionless	0.9	Hiatt and Grady (2008)
η_{g1}	Anoxic growth factor for nitrate reduction, dimensionless	0.28	Hiatt and Grady (2008)
η_{g2}	Anoxic growth factor for nitrite reduction, dimensionless	0.16	Hiatt and Grady (2008)
η_{g3}	Anoxic growth factor for NO reduction, dimensionless	0.35	Hiatt and Grady (2008)
η_{g4}	Anoxic growth factor for N ₂ O reduction, dimensionless	0.35	Hiatt and Grady (2008)
K_{S1}	Affinity constant for Ss in nitrate reduction, g-COD m ⁻³	20	Hiatt and Grady (2008)
K_{S2}	Affinity constant for Ss in nitrite reduction, g-COD m ⁻³	20	Hiatt and Grady (2008)
K_{S3}	Affinity constant for Ss in NO reduction, g-COD m ⁻³	20	Hiatt and Grady (2008)
K_{S4}	Affinity constant for Ss in N ₂ O reduction, g-COD m ⁻³	40	Hiatt and Grady (2008)
K_{NO3}^{HB}	Affinity constant for nitrate, g N m ⁻³	0.2	Hiatt and Grady (2008)
K_{NO2}^{HB}	Affinity constant for nitrite, g N m ⁻³	0.2	Hiatt and Grady (2008)
K_{NO}^{HB}	Affinity constant for NO, g N m ⁻³	0.05	Hiatt and Grady (2008)
K_{N2O}^{HB}	Affinity constant for N ₂ O, g N m ⁻³	0.05	Hiatt and Grady (2008)
$K_{I,NO,2}$	NO inhibition coefficient for nitrite reduction, g N m ⁻³	0.5	Hiatt and Grady (2008)
$K_{I,NO,3}$	NO inhibition coefficient for NO reduction, g N m ⁻³	0.3	Hiatt and Grady (2008)
$K_{I,NO,4}$	NO inhibition coefficient for N ₂ O reduction, g N m ⁻³	0.075	Hiatt and Grady (2008)

Model H – ASM-ICE of heterotrophic denitrifiers

$r_{COD,max}$	Maximum carbon source oxidation rate, mmol COD/(L*h)	0.34	Pan et al. (2015)
$r_{NO3,max}$	Maximum nitrate reduction rate, mmol NO ₃ ⁻ /(mmol biomass*h)	0.045	Pan et al. (2013b)
$r_{NO2,max}$	Maximum nitrite reduction rate, mmol NO ₂ ⁻ /(mmol biomass*h)	0.059	Pan et al. (2013b)
$r_{NO,max}$	Maximum NO reaction rate, mmol NO/(mmol biomass*h)	0.56	Pan et al. (2013b)
$r_{N2O,max}$	Maximum N ₂ O reaction rate, mmol N ₂ O/(mmol biomass*hour)	0.23	Pan et al. (2013b)

K_S	Affinity constant for Ss, mmol COD/L	0.1	Pan et al. (2013b)
$K_{NO_3}^{HB}$	Affinity constant for nitrate, mmol NO_3^- /L	0.018	Pan et al. (2013b)
$K_{NO_2}^{HB}$	Affinity constant for nitrite, mmol NO_2^- /L	0.0041	Pan et al. (2013b)
K_{NO}^{HB}	Affinity constant for NO, mmol NO/L	0.000011	Pan et al. (2013b)
$K_{N_2O}^{HB}$	Affinity constant for N_2O , mmol N_2O /L	0.0025	Pan et al. (2013b)
K_{Mox}	Affinity constant for S_{Mox} for Ss oxidation, mmol/mmol biomass	0.0001	Pan et al. (2013b)
$K_{Mred,1}$	Affinity constant for S_{Mred} in nitrate reduction, mmol/mmol biomass	0.0046	Pan et al. (2013b)
$K_{Mred,2}$	Affinity constant for S_{Mred} in nitrite reduction, mmol/mmol biomass	0.0004	Pan et al. (2013b)
$K_{Mred,3}$	Affinity constant for S_{Mred} in NO reduction, mmol/mmol biomass	0.00001	Pan et al. (2013b)
$K_{Mred,4}$	Affinity constant for S_{Mred} in N_2O reduction, mmol/mmol biomass	0.0032	Pan et al. (2013b)
Y_H	Yield coefficient for heterotrophs, mmol/mmol	0.5	Pan et al. (2013b)
C_{tot}	Total electron carrier concentration, mmol/mmol biomass	0.01	Pan et al. (2013b)