lifecoursecentre

ARC Centre of Excellence for Children and Families over the Life Course

Review of Small Area Estimation of Disadvantage in Australia

Bernard Baffour,
Institute for Social Science Research, The University of Queensland

Arturo Martinez Jr.
Institute for Social Science Research, The University of Queensland

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

THE UNIVERSITY OF MELBOURNE

THE UNIVERSITY OF WESTERN AUSTRALIA

THE UNIVERSITY OF SYDNEY

Australian Government
Australian Research Council

NON-TECHNICAL SUMMARY

Survey samples are typically designed to produce estimates of population characteristics of planned areas. The sample sizes are calculated so that the survey estimator for each of the planned areas satisfies a certain level of precision. We call this estimator the direct survey estimator. In practice, users of survey data also utilize direct estimators to measure population characteristics for unplanned areas. However, it is possible that sample sizes for these unplanned areas to be very small. Having few observations in unplanned areas increases sampling error and tend to make direct survey estimators unreliable. There are two ways to solve this problem. The first solution is to increase the sample size so that the 'unplanned' areas will have enough observations. However, this strategy may be impractical because it also inflates the cost of data collection. The second solution is to apply small area estimation techniques, a class of statistical tools that borrow strength from auxiliary data sources to improve the precision of survey estimators. Ideally, candidate auxiliary data sources should have (i) wider coverage and not be prone to large sampling error, and (ii) strong theoretical relationship with population characteristic of interest. Examples of good auxiliary data are census and administrative records.

Poverty estimation can be contextualized as a small area estimation problem because sample sizes of survey datasets used for poverty estimation are rarely large enough to provide reliable estimates for disaggregated analysis. This paper reviews the small area estimation methodology and discusses the important developments in the context of measurement of poverty and disadvantage in Australia.

The literature review reveals that there are various ways of implementing small area estimation and the choice of strategy to be adopted needs to be contextualized on the type of population characteristic to be estimated and the types of auxiliary data available. In some cases, auxiliary data exist in the form of unit-level records while in other occasions, they are only available at a certain aggregated-level. In general, the level at which the auxiliary information is available is an important consideration for identifying the appropriate small area estimation technique to be used.

The literature review also highlights that there have been several methodological and substantive developments in poverty measurement. In Australia, the measurement of social exclusion and disadvantage has progressively moved towards the adoption of a multidimensional lens. Nevertheless, there are still areas in poverty measurement research where novelty can be introduced. For instance, very few studies take into account the spatial and temporal dynamics of disadvantage, simultaneously. Examining these unique patterns is important because the policy response necessary to address these problems may be very different from each other. Hence, further research is needed to seamlessly integrate multidimensional measurement of temporal dynamics of poverty with the statistical technique of small area estimation.
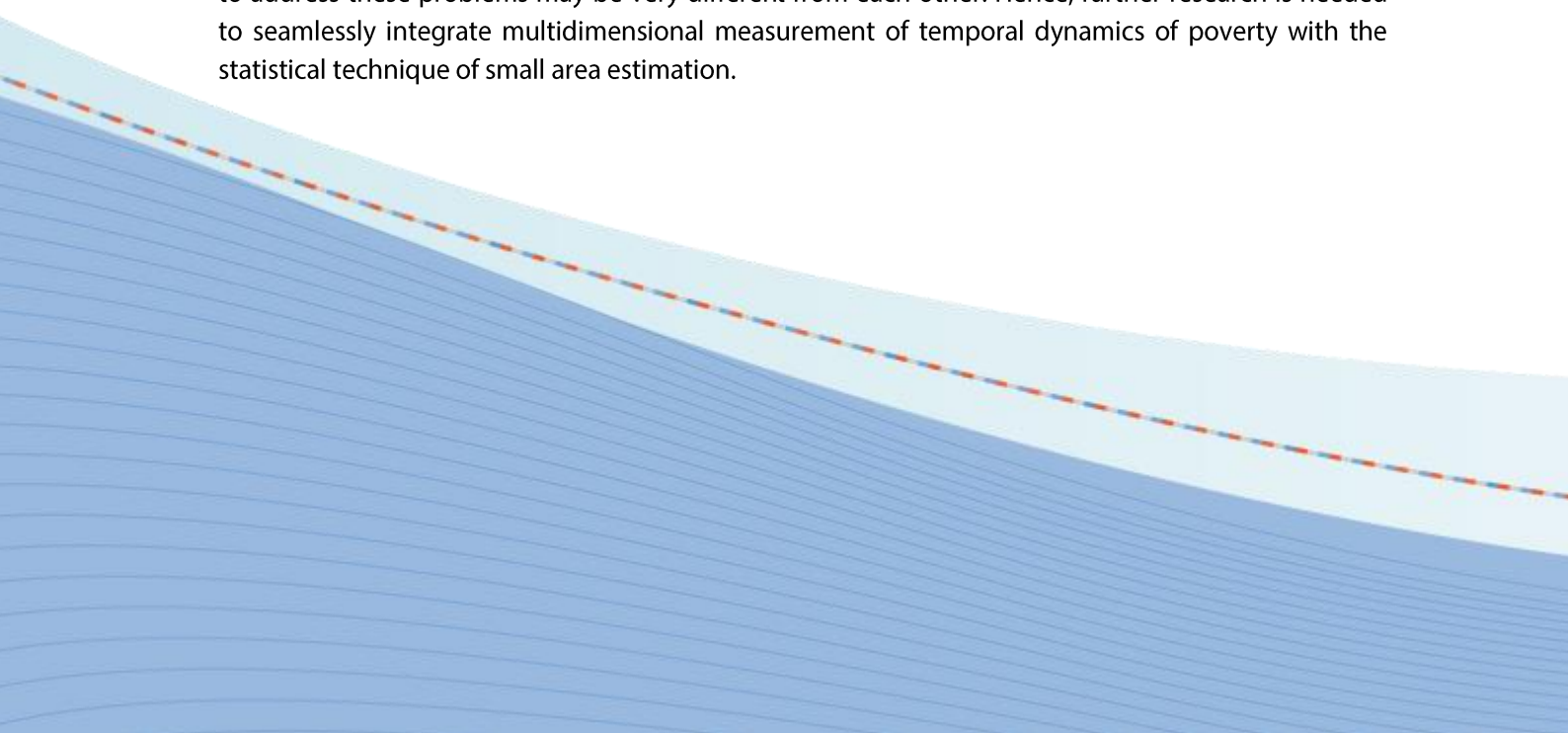
ABOUT THE AUTHORS

Bernard Baffour is Research Fellow in Social Statistics at the Institute for Social Science Research at the University of Queensland. His recent research has been on census and survey methodology, including the areas of non-response, weighting, survey quality and dual frame sampling. Recent publications include articles in *Journal of the Royal Statistical Society: Statistics in Society* on forecasting hospital admissions using meteorological information and *International Statistical Review* on modern censuses. Email: b.baffour@uq.edu.au

Arturo Martinez is Research Fellow in Social and Economic Inequality and Mobility at the Institute for Social Science Research at The University of Queensland. His research interests cover the application of statistical methods for measuring poverty and disadvantage in developing and developed countries. Recent publications include articles in *Research in Social Stratification and Mobility* and *Asian-Pacific Economic Literature* on employment quality and income mobility in Asian countries. Email: a.martinez2@uq.edu.au

Abstract

Describing and examining the prevalence of poverty and disadvantage, how they change over time and how these differ across geographic and population clusters is important to enable better use of poverty and disadvantage research findings in evidence-based policymaking. This study reviews some of the methodological and substantive contributions to understanding the spatial and temporal dynamics of poverty within Australia. The review reveals one important methodological gap: there is limited research in developing statistical indicators that simultaneously account for the (i) prevalence, nature and geographical heterogeneity of multidimensional poverty and disadvantage in Australia, and (ii) its temporal dynamics at the micro-level. Hence, further research on seamlessly integrating multidimensional measurement of temporal dynamics of poverty with the statistical technique of small area estimation is warranted.

Keywords: small area estimation; multidimensional poverty; poverty dynamics; Australia

JEL classification codes: D31, I32, O15

1. Introduction

Social, economic and other indicators of disadvantage provide useful inputs for targeting poverty policies. The data used to measure poverty are typically derived from household surveys of living standards. However, sample sizes of survey datasets used for poverty estimation are rarely large enough to provide reliable estimates for disaggregated analysis. For example in Australia, certain population groups, such as those disadvantaged or at higher risk of being socially-excluded, are not adequately represented in surveys (Productivity Commission 2013). Similarly, sample sizes beyond state/territorial level, are often too small to allow reliable estimation of poverty and disadvantage. Hence, solely relying on survey data to provide localised poverty estimates is contentious due to the limited sample size at finer levels of disaggregation. This issue is problematic if we assume that there is a wide spatial disparity in poverty rates and that policies can be implemented more effectively when adequate information is available to give a detailed localised picture.

National statistical systems are often confronted by both financial and administrative constraints and thus, only few of them have implemented surveys that can yield reliable estimates at fine levels of disaggregation (Marhuenda et al. 2013; Martinez et al. 2014). Small area estimation methodology provides an alternative approach for producing precise estimates at the local-level instead of inflating the sample sizes of household surveys. Over the years, we have seen a number of applications of various small area estimation tools in the measurement of disadvantage. In fact, some national statistical agencies compile small area estimates regularly. For example, the United States Census Bureau produce small area income and poverty estimates for school districts, counties, and states annually (US Census Bureau 2014). Similarly, United Kingdom's Office for National Statistics publishes estimates of average household income for local areas since 2004 (Bond and Campos 2010; Rahman et al. 2013). Additionally, the World Bank has conducted several poverty mapping projects in many developing countries such as Cambodia, Lao PDR, the Philippines, Thailand and Vietnam (Haslett et al. 2010).

In Australia, there are numerous studies that have examined different indicators of disadvantage across different population groups at the national level. There have been several initiatives to examine these measures of disadvantage at the local level (e.g., income poverty – Miranti et al. 2011; unemployment rate – ABS 2010; disability – Elazar 2004; ABS 2005 and 2006). The effort exerted to examine the multiple facets of poverty at the local level is the response to the need to understand disadvantage in terms of other dimensions aside from

income deprivation. More recently, experts of poverty measurement such as Alkire, Foster and Santos (Alkire and Foster 2011; Alkire and Santos 2014) and proposed a composite measure of poverty and have actively encouraged many countries to compile such type of measures. In Australia, Scutella et al. also proposed a composite index of disadvantage (Scutella and Wilkins 2010; Scutella, Wilkins and Kostenko 2013). Using this measure, Azpitarte (2014) finds that Australia's economic growth between 2001 and 2008 can be considered more pro-income-poor than pro-multidimensionally-poor. In other words, the income gains of the people identified as multidimensionally-poor were far below the average income gains of the income poor. On the other hand, one of the important gaps in our current understanding of these phenomena is in examining the spatial and temporal dynamics of social exclusion and disadvantage. In this context, spatial dynamics refers to the variation of poverty status across different population groups or geographic areas while temporal dynamics refers to how a person moves into and out of poverty over time. Some of the existing small area estimates of socioeconomic disadvantage in Australia do not carefully distinguish those who are trapped in an uninterrupted spell of poverty from those who move into and out of the different dimensions of disadvantage periodically. These limitations have undermined the influence of poverty research on policy planning (Saunders and Naidoo 2009).

This study is a literature review that provides some methodological background and motivation behind SAE. It focuses on the application of SAE in the measurement of socioeconomic disadvantage in the Australian context and reviews the current developments in this area. We find that there is a relative paucity of the use of this statistical approach outside official statistics and there is also limited application in the analysis of multidimensional poverty dynamics. Thus, we conclude by discussing a general framework that can be used for measuring the dynamics of multiple dimensions of socioeconomic disadvantage at the local level in Australia.

2. Small Area Estimation Concepts

2.1 What is Small Area Estimation?

Small area or *small domain* are terms used by statisticians to refer to any sampling grouping in which surveys provide insufficient sample data to accurately estimate a specific characteristic of interest for the target population (Pfeffermann 2002; Rao 2003:1). In this context, small areas may correspond to any socioeconomic, demographic or geographic

groups that are not well-represented in surveys. Over the years, a class of estimation tools commonly referred as small area estimation (SAE) methodology, has been proposed to address the issue of survey estimation with small sample sizes. It is a methodology for producing estimates at a greater level of precision than can be obtained from solely relying on survey data. To accomplish this, SAE relies on the concept of 'borrowing strength' from auxiliary data. In particular, SAE techniques take advantage of various relationships in the various data sources and make efficient use of the additional information available to be able to increase the sample size artificially. For example, there is routinely collected information from census or other administrative large data source that is of a sufficient size to allow disaggregation, but this usually either contains no information on the outcome of interest, or measures this information poorly. Some SAE techniques combine the information from (small sample) survey data with (large sample) census data to take advantage of the detail in the household sample surveys, and the comprehensive coverage of the census.

There are several ways to borrow strength from auxiliary data in order to improve the survey estimators at the local-level. For instance, some use implicit methods which basically rely on simpler statistical tools when the small areas share similar characteristics and such information can be easily derived from an existing auxiliary data. Other researchers resort to explicit models to take unobserved heterogeneity into account. Furthermore, many analytical tools for SAE are specifically designed for a certain type of characteristics of interest for the target population, e.g., totals, counts, means, and proportions while other SAE techniques can be applied more generally (Rao 2005; Pfeffermann 2013). Nevertheless, all SAE methodologies aim to produce statistically valid and reliable estimators for a characteristic of interest. Statistically speaking, an estimator is considered valid if the average value of this estimator under repeated sampling is equal to the value of the target population's characteristic of interest. Analogously, an estimator is said to be reliable if it yields approximately the same value if one takes repeated samples of the population. Precise estimators are those that satisfy both validity and reliability conditions. The performance of a small area estimator can be evaluated by comparing its statistical precision with that of the direct survey estimator (Ghosh and Rao 1994; Rao 2003; Pfeffermann 2002; Pfefferman 2013).

2.2 Where do we get auxiliary data?

In the SAE literature, the term *auxiliary data* is used to refer to data from which strength is borrowed to be able to improve the statistical precision of survey estimators at the local-level. The quality of small area estimates can be greatly enhanced when the auxiliary data have the following characteristics. Firstly, the auxiliary data comprehensively cover the entire population for which the small area estimates are required. Secondly, the auxiliary data are of high quality to reliably allow the identification of all units in the small areas. Thirdly, is correspondence between the auxiliary data and small area population characteristic of interest being estimated. Finally, there is a strong correlation between the auxiliary data and the small area characteristic of interest. The greater this correlation, the more 'strength' can be borrowed through exploiting this relationship (Gonzalez 1973).

ABS (2006) schematically provides a detailed description of the relationships between the small area and auxiliary information. These are

a. Cross-sectional relationships – correlations between units with similar characteristics, even though they are not observed in the same area. For example, although there may be localised differences between small areas, the age-sex characteristics will be similar across areas.

b. Time series relationships – strength can be borrowed through pooling small area sample data across time, and then using the series autocorrelations across time to increase the effective sample size in each area. For example, repeated or longitudinal surveys, or censuses across time

c. Spatial relationships – most small areas are divided by arbitrary boundaries and therefore different units bear a relationship to each other based on the distance and direction between them, and this can be used for increasing the effective sample size of the broader areas. For example, in the context of health and environmental effects of ozone pollution, exposure to pollutants is related to the location and will vary with space.

d. Multivariate relationships – by jointly modelling two or more variables simultaneously, we can borrow information between units and take advantage of this additional information to obtain more robust and reliable estimates. For example, in modelling social class and income, a multivariate approach is more efficient in terms of producing more accurate predictions due to the strong correlations between the constituent variables.

3.   What are the approaches to small area estimation?

3.1   Notation

Following on from Ghosh and Rao (1994), Rao (2003), and Pfeffermann (2002; 2013) and others, let U be the population of size N. Consider that U is divided into M exclusive and exhaustive areas, such that $U_1 \cup U_2 \cup \dots \cup U_M$, with N_i units in area i. This implies that $\sum_{i=1}^{M} N_i = N$.

Next suppose we select a sample (randomly) S of size n from this population. Also suppose that samples are available for m ≤ M of these areas. Let $S_1 \cup S_2 \dots \cup S_m$ define the overall sample, where S_i of size n_i is the sample from area i. Following on from this, we have $\sum_{i=1}^{m} n_i = n$.

Let us now define y to be the characteristic of interest, and denote y_{ij} to be the response value for unit j belonging to area i, where $i = 1, 2, \dots, M$ and $j = 1, 2, \dots, N_i$. For this characteristic of interest, we can see that the sample mean for area i is given by $\bar{y}_i = \sum_{j=1}^{n_i} \frac{y_{ij}}{n_i}$.

For the 'small' area, let us assume we are interested in the area quantity, θ_i, for example the area mean, given by $\theta_i = \bar{Y}_i = \sum_{j=1}^{N_i} \frac{y_{ij}}{N_i}$.

3.2   The Direct Estimator

The direct estimator is the (unbiased) population quantity using only the data available for the sample target area. For our area quantity, the mean area response, the direct estimator is estimated by

$$\theta_D = \bar{y}_i = \sum_{j=1}^{n_i} \frac{y_{ij}}{n_i} \tag{1}$$

with variance $V_D[\bar{y}_i | n_i] = (S_i^2 / n_i)\left[1 - \left(\frac{n_i}{N_i}\right)\right]$, where S_i^2 is the sample variance, and given by $S_i^2 = \frac{\sum_{j=1}^{N_i}(y_{ij} - \bar{Y}_i)^2}{N_i - 1}$.

The issue is that this variance $V_D[\bar{y}_i | n_i]$ is large, since the sampling variances S_i^2 are usually large, especially for those small areas with small samples, n_i.

Direct estimators are often referred to as design-based small area techniques because they rely on survey sampling theory and inference (Pfeffermann 2002; 2013).

3.3 The Synthetic Estimator

The synthetic estimator can be derived by partitioning the sample S into B 'broad' areas, such that B ≤ M. These broad areas, b =1, 2, .. B, are chosen so that they are mutually exclusive and exhaustive, and a reliable direct estimator can be found for this larger, broad area. Let us assume that each broad area b is made up of constituent smaller areas.

We now need to 'link' the small areas to the larger broad areas. Let us denote $X_{ijk} = \left(x_{1ijb}, x_{2ijb}, ..., x_{pijb} \right)$ to be covariate values associated with unit j belonging to area i, in broad area b.

Without loss of generality, we can assume that the broad area covariates can be derived from summing the small areas.

Then a synthetic estimator for the small area i, in broad area b is given by

$$\theta_S = \bar{y}_{iS} = \frac{\sum_{j=1}^{N_i} \sum_{b=1}^{B} \frac{x_{ijb}}{x_{bj}} y_{bj}}{N_i} \tag{2}$$

This form of the synthetic estimator is called the synthetic-ratio estimator (Ghosh and Rao 1994).

More common is to write this as a regression-type estimator, of the ith small area population quantity, where

$$Y_{iS} = X_i^{T'} \beta \tag{3}$$

where X_i is the known auxiliary information in a small area i, and β is the estimate of the population regression coefficients.

The synthetic estimator is more biased than the direct estimator, and Ghosh and Rao (1994) showed that this bias is given by

$$E(\bar{y}_{iS}) - \bar{Y}_i \cong \sum_{b=1}^{B} X_{ib} \left(\frac{Y_b}{X_b} - \frac{Y_{ib}}{X_{ib}} \right) \tag{4}$$

which is only zero (meaning unbiased) if $\frac{Y_b}{X_b} = \frac{Y_{ib}}{X_{ib}}$.

However, the variance of this (synthetic) estimator is much smaller than the direct estimator, since it only depends on the variances of the reliable information from the broader areas.

Therefore, the synthetic estimator is fundamentally a biased estimator, but when the small areas within each broader area are homogenous with respect to the quantity being measured it will be more efficient, through having a smaller mean squared error.

Rao (2003) and Pfeffermann (2002) derive two forms of the synthetic estimator, namely the ratio estimator and the calibration estimator. Under general conditions, both estimators are equivalent (Pfeffermann 2002; 2013). The ratio estimator works by proportionately sharing the estimated quantity computed using a larger broad area across the small areas contained within this broad area. This apportioning is done according to some ratio based on auxiliary information. On the other hand, the calibration estimator works by adjusting the original survey design weights (usually given to be the inverse of the sample selection probabilities). These design weights are then replaced by adjusted weights that are close to the original design weights but are calibrated to some auxiliary variable for the population (Deville and Särndal 1992; Pfeffermann 2002; Estevao and Särndal 2006). In the simplest example, the calibration works by adjusting the survey design weights to known population age by gender totals for each small area (see for example ABS 2005; Brown et al 1999). The survey weights are then calibrated so that the estimate of the population count by age and gender agree with the known population totals. The calibration estimator is equivalent to the (generalized) regression estimator since it ensures consistency with auxiliary totals (Deville and Särndal 1992; Singh and Mohl 1996, Bell 2000).

More specifically, noting that the overall objective of the small area estimation strategy is to estimate a true quantity of a small area, say, the area mean $\bar{Y}_i = \sum_{j=1}^{N_i} y_{ij} / N_i$.

If there is no auxiliary information available the ordinary direct estimator results, using only the available sample data

$$\bar{y}_i = \sum_{j=1}^{n_i} y_{ij} / n_i \text{ and variance } Var_D[\bar{y}_i | n_i] = \left(\tilde{S}_i^2 / n_i \right) \left[1 - \frac{n_i}{N_i} \right] = S_i^{D^2}, \qquad (5)$$

where

$$\tilde{S}_i^2 = \sum_{j=1}^{N_i} (y_{ij} - \bar{Y}_i)^2 / (N_i - 1) \qquad (6)$$

For small n_i the variance will be large, unless the variability in the y_{ij} values is sufficiently small.

Let us now suppose that in addition to measuring the y_{ij} values we have some auxiliary variables x_{ij} collected from alternative sources such that the small area population means $\bar{X}_i = \sum_{j=1}^{N_i} x_{ij}/N_i$ are known. Pfeffermann (2002) showed that an efficient unbiased estimator using this auxiliary information is the regression estimator,

$$\bar{y}_{reg,i} = \bar{y}_i = (\bar{X}_i - \bar{x}_i)^T \beta_i \tag{7}$$

$$Var_{D(\bar{y}_{reg,i})} = S_i^{D^2}(1 - R_i^2) \tag{8}$$

Where $\bar{x}_i = \sum_{j=1}^{n_i} x_{ij}/n_i$, β_i is the vector of regression coefficients, and R_i^2 is the vector of multiple correlation coefficients between the y-values and the auxiliary variables $X_1, X_2,\ldots, X_p$. Adding the auxiliary information in the estimation of the population quantities has the benefit of reducing the variability by a factor of $(1 - R_i^2)$, and illustrates the importance of having auxiliary information with good predictive power (Pfeffermann 2002). In practice, however, the coefficients β_i are not known and the alternative of replacing them with $\hat{\beta}_i$s computed by using the sample information is not effective because of the lack of adequate sample sizes for small areas. If the assumption is made that the β_i coefficients are similar across groups of small areas, then the synthetic regression estimator is given by

$$\bar{y}_{reg,i}^{syn} = (\bar{y} - \bar{x}^T b) + \bar{X}_i^T b \tag{9}$$

where $b = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n}(x_{ij}-\bar{x}_i)(y_{ij}-\bar{y}_i)}{\sum_{i=1}^{m} \sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^T(x_{ij}-\bar{x}_i)}$ is a pooled estimator computed from all the sample information.

When there is a single auxiliary variable, and zero intercepts, then if X and Y are highly correlated, the ratio relationship

$$\frac{\bar{y}_i}{\bar{X}_i} = \frac{\mu_y}{\mu_x} = \frac{\bar{y}}{\bar{x}} \tag{10}$$

And the synthetic ratio estimator is derived as

$$\bar{y}_{ratio,i}^{syn} = \bar{X}_i \frac{\bar{y}}{\bar{x}}. \tag{11}$$

The ratio estimator can be written as a generalized regression estimator,

$$\bar{y}_{ratio,i}^{syn} = \bar{y} + (\bar{X}_i - \bar{x})b^r, \text{ where } b^r = \frac{\bar{y}}{\bar{x}}. \tag{12}$$

Both of these synthetic estimators rely on the assumption that reliable direct population estimates of the characteristic of interest for the larger, broad area (with sufficiently larger sample size) from the survey exists, and this can be used to derive the small area quantities. The estimators are unbiased if the relationship applies to both the broad area and the small areas within.

3.4 Composite Estimators

A composite estimator is essentially a way to balance between the bias of synthetic estimator with instability of the direct estimator. The approach is based on the theoretical result that as the sample size in a small area increases, the direct estimate becomes more appealing (Purcell and Kish 1979). This is because, on the one hand, when the area level sample sizes are too small, the synthetic estimator will do better than the traditional direct estimator. On the other hand, when the sample sizes are large enough, then the direct estimator is better performing when compared to the synthetic estimator. A suitable compromise between these two is to take a weighted average of both estimators. This is similar to taking a linear combination of the direct estimator ($\bar{y}_{iD}$) and synthetic estimator ($\bar{y}_{iS}$), and hence it is sometimes referred to as a combined estimator (Pfeffermann 2013).

This composite estimator is given by

$$\bar{y}_{iC} = w_i \bar{y}_{iD} + (1 - w_i)\bar{y}_{iS} \tag{13}$$

and the compositing weight, $0 \leq w_i \leq 1$, is suitably chosen to according to some optimal criteria (Ghosh and Rao 1994; Rao 2003). Ideally, this optimal value, can found through minimizing the mean square error (MSE) of the composite estimator. This is given by,

$$w_i^{opt} = \frac{MSE(\bar{y}_{iS})}{MSE(\bar{y}_{iS})+MSE(\bar{y}_{iD})}, \quad (14)$$

provided the assumption is made that the covariance between the synthetic and direct estimators is negligible.

Although composite estimation appears to be a good way of compromising between the potential bias of synthetic estimation and the imprecision of direct estimation, there are some issues. Firstly, there is no general consensus as to the choice of weights. Also, the computation of the mean square error of the composite estimator is not straightforward (Pfeffermann 2013). Secondly, the weights do not take account of the size of between area variation relative to the within area variation for the characteristic of interest. In fact, all population characteristics are given the same weight, regardless of their differences with respect to the between area homogeneity.

These limitations, combined with the other limitations of the implicit small area estimation approaches, can be avoided through models that incorporate the random area-specific effects. Such models are called explicit small area estimation models, and are considered in detail next.

Small area models that include random effects to cope with the between area variation offer some specific advantages. First, differences in the small areas that cannot be fully explained by the auxiliary data can be accounted for specifically and this can lead increased reliability and improved accuracy. Second, framing the small area problem in this approach allows the use of model diagnostics that can be used to find suitable models that fit the data well, and also allow the use of selection criteria to choose between competing models. Third, area specific measures of precision can be used to detect departures from the model, or where the model is poorly estimating particular small areas. Fourth, these small area modeling concepts are flexible to deal with both linear and nonlinear data structures, and can be extended to logistic and other generalized models with random area effects.

There are a number of recent advances for small area modelling approaches (Chambers and Tzavidis 2006; Chandra and Chambers 2009) to estimate small areas with complex data structures and dependence. However, the success of the models are strongly influenced by the choice of auxiliary variables and availability of good quality auxiliary data (Pfeffermann 2013). Rao (2013: 75) state that "attention should be given to the compilation of auxiliary variables that are good predictors of the study variables".

Explicit small area models can be classified into two main types: (1) Aggregate (or area) level models that relate the small area parameters of interest to the area-specific auxiliary variables. Or (2) Unit level (nested error) model that relate the unit values of the response variable to unit specific auxiliary variables.

3.5   Area level small area models

This model is used when the covariate information is only known at the area level. These models include area level random effects to relate the direct estimates to area-specific covariates (Fay and Herriot 1979).

This is defined as

$$\tilde{y}_i = \theta_i + e_i \text{ (known as the sampling model)};$$
$$\theta_i = X_i^T \beta + u_i \text{ (known as the linking model).} \tag{13}$$

And put together we have

$$\tilde{y}_i = \tilde{y}_i =; \ \tilde{y}_i = X_i^T \beta + u_i + \ e_i, \ for \ i = 1, 2, \dots, m. \tag{14}$$

Here $\tilde{y}$ denotes the direct sample estimate of the population characteristic of interest, θ_i (for instance, the sample mean of the small area i, $\bar{y}_i$) ,and e_i are the sampling errors and the u_i are the area level random effects.

These sampling errors are assumed to be independent with zero mean and known variance, such that $E(e_i|\ \theta_i) = 0$ and $V(e_i|\ \theta_i) = \sigma_e^2$.
The area level random effects are assumed to be also independent with zero mean and known variance, such that $E(u_i) = 0$ and $V(u_i) = \sigma_u^2$.

Pfeffermann (2013) showed that under general conditions, this model yields a best linear unbiased estimator of the small area characteristic of interest, θ_i, given by

$$\hat{\theta}_i = \gamma_i \tilde{y}_i + (1 - \ \gamma_i) X_i^T \hat{\beta} = X_i^T \ \hat{\beta} + \gamma_i (\tilde{y}_i - X_i^T \ \hat{\beta}) = \ X_i^T \ \hat{\beta} + \ \hat{u}_i. \tag{15}$$

Fay and Herriot (1979) showed that the coefficient γ_i is given by

$$\gamma_i = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2} = \frac{\left(\frac{\sigma_u^2}{\sigma_e^2}\right)}{\left(\frac{\sigma_u^2}{\sigma_e^2}\right) + 1} \tag{16}$$

which is a function of the ratio $\left(\frac{\sigma_u^2}{\sigma_e^2}\right)$ of the respective variances of the prediction errors of $X_i^T \hat{\beta}$ and $\tilde{y}_i$, and is sometimes referred to as a shrinkage coefficient. The model is derived from area specific auxiliary data to relate to the small areas, and then combine with these with the underlying sampling model. This is analogous to the composite estimator discussed above, but here the optimal weight is γ_i and is based on the (known) variances of the sampling and linking models (Fay and Heriot 1979; Datta 2009).

The assumption that these model variances (σ_e^2 and σ_u^2) are known is considered a limitation of the basic area level model (Rao 2003). The reason given is that assuming that the sampling variances are known can in fact be too restrictive. In addition, the linking model assumes a linear relationship, but this often not true and defined for a transformation of $\hat{\theta}_i$. The effect this has is that for small areas with particularly small sample sizes, the relationship will not hold. In the original development, Fay and Heriot (1979) replaced θ_i with the logarithmic transformation. Extensions to these area models that allow correlated sampling errors, and also allow correlations to exist amongst the areas are available (e.g. the 1990 US Census Coverage Adjustment (Isaki, Tsay and Fuller 2000)).

3.6 Unit level small area models

This model uses individual observations y_{ij} to relate to unit specific covariates (auxiliary information) X_{ij}, originally proposed by Batesse, Harter and Fuller (1988).

The model has the form

$$\tilde{y}_{ij} = X_{ij}^T \beta + u_i + \epsilon_{ij} \tag{17}$$

where the u_i's are random area effects and the ϵ_{ij} are residual terms, which are mutually independent identically distributed, such that $E(\epsilon_{ij} | y_{ij}) = 0$ and $V(\epsilon_{ij} | y_{ij}) = \sigma_\epsilon^2$; $E(u_i) = 0$ and $V(u_i) = \sigma_u^2$.

These X_{ij} unit-specific auxiliary data are available for all areas, $i = 1, 2, \dots, m$ and units, $j = 1, 2, \dots N_i$ since, N_i is the number of population units in the i$^{\text{th}}$ area.

β represents the vector of regression parameters.

The unit responses, y_{ij} , are related to the auxiliary covariates X_{ij} through the above nested error regression equation.

Under this nested error model, the true small area estimates (let us assume we are interested in the area means) are specified as

$$\bar{Y}_i = \bar{X}_i^T \beta + u_i + \bar{\epsilon}_i. \tag{18}$$

Now since the mean error terms, $\bar{\epsilon}_i = \sum_{j=1}^{N_i} {\epsilon_{ij}}/{N_i} \cong 0$, for large N_i then these area means become

$$\theta_i = \bar{X}_i^T \beta + u_i = E[\bar{Y}_i | u_i]. \tag{19}$$

For known variances $(\sigma_u^2, \sigma_\epsilon^2)$ the best linear unbiased predictor (BLUP) of θ_i is given by

$$\hat{\theta}_i = \gamma_i \{\bar{y}_i + (\bar{X}_i - \bar{x}_i)^T \hat{\beta}\} + (1 - \gamma_i) \hat{X}_i^T \hat{\beta} \tag{20}$$

where $\bar{x}_i = \sum_{j=1}^{n_i} x_{ij}/n_i$ represent the sample means of the auxiliary variable information for the small area i, and $\hat{\beta}$ is the estimate of the regression parameters computed from all the observations. Additionally, the 'shrinkage' term for this model is

$$\gamma_i = \frac{\sigma_u^2}{\sigma_u^2 + {\sigma_\epsilon^2}/{n_i}}. \tag{21}$$

The benefit of this model specification is that for a small area k with no sample, but with known covariate information, then small area model-based estimator is simply

$$\hat{\theta}_k = \bar{X}_k^T \hat{\beta}. \tag{22}$$

This BLUP, $\hat{\theta}_i$ has some fine properties that simplify the estimation and aid interpretation. It can be seen that $\hat{\theta}_i$ is in fact a composite estimator with the direct estimator given by $\{\bar{y}_i + (\bar{X}_i - \bar{x}_i)^T \hat{\beta}\}$ and the model-based synthetic estimator, $\hat{X}_i^T \hat{\beta}$. The weights are assigned more optimally based on the ratio, $\sigma_u^2/\sigma_\epsilon^2$, unlike the design-based composite estimators which have an ad hoc choice of weights (Pfeffermann 2013). Another property of the BLUP in this format is that there is no need to specify the distribution of the variance terms. This is because in practice the BLUP is replaced by the empirical best linear unbiased predictor (EBLUP) which is found by replacing the unknown variances with their sample estimates everywhere they appear in the expression of the BLUP. Under a Bayesian approach the BLUP is the linear Bayes predictor, and we have the empirical Bayes (EB) predictor when we substitute the unknown terms with sample estimates. Note that when the weight γ_i is zero the resulting estimator, $\hat{\theta}_i = X_i^T \hat{\beta}$, is the synthetic estimator which does not account for local variation other than the variation reflected in the auxiliary variables X_i.

3.7 Spatial (Geographically-based) Small Area Estimation

There are alternative ways of deriving small area estimates, using spatial or geographical techniques. These spatial techniques use data from similar areas (or domains) to estimate the statistics in the small areas through an indirect process that seeks to link the outcomes to a set of explanatory (auxiliary) variables through assuming a model that relates the small areas (Tanton et al., 2011). The ideas of borrowing strength from similar areas as used in statistical small area estimation also underlie the spatial small area estimation.

In contrast to the statistical approaches discussed above, spatial techniques are grounded in economic and geographic theories, and their methodologies are based on trying to essentially reconstruct the small area data in an indirect or synthetic manner so that they have the specific characteristics (usually based on larger area data as constraints, referred to as benchmarks (Tanton et al. 2011)). Spatial microsimulation has been widely used in Australia in estimating the effects of policy change on the size and distributions of different population characteristics such income deprivation (Tanton et al. 2009; Tanton 2011; Miranti et al. 2011), housing stress (MacNamara et al. 2007), disability (Elazar 2004; ABS 2005), at small area level when contemporaneous census and/or survey information is unavailable.

This can be done through synthetic reconstruction or reweighting methods. Synthetic reconstruction attempts to recreate indirect synthetic micro-populations at the small area level in such a way that known higher level constraints are met. This is similar in approach to the

iterative proportional fitting algorithm (Stephan and Deming 1940). In the classical iterative fitting algorithm, the (population) cell estimates in a contingency table are found by adjusting the sample data subject to marginal control totals. Reweighting is a fairly newer approach and works by using reliable data to calibrate the sampling design weights to a new set of weights based on some distance criteria. This is similar to the generalized regression (GREG) estimator which is also a calibration (synthetic) estimator (Deville and Särndal 1992; Singh and Mohl 1996). In Australia, Bell (2000) developed an algorithm (known as GREGWT) that provides calibrated weights for the creation of synthetic micro-populations. This works by minimizing the chi-squared distance between the basic design weights and the new adjusted weights so that the new weights modify the design weights as little as possible, subject to the calibration constraints or benchmarks. This has been used by the ABS for estimating small area statistics in a number of surveys (such as the Labour Force Survey (Department of Education Employment and Workplace Relations 2009), the Household Expenditure Survey (ABS 2002); and the Survey of Disability Ageing and Carers (ABS 2005)).

Data fusion or matching is also another approach of microsimulation, although rarely referred to as a small area estimation technique. In this approach data from multiple sources are brought together into a single source through matching on the basis of variables which uniquely identify individuals (and households). This exact matching is not possible when these unique identifiers are not available. In reality due reasons of confidentiality and privacy, for instance, records from different sources are probabilistically matched if they share a set of common characteristics (e.g. sex, age, marital status, geographic location). The Australian Census Longitudinal Dataset (ACLD) (ABS 2013) is an example of a microsimulation through data fusion/matching.

3.8 A prediction approach – differences between model based and design based approaches in statistics

There are two broad schools of inferential and sampling statistics, which until recently were diametrically opposites. From a design of experiments point of view, random variables in the randomization were not overly concerned with the responses, y_i. But of interest here are the indicator variables, Z_i, that tell us whether a unit i is in the sample or not. In a randomization theory (i.e. design based) approach to inferential statistics, the only relationship between units sampled and units not sampled is through the sampling process. In other words, the assumption is that if we were to use a different random starting point to generate our sample, we could have the included the previously non-sampled units.

The other approach is to assume that the random variables follow a probability distribution. Then the sample values y_i are realisations of the random variables. Therefore, we would usually assume that $Y_i \sim Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed from a normal distribution. Subsequently we can use the properties of independent random variables and the normal distribution to find expected values of various statistics. The other approach to sampling is based on assuming that the sample values y_i are realisations of the random variables Y_i, and these follow a probability distribution. These random variables are therefore generated from some model, and the actual values are one realization of the random variables (for a finite population). The joint probability distribution of $Y_1, Y_2, \ldots, Y_N$ is the link between the units in the sample, $\{y_i, i \in S\}$ and the non-sampled units $\{y_i, \ i \in S^{'}\}$. The difference here is that a sample is drawn, and we can use the sampled data to predict the unobserved values of the nonsampled units. Although, both approaches can be theoretically equivalent, the model-based strategies can perform poorly when the model is not properly specified, whereas the design-based approaches rely on knowing the sampling probabilities of every unit in the population (or at every unit having a nonzero chance of being included).

3.9 Issues around the Empirical Best Linear Unbiased Prediction (EBLUP) and Empirical Bayes (EB) Estimation

The implicit techniques using the auxiliary data to link related small areas are fairly simple to estimate using standard approaches that explain the differences between the areas. But in regards to explicit small area models that make specific allowance for the heterogeneity between areas, there is usually the need to introduce random effects at each specific area to account for the between area variation and borrow strength across related areas. These models rely on empirical best linear unbiased prediction (EPLUP) or empirical Bayes (EB) estimation of the unknown unobserved heterogeneity terms, or random effects. The case of unknown random effect terms is difficult to handle in the context of small area models, without additional assumptions. First, there is the assumption of linearity, but this might not necessarily hold if there is a nonlinear relationship, particularly when the area sample size is small. Second, there is the assumption of uncorrelated errors, but in the case when the small areas straddle more than one boundary in the sample design, there is a violation of independence. Thirdly, normality of the random effects is assumed, but this usually depends on transforming the y_is, e.g. by modelling $\log(y_i)$ (see Fay and Heriot 1979; Rao 2003: 77).

However, the main issue is that the EBLUP/EB assume known variances and hence, the mean square error (MSE) estimator, which gives a measure of the prediction accuracy, obtained by replacing the unknown random effect terms by their sample estimates ignores the error resulting from the variance estimation. This estimator therefore underestimates the true mean square error (Pfeffermann 2002; 2013). These model-based procedures do overcome the problems associated with design-based procedures since the randomization based estimation of precision e.g. confidence intervals and standard errors, rely on large sample normality assumptions, but this will not hold in areas with small or no sample size. However, these models can only produce sufficiently accurate small area predictors when there is available auxiliary data with good predictive power. The robustness of inferences to model mis-specification and the correct estimation of the mean square error has been where there have been continued developments in small area modelling. Prasad and Rao (1990), Datta and Lahiri (2000), Das, Jiang and Rao (2004), Datta, Rao and Smith (2005) have developed MSE estimators that are approximately unbiased through simplifying the model assumptions, making the model more robust to non-normality, and computing an additional term to correct for bias. However, these still rely on making some distributional assumptions.

4. Small Area Estimation of Disadvantage

4.1 State of the art of localised poverty measurement

A new development of small area modelling that has received wide attention in recent years has been in poverty estimation. This is because the estimation of poverty indicators is small regions is important for both national and international policy (Fabrizi et al. 2014; Molina and Rao 2010). But the information that is available for the estimation of these indicators is mostly available from national surveys that are designed to provide accurate estimates at the national and regional level. In order to be able to target policies and programs to alleviate poverty, what is routinely required is lower level information which is often limited, at best, or simply unavailable from these national official surveys. Therefore, poverty estimation uses small area techniques to borrow strength from different areas through linking models based on auxiliary information that is available from census or administrative data (Molina and Rao 2010).

Since as far back as the 1970s, the US has provided small area statistics for poverty estimates at a local government (county) level. This has been primarily been through the Fay Heriot model (Fay and Herriot 1979) which was used to provide updated income per capita estimates for small places. This information was used by the US Census Bureau to determine

the fund allocations for local government places and administer federal programs (Bell et al. 2007). Currently an area based model has been used to produce model-based estimates of the number of school age children living in poverty under the Small Area Income and Poverty Estimates (SAIPE) program (Citro and Kalton 2000; Huang and Bell (2006)). In Europe, the EURAREA project aimed to provide small area statistics of key indicators of poverty – the proportion of unemployed people, the proportion of singe person households, and the average equivalized household net income at a county or provincial level across six participating European countries (Poland, Finland, Italy, Spain, United Kingdom, and Norway) (www.statistics.gov.uk/euarea). Both projects use outcomes of poverty that are linear (or more realistically a logarithmic transformation is performed to linearize the target parameter). This assumption of linearity has been a cause for concern, especially amongst researchers and policy makers evaluating and measuring poverty and/or inequality. Elbers, Lanjouw and Lanjouw (2003) developed estimators of population parameters that are nonlinear functions of the underlying variable, and produce small area estimates based on a unit level model that combines both survey and census data through taking advantage of the detail in the survey and the comprehensive coverage of the census. They then used the recent advances in geographical information systems (GIS) software to produce finely disaggregated maps which describe the spatial patterns and distribution of poverty and inequality (i.e. poverty mapping). This work formed part of a programme of estimation of poverty indicators in small regions in different countries and was sponsored by the World Bank. Hence the Elbers, Lanjouw and Lanjouw (2003) approach is often referred to as the World Bank method.

Underlying the work of Elbers, Lanjouw and Lanjouw (2003) is a class of poverty measures defined by Foster, Green and Thorbecke (FGT) (1984; 2010). Denote t to be the poverty line (fixed). This is defined as the threshold under which a person is considered to be 'under poverty'. Then the family of FGT poverty measures for each small area i, are defined by

$$F_{\alpha i} = \sum_{j=1}^{N_i} F_{\alpha i j}, \qquad\qquad \text{for } i = 1, \dots, m \qquad\qquad (23)$$

$$\text{and} \quad F_{\alpha i j} = \left[\frac{t - y_{ij}}{t}\right]^{\alpha} I(y_{ij} \leq t), \text{ for } j = 1, 2 \dots N_i, \alpha = 0, 1, 2, \qquad (24)$$

where $I(y_{ij} \leq t)$ is an indicator variable that takes value one if $y_{ij} \leq t$ (person is under poverty), or value zero if $y_{ij} > t$ (person is not under poverty).

When $\alpha = 0$, we have the head count ratio (or poverty incidence measure). This gives the proportion of individuals that are under poverty in a given small area i. For $\alpha = 1$, the

measure of poverty is called the poverty gap. This gives the relative distance of the each individual to the poverty line. Finally, when $= 2$, the measure is called the poverty severity. This gives the squared distance from the poverty line, and has the effect of giving larger weight to individuals that are far from the poverty line.

Elbers, Lanjouw and Lanjouw (2003) assume that y_{ij} satisfy the unit level (nested error model (Battese, Harter and Fuller 1988) such that $y_{ij} = X_{ij}^T \beta + u_i + \epsilon_{ij}$, with u_i random effects and ϵ_{ij} residual terms, which are mutually independent with zero means and variances σ_u^2 and σ_ϵ^2 respectively. For sampled units $j \in s_i$, the $F_{\alpha ij}$ poverty measure is known. However, for the non-sampled units $k \in r_i$, the missing values have to be imputed, and then use the decomposition

$$F_{\alpha i =} \sum_{j \in s_i} F_{\alpha ij} + \sum_{k \in r_i} F_{\alpha ik}. \tag{25}$$

These predictions are then obtained by Monte Carlo simulation from the conditional normal distribution of the unobserved outcomes given the observed outcomes under the nested error model. Molina and Rao (2010) proposed an empirical best predictor method that accounts for the random effects of the non-sampled units, and these resulting estimators have smaller predicted mean squared errors than those using the Elbers, Lanjouw and Lanjouw (2003) method.

These two methods still rely on normality assumptions and can be less robust. M-quantile estimation has been proposed to provide small area estimates of poverty measures by relaxing the parametric model assumptions, and is robust to outliers and model mis-specification (Breckling and Chambers, 1988; Chambers and Tzavidis 2006; Tzavidis et al 2010). Further extensions are presented by (Esteban et al. 2012; Marhuenda et al. 2013) by using spatio-temporal models with random effects that take account of time and space variability between small areas. These models extend on the original model proposed by Rao and Yu (1994) and Singh et al. (2005) that borrows information across areas and over time, and which includes terms to account for the unexplained area and temporal variation. In all these cases, the most important, and difficult aspect of the modelling lies in the estimating of the uncertainty since for any statistic to be useful it has to be accompanied by a measure of the variability.

The current development in the small area estimation literature is moving away from the estimation of the mean square error (i.e. the variability) using linearization under normality assumptions (Prassad and Rao 1990; Datta and Lahiri 2000), since no closed-form solution

exists. Owing to the developments in computing technology, resampling approaches are being developed. Jiang, Lahiri and Wan (2002) proposed using a jack-knife procedure to estimate the mean squared error with relying on linearizing the function. Other jack-knife based resampling extensions have been proposed by Chen and Lahiri (2003) and Lohr and Rao (2009) to simplify the estimation process. Bootstrapping procedures have also been developed initially by Hall and Maiti (2006). These can either be with a parametric bootstrap (Molina and Rao 2010) or non-parametric bootstrap (Tzavidis et al 2008; 2010). But, all these resampling methods are in fact model-dependent since they rely on the repeated computation of the empirical best predictors under the chosen model (Pfeffermann 2013).

4.2 Measuring Disadvantage in Australia

While Australia prides itself as an egalitarian nation due to relatively high levels of economic mobility (Martinez and Perales 2014), a recent global report shows that the proportion of Australians who live with less than half of the median income, which is estimated to be 14% is higher than the OECD average of 11% (OECD 2014). Additionally, some studies suggest that economic inequality in the country is rising and the country may soon enter economic slowdown due to its jobless growth (Martinez and Perales 2014; OECD 2014). Together, these issues could have adverse consequences on the lives of the most vulnerable.

From a policy perspective, it is important to have a good understanding of social exclusion and disadvantage because such knowledge can be used as important inputs for policy planning. To gain a holistic and more nuanced insight into poverty, there are several things that need to be taken into consideration. First, it is important to note that poverty, social exclusion or disadvantage goes beyond income deprivation. There are several initiatives that have responded to the need to probe beyond conventional income-based measures of disadvantage. For instance, researchers from the Melbourne Institute of Applied Economic and Social Research in collaboration with the Brotherhood of St Laurence have developed a method to measure disadvantage through focusing on 'social exclusion' (Scutella et al. 2009; Scutella and Wilkins 2010; Scutella, et al. 2013). Their measure of social exclusion is based on Amartya Sen's capability framework of inequality (Sen 1976; 1979). This measure identifies disadvantage as an accumulation of the lack of resources in different domains across life (UNDP 2004; Sen 2006). A person is therefore deprived (or socially excluded) through being unable to participate in life fully as a result of the lack these circumstances. In particular, the measure of social exclusion devised divides disadvantage into seven, inter-related, domains namely – material resources; employment; education and skills; health and

disability; social connection; community and personal safety. This is similar conceptualization and operationalization of index of multidimensional poverty in other countries (Alkire and Foster 2011; Alkire and Santos 2014). Each life domain is then captured through a series of indicators based on information from the Households, Income and Labour Dynamics in Australia (HILDA) survey. These indicators are given below in Table 1. The data from these individual indicators and life domains are then transformed into a single measure of multidimensional poverty (or social exclusion) through weighting the different constituent indicators and summing across the various domains (using an approach originally devised by Scutella, Wilkins and Horn (2009)). Based on this, individuals can be classified into different groups according to the number of dimensions in which they are disadvantaged (Martinez and Perales 2014).

Another important development in poverty research in Australia is the application of small area estimation. Differences in terms of the magnitude of economic disadvantage exist between population clusters and across spatial areas exist. It is essential that policy makers can access reliable data at the local-level to be able to target areas where significant pockets of poverty exist. However, the household surveys that are commonly used to provide such measures are rarely representative at the local-level. As discussed in the previous sections, small area estimation is a statistical technique that can be used to address this methodological problem.

There are several studies that have attempted to provide measures of poverty and disadvantage at the level of administrative regions or for hard-to-reach population clusters (e.g. indigenous or culturally or linguistically diverse (CALD) people). For instance, the ABS has done some statistical small area modelling of disability (Elazar 2004; ABS 2005) and labour force participation (Department of Education, Employment and Workplace Relations 2009) but to some extent, these can be considered as 'experimental' statistics (ABS 2010) and hence, they are not regularly undertaken. More recently, the National Centre for Social and Economic Modelling (NATSEM) has produced small area statistics of income poverty in Australia. However, although small area estimation has been used in this context for Australian research, the focus has been on the use of spatial (geographically-based) techniques through spatial microsimulation modeling (see for example, Chin et al., 2005; Tanton et al 2007; Harding et al 2006; Vidyattama et al 2012; Tanton 2014). As discussed earlier, these methods are different from the statistical modelling approaches that relate the small area information to larger areas through borrowing strength across areas.

It is also important to note that poverty is a highly dynamic phenomenon because people move into and out of it (Martinez 2015). For some groups of people, social exclusion and disadvantage are temporary obstacles while for others these are persistent features of their living conditions. It is important to distinguish these groups because an efficient policy intervention needs to be contextualized on how a person experiences poverty over time. However, most of the indicators described above are usually presented as static measures and do not capture the temporal dynamics of poverty. A restrictive formulation of disadvantage prevents the careful examination of the concept of persistence through being able to capture the dynamics of people's experiences of disadvantage (Productivity Commission 2013).

In summary, while the poverty measurement literature in Australia recognizes the importance of multiple and inter-related factors that determine a person's capacity to fully participate in society (Townsend 1987), there are areas that warrant further investigation. In particular, there is a need to do more research on how to incorporate temporal and spatial dynamics simultaneously when measuring multidimensional poverty.

4.3   Issues facing small area estimation of disadvantage in Australia

There are two key issues that need to be considered when measuring small area statistics of disadvantage – (i) methodological concerns and (ii) data limitations. Firstly, from a methodological viewpoint, how to best measure the multiple dimensions of poverty remains debatable. In particular, while the multi-dimensional approach set out by the Melbourne Institute and Brotherhood of St Laurence is a step in the right direction, the various domains are conveniently given the same (equal) weight. Despite its simplicity, there is evidence that the measure of social exclusion is an unequally weighted sum of the level of exclusion in each of the dimensions (Townsend 1987; Hick 2012). In fact, some poverty measurement experts even contend the developing composite indices of disadvantage will always be problematic due to the absence of any objective valuation method that can be used to come up with the optimal set of weights (Ravallion 2012). Obviously, there is a lot of work that has to be done to address these theoretical concerns. This may require a careful examination of how existing variables collected in surveys, censuses and administrative registers can be used to construct a measure of disadvantage.

Table 1. Dashboard of Social Exclusion and Disadvantage Indicators Available in HILDA Survey

Domain	Indicator	Description
Material resources	• Household income	1 if income is less than 60% of median equivalised income, 0 otherwise
	• Financial hardship	1 if experienced 3+ indicators of financial hardship (*could not pay electricity, gas or telephone bills on time; could not pay the mortgage or rent on time; pawned or sold something; went without meals; was unable to heat the home; asked for financial help from friends or family; asked for help from welfare or community organization*), 0 otherwise
	• Household net worth	1 if net worth is less than 60% of median equivalised household net worth, 0 otherwise
	• Household consumption expenditure	1 if consumption expenditure is less than 60% of median equivalised household consumption expenditure, 0 otherwise
Employment	• Long-term unemployment	1 if currently unemployed, looked for work for the past 4 weeks and has been unemployed for the preceding 12 months, 0 otherwise
	• Unemployment	1 if unemployed, 0 otherwise
	• Marginal attachment to the labour force	1 if not employed but looking for work or not employed and not looking for work because of the belief of being unlikely to find work, 0 otherwise
	• Underemployment	1 if working for less than 35 hours per week, 0 otherwise
	• Living in jobless household	1 if no household member is employed and at least one household member is aged 15 to 64, 0 otherwise
Education and skills	• Low literacy	1 if respondent scored low in literacy, 0 otherwise
	• Low numeracy	1 if respondent scored low in numeracy, 0 otherwise
	• Poor English-language proficiency	1 if respondent speaks a language other than English at home and reports that he/she does not speak English well, 0 otherwise
	• Low level of formal education	1 if respondent is not currently studying full-time and her highest educational qualification is less than high school completion, 0 otherwise
	• Limited work experience	1 if respondent has spent fewer than three years in paid employment, 0 otherwise
Health and disability	• Poor general health	1 if respondent indicated that he/she has poor general health (0-50 on a 0-100 scale), 0 otherwise
	• Poor physical health	1 if respondent indicated that he/she has poor physical health, (0-50 on a 0-100 scale), 0 otherwise
	• Poor mental health	1 if respondent indicated that he/she has poor mental health, (0-50 on a 0-100 scale), 0 otherwise
	• Presence of disable child	1 if respondent is living in a household that has a disabled (0-50 on a 0-100 scale), 0 otherwise
Social support	• Little social support	1 if respondent reported that he/she receives little social support (0-30 on a 0-70 scale), 0 otherwise
	• Low participation in common social activities	1 if respondent gets together with friends / relatives less than once a month, 0 otherwise
Community participation	• Low neighbourhood satisfaction	1 if respondent satisfaction with his neighbourhood was low (0-5 on a 0-10 scale), 0 otherwise
	• Low community connection	1 if respondent satisfaction with feeling part of local community was low (0-5 on a 0-10 scale), 0 otherwise
	• Non-participation to community activities	1 if respondent is not currently a member of a sporting, hobby or community-based club or association, 0 otherwise
	• Non-participation to voluntary work	1 if respondent is not engaged in any voluntary activity in a typical week, 0 otherwise
Personal safety	• Victim of violent crime	1 if respondent reported being a victim of physical violence in the last 12 months, 0 otherwise
	• Poor perceived personal safety	1 if respondent reported being a victim of property crime in the last 12 months, 0 otherwise
		1 if respondent satisfaction with safety feelings low (0-5 on a 0-10 scale), 0 otherwise

Notes: Adapted from Scutella et al. (2013).

Regarding data availability, this review of literature has shown that the type of data available to researchers is an important factor in deciding which small area estimation technique should be used. Subsequently, this choice affects the estimation and interpretation of results. Furthemore, the success of any small area estimation initiative depends on the availability of good auxiliary data. Hence, it is important to devote a great deal of attention to the compilation of the auxiliary variables. Related to this, since small area estimation relies on model-based inference, it is important to establish what constitutes a good prediction model. This depends on being able to have tools in model selection and validation, particularly in terms of providing estimates that are robust to model misspecification. The difficulty lies in the fact that it is difficult to verify the soundness of model assumptions and it is not easy to ascertain goodness of fit. Specifically, the majority of small areas estimation models contain assumptions of unobservable random effects which are difficult to verify in practice.

As research on small area estimation continues to flourish, it is important that effort is also exerted on the development of analytical tools that can facilitate effective communication between producers of statistics and policymakers. Poverty mapping has facilitated this process through visualizing the spatial distribution and providing a detailed description of the small area measures. Nevertheless, more research is needed so that this statistical technique can be more useful for formulating policies and programs, allocation and evaluation of localised expenditure, regional planning and decision making.

5. Summary and Conclusion

While the literature of social exclusion and disadvantage in Australia has progressively moved towards the adoption of a multidimensional lens (Martinez and Perales 2014), there are very few studies that take into account the spatial and temporal dynamics of disadvantage, simultaneously. The concept of spatial dynamics takes the variations in poverty levels between different geographic and socio-demographic groups into consideration while the concept of temporal dynamics differentiates the transiently poor from the chronically poor. Examining these unique patterns is important because the policy response necessary to address these problems may be very different from each other.

Small area estimation of multidimensional poverty dynamics provides an analytical tool to better describe and examine the prevalence of social exclusion and disadvantage, change over time and how these differ across geographic and population clusters. These measures simultaneously take into account the multifaceted and dynamic components of socioeconomic disadvantage. While there are numerous studies that applied various small

area estimation strategies to come up with localised poverty estimates in Australia (e.g. ABS 2005; Harding, et al. 2006; ABS 2010; Miranti et al. 2011), this review of literature has identified very few studies that looked into the dynamics of multidimensional nature of disadvantage. Thus, a seamless integration of multidimensional measurement of poverty dynamics with small area estimation, in the Australian context, is an area that needs further research.

There are several opportunities and challenges that further research on small area estimation of multidimensional poverty dynamics needs to theorize, address and capitalize on. For instance, there are several large-scale longitudinal datasets that can be exploited because they are collected regularly. The HILDA Survey is an example of such data source, it is conducted annually and contains rich set of information on various measures of social exclusion and disadvantage for a panel sample of households and individuals. Together with the (longitudinal) Census and other administrative datasets collected by the Commonwealth Government, these data sources provide opportunities to study multidimensional poverty at the local level. However, access to unit-level records of some of these datasets is restricted due to confidentiality issues. Thus, many of the small area estimation of poverty studies in Australia has relied on either simplistic survey calibration techniques or microsimulation-based methods that do not require unit-level records for the auxiliary data. However, some of the more sophisticated small area estimation techniques that may be more appropriate for the local-level measurement of multidimensional poverty dynamics are premised on the availability of unit-record files. Hence, this is an area that future research needs to navigate its way around. The development of hierarchical spatial-temporal models is an area that can be explored. Intuitively, panel responses will be correlated over time, and previous research suggests that such temporal correlation can be exploited to achieve further efficiency gains in small area estimates (Ferrante and Pacei 2004; Martinez 2012). Additionally, this strategy can be supplemented with survey reweighting to known population benchmarks (Pfeffermann 2013; Martinez et al. 2014).

Finally, when examining multidimensional poverty dynamics at the small area-level, aside from measuring the magnitude of poverty, it is also important to identify the proximate determinants of movements into and out of the multiple dimensions of social exclusion and disadvantage. This topic can be potentially reframed within the life course paradigm as the latter offers a rich and encompassing conceptual framework for identifying life events that may precede or coincide with movements into and out of the states of poverty and non-poverty.

References

Australian Bureau of Statistics (2002), The 1998-99 Household Expenditure Survey, Australia: Confidentialised Unit Record Files (CURF), Technical Manual (2nd ed.), ABS Catalogue no. 6544.0, Canberra, Australian Bureau of Statistics.

Australian Bureau of Statistics (2005), Small Area Estimation of Disability Research Paper. Cat no. 1351.0.55.006, Canberra, Australian Bureau of Statistics. Available from: http://www.abs.gov.au/ausstats/abs@.nsf/mf/1351.0.55.006

Australian Bureau of Statistics (2010) - Research Paper: Small Area Estimation with Simulated Samples from the Population Census (Methodology Advisory Committee). Cat. No. 1352.0.55.106. Canberra, Australian Bureau of Statistics. Available from: http://www.abs.gov.au/ausstats/abs@.nsf/mf/1352.0.55.106

Australian Bureau of Statistics (2011), State and regional indicators. Cat no. 1367.2. Canberra, Australian Bureau of Statistics. Available from: http://www.abs.gov.au/ausstats/abs@.nsf/lookup/238C9905341645B9CA25783B000ECF29?opendocument

Australian Bureau of Statistics (2013), The Australian Longitudinal Census Dataset, 2006-2011 – Microdata. ABS cat no 2080.0. Canberra. Australian Bureau of Statistics. Available from: http://www.abs.gov.au/ausstats/abs@.nsf/mf/2080.0

Alkire, S. and Foster, J. (2011), Counting and multidimensional poverty measurement. *Journal of public economics*, 59(7/8), pp. 476-487.

Alkire, S. and Santos, E. (2014), A Multidimensional Approach: Poverty Measurement and Beyond. *Social Indicators Research*, 112(2), pp. 239-257.

Azpitarte, F. (2014), Has Economic Growth in Australia Been Good for the Income-Poor? And for the Multidimensionally-Poor? *Social Indicators Research*, 117(3), pp. 871-905.

Bell, W., Basel, W.,Cruse, C., Dalzell, L., , Maples, J., O'Hara, B., and Powers, D. (2007), Use of ACS data to provide SAIPE model-based estimates for counties. US Census Bureau. Available from: https://www.census.gov/did/www/saipe/publications/files/report.*pdf*.

Bell (2000), GREGWT and TABLE macros - User guide, ABS, Canberra, unpublished. Available from Author.

Breckling, J. and Chambers, R. (1988), M-quantiles, *Biometrika*, 75, pp. 761-771.

Bond, S. and Campos, C., (2010). Understanding income at small area level. *Regional Trends*, 42, pp.80–94.

Brown, J. J., Diamond, I. D., Chambers, R. L., Buckner, L. J., and Teague, A. D. (1999). A methodological strategy for a one-number census in the UK. *Journal of the Royal Statistical Society. Series A (Statistics in Society)*, pp. 247-267.

Breckling, J. and Chambers, R. (1988), M-quantiles, *Biometrika*, 75, pp. 761-771.

Chambers, R. and Tzavidis, N. (2006). M quantile models for small area estimation. *Biometrika*, 93, pp. 255-268.

Chandra, H. and Chambers, R. (2009), Multipurpose small area estimation. *Journal of Official Statistics*, 25, pp. 379-395

Chambers, R. and Tzavidis, N. (2006), M quantile models for small area estimation. *Biometrika*, 93, pp. 255-268.

Chandra, H. and Chambers, R. (2009), Mutlipurpose small area estimation. *Journal of Official Statistics*, 25, pp. 379-395

Chen, S. and Lahiri, P. (2002), On mean squared prediction error estimation in small area estimation problems. In Proceedings of the Survey Research Methods Section 473-477. American Statistical Association, Alexandria, VA.

Chin, SF, Harding, A, Lloyd, R, McNamara, J, Phillips, B and Quoc, NV (2005), Spatial Microsimulation Using Synthetic Small-area estimates of Income, Tax and Social Security Benefits, *Australasian Journal of Regional Studies*, 11(3), pp. 303 – 335

Citro,C.F. and Kalton, G. (2000), Small Area Estimates of School-Age Children in Poverty: Evaluation of Current Methodology. Committee on National Statistics, National Research Council, Washington, DC: National Academy Press.

Das, K. Jiang, J., and Rao, J.N.K. (2004), Mean squared error of empirical predictor. *Annals of Statistics*, 32, pp. 818-840

Datta, G.S. and Lahiri, P. (2000), A unified measure of uncertainty of estimated best linear unbiased predictors in small area estimation problems. *Statistica Sinica*, 613-627

Datta, G.S., Rao, J.N.K. and Smith, D.D. (2005), On measuring the variability of small area estimators under a basic area level model.

Datta, G.S. (2009), Model based approach to small area estimation. In *Sample Surveys: Inference and Analysis*, (Pfeffermann, D, and Rao, J.N.K (eds)), *Handbook of Statistics* 29B, 251-288. North Holland, Amsterdam.

Department of Education Employment and Workplace Relations. (2009), Small Area Labour Markets Australia. Canberra: Australian Federal Government.

Deming, W.E. and Stephan, F.F. (1940), On a least squares adjustment of a sampled frequency table when the expected marginal totals are known. *The Annals of Mathematical Statistics*, 11, pp. 427-444.

Deville, J.C. and Särndal, C.E. (1992), Calibration estimators in survey sampling, *Journal of the American Statistical Association*, 87, pp. 376-382.

Elazar, D.N. (2004), Small Area Estimation of Disability in Australia. *Statistics in Transition*, 6, pp. 667-684.

Elbers, C, Lanjouw, J. and Lanjouw, P. (2003), Micro-level estimation of poverty and inequality. *Econometrica*, 71, pp. 355-364.

Esteban, M.D., Morales, D., Perez, A. and Santamaria, L. (2012), Small area estimation of poverty proportions under area-time models. *Computational Statistics and Data Analysis*, 56, pp. 2840-2855.

Fabrizi, E., Salvati, N., Pratesi, M. and Tzavidis, N. (2014), Outlier robust model-assisted small area estimation. Biometrical Journal, 56, (1), 157-175.

Fay, R.E., Herriot, R.A., (1979), Estimates of income for small places: an application of James–Stein procedures to census data. *Journal of the American Statistical Association* 74, pp. 269–277.

Ferrante, M. and Pacei, S. (2004), Small area estimation for longitudinal surveys. *Statistical Methods and Applications*, 13(3), pp. 327-340.

Foster, J., Greer, J. and Thorbecke (1984), A class of decomposable poverty measures. *Econometrica*, 52, pp. 761-766

Foster, J., Greer, J. and Thorbecke (2010), The Foster-Greer-Thobecke (FGT) poverty measures; 25 years later. *Journal of Economic Inequality*, 8, pp. 491-524

Ghosh, M., Rao, J.N.K. (1994), Small area estimation: an appraisal. *Statistical Science* 9, pp. 55–93.

Gonzalez, M.E. (1973), Use and Evaluation of synthetic Estimates. Proceedings of the Social Statistics Section, 33-36, American Statistical Association, Alexandria, VA.

Hall, P. and Maiti, T. (2006), On parametric bootstrap methods for small area prediction. *Journal of the Royal Statistical Society: Series B (Statistical Methodology),* 68, pp. 221-238.

Harding, A., Lloyd, R., Bill, A., and King, A. (2006), Assessing Poverty and Inequality at a Detailed Regional Level: New Advances in Spatial Microsimulation. In McGillivray and Clarke (Eds), *Understanding Human Well-Being*, United Nations University Press, Helsinki, pp. 239-261.

Haslett, S., Isidro, M. and Jones, G. (2010), Comparison of survey regression techniques in the context of small area estimation of poverty. *Survey Methodology*, 36(2), pp. 157-170.

Haslett, S. and Jones, G. (2005), Small area estimation using surveys and some practical and statistical issues. *Statistics in Transition*, 7, pp. 541-555.

Hick, R. (2012), The capability approach: insights for a new poverty focus. *Journal of Social Policy*, *41*, pp. 291-308.

Huang, E.T. and Bell, W.R. (2006), Using the t-distribution in small area estimation: An application to the SAIPE state poverty models. In Proceedings of the Survey Research Methods Section 3142-3149. American Statistical Association, Alexandria, VA.

Jiang, J. and Lahiri, P., (2006), Estimation of finite population domain means: a model-assisted empirical best prediction approach. *Journal of the American Statistical Association*, 101, pp. 301-311.

Lohr, S.L. and Rao, J.N.K (2009), Jack-knife estimation of mean squared error of small area predictors in nonlinear mixed models. *Biometricka*, 96, pp. 457-468

McNamara, J., Tanton, R. and Phillips, B. (2007), The regional impact of housing costs and assistance on financial disadvantage: final report. Report 109. Australia Housing and Urban Research Institute, Melbourne.

Marhuenda, Y., Molina, I. and Morales, D. (2013), Small area estimation with spatio-temporal Fay Heriot models. *Computational Statistics and Data Analysis*, 58, pp. 308-325

Martinez, A. (2012), Small Area Estimation with a Multivariate Spatial-Temporal Model. *The Philippine Statistician*, 61(2), pp. 1-17.

Martinez, A. (2015), Analytical Considerations when Measuring Income Mobility. ARC Life Course Centre Working Paper No. 2015-04.

Martinez, A., Lucio, E. and Villaruel, M. (2014), Examining a weight reallocation method for small area estimation of poverty. *Electronic Journal of Applied Statistical Analysis*. DOI: 10.1285/i20705948v7n2p417

Martinez, A. and Perales, F. (2014), The Dynamics of Multidimensional Poverty in Contemporary Australia. ARC Life Course Centre Working Paper Series No. 2014-08.

Miranti, R., McNamara, J., Tanton, R. and Harding, A. (2011), Poverty at the local level: National and small area poverty estimates by family type for Australia in 2006. *Applied Spatial Analysis and Policy*, 4(3), pp. 145-171.

Molina, I., Saei, A. and Lombardía, J.M. (2007), Small area estimates of labour force participation under a multinomial logit mixed model'. *Journal of the Royal Statistical Society: Series A*, 170(4), pp. 975-1000.

Molina, I. and Rao, J.N.K. (2010), Small area estimation of poverty indicators. *The Canadian Journal of Statistics*, 38, pp. 365-389.

OECD (2014). Society at a glance – 2014 – OECD Social Indicators – Australia. Available from: www.oecd.org/social/societyataglance.htm

Pfeffermann, D. (2002), Small area estimation — new developments and directions. *International Statistical Review,* 70 (1), pp. 125–143.

Pfeffermann, D. (2013), New important developments in small area estimation. *Statistical Science*, 28, pp. 40-68.

Prasad, N.G.N., Rao, J.N.K., (1990), The estimation of the mean squared error of small-area estimators. *Journal of the American Statistical Association* 85, pp. 163–171.

Productivity Commission. (2013), Deep and Persistent Disadvantage in Australia. Productivity Commission Staff Working Paper.

Purcell, N.J. and Kish, L. (1979), Estimates for Small Domains, *Biometrics*, 35, pp. 365-384

Rahman, N., Clarke, P., McGrath, K., and Silvia, D. (2013), Small Area Estimates of Households in Poverty in England and Wales. UK Office for National Statistics: Titchfield

Rao, J.N.K. and Yu, M., (1994), Small area estimation by combining time series and cross sectional data. *Canadian Journal of Statistics* 22, pp. 511–528.

Rao, J.N.K. (1999), Some recent advances in model based estimation. *Survey Methodology*, 25, pp. 175-186

Rao, J.N.K. (2003), *Small Area Estimation*. New York: Wiley.

Rao, J.N.K. (2005), Inferential issues in small area estimation: some new developments. *Statistics in Transition*, 7, pp. 513-526.

Ravallion, M. (2012), On multidimensional indices of poverty. *Journal of economic inequality*, 9(2), pp. 235-248.

Saunders, P., and Naidoo, Y. (2009), Poverty, Deprivation and Consistent Poverty. *Economic Record*, 85(271), 417-432.

Scutella, R, Wilkins, R and Horn, M (2009), Measuring poverty and social exclusion in Australia: a proposed multi-dimensional framework for identifying socio-economic disadvantage (PDF file, 494 KB), Melbourne Institute of Applied Economic and Social Research and Brotherhood of St Laurence, Melbourne.

Scutella, R. and Wilkins, R. (2010), Measuring social exclusion in Australia: assessing existing data sources. *Australian Economic Review*, 43(4), pp. 449-463.

Scutella, R. Wilkins, R. and Kotsenko, W. (2013), Intensity and persistence of individuals' social exclusion in Australia.

Sen, A. (1976), Poverty: an ordinal approach to measurement. *Econometrica*, 44, pp. 219-231

Sen, A. (1979), Issues in the measurement of poverty. *Scandinavian Journal of Economics*, 81, pp. 285-307.

Sen, A. (2006), Conceptualizing and measuring poverty. In Grunky, D.B., Kanbur, S.M.R. (Eds) *Poverty and Inequality*, pages 30-46. Stanford University Press: Palo Alto

Singh A and Mohl C. (1996), Understanding calibration estimators in survey sampling, *Survey Methodology*, 22, pp. 107 - 115.

Singh, B., Shukla, G., Kundu, D., (2005), Spatio-temporal models in small area estimation. *Survey Methodology* 31, pp. 183–195.

Tanton R, Vidyattama Y, McNamara J, Vu Q and Harding A (2009), Old, Single and Poor: Using Microsimulation and Microdata to Analyse Poverty and the Impact of Policy Change among Older Australians, *Economic Papers: A journal of applied economics and policy*, 28(2), pp. 102-120.

Tanton R (2011), Spatial microsimulation as a method for estimating different poverty rates in Australia. *Population, Space and Place*, 17(3), pp. 222 - 235.

Tanton, R. (2014), A Review of Spatial Microsimulation Methods. *International Journal of Microsimulation*, 7(1), pp. 4-25

Tanton R, Vidyattama Y, Nepal B and McNamara J. (2011), Small area estimation using a reweighting algorithm. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 174(4), pp. 931 – 951.

The Scottish Government (2012), Scottish Index of Multiple Deprivation. Available from: http://www.gov.scot/Topics/Statistics/SIMD

Townsend, P. (1987), Deprivation. *Journal of Social Policy*, 16, pp. 125-146

Tzavidis, N., Salvati, N. Pratesi, M. and Chambers, R. (2008), M quantile models for poverty mapping. Statistical Methods and Applications, 17, pp. 393-411.

Tzavidis, N., Marchetti, S,. and Chambers, R. (2010), Robust estimation of small area means and quantiles. *Australia New Zealand Journal of Statistics*, 52, pp. 167-186.

UK Department for Communities and Local Government (2011), English Indices of Multiple Deprivation. Available from; https://www.gov.uk/government/statistics/english-indices-of-deprivation-2010

United Nations Development Programme. (2004), *UN Human Development Report 2004*. United Nations Development Programme. (Available from: http://hdr.undp.org/en/content/human-development-report-2004.

US Census Bureau (2014). Small Area Income and Poverty Estimates. Available from; http://www.census.gov/did/www/saipe/data/highlights/2013.html

Vidyattama Y, Cassells R, Harding A and McNamara, J. (2011), Rich or poor in retirement? A small area analysis of Australian superannuation savings in 2006 using spatial microsimulation. *Regional Studies*, 47(5), pp. 722 – 739. doi: 10.1080/00343404.2011.589829

Welsh Government (2014), Welsh Index of Multiple Deprivation. Available from: http://gov.wales/statistics-and-research/welsh-index-multiple-deprivation/?lang=en