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Heralded noiseless linear amplification and quantum channels

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The employ of a heralded noiseless linear amplifier has been proven as a useful tool for mitigating imperfections in quantum channels. Its analysis is usually conducted within specific frameworks, for which the set of input states for a given protocol is fixed. Here we obtain a more general description by showing that a noisy and lossy Gaussian channel followed by a heralded noiseless linear amplifier has a general description in terms of effective channels. This has the advantage of offering a simpler mathematical description, best suited for mixed states, both Gaussian and non-Gaussian. We investigate the main properties of this effective system, and illustrate its potential by applying it to loss compensation and reduction of phase uncertainty.

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I. INTRODUCTION

Deterministic phase-insensitive quantum amplifiers, which amplify equally any quadrature of light, are fundamentally limited by quantum physics and must add a minimal amount of quantum noise [1]. A heralded noiseless linear Amplifier (NLA), on the other hand, can in theory achieve a phase-insensitive amplification which does not add any noise, and more surprisingly which does not amplify the quantum noise, but at the expense of a necessarily probabilistic but heralded transformation [2].

Noiseless linear amplification has been actively studied from various perspectives. The first one concerns the implementation of the NLA itself, since a perfect noiseless amplification can only occur with a zero probability of success [3]. However, one can obtain an output with a very high fidelity and nonzero probability if the output state is well approximated within a N dimensional Hilbert space. Increasing this value, and hence the working range of the approximate NLA, inevitably decreases the probability of success. Several methods have been proposed and experimentally realized to implement an approximate NLA [2,4–10]. Some implementations have also been proposed in order to increase the probability of success [11], or to avoid the use of non-Gaussian resources [12,13] when restricted to the amplification of coherent states. The NLA has also been studied from a more abstract point of view [14], and with a focus on optimal design and probability of success [15,16].

The second perspective has focused on the use of the NLA for various applications, such as quantum information protocols or quantum state preparation [17], either considering a perfect NLA as a theoretical limit, or an approximated one. The NLA has for instance been shown to be useful in quantum key distribution, for continuous variable [18–20] as well as discrete variable [21–23]. It can also be used for loss suppression [24,25], Bell-inequality violation [26,27], entanglement distillation [2,28], quantum cloning [29], phase-insensitive squeezing [30], or error correction [31].

A promising result towards a practical use of the NLA is the possibility to implement it virtually, using only postselection [19,20], as experimentally demonstrated for entanglement distillation [32].

Most of the analyses mentioned above start by considering specific protocols, hence addressing a specific class of input states, such as coherent states. This is an effective approach, but clearly lacks generality, in particular when one is interested in using the NLA with non-Gaussian states. In this paper we present a generalization which allows one to describe the NLA acting after a Gaussian channel as an effective channel. The usefulness of such a description is twofold. First, the noiseless amplification from the effective system is usually simpler to compute, especially if the input state is pure, as assumed in most protocols. Second, it gives a physical insight in the transformation produced by the noiseless amplification, and allows one to find new protocols and applications.

The outline is as follows. In the first part, we show that a linear symmetric lossy and noisy Gaussian quantum channel followed by a NLA produces the same transformation as an effective NLA of different gain, followed by an effective linear symmetric Gaussian quantum channel, as shown in Fig. 1. The effective quantum channel is then studied in detail. We analyze some behaviors and physical constraints on the effective parameters, and show that the effective channel can be reduced to a simpler one in some cases.

In the second part, we use those results to present two potential applications of the NLA. We first generalize the results of [24], and show that an exact loss reduction is achievable using the effective system. The second application concerns the "phase concentration" and the signal-to-noise ratio (SNR). We show that the addition of thermal noise can improve both of them, thanks to a nontrivial behavior of the effective parameters.

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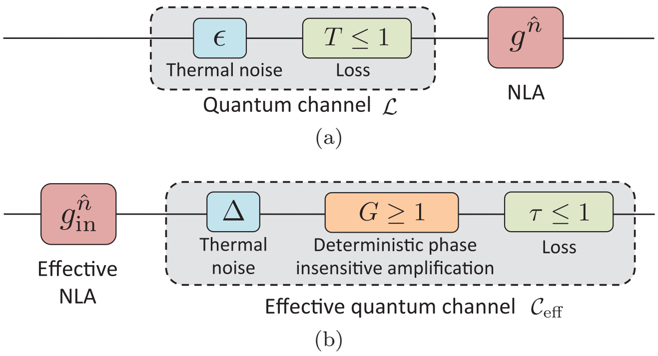

FIG. 1. (Color online) Two equivalent systems, up to a global state-independent normalization factor: (a) NLA placed after a linear symmetric lossy and noisy Gaussian quantum channel. (b) Effective NLA placed before an effective linear symmetric Gaussian channel, composed of input noise, deterministic phase insensitive amplification, and loss. See main text for more details.

II. EFFECTIVE SYSTEM

Let us start by introducing the main ideas leading to the effective system, while leaving the detailed calculation in the Appendixes. We consider a perfect NLA, described by the operator $g^{\hat{n}}$ when the probabilistic implementation is successful. This operator transforms a coherent state $|\alpha\rangle$ to

$$g^{\hat{n}}|\alpha\rangle = e^{\frac{|\alpha|^2}{2}(g^2-1)}|g\alpha\rangle. \tag{1}$$

We stress that any physical amplified state needs to be normalized, and that our approach focuses on a linear state-independent regime which is to be understood as a theoretical limit for any physical implementation of the NLA. In practice, a given physical implementation will act as a NLA only for a limited range of input states, which can however be made arbitrarily large, e.g., by increasing the number of stages in the quantum scissors scheme [2].

The effective system is obtained by computing the output state with two methods, for an arbitrary input state. The first method corresponds to the quantum channel followed by the NLA. The second method corresponds to the effective system, where the input state is first noiselessly amplified, and then sent through an effective quantum channel. By comparing the two outputs, we can get the expressions of the parameters of the effective system such that the two transformations are equal.

A. Effective parameters

Let $\hat{\rho}_{in}$ be an arbitrary quantum state, which we express using the P function [33]:

$$\hat{\rho}_{in} = \int d^2\gamma\, P_{in}(\gamma)|\gamma\rangle\langle\gamma|. \tag{2}$$

Note that P_{in} may in general be ill behaved for nonclassical states, including squeezed and non-Gaussian states; however, we will not need its explicit expression, but simply use the linearity of the transformations in the coherent states basis to obtain the expression of the effective system.

1. Output state after the initial channel and the NLA

The action of the initial channel $\mathcal{L}$ on the input state $\hat{\rho}_{in}$ can be described by a linear quantum operation $\mathfrak{L}$, which transforms a coherent state of mean amplitude γ to a thermal state of parameter λ_{ch} and mean amplitude $\sqrt{T}\gamma$. As shown in Appendix A, the action of a NLA on such a displaced thermal state produces another displaced thermal state

$$\hat{\sigma}(\gamma) = \hat{D}(\tilde{g}\sqrt{T}\gamma)\hat{\rho}_{th}(g\lambda_{ch})\hat{D}^{\dagger}(\tilde{g}\sqrt{T}\gamma) \tag{3}$$

of parameter $g\lambda_{ch}$ and mean amplitude $\tilde{g}\sqrt{T}\gamma$, where the gain $\tilde{g}$ is given by

$$\tilde{g} = g\frac{1-\lambda_{ch}^2}{1-g^2\lambda_{ch}^2}. \tag{4}$$

This allows us to obtain the output state $\hat{\rho}_{out}^{NLA}$ produced by the system depicted in Fig. 1(a),

$$\hat{\rho}_{out}^{NLA} \propto \int d^2\gamma\, P_{in}(\gamma)\hat{\sigma}(\gamma)e^{|\gamma|^2 T\frac{(g^2-1)(1-\lambda_{ch}^2)}{1-g^2\lambda_{ch}^2}}. \tag{5}$$

2. Output state after the effective system

We now consider the case depicted in Fig. 1(b), where a NLA of gain g_{in} is directly applied to the input state $\hat{\rho}_{in}$. Using again the decomposition (2), the action of this NLA on a coherent state $|\gamma\rangle\langle\gamma|$ can be directly obtained from (1). In order to obtain the same exponential factor as in (5), g_{in} needs to satisfy

$$g_{in}^2 - 1 = T\frac{(g^2-1)\left(1-\lambda_{ch}^2\right)}{1-g^2\lambda_{ch}^2}. \tag{6}$$

We seek for a more general channel $\mathcal{C}_{eff}$ after the effective NLA, as depicted in Fig. 1(b). In the most general case, a deterministic linear symmetric Gaussian channel is composed of three elements: an addition of thermal noise Δ at its input, a deterministic phase-insensitive amplifier of intensity gain $G \geqslant 1$, limited to the quantum noise, and a noiseless lossy channel of transmission $\tau \leqslant 1$. As discussed below, the effective channel can be reduced to an addition of input noise followed by loss or by a deterministic amplification; however, we consider those two elements here to stay in a more general case.

As shown in Appendix B, an amplified coherent state $|g_{in}\gamma\rangle\langle g_{in}\gamma|$ is therefore also transformed to a displaced thermal state

$$\hat{\sigma}_{eff}(\gamma) = \hat{D}(g_{in}\sqrt{\tau G}\gamma)\hat{\rho}_{th}\left(\lambda_{ch}^{g}\right)\hat{D}^{\dagger}(g_{in}\sqrt{\tau G}\gamma) \tag{7}$$

of parameter λ_{ch}^{g} and mean amplitude $g_{in}\sqrt{\tau G}\gamma$, leading to the output state

$$\hat{\rho}_{out}^{eff} = \int d^2\gamma\, P_{in}(\gamma)\hat{\sigma}_{eff}(\gamma)e^{(g_{in}^2-1)|\gamma|^2}. \tag{8}$$

The output states (5) and (8) will be proportional, with a state-independent factor, if the condition (6) is satisfied, and if $\hat{\sigma}(\gamma)$ and $\hat{\sigma}_{eff}(\gamma)$ are equal, that is if

$$g_{in}\sqrt{\tau G} = \tilde{g}\sqrt{T}, \tag{9}$$

$$\lambda_{ch}^{g} = g\lambda_{ch}. \tag{10}$$

The resolution of this set of equations gives the following effective parameters:

$$g_{\text{in}} = \sqrt{\frac{2 + (g^2 - 1)(2 - \epsilon)T}{2 - (g^2 - 1)\epsilon T}}, \tag{11}$$

$$\tau G = \frac{g^2 T}{1 + (g^2 - 1)T\left[\frac{1}{4}(g^2 - 1)(\epsilon - 2)\epsilon T - \epsilon + 1\right]} := \eta, \tag{12}$$

$$\Delta = \frac{2}{G} + \frac{2 - \epsilon}{2}[(g^2 - 1)T\epsilon - 2]. \tag{13}$$

B. Properties of the effective channel

Let us first comment on some properties of the effective channel. First, there is only a condition on the product $\tau G = \eta$, and not on τ and G separately. The input noise Δ also depends on G, since when G increases, for given values of η and of the output noise, more noise is added by the deterministic amplification, and hence less input noise is needed.

1. Added noise

There is a channel degeneracy: several combinations (Δ, G, τ) can be equivalent to the same initial channel $\mathcal{L}$ followed by the real NLA. Indeed, a state of variance V is transformed to an output state of variance [34]

$$V_{\text{out}} = \tau[G(V + \Delta) + (G - 1)] + 1 - \tau, \tag{14a}$$

$$= \tau G\left(V + \Delta + \frac{G - 1}{G} + \frac{1 - \tau}{\tau G}\right), \tag{14b}$$

and one can define a *total* added noise at the input

$$\chi_{\text{tot}} = \Delta + \chi_{\text{ch}}, \tag{15}$$

composed of the input noise Δ, and of the noise due to the deterministic amplification and to the loss:

$$\chi_{\text{ch}} = \frac{G-1}{G} + \frac{1 - \tau}{\tau G} = \frac{\tau(G - 2) + 1}{\tau G}. \tag{16}$$

We stress that g_{in} defined by (11) does not depend on the choice of $\mathcal{C}_{\text{eff}}$, as well as χ_{tot}:

$$\chi_{\text{tot}} = \frac{1}{g^2 T} + \frac{\epsilon[4 - (g^4 - 1)T(\epsilon - 2)] - 4}{4g^2}. \tag{17}$$

2. Three kinds of effective channels

Using the effective channel degeneracy, one can find the simplest one, depending on the value of η as follows.

$\eta \leqslant 1$. One can set $G = 1$ and $\tau = \eta$. In that case, one recovers the effective parameters of [18], and the effective channel $\mathcal{C}_{\text{eff}}$ is composed of a lossy channel of transmission η, with an input noise $\Delta_{G=1} = \epsilon^g$ and $\chi_{\text{ch}} = \frac{1-\eta}{\eta}$.

$\eta = 1$. One can set $G = \tau = 1$, and the effective channel $\mathcal{C}_{\text{eff}}$ is simply composed of an input noise addition $\Delta_{G=1} = \epsilon^g$.

$\eta \geqslant 1$. One can set $\tau = 1$ and $G = \eta$. In that case, the effective channel $\mathcal{C}_{\text{eff}}$ is composed of a deterministic phase insensitive amplifier of gain $G = \eta$, with an input noise Δ and $\chi_{\text{ch}} = \frac{\eta-1}{\eta}$.

We stress that the effective parameters are obtained by a general method without involving any normalization; hence they are independent of the input state. The equivalence shown is this paper is also still valid if the input state has several modes, with one sent through the channel.

C. Properties of the effective parameters

Since the perfect NLA is theoretically described by an unbounded operator, it can lead to nonphysical amplified states. For the same reason, it can lead to nonphysical effective parameters when the gain of the real NLA is too large. Thus the following constraints must be satisfied: the effective gain must be real and nondivergent; each displaced thermal state given by (3) must not diverge; the global transmission η must not diverge; and the input noise Δ must be positive.

Remarkably, each of all those constraints leads to the same single condition on g, given by

$$g < g_{\text{lim}} = \sqrt{1 + \frac{2}{T\epsilon}}. \tag{18}$$

As long as (18) is satisfied, the effective parameters have a physical meaning. However, one has to be careful that this does not ensure that the amplified output state will be physical, as this depends on the input state. One can also define the maximum amount of noise for a given gain of the NLA from g_{lim}:

$$\epsilon_{\text{lim}} = \frac{2}{(g^2 - 1)T}. \tag{19}$$

When the channel is noiseless, i.e., when $\epsilon = 0$, g is not constrained by the effective channel, as pointed out by several prior studies (see, e.g., Ref. [2]).

As shown in [18], η is smaller than 1 as long as g is smaller than a value g_{max} which depends on T and ϵ. It is straightforward to see that g_{max} is always smaller than g_{lim}, and therefore the physicality constraints are always fulfilled if the effective channel is restricted to a noisy and lossy channel.

Let us now highlight an important property of those effective parameters, coming from the fact that we consider the global transformation composed of the initial quantum channel and the NLA. Generally speaking, they increase with all parameters g, T, or ϵ. In particular, as soon as $\epsilon > 0$, g_{max} will not be infinite and there will be a value of g such that $\eta = 1$. On the contrary, for a fixed value of g, the value of η increases with ϵ. By adding thermal noise on purpose, it is thus possible to convert the initial channel to a lossless channel with $\eta = 1$, for any gain g of the NLA greater than 1. Naturally, the smaller the gain g, the greater the noise to add. This property will be analyzed in the next section.

III. APPLICATION TO QUANTUM COMMUNICATIONS

In this section, we present two applications of our results, for loss suppression and phase concentration. We note that we can also recover the results of [18], since when the input state is an EPR state of parameter λ, the effective NLA transforms it to another EPR state of parameter $g_{\text{in}}\lambda$. If the gain of the NLA is smaller than g_{max}, we can use the effective channel degeneracy and set $G = 1$.

A. Loss suppression

Mičuda *et al.* have introduced the concept of noiseless attenuation, which allows one to reduce the loss from a channel, when used with a NLA of appropriate gain [24]. This attenuator is a NLA of gain $\nu < 1$, which can be implemented by sending the state to be attenuated through a beam splitter of amplitude transmission ν, and conditioning on the vacuum for the reflected mode.

The principle of their protocol is the following: the initial state is first noiselessly attenuated with a factor ν. It is then sent through the quantum channel, assumed noiseless in [24], which reduces its amplitude by $\sqrt{T}$. Finally, the state is noiselessly amplified with a NLA of gain $g = \frac{1}{\nu\sqrt{T}}$. In the limit $\nu \to 0$, the protocol tends to the identity operation, and the input state does not undergo any loss. On the contrary, for a nonzero ν, the output state is also "contaminated" by noisy terms.

Apart from the fact that $\nu = 0$ corresponds to an infinite value of g, and hence a zero probability of success, this suppression of loss does not take a simple form for $\nu > 0$. The results of [24] are also valid only for a noiseless channel. Using the equivalent system presented in this paper, the generalization of loss suppression is straightforward not only for a nonzero ν, but also for a noisy channel. As shown below, by using the appropriate gain for the noiseless attenuation, it is possible to exactly reduce loss, even if this gain does not tend to zero.

Indeed, we have shown that a NLA after a quantum channel $\mathcal{L}$ is equivalent to an effective NLA of gain g_{in} before an effective channel $\mathcal{C}_{\text{eff}}$. Therefore, an attenuator of gain $1/g_{\text{in}}$ completely compensates the action of the effective NLA, since

$$(1/g_{\text{in}})^{\hat{n}} g_{\text{in}}^{\hat{n}} = \hat{\mathbb{I}}. \tag{20}$$

There remains only the effective channel $\mathcal{C}_{\text{eff}}$, as depicted in Fig. 2.

For a noiseless channel ($\epsilon = 0$), the effective gain is given by $g_{\text{in}} = \sqrt{1+(g^2-1)T}$, and the effective parameter η, given by

$$\eta = \frac{g^2 T}{1+(g^2-1)T}, \tag{21}$$

always satisfies $T \leqslant \eta \leqslant 1$ for $g \geqslant 1$. One can thus *exactly* obtain a channel with smaller loss, using an attenuator of gain

$$\nu = \frac{1}{g_{\text{in}}} = \frac{1}{\sqrt{1+(g^2-1)T}}. \tag{22}$$

For a gain $g \gg 1$, Eq. (22) becomes

$$\nu \simeq \frac{1}{g\sqrt{T}}, \tag{23}$$

which corresponds to the gain used in [24]. We see here that using a gain (22) instead always leads to an exact channel with lower loss for any value of g.

When the initial channel is noisy, the effective channel can always be transformed to a lossless channel with $\eta = 1$. For a given value of g, this can be achieved by adding some noise between the attenuator and the channel, which allows one to fully suppress loss, with a finite gain, but at the price of having more noise.

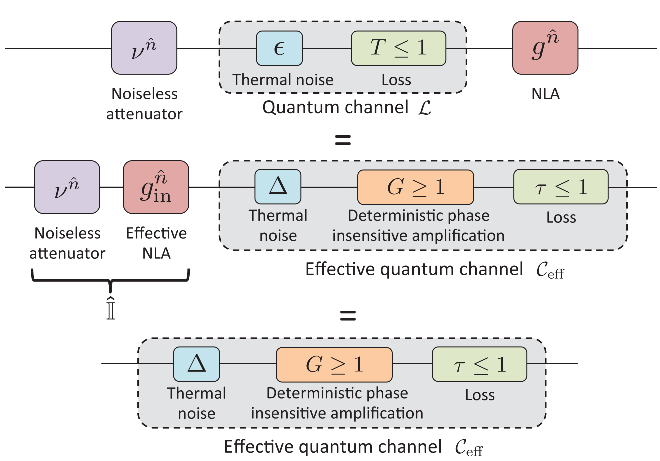

FIG. 2. (Color online) Loss suppression using a noiseless attenuator. For a noisy channel, a perfect loss suppression can be achieved with a finite gain g.

B. Phase concentration and SNR augmentation

The experimental implementation of a NLA is very demanding on resources, especially on single photon for most of the schemes. A protocol proposed by Marek and Filip [12] allows a particularly simple setup, as experimentally demonstrated by the group of Andersen [10]. The principle is the following: the coherent state is randomly displaced around its mean value, which corresponds to thermal noise addition. A photon is then subtracted from the noisy state. Although this scheme does not strictly produce an amplified coherent state, the photon subtraction will "select" high amplitudes $|\beta\rangle\langle\beta|$ with a weight $|\beta|^2$, leading to a reduction of the phase variance, hence the appellation of phase concentration.

Following the same idea, but replacing the photon subtraction by a NLA, high amplitude coherent states will also be selected, but with an exponential factor $\exp[(g^2-1)|\beta|^2]$. For a NLA of given gain g, it thus appears that adding noise before the noiseless amplification can also lead to phase concentration. We use a simple criteria to define the phase uncertainty ϕ by

$$\tan\frac{\phi}{2} = \frac{\text{standard deviation}}{\text{mean amplitude}} = \frac{\sqrt{\frac{1+g^2\lambda_{\text{ch}}^2}{1-g^2\lambda_{\text{ch}}^2}}}{2\tilde{g}\alpha}, \tag{24}$$

as shown in Fig. 3(a). This quantity therefore corresponds to the inverse of the square root of the output SNR, and can be easily calculated using the equivalent system, since the noise addition corresponds to a channel with $T = 1$. Note that there is no loss in the effective channel $\mathcal{C}_{\text{eff}}$ in that case ($\tau = 1$), since $\eta > 1$ as soon as $T = 1$ and $\epsilon \neq 0$. In that picture, the effective NLA transforms the initial coherent state of mean amplitude α to a coherent state of mean amplitude $g_{\text{in}}\alpha$. The effective channel then degrades the SNR by adding the equivalent input noise $\chi_{\text{tot}} = \Delta_{\tau=1} + \frac{\eta-1}{\eta}$ (15). The phase uncertainty (24) is

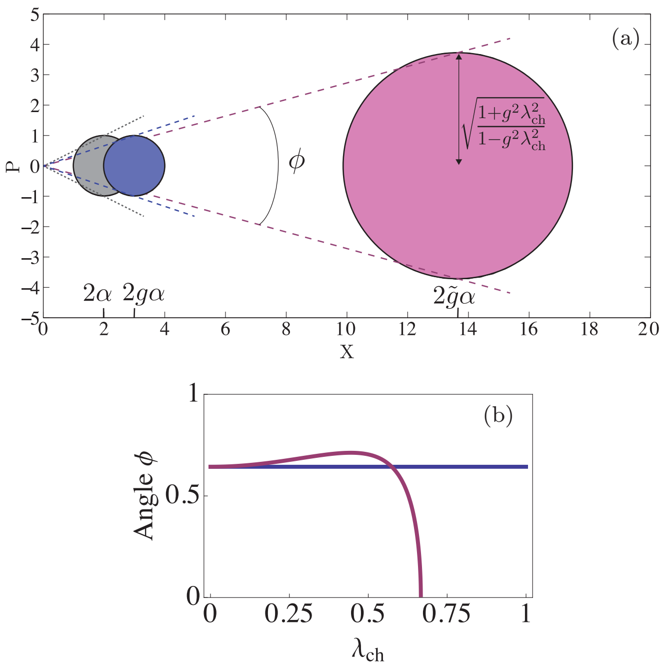

FIG. 3. (Color online) Phase concentration by adding noise before a noiseless amplification. (a) Example of Wigner functions (with one standard deviation of radius) of the initial coherent state with $\alpha = 1$, of the amplified state obtained with a NLA of gain $g = 1.5$, and of the amplified state when thermal noise is added before the NLA. The values of the quadratures X and P are dimensionless. (b) Angle ϕ (in rad) as defined by (24). The input coherent state has an amplitude $\alpha = 1$, and the NLA has a gain $g = 1.5$. Note that the factor 2 comes from the convention used for N_0. The blue curve is without thermal noise, and the pink curve is for a thermal noise ϵ defined by $\lambda_{ch}^2 = \frac{\epsilon}{2+\epsilon}$. The parameter λ_{ch} is dimensionless.

therefore also given by

$$\tan\frac{\phi}{2} = \frac{\sqrt{1+\chi_{tot}}}{2g_{in}\alpha}. \quad (25)$$

Figure 3(b) shows that ϕ can be theoretically reduced to an arbitrarily low value with the noise addition. The parameter λ_{ch} goes to the maximal corresponding value ϵ_{lim}. From this result, we can also conclude that the SNR can be arbitrarily increased, for a NLA of fixed gain, by adding thermal noise to the state.

Note also that the importance of thermal noise with noiseless amplification was also observed in [18], since the NLA does not improve the key rate when the channel has loss only.

IV. DISCUSSION AND CONCLUSION

We have discussed a general equivalence when a NLA is used after a noisy and lossy quantum channel, and shown that the transformation is equal to an effective NLA followed by an effective channel, up to a state-independent proportionality factor. This equivalence is valid regardless of the nature or the input state, which may be non-Gaussian, or part of a multimode state.

Using this picture, we have analyzed several applications: when a suitable noiseless attenuation is used before the quantum channel, it is possible to obtain an exact effective quantum channel with smaller loss, but with larger noise if the initial noise is nonzero. Increasing the gain of the NLA, or deliberately adding noise before the quantum channel (and after the noiseless attenuation) can lead to a perfectly noisy lossless quantum channel. We have also shown that this noise addition can be used to reduce the phase uncertainty of input coherent states.

As shown with those two applications, our results not only allow for a simpler calculation of the amplified states, but they also provide a detailed physical explanation, which is likely to be useful for future applications.

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APPENDIX A: AMPLIFICATION AFTER THE QUANTUM CHANNEL

In this Appendix, we detail the derivation of the effective system. Let us begin by studying the action of a NLA placed after a linear symmetric lossy and noisy Gaussian quantum channel $\mathcal{L}$, as pictured on Fig. 1(a). This channel has a transmission T, and an input noise ϵ. For the sake of simplicity, one can consider that such a channel is composed of the addition of thermal noise ϵ at its input, followed by a lossy noiseless channel of transmission T. An input state having a quadrature variance V is thus transformed to a state of variance $T(V+\epsilon)+1-T$.

We associate an operation $\mathfrak{L}$ to this quantum channel. Since the amplification of a coherent state is simply given by (1), the P function is a very useful tool to compute the amplification of an arbitrary state.

1. Action of $\mathcal{L}$

Let us consider an arbitrary quantum state given by (2). Using the linearity of $\mathfrak{L}$, the output state of the channel, before the NLA, is given by

$$\hat{\rho}_{out} = \mathfrak{L}[\hat{\rho}_{in}] = \int d^2\gamma\ P_{in}(\gamma)\mathfrak{L}[|\gamma\rangle\langle\gamma|]. \quad (A1)$$

The NLA then produces an (unnormalized) amplified state $\hat{\rho}_{out}^{NLA}$:

$$\hat{\rho}_{out}^{NLA} = g^{\hat{n}}\hat{\rho}_{out}g^{\hat{n}} \quad (A2a)$$

$$= \int d^2\gamma\ P_{in}(\gamma)g^{\hat{n}}\mathfrak{L}[|\gamma\rangle\langle\gamma|]g^{\hat{n}}. \quad (A2b)$$

Therefore, due to the channel linearity, it is sufficient to know the evolution of a coherent state $|\gamma\rangle\langle\gamma|$ in order to obtain the evolution of an arbitrary state. The transformation of a coherent state by the lossy and noisy channel is trivial: first, the mean amplitude γ is transformed to $\sqrt{T}\gamma$. Then, the variance of the quadratures is transformed to $T(1+\epsilon)+1-T=1+T\epsilon$. Since the channel is assumed to be symmetric and Gaussian, the state $\mathfrak{L}[|\gamma\rangle\langle\gamma|]$ is therefore a thermal state $\hat{\rho}_{\text{th}}(\lambda_{\text{ch}})$ displaced by $\sqrt{T}\gamma$:

$$\mathfrak{L}[|\gamma\rangle\langle\gamma|]=\hat{D}(\sqrt{T}\gamma)\hat{\rho}_{\text{th}}(\lambda_{\text{ch}})\hat{D}^{\dagger}(\sqrt{T}\gamma). \tag{A3}$$

The parameter λ_{ch} is such that the variance of $\hat{\rho}_{\text{th}}(\lambda_{\text{ch}})$ equals $1+T\epsilon$, which gives

$$\lambda_{\text{ch}}^2=\frac{T\epsilon}{2+T\epsilon}. \tag{A4}$$

2. Amplification of a displaced thermal state

In order to compute the action of the NLA, one can also express the displaced thermal state $\mathfrak{L}[|\gamma\rangle\langle\gamma|]$ using the P function:

$$\mathfrak{L}[|\gamma\rangle\langle\gamma|]=\int d^2\alpha\, P_\gamma(\alpha)|\alpha\rangle\langle\alpha|. \tag{A5}$$

As shown in [18], $P_\gamma(\alpha)$ can be expressed as

$$P_\gamma(\alpha)=P_{\gamma_x}(\alpha_x)P_{\gamma_y}(\alpha_y), \tag{A6}$$

where

$$P_{\gamma_x}(\alpha_x)=\frac{1}{\sqrt{\pi}}\sqrt{\frac{1-\lambda_{\text{ch}}^2}{\lambda_{\text{ch}}^2}}\,e^{-\frac{1-\lambda_{\text{ch}}^2}{\lambda_{\text{ch}}^2}(\alpha_x-\sqrt{T}\gamma_x)^2}, \tag{A7a}$$

$$P_{\gamma_y}(\alpha_y)=\frac{1}{\sqrt{\pi}}\sqrt{\frac{1-\lambda_{\text{ch}}^2}{\lambda_{\text{ch}}^2}}\,e^{-\frac{1-\lambda_{\text{ch}}^2}{\lambda_{\text{ch}}^2}(\alpha_y-\sqrt{T}\gamma_y)^2}. \tag{A7b}$$

Using again the linearity of the NLA, the amplification of a displaced thermal state is given by

$$g^{\hat{n}}\mathfrak{L}[|\gamma\rangle\langle\gamma|]g^{\hat{n}}=\int d^2\alpha\, P_\gamma(\alpha)g^{\hat{n}}|\alpha\rangle\langle\alpha|g^{\hat{n}} \tag{A8a}$$

$$=\int d^2\alpha\, P_\gamma(\alpha)e^{(g^2-1)|\alpha|^2}|g\alpha\rangle\langle g\alpha|. \tag{A8b}$$

Then, the change of variable $u=g\alpha=u_x+iu_y$ gives $d^2\alpha=d^2u/g^2$, and

$$g^{\hat{n}}\mathfrak{L}[|\gamma\rangle\langle\gamma|]g^{\hat{n}}=\int\frac{1}{g^2}d^2u\, P_\gamma(u/g)e^{\frac{g^2-1}{g^2}|u|^2}|u\rangle\langle u|. \tag{A9}$$

As before, one can separate the variables u_x and u_y. We now focus on u_x, the results being similar for u_y. We first highlight that

$$\frac{1}{g}P_{\gamma_x}(u_x/g)e^{\frac{g^2-1}{g^2}u_x^2} \tag{A10a}$$

$$=\frac{1}{g}\frac{1}{\sqrt{\pi}}\sqrt{\frac{1-\lambda_{\text{ch}}^2}{\lambda_{\text{ch}}^2}}\,e^{-\frac{1-\lambda_{\text{ch}}^2}{\lambda_{\text{ch}}^2}(\frac{u_x}{g}-\sqrt{T}\gamma_x)^2+\frac{g^2-1}{g^2}u_x^2} \tag{A10b}$$

$$=\sqrt{\frac{1-\lambda_{\text{ch}}^2}{1-g^2\lambda_{\text{ch}}^2}}\frac{1}{\sqrt{\pi}}\sqrt{\frac{1-g^2\lambda_{\text{ch}}^2}{g^2\lambda_{\text{ch}}^2}}\,e^{-\frac{1-\lambda_{\text{ch}}^2}{\lambda_{\text{ch}}^2}(\frac{u_x}{g}-\sqrt{T}\gamma_x)^2+\frac{g^2-1}{g^2}u_x^2}. \tag{A10c}$$

The argument of the exponential can be easily put in the form

$$-\frac{1-\lambda_{\text{ch}}^2}{\lambda_{\text{ch}}^2}\left(\frac{u_x}{g}-\sqrt{T}\gamma_x\right)^2+\frac{g^2-1}{g^2}u_x^2 \tag{A11}$$

$$=\underbrace{-\frac{1-g^2\lambda_{\text{ch}}^2}{g^2\lambda_{\text{ch}}^2}}_{\substack{\text{Thermal state}\\ \text{of parameter } g\lambda_{\text{ch}}}}\left(u_x-\sqrt{T}\gamma_x\,\underbrace{g\frac{1-\lambda_{\text{ch}}^2}{1-g^2\lambda_{\text{ch}}^2}}_{\text{Gain}}\right)^2 \tag{A12}$$

$$\underbrace{+T\gamma_x^2\frac{(g^2-1)(1-\lambda_{\text{ch}}^2)}{1-g^2\lambda_{\text{ch}}^2}}_{\substack{\text{Normalization term}\\ \text{independent of } u_x}}. \tag{A13}$$

Apart from the normalization term, one easily recognizes the signature of a thermal state of parameter $g\lambda_{\text{ch}}$ and of variance

$$\frac{1+g^2\lambda_{\text{ch}}^2}{1-g^2\lambda_{\text{ch}}^2}=\frac{2+T\epsilon(1+g^2)}{2+T\epsilon(1-g^2)}, \tag{A14}$$

displaced by $g\frac{1-\lambda_{\text{ch}}^2}{1-g^2\lambda_{\text{ch}}^2}\sqrt{T}\gamma_x$. The NLA thus amplifies the mean amplitude of the state with a gain

$$\tilde{g}=g\frac{1-\lambda_{\text{ch}}^2}{1-g^2\lambda_{\text{ch}}^2} \tag{A15}$$

greater than g, since $g\lambda_{\text{ch}}$ must remain smaller than 1 for the amplified state to be physical.

In conclusion, the (unnormalized) amplification of a displaced thermal state is given by

$$g^{\hat{n}}\mathfrak{L}[|\gamma\rangle\langle\gamma|]g^{\hat{n}}=\hat{D}(\tilde{g}\sqrt{T}\gamma)\hat{\rho}_{\text{th}}(g\lambda_{\text{ch}})\hat{D}^{\dagger}(\tilde{g}\sqrt{T}\gamma)\times\left(\frac{1-\lambda_{\text{ch}}^2}{1-g^2\lambda_{\text{ch}}^2}\right)e^{T|\gamma|^2\frac{(g^2-1)(1-\lambda_{\text{ch}}^2)}{1-g^2\lambda_{\text{ch}}^2}}. \tag{A16}$$

Finally, by inserting (A16) in (A2b), the amplified state produced by the system depicted in Fig. 1(a) is given by

$$\hat{\rho}_{\text{out}}^{\text{NLA}}=\left(\frac{1-\lambda_{\text{ch}}^2}{1-g^2\lambda_{\text{ch}}^2}\right)\int d^2\gamma\, P_{\text{in}}(\gamma)\hat{\sigma}(\gamma)e^{|\gamma|^2T\frac{(g^2-1)(1-\lambda_{\text{ch}}^2)}{1-g^2\lambda_{\text{ch}}^2}}, \tag{A17}$$

where

$$\hat{\sigma}(\gamma)=\hat{D}(\tilde{g}\sqrt{T}\gamma)\hat{\rho}_{\text{th}}(g\lambda_{\text{ch}})\hat{D}^{\dagger}(\tilde{g}\sqrt{T}\gamma). \tag{A18}$$

APPENDIX B: EFFECTIVE SYSTEM

1. Amplification by the effective NLA

The effective channel $\mathcal{C}_{\text{eff}}$ following the effective NLA is described by an operation $\mathfrak{L}_g$. As explained in the main text, we look for parameters g_{in}, Δ, G, and τ such that

$$g^{\hat{n}}\mathfrak{L}[\hat{\rho}_{\text{in}}]g^{\hat{n}}=\mu\,\mathfrak{L}_g\big[g_{\text{in}}^{\hat{n}}\hat{\rho}_{\text{in}}g_{\text{in}}^{\hat{n}}\big], \tag{B1}$$

where $g_{\text{in}}^{\hat{n}}$ is the operator associated to the effective NLA, and μ is a constant factor, independent of $\hat{\rho}_{\text{in}}$.

Let us start by writing the noiseless amplification of $\hat{\rho}_{\text{in}}$, using the P function:

$$g_{\text{in}}^{\hat{n}}\hat{\rho}_{\text{in}}g_{\text{in}}^{\hat{n}} = \int d^2\gamma\ P_{\text{in}}(\gamma)g_{\text{in}}^{\hat{n}}|\gamma\rangle\langle\gamma|g_{\text{in}}^{\hat{n}} \tag{B2a}$$

$$= \int d^2\gamma\ P_{\text{in}}(\gamma)|g_{\text{in}}\gamma\rangle\langle g_{\text{in}}\gamma|e^{(g_{\text{in}}^2-1)|\gamma|^2}. \tag{B2b}$$

2. Output state after the effective channel

Since $\mathcal{C}_{\text{eff}}$ is a symmetric and Gaussian channel, a coherent state $|g_{\text{in}}\gamma\rangle$ is simply transformed to a state of mean amplitude $g_{\text{in}}\sqrt{\tau G}\gamma$, with a variance

$$V_{\text{out}} = \tau[G(1+\Delta)+G-1]+1-\tau \tag{B3a}$$

$$= 1+\tau G\Delta+2\tau(G-1) \tag{B3b}$$

for both quadratures. It can thus be written as a displaced thermal state $\hat{\sigma}_{\text{eff}}(\gamma) = \hat{D}(g_{\text{in}}\sqrt{\tau G}\gamma)\hat{\rho}_{\text{th}}(\lambda_{\text{ch}}^g)\hat{D}^\dagger(g_{\text{in}}\sqrt{\tau G}\gamma)$, where λ_{ch}^g is such that $V_{\text{out}} = \frac{1+(\lambda_{\text{ch}}^g)^2}{1-(\lambda_{\text{ch}}^g)^2}$, which gives

$$\lambda_{\text{ch}}^g = \sqrt{\frac{\tau(\Delta G+2G-2)}{\Delta\tau G+2\tau G+2(1-\tau)}}. \tag{B4}$$

After the effective quantum channel $\mathcal{C}_{\text{eff}}$, Eq. (B2b) finally becomes

$$\mathfrak{L}_g\big[g_{\text{in}}^{\hat{n}}\hat{\rho}_{\text{in}}g_{\text{in}}^{\hat{n}}\big] = \int d^2\gamma\ P_{\text{in}}(\gamma)\hat{\sigma}_{\text{eff}}(\gamma)e^{(g_{\text{in}}^2-1)|\gamma|^2}. \tag{B5}$$

3. Conditions for the effective parameters

Comparing the states (A17) and (B5), one can identify a set of equations for the effective parameters. The first condition is given by comparing the exponential factors:

$$g_{\text{in}}^2-1 = T\frac{(g^2-1)\big(1-\lambda_{\text{ch}}^2\big)}{1-g^2\lambda_{\text{ch}}^2}. \tag{B6}$$

The second and third conditions are given by comparing the displaced thermal states $\hat{\sigma}(\gamma)$ and $\hat{\sigma}_{\text{eff}}(\gamma)$, and by imposing the same mean amplitude and variance:

$$g_{\text{in}}\sqrt{\tau G} = \tilde{g}\sqrt{T}, \tag{B7}$$

$$\lambda_{\text{ch}}^g = g\lambda_{\text{ch}}. \tag{B8}$$

One can easily solve this system of equations, obtaining the following effective parameters:

$$g_{\text{in}} = \sqrt{\frac{2+(g^2-1)(2-\epsilon)T}{2-(g^2-1)\epsilon T}}, \tag{B9}$$

$$\tau G = \frac{g^2T}{1+(g^2-1)T\big[\frac{1}{4}(g^2-1)(\epsilon-2)\epsilon T-\epsilon+1\big]} := \eta, \tag{B10}$$

$$\Delta = \frac{2}{G}+\frac{2-\epsilon}{2}[(g^2-1)T\epsilon-2]. \tag{B11}$$

The constant factor μ is given by

$$\mu = \frac{1-\lambda_{\text{ch}}^2}{1-g^2\lambda_{\text{ch}}^2}, \tag{B12}$$

which is independent of the input state $\hat{\rho}_{\text{in}}$.

4. Verification without using the P function

The effective system has been obtained using a P function decomposition, which provides interesting insights on the transformation and a convenient way to perform the calculations. Here we provide a different proof of our results which does not rely on the P function. This excludes that difficulties arise when using nonregular P functions.

As in the main part of this paper, we consider an arbitrary input state $\hat{\rho}_{\text{in}}$. The verification will be performed for a single-mode state, but can be easily generalized to multimode states. We begin by inserting two times the closure relation with coherent states $1/\pi\int d^2\gamma|\gamma\rangle\langle\gamma| = \mathbb{I}$,

$$\hat{\rho}_{\text{in}} = \frac{1}{\pi^2}\int d^2\gamma\, d^2\beta\langle\gamma|\hat{\rho}_{\text{in}}|\beta\rangle|\gamma\rangle\langle\beta|. \tag{B13}$$

Using the linearity of the transformations, one can restrict the analysis to the evolution of a term $|\gamma\rangle\langle\beta|$ separately.

In the following, we show that the direct and the effective systems produce the same transformation of this general term, that is, as given by (B1),

$$g^{\hat{n}}\mathfrak{L}[|\gamma\rangle\langle\beta|]g^{\hat{n}} = \mu\,\mathfrak{L}_g\big[g_{\text{in}}^{\hat{n}}|\gamma\rangle\langle\beta|g_{\text{in}}^{\hat{n}}\big]. \tag{B14}$$

a. Direct system

Let us first consider the direct system corresponding to the left-hand side of the equation above. For the sake of simplicity, we model the Gaussian quantum channel with a slightly different but equivalent notation. An input state is first attenuated with pure loss $\sqrt{T}$ and a thermal noise of variance $T\epsilon$ is then added to its quadratures. The pure loss produces the transformation

$$|\gamma\rangle\langle\beta| \to |\sqrt{T}\gamma\rangle\langle\sqrt{T}\beta|e^{-\frac{1}{2}(1-T)|\gamma-\beta|^2}e^{i(1-T)\Im[\beta^*\gamma]}$$

$$:= \hat{\sigma}(\gamma,\beta,T). \tag{B15}$$

The addition of thermal noise or variance $T\epsilon$ is then described as a random displacement $\hat{D}(\alpha)$ in the phase space [35]:

$$\hat{\sigma}(\gamma,\beta,T) \to \mathfrak{L}[|\gamma\rangle\langle\beta|]$$

$$= \frac{1}{\pi\epsilon_{\text{ch}}}\int d^2\alpha\, e^{-\frac{|\alpha|^2}{\epsilon_{\text{ch}}}}\hat{D}(\alpha)\hat{\sigma}(\gamma,\beta,T)\hat{D}^\dagger(\alpha), \tag{B16}$$

where $\epsilon_{\text{ch}} = T\epsilon/2$. We finally apply the NLA $g^{\hat{n}}$ to obtain the total transformation, and use again the coherent-state closure relation two times:

$$g^{\hat{n}}\mathfrak{L}[|\gamma\rangle\langle\beta|]g^{\hat{n}}$$

$$= \frac{1}{\pi^2}\int d^2\xi_1 d^2\xi_2\langle\xi_1|g^{\hat{n}}\mathfrak{L}[|\gamma\rangle\langle\beta|]g^{\hat{n}}|\xi_2\rangle|\xi_1\rangle\langle\xi_2|, \tag{B17}$$

where

$$\langle\xi_1|g^{\hat{n}}\mathfrak{L}[|\gamma\rangle\langle\beta|]g^{\hat{n}}|\xi_2\rangle$$

$$= \frac{1}{\pi\epsilon_{\text{ch}}}\int d^2\alpha\, e^{-\frac{|\alpha|^2}{\epsilon_{\text{ch}}}}\langle\xi_1|g^{\hat{n}}\hat{D}(\alpha)\hat{\sigma}(\gamma,\beta,T)\hat{D}^\dagger(\alpha)g^{\hat{n}}|\xi_2\rangle. \tag{B18}$$

Note that we can apply $g^{\hat{n}}$ on $\langle\xi_1|$ and $|\xi_2\rangle$, and hence the term $\langle\xi_1|g^{\hat{n}}\hat{D}(\alpha)\hat{\sigma}(\gamma,\beta,T)\hat{D}^\dagger(\alpha)g^{\hat{n}}|\xi_2\rangle$ is simply given by coherent state overlaps and Gaussian functions.

The term (B18) is therefore finally obtained by computing straightforward Gaussian integrals.

b. Effective system

We now follow the same approach to compute the right-hand side of (B14), using the effective parameters previously obtained with the P function method. Applying first the effective NLA on $|\gamma\rangle\langle\beta|$ leads to

$$g_{\rm in}^{\hat{n}}|\gamma\rangle\langle\beta|g_{\rm in}^{\hat{n}} = e^{\frac{1}{2}(g_{\rm in}^2-1)(|\gamma|^2+|\beta|^2)}|g_{\rm in}\gamma\rangle\langle g_{\rm in}\beta|. \tag{B19}$$

The term $|g_{\rm in}\gamma\rangle\langle g_{\rm in}\beta|$ is then transformed by the effective channel of transmission η and input added noise Δ, producing the output

$$\mathfrak{L}_g[|g_{\rm in}\gamma\rangle\langle g_{\rm in}\beta|] = \frac{1}{\pi\Delta_{\rm ch}}\int d^2\alpha\, e^{-\frac{|\alpha|^2}{\Delta_{\rm ch}}}\hat{D}(\alpha)\hat{\sigma}(g_{\rm in}\gamma,g_{\rm in}\beta,\eta)\hat{D}^\dagger(\alpha), \tag{B20}$$

where $\Delta_{\rm ch} = \eta\Delta/2$. Using these results and inserting two times the closure relation, we obtain

$$\mu\,\mathfrak{L}_g\big[g_{\rm in}^{\hat{n}}|\gamma\rangle\langle\beta|g_{\rm in}^{\hat{n}}\big] = \frac{1}{\pi^2}\int d^2\xi_1 d^2\xi_2 \mu\langle\xi_1|\mathfrak{L}_g\big[g_{\rm in}^{\hat{n}}|\gamma\rangle\langle\beta|g_{\rm in}^{\hat{n}}\big]|\xi_2\rangle|\xi_1\rangle\langle\xi_2|, \tag{B21}$$

where

$$\mu\langle\xi_1|\mathfrak{L}_g\big[g_{\rm in}^{\hat{n}}|\gamma\rangle\langle\beta|g_{\rm in}^{\hat{n}}\big]|\xi_2\rangle = \mu\frac{1}{\pi\Delta_{\rm ch}}\int d^2\alpha\, e^{-\frac{|\alpha|^2}{\Delta_{\rm ch}}}e^{\frac{1}{2}(g_{\rm in}^2-1)(|\gamma|^2+|\beta|^2)} \times\langle\xi_1|\hat{D}(\alpha)\hat{\sigma}(g_{\rm in}\gamma,g_{\rm in}\beta,\eta)\hat{D}^\dagger(\alpha)|\xi_2\rangle. \tag{B22}$$

Here again, Eq. (B22) involves only Gaussian integrals and is straightforward to compute.

The calculations show that the coefficients given by (B18) and (B22) are strictly equal. We can conclude that $g^{\hat{n}}\mathfrak{L}[|\gamma\rangle\langle\beta|]g^{\hat{n}}$ and $\mu\,\mathfrak{L}_g[g_{\rm in}^{\hat{n}}|\gamma\rangle\langle\beta|g_{\rm in}^{\hat{n}}]$ have exactly the same coefficients for every element $|\xi_1\rangle\langle\xi_2|$ in (B18) and (B22), and we have therefore proven that these two transformations are equal, without using a P function decomposition.

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