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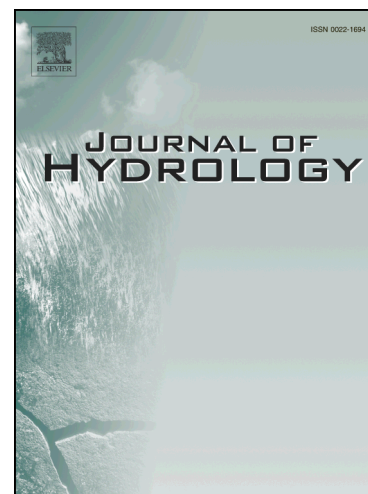
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**Periodic seepage face formation and water  
pressure distribution along a vertical  
boundary of an aquifer**

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23 **Abstract**

24 Detailed measurements of the piezometric head from sand  
25 flume experiments of an idealized coastal aquifer forced by a  
26 simple harmonic boundary condition across a vertical boundary  
27 are presented. The measurements focus on the pore pressures  
28 very close to the interface ( $x = 0.01 \text{ m}$ ) and throw light on the  
29 details of the boundary condition, particularly with respect to  
30 meniscus suction and seepage face formation during the falling  
31 tide. Between the low and the mean water level, the response is  
32 consistent with meniscus suction free models in terms of both  
33 the vertical mean head and oscillation amplitude profiles and is  
34 consistent with the observation that this area of the interface  
35 was generally within the seepage face. Above the mean water  
36 level, the influence of meniscus formation is significant with  
37 the mean pressure head being less than that predicted by  
38 capillary free theory and oscillation amplitudes decaying faster  
39 than predicted by suction free models. The reduced hydraulic  
40 conductivity in this area due to partial drainage of pores on the  
41 falling tide also causes a delay in the response to the rising tide.  
42 The combined influence of seepage face formation, meniscus  
43 suction and reduced hydraulic conductivity generate higher  
44 harmonics with amplitudes of up to 26% of the local main  
45 harmonic. To model the influence of seepage face formation  
46 and meniscus suction a numerical solution of the Richards  
47 equation was developed and evaluated against the data. The

48 model-data comparison shows a good agreement with the  
49 behavior high above the water table sensitive to the choice of  
50 moisture retention parameters.

51

52 **Keywords:** groundwater; seepage face; capillary fringe;  
53 Richards' equation

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75 **1. Introduction**

76 The interaction between surface and sub-surface water plays an  
77 important role in a variety of coastal zone processes including  
78 salt-water intrusion and contaminant transport in coastal  
79 aquifers (e.g. Cartwright et al., 2004a, b; Cartwright and  
80 Nielsen, 2001a,b, 2003; Isla and Bujalesky, 2005; Nielsen,  
81 1999; Nielsen and Voisey, 1998; Robinson et al., 2006; Turner  
82 and Acworth, 2004; Xin et al., 2010) and beach profile  
83 morphology (e.g. Emery and Foster, 1948; Grant, 1946, 1948).  
84 Oceanic forcing of coastal aquifers across the beach face is  
85 highly dynamic occurring over a wide range of magnitude and  
86 frequency scales (i.e. tide, wave, storm surge, etc.). A number  
87 of oceanic and atmospheric mechanisms which have been  
88 involved with observed beach water table fluctuations  
89 identified by Turner (1998). The majority of studies have  
90 described beach groundwater fluctuations due to tidal forces  
91 (e.g., Emery and Foster, 1948; Ericksen, 1970; Lanyon et al.,  
92 1982; Nielsen, 1990; Turner, 1993a; Turner et al., 1997). A  
93 limited number of studies have observed wave-induced the  
94 beach water table oscillations (Bradshaw, 1974; Cartwright et  
95 al., 2002, 2006; Hegge and Masselink, 1991; Kang et al., 1994;  
96 Lewandowski and Zeidler, 1978; Turner and Nielsen, 1997;  
97 Turner and Masselink, 1998; Waddell, 1973, 1976, 1980).  
98 Understanding the behavior of this periodic boundary condition

99 is thus important for accurate modeling of coastal groundwater  
100 dynamics and associated issues.

101 Existing analytical models of ground water dynamics are based  
102 on the one or two-dimensional solution of the Boussinesq  
103 equation under the Dupuit-Forchheimer assumption, (e.g. Baird  
104 et al., 1998; Li et al., 2002; Nielsen et al., 1997; Nielsen, 1990)  
105 with corrections for vertical flow effects and also capillary  
106 fringe effects by only considering the additional water mass  
107 above the water table (e.g. Barry et al., 1996; Cartwright et al.,  
108 2005; Li et al., 2000; Nielsen and Perrochet, 2000; Nielsen and  
109 Turner, 2000). None of the analytical models consider  
110 unsaturated flow or seepage face and meniscus formation at the  
111 boundary.

112 In the natural system, the interface between surface and  
113 groundwater is generally sloping; however, in order to simplify  
114 the problem, a vertical interface is considered here. This paper  
115 presents detailed measurements of the piezometric head close  
116 to the vertical interface ( $x = 0.01 \text{ m}$ ) of a non-shallow  
117 laboratory aquifer forced by simple harmonic oscillations. The  
118 data provides insight into the influence of meniscus suction and  
119 seepage face formation in and around the inter-tidal zone. The  
120 data is then used to evaluate a 2D vertical numerical model  
121 based on the Richards equation (Richards, 1931) with due  
122 consideration of the mixed periodic boundary condition to

123 simulate the formation of the seepage face and meniscus  
124 suction.

125

## 126 **2. Capillary suction and seepage face formation on the** 127 **interface**

128 Figure 1 provides a schematic illustration of the pressure  
129 distribution along a beach face when the water table exit point  
130 becomes decoupled from the ocean level. Note similar  
131 scenarios will exist in systems with periodic forcing of  
132 groundwater systems such as tidal rivers and lakes where  
133 seiching may occur. When decoupling occurs, two distinct  
134 pressure zones become apparent. Below the exit point and  
135 above the ocean level (i.e. in the seepage face), the surface has  
136 a glassy appearance indicating that the water table is at the  
137 surface and that the gauge pressure  $p(x, z) = 0$ . Above the exit  
138 point, the surface has a matt appearance due to the presence of  
139 meniscuses and as such  $p(x, z) < 0$ .

140 The capillary suction gets stronger with increasing elevation  
141 above the water table, but upwards of a certain level this  
142 suction will not have a significant effect on watertable  
143 dynamics due to a lack of connectivity in sand with low  
144 moisture content and hence very low permeability. Some a-  
145 priori insight into vertical and horizontal flow in the capillary

146 fringe might be gained from the steady flow study of Silliman  
147 et al. (2002).

148 Several numerical and experimental studies have been  
149 conducted which consider the exit point location and seepage  
150 face formation. Turner (1993b, 1995) adapted a numerical  
151 model from the governing equations of Dracos (1963) to  
152 simulate exit point movement across a saturated beach face.  
153 The model is based solely on the force balance on a water  
154 particle at the sand surface and neglects the sub-surface  
155 pressure distribution. In addition, Turner (1993b, 1995)  
156 assumed that, during the decoupled phase, the movement of the  
157 exit point is independent of the tide level.

158 Clement et al. (1994) developed a 2D finite-difference  
159 algorithm to solve Richards' (1931) variably saturated flow  
160 equation for porous media which was then applied to solve  
161 steady state and transient seepage face problems. Clement et al.  
162 (1994) used three kinds of boundary conditions including  
163 Dirichlet boundary condition for nodes with known pressure  
164 head, Neumann boundary condition for nodes where the values  
165 of normal fluxes are known and a seepage face boundary  
166 condition. During simulation of the variably saturated flow, the  
167 length of seepage face is unknown until the problem is solved;  
168 however, the problem cannot be completely solved unless the  
169 length of seepage face is determined. Hence, an iterative  
170 process is needed to determine the seepage face length at each



171 time step. Clement et al. (1994) used Cooley's (1983) modified  
172 version of Neuman's (1973) iterative-search procedure which is  
173 based on the following. During the first iteration, an initial  
174 guess of the location of the exit point (i.e. the length of seepage  
175 face) is used to solve the flow equation. Based on the solution's  
176 results for pressure head and flow along the boundary, it is  
177 possible to understand whether the location of exit point is  
178 correct or it needs modification. One of three different  
179 conditions may exist. First, the solution gives a zero pressure  
180 and a net outflow for all nodes along the seepage face which  
181 means that the guessed location of exit point is correct. The  
182 nodes above the seepage face are considered as a no-flow  
183 boundary condition with negative pressure. Second, if the  
184 results show non-zero inflow for some of the nodes along the  
185 seepage face which have zero pressure, the height of exit point  
186 is overestimated. Third, if some of the nodes above the seepage  
187 face which are located on no-flow boundary condition get  
188 positive flux, the height of the seepage face is underestimated.  
189 The seepage face height is then adjusted as required and the  
190 flow equation solved again with the new interface pressure  
191 profile. This iterative method is repeated until finding the  
192 correct length of the seepage face is produced. This model was  
193 later validated by Simpson et al. (2003) against laboratory  
194 observations in a radial sand tank. Ataie-Ashtiani et al. (1999)  
195 also adopted this approach when simulating periodic seepage

196 face formation with the density dependent variably saturated  
197 groundwater flow model SUTRA (Voss, 1984).

198 Li et al. (1997) presented a Boundary Element Method (BEM)  
199 model to solve a 2D flow equation to simulate the groundwater  
200 fluctuations and seepage face dynamics under tidal forcing for  
201 saturated flow conditions using a moving boundary condition.

202 On the water free surface profile (i.e. the water table), the  
203 potential head is unknown, but by applying a kinematic  
204 boundary condition on the free surface (Liggett and Liu, 1983),  
205 the potential head and consequently the water table head  
206 elevation profile can be determined. The elevation of the water  
207 table exit point can be obtained as the intersection of the water  
208 table and beach face profile and the shoreline elevation is the  
209 tidal elevation. If the exit point becomes decoupled from the  
210 tide then a seepage face exists between shoreline and exit point  
211 and the boundary condition on the seepage face is set to  
212 atmospheric pressure (i.e. the potential head is equal to  
213 elevation head), otherwise, the potential head is calculated  
214 based on the tidal elevation.

215 Baird et al. (1998) developed a numerical solution of the 1D  
216 Boussinesq equation including seepage formation. In the  
217 numerical model, if the landward computational cell (i.e. cells  
218 are located before shoreline) is completely filled with the water,  
219 it can be assumed that a seepage face is exist and the most  
220 landward cell with this condition will be considered as the exit

221 point. Baird et al. (1998) defined a condition in their numerical  
222 Boussinesq model to consider the presence of seepage face.  
223 Based on that condition, if at any computational cells the  
224 summation of water table elevation and the net rate of  
225 groundwater discharge into and out the cell during the time step  
226 per cross-shore width of computational cell is greater than the  
227 cell's elevation, water level is considered on the ground surface  
228 for that cell and decoupling is happened.

### 229 **3. Experimental setup and procedures**

#### 230 3.1 The flume and sand

231 The experimental setup is illustrated in Figure 2 where a 9.2 m  
232 long, 0.15 m wide and 1.5 m high unconfined sand flume  
233 aquifer is subject to simple harmonic forcing across a vertical  
234 boundary at the “ocean” end of the flume and a no-flow  
235 boundary condition was used at the “landward” end of the  
236 flume. The vertical interface between the external driving head  
237 reservoir and the aquifer consisted of a filter made up of  
238 stainless steel wire mesh with 0.15 mm openings supported by  
239 a coarser grid with 2 cm openings. The top of the flume is open  
240 to atmosphere, but it was covered by a loose plastic to  
241 minimize any evaporation. To reduce air encapsulation during  
242 the sand packing process, the sand was added in ~10cm  
243 thickness layers to the water-filled flume and the layers packed  
244 by allowing them to settle by gravity. Subsequent layers where

245 then added and manually mixed with the preceding layer so as  
246 to avoid layering due to differential sedimentation.

247 Locally mined dune sand containing more than 99% quartz  
248 content was used in the flume and Table 1 presents the sand's  
249 physical and hydraulic properties which were investigated by  
250 Nielsen and Perrochet (2000).

### 251 3.2 The driving head

252 The driving head in the clear water reservoir  $h_o(t)$ , was simple  
253 harmonic such that,

$$254 \quad h_o(t) = d + A_o \cos(\omega t) \quad (1)$$

254 where  $d$  is the mean elevation,  $A_o$  is the amplitude and  
255  $\omega = 2\pi/T$  is the angular frequency and  $T$  is the oscillation  
256 period. The data presented here is for the following forcing  
257 parameters:  $T = 567$  s,  $A_o = 0.215$  m and  $d = 0.92$  m.

### 258 3.3 Monitoring of piezometric head

259 The piezometric head was measured using UMS-T5  
260 tensiometers installed horizontally into the aquifer through the  
261 wall of the flume. The focus of the experiments was on the  
262 physics close to the hydrostatic reservoir and so tensiometers  
263 were installed at  $x = 0.01$  m at each of the following  
264 elevations:  $z = 0.6, 0.7, 0.8, 0.9, 1.0, 1.1$  m.

265

266 **4. Numerical modeling**

267 As will be demonstrated later, the experimental observations  
 268 show significant influence of meniscus suction and seepage  
 269 face formation on the aquifer response. Neither of these  
 270 processes are considered by the analytic solutions outlined  
 271 previously in section 1 and so a numerical modelling approach  
 272 was developed.

273 4.1 Governing Equations

274 To simulate the influence of meniscus formation at the  
 275 interface above the water table requires consideration of  
 276 variably saturated flow which is governed by the Richards'  
 277 equation (Richards, 1931),

278

$$\left(\frac{C_m}{\rho g} + S_e S\right) \frac{\partial H_p}{\partial t} + \nabla \left( -\frac{\kappa_s}{\mu} k_r (\nabla H_p + \nabla z) \right) = 0 \quad (2)$$

279

280 where  $H_p$  is the pressure head which is the dependent  
 281 variable,  $C_m$  is the specific moisture capacity,  $S_e$  is the effective  
 282 saturation,  $S$  is the storage coefficient,  $\nabla$  is the gradient  
 283 operator,  $\kappa_s$  is the intrinsic permeability which is related to the  
 284 hydraulic conductivity ( $K$ ) as  
 285  $\kappa_s = K\mu/\rho g$ ,  $\mu$  is the fluid dynamic viscosity,  $k_r$  is the relative  
 286 permeability,  $z$  is the vertical elevation.

287 Richards' equation (2) is solved here using the finite element  
 288 method using two commercially available software packages,  
 289 COMSOL 4.3b (COMSOL, 2013) and FEFLOW 6.0  
 290 (FEFLOW, 2012). The two packages were used in order to  
 291 evaluate differing approaches for modelling seepage face  
 292 formation as will be described later in section 4.2.

293 Solution of Richards' equation (2) requires prior knowledge of  
 294 the specific moisture capacity  $C_m$  and the relative  
 295 permeability  $k_r$  which are both dependent on the soil moisture  
 296 retention properties. Here, the soil moisture retention properties  
 297 are quantified using the van Genuchten (1980) formulae,

$$\theta = \begin{cases} \theta_r + \frac{\theta_s - \theta_r}{\left[1 + |\alpha H_p|^\beta\right]^m} & H_p < 0 \\ \theta_s & H_p \geq 0 \end{cases} \quad (3)$$

298 where  $\theta_r$  and  $\theta_s$  are the residual and saturated liquid volume  
 299 fractions.

300 The van Genuchten relative permeability is,

$$k_r = \begin{cases} S_e^l \left[1 - \left(1 - S_e^{\frac{1}{m}}\right)\right]^2 & H_p < 0 \\ 1 & H_p \geq 0 \end{cases} \quad (4)$$

301 where the effective saturation is,

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad (5)$$

302

303 The specific moisture capacity is defined as,

$$C_m = \frac{d\theta}{dH_p} = \begin{cases} \frac{\alpha m}{1-m} (\theta_s - \theta_r) S_e^{\frac{1}{m}} \left(1 - S_e^{\frac{1}{m}}\right)^m & H_p < 0 \\ 1 & H_p \geq 0 \end{cases} \quad (6)$$

304

305 where  $\alpha, \beta, l = 0.5$  and  $m = 1 - 1/\beta$  are empirical curve

306 fitting parameters and  $H_p = 0$  is the atmospheric pressure

307 distinguishes saturated and unsaturated flow. Table 1 provides

308 hydraulic and moisture parameters of the sand which were used

309 in the numerical simulation.

310

#### 311 4.2 Boundary condition implementation

312 Two different methods were applied to simulate the simple

313 harmonic “ocean” boundary condition with seepage face

314 formation. A Cauchy boundary condition was implemented in

315 the COMSOL simulations and a prescribed head boundary

316 condition combined with flux constraints was used in the

317 FEFLOW simulations. The principle of these two methods is

318 similar to the methods described in 2 i.e. dividing the boundary

319 to three separated parts and changes from Dirichlet to Neumann

320 boundary condition. However, the Cauchy boundary condition

321 uses the logical statements based on the saturation condition

322 and changing the thickness of an arterial layer between external

323 fluid source and the domain to switch between Dirichlet to

324 Neumann boundary condition. The prescribed head with flux

325 constraint method switches the boundary condition between

326 Dirichlet and Neumann based on the flow direction on each  
 327 part, similar to Clement et al.'s (1994) method.

328

#### 329 4.2.1 Cauchy boundary condition

330 The Cauchy boundary condition is given by,

331

$$\mathbf{n} \cdot \rho K \nabla (H_p + z) = \rho R_b [(H_{pb} - H_p) + (z_b - z)] \quad (7)$$

332 where  $H_{pb}$  and  $z_b$  are the pressure and elevation of the distant  
 333 fluid source, respectively and  $R_b$  is the conductance of the  
 334 material between the source and the model domain.

335 Typically  $R_b = K'/B'$ , where  $K'$  is hydraulic conductivity of the  
 336 layer and  $B'$  is its thickness, which were assumed here to be  
 337  $4.7 \times 10^{-4} \text{ m/s}$  and  $0.001 \text{ m}$ , respectively.

338 The Cauchy type boundary condition is used in conjunction  
 339 with appropriate logical statements in order to switch between a  
 340 Dirichlet boundary condition for nodes below the ocean level  
 341 and in the seepage face and a Neumann boundary condition  
 342 above of the water table exit point (cf. Figure 3). Following the  
 343 work of Chui and Freyberg (2009), at the start of each time  
 344 step, all nodes below the external driving head level ( $h_o$ ) are  
 345 assigned pressures assuming a hydrostatic external pressure  
 346 distribution and for nodes above the driving head level the  
 347 pressure head is zero,



348

$$H_{pb} = \begin{cases} h_o - z & \text{for } z \leq h_o \\ 0 & \text{for } z > h_o \end{cases} \quad (8)$$

349 The flow equation (2) is then solved and the position of the  
 350 water table (i.e.  $p = 0$ ) is determined. Below the water table in  
 351 the saturated zone ( $p > 0$ ) the conductance  $R_b$  is modified to a  
 352 large number ( $R_b = K'/B'$ ) thus creating a flow condition and  
 353 above the water table the conductance is set to zero ( $R_b = 0$ )  
 354 thus creating a no-flow boundary condition. That is,

$$R_b = \begin{cases} \frac{K'}{B'} & \text{for } H_p \geq 0 \Rightarrow \text{flow B.C.} \\ 0 & \text{for } H_p < 0 \Rightarrow \text{no-flow B.C.} \end{cases} \quad (9)$$

355

356 During the same time step, the new boundary condition  
 357 (equation (9)) is applied and equation (2) is solved again and  
 358 the position of exit point adjusted. This iterative procedure  
 359 continues until the correct position of the exit point is found  
 360 such that above the exit point there is no flow and pressure  
 361 head is negative and that along the seepage face, flow drains  
 362 the domain and pressure head is zero.

363

#### 364 4.2.2 Prescribed head with flux constraint

365 In the FEFLOW model, seepage face formation is modelled  
 366 using a prescribed head boundary condition in conjunction with  
 367 a constrained flux condition. For boundary nodes below the

368 minimum driving head level the head is prescribed to be the  
369 same as the driving head (i.e.  $h = h_o$  for  $z \leq \min h_o$ ). For  
370 boundary nodes above the maximum driving head level a no-  
371 flow condition is applied (Figure 4).

372 For boundary nodes between the minimum and maximum of  
373 driving head level the prescribed head with flux constraint is  
374 implemented as illustrated in Figure 5. In each time step, if the  
375 driving head level is above the node then the head is prescribed  
376 as  $h(z) = h_o$  and the flux is unconstrained. If the driving head  
377 level is below the node then the node will either be in the  
378 seepage face (outflow from the domain) or above the exit point  
379 (no-flow). If the flow at the node is positive (i.e. into the  
380 domain) then the flux is constrained to  $q < 0 \text{ m}^3/d$  and the  
381 prescribed head condition is relaxed and the pressure head is  
382 allowed to be negative. The model then iterates and adjusts the  
383 water table position until the solution converges.

384

## 385 **5. Results and Discussion**

### 386 **5.1 Piezometric head distribution**

387 Figure 6 compares the measured and predicted piezometric  
388 head time series at different intertidal elevations very close to  
389 the boundary,  $h^*(x = 0.01 \text{ m}, z, t)$ . While the driving head is  
390 simple harmonic, the piezometric head at higher elevations  
391 indicates the influence of the generation of higher harmonic

392 components due to a combination of seepage face formation  
393 and the non-linear relationship between moisture content and  
394 pore pressure (cf. equation (3)).

395 Typically, the intertidal time series separate from the driving  
396 head when the driving head drops below the measurement  
397 elevation because of seepage face formation with the falling  
398 water level and also due to the draining of pore water which  
399 leads to a lower hydraulic conductivity. At  $z = 1.1 \text{ m}$ , a  
400 significant delay during the rising of driving head is also seen  
401 because the sand surrounding the probe becomes partly drained  
402 and hence has a lower hydraulic conductivity until it becomes  
403 re-saturated and returns to a saturated hydraulic conductivity.  
404 The measurements below the low level of driving head are not  
405 shown, but they all follow the driving head very closely as  
406 shown by the probe at the low level of driving head ( $z =$   
407  $0.7 \text{ m}$ ).

408 The numerical results show that the two different methods  
409 applied to simulate seepage face formation produce identical  
410 results. In addition, the comparison between the results of both  
411 models and laboratory data shows a good agreement for  
412  $z \leq 0.9 \text{ m}$ . However, at higher elevations there are some  
413 obvious discrepancies, especially at the highest elevation  
414 ( $z = 1.1 \text{ m}$ ), where the model underestimates the hydraulic  
415 head. This is because model performance in the unsaturated  
416 zone will be more sensitive to any uncertainty in the adopted

417 van Genuchten (1980) moisture retention curve parameters ( $\alpha$   
418 and  $\beta$ ).

419

420 Many previous studies (e.g. Lehman et al., 1998; Stauffer and  
421 Kinzelbach, 2001; Werner and Lockington, 2003) show that  
422 consideration of hysteresis can significantly improve the  
423 predictive ability of the Richards' equation under periodic flow  
424 conditions.

425 Cartwright et al. (2005) found that using a single non-hysteretic  
426 moisture retention curve with  $\beta = 3$  captured the observed  
427 water table dynamics in periodic sand column experiments.

428 Cartwright (2014) demonstrated that this is due to the fact that  
429 the  $\beta = 3$  moisture retention curve has a specific moisture  
430 capacity ( $C_m = d\theta/dH_p$ ) which more closely resembles the  
431 observed moisture-pressure scanning loops compared to the  
432 specific moisture capacity found using the first drying curve  
433 data ( $\beta = 9$ ).

434

435 To examine this further, the model was run using a modified  
436 moisture retention curve with  $\beta = 3$  that was fit to the ( $\beta = 9$ )  
437 wetting and drying curves (cf. Figure 7). Note that the wetting  
438 curve was estimated based on the observed drying curve  
439 ( $\beta = 9$ ) and a hysteresis ratio,  $\xi = \alpha_w/\alpha_d = 2$  after Kool and  
440 Parker (1987). Figure 8 shows the new comparison of  
441 numerical prediction using  $\beta = 3$  with the experimental data

442 for different elevations at  
443  $x = 0.01 \text{ m}$ . It is apparent that the modified  $\beta = 3$  moisture  
444 retention curve significantly improves the numerical results,  
445 especially at upper elevations ( $z = 1.0, 1.1 \text{ m}$ ) in the  
446 unsaturated zone where the specific moisture capacity plays a  
447 greater role.  
448  
449 Table 2 summarises the harmonic components for laboratory  
450 data and numerical results further demonstrating the generation  
451 of higher harmonics due to seepage face formation and  
452 meniscus suction at the boundary. Above the minimum water  
453 elevation ( $z = 0.7 \text{ m}$ ), the higher order harmonic amplitudes  
454 phases are seen to increase with elevation. The maximum ratio  
455 of the second harmonic to the fundamental mode is  
456  $R_2/R_1 = 0.26$  at  
457 ( $x = 0.01, z = 1.1 \text{ m}$ ). For the third harmonic, the  
458 corresponding maximum is  $R_3/R_1 = 0.07$  at ( $x = 0.01, z =$   
459  $1.1 \text{ m}$ ).

## 461 5.2 Pressure head range

462 Measured and simulated pressure head ranges very close to the  
463 boundary ( $x = 0.01 \text{ m}$ ) are shown in Figure 9. Since the  
464 results of the other simulations (cf. *Range* in Table 2) were  
465 almost similar, only the result of FEFLOW simulation with the  
466 modified retention curve

467 (i.e.  $\alpha = 3 \text{ m}^{-1}$  and  $\beta = 3$ ) are shown in this figure. The solid  
468 line shows the pressure head range in the reservoir. For  
469 elevations below the low water level the head range is similar  
470 to reservoir head because of hydrostatic pressure distribution.  
471 For  $0.7 < z(m) < 0.8$  the head range very close to the  
472 reservoir head which means the negative pressure due to  
473 meniscuses formation is negligible. For  $z > 0.8 \text{ m}$  the pressure  
474 head range is separated from the reservoir head because no  
475 negative pressure can exist in the reservoir while inside the  
476 aquifer at  $x = 0.01 \text{ m}$ , formation of meniscuses at the sand  
477 surface act to generate negative pressures and hence the  
478 pressure head range reduces for higher elevations. A good  
479 agreement between measured and predicted data can be seen in  
480 Figure 9.

481

482 5.3 Phase variation of the pressure through various verticals

483 Figure 10 shows the comparison of measured and predicted  
484 phase lag at  $x = 0.01 \text{ m}$ . In both numerical models the best  
485 agreement can be obtained by using modified retention curve  
486 i.e. van Genuchten parameters of  $\alpha = 3 \text{ m}^{-1}$  and  $\beta = 3$ . The  
487 phase lag relative to the driving head ( $x = 0, z = 0$ ) is almost  
488 zero (i.e. constant phase) below the mean water level  
489 ( $z = 0.92 \text{ m}$ ) indicates hydrostatic behaviour in this range. At  
490 higher elevations, the phase lag increases due to non-  
491 hydrostatic behaviour in upper elevations which is the result of

492 existence of higher harmonics because of seepage face  
 493 formation and meniscus suction.

494

495 5.4 Mean pressure head profile

496 Philip (1973) used time averaging of the Boussinesq equation  
 497 to predict the asymptotic inland overheight of the watertable in  
 498 the absence of meniscus formation and/or seepage formation.

$$\bar{\eta}_{\infty} = \sqrt{h^2 + \frac{A^2}{2}} - h \approx \frac{A^2}{4h} \quad (10)$$

499

500 Cartwright et al. (2003) observed that the asymptotic (landward  
 501 boundary) value of the time-averaged head profile  $\bar{h}_{\infty}$  is less  
 502 than the ‘Boussinesq’ value predicted by Philip (1973)  
 503 (equation (10)). The present experiments also showed the same  
 504 results i.e. a lower measured value of  $\bar{h}_{\infty} = 0.924 \text{ m}$  compared

505 with Philip’s  $\bar{h}_{\infty} = \sqrt{d^2 + \frac{1}{2}A^2} = 0.932 \text{ m}$ , corresponding to a

506 measured overheight of 4 mm and a predicted of 12 mm. Knight

507 (1982) showed Philip’s result is valid even for non-shallow

508 aquifer, hence this difference is likely due to negative pressure

509 above the driving head and capillary fringe effects which are

510 not accounted by Philip’s theory.

511 The time-averaged pressure head distribution above the low  
 512 water level without considering the capillarity effects can be  
 513 expressed as (see appendix A for detail),

$$\frac{\bar{p}}{\rho g} = \frac{1}{\pi} \left[ (d - z) \cos^{-1} \left( \frac{z - d}{A} \right) + A \sqrt{1 - \left( \frac{z - d}{A} \right)^2} \right] \quad (11)$$

514 Figure 11 compares the measured and predicted time-averaged  
 515 pressure head at different elevations at  $x = 0.01 \text{ m}$ . For clarity,  
 516 only the FEFLOW results using modified retention curve (i.e.  
 517  $\alpha = 3 \text{ m}^{-1}$  and  $\beta = 3$ ) are shown in the figure. The results of  
 518 other simulations show the same trend and they are  
 519 summarized in Table 2 (cf.  $\bar{p}/\rho g$ ). The time-averaged  
 520 pressure head distribution calculated by equation (11) is also  
 521 shown in the figure as a reference. A good agreement between  
 522 model results and laboratory data can be seen in this figure.

523 As expected, the mean water pressure head is hydrostatic below  
 524 the minimum water level ( $z \leq 0.7 \text{ m}$ ). For  $0.7 < z \text{ (m)} < 0.8$   
 525 the trend still follows the theoretical curve suggesting that the  
 526 meniscuses and capillary effects are not significant in this range  
 527 due to the presence of a seepage face during the falling stage of  
 528 driving head. For  $z > 0.8 \text{ m}$ , the mean water pressure head is  
 529 lower than the theoretical curve demonstrating the significance  
 530 of negative pressures at the boundary (i.e. meniscus formation  
 531 and capillarity effects).



532 **6. Conclusion**

533 A laboratory sand flume has been used to observe the  
534 piezometric head in an idealised unconfined aquifer bordering a  
535 tidal (simple harmonic) reservoir with a vertical interface. The  
536 data demonstrate the influence of seepage face and meniscus  
537 formation at the boundary which lead to the generation of  
538 higher harmonics in the pore pressure time series at locations  
539 above the water table. The data also show that the formation of  
540 meniscuses and capillary suction has a significant effect on  
541 reduction of mean pressure head and pressure head range in  
542 upper elevation above minimum water level where located in  
543 unsaturated zone and have lower hydraulic conductivity related  
544 to saturated part. At higher elevations, the phase lag related to  
545 the tide is also increased due to non-hydrostatic behaviour  
546 which is the result of existence of higher harmonics because of  
547 seepage face formation and meniscus suction. The laboratory  
548 data indicate that the seepage face formation and capillary  
549 suction due to meniscuses play an important role in ground  
550 water flow and should be consider in the numerical models by  
551 using unsaturated flow models.

552 The experimental data was then used to evaluate the predictive  
553 capabilities of a numerical solution of the Richards equation.  
554 Two approaches to the boundary condition were evaluated.  
555 The first method used a mixed (Cauchy) type boundary  
556 condition with appropriate logic statements to switch between a

557 Dirichlet boundary condition below the ocean level and in the  
558 seepage face and a Neumann boundary condition above of the  
559 water table exit point. The second method was a combination  
560 of a prescribed head and the flux constraint condition to  
561 activate a Dirichlet boundary condition below the ocean level  
562 and along the seepage face and a Neumann boundary condition  
563 above the exit point. The results show that both methods were  
564 equal in capturing the influence of seepage face and meniscus  
565 formation on the pressure along the boundary.

566 The comparison between the simulated and measured pressure  
567 head distribution along the boundary revealed significant  
568 discrepancies, especially in higher elevations (located in the  
569 unsaturated zone). These discrepancies were overcome by  
570 adopting a modified moisture retention curve with a specific  
571 moisture capacity ( $C_m = d\theta/dH_p$ ) more closely related to the  
572 moisture-pressure scanning loops observed by Cartwright  
573 (2014) using the same sand type.

574 In terms of the mean pressure head profile near the boundary,  
575 the simulated results are in a good agreement with the  
576 laboratory data. The results also show the effect of capillary  
577 suction and meniscuses formation in reducing the mean  
578 pressure head in upper elevations near the boundary. In  
579 addition, comparison of harmonic components of laboratory  
580 data and numerical results show the ability of numerical models

581 to reproduce the generation of higher harmonic in hydraulic  
582 head time series in upper elevation located in capillary fringe.

583 It is noted that the present study considers the simple case of a  
584 vertical boundary. However, for natural systems such as  
585 beaches and river banks, the interface is generally sloped. The  
586 methods demonstrated in this paper to simulate the effects of  
587 seepage face and meniscus formation can readily be applied on  
588 sloped surface and is the focus of ongoing work.

589 The interaction of surface and subsurface water at the beach  
590 face plays a vital role in changing the hydraulic gradients and  
591 controlling the in/exfiltration across the interface.

592 In/exfiltration across the beach face is linked to both sediment  
593 transport (e.g. Elfrink and Baldock, 2002) and also  
594 contaminant transport and saltwater intrusion (e.g. Xin et al,  
595 2010). The data and modelling approaches discussed in this  
596 paper will thus provide some useful insights into more accurate  
597 modelling of these types of problems.

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873

874 **Appendix A: Capillary free time-averaged pressure head**  
 875 **profile**

876

877 Below the low level of driving head the time averaged  
 878 piezometric head in the reservoir is  $d$  . Above the low level of  
 879 driving head, in a capillarity free scenario (zero pressure at all  
 880 points above water), it is given by

$$881 \quad \overline{h^*} = \frac{1}{T} \int_{t_u}^{t_d} (d + A \cos \omega t) dt + [T - (t_d - t_u)]z$$

882 A(1)

883 where  $t_u(z)$ , respectively  $t_d(z)$  are the time of zero upcrossing  
 884 and downcrossing for the water surface through the level  $z$ , i

885 .e.,  $t_u = -(T/2\pi) \cos^{-1}(z/A)$  and  $t_d = (T/2\pi) \cos^{-1}(z/A)$

886  $t_d = (T/2\pi) \cos^{-1}(z/A)$ . This leads to

887

888 A(2)

$$\bar{h}^* = \frac{1}{\pi} \int_{\omega t=0}^{\cos^{-1}\frac{z-d}{A}} (d + A \cos \omega t) d\omega t + \left[ 1 - \frac{1}{\pi} \cos^{-1} \left( \frac{z-d}{A} \right) \right] d$$

889

A(3)

$$\bar{h}^* = z + \frac{1}{\pi} \left[ (d - z) \cos^{-1} \left( \frac{z-d}{A} \right) + A \sqrt{1 - \left( \frac{z-d}{A} \right)^2} \right]$$

890

891 and the corresponding mean pressure head

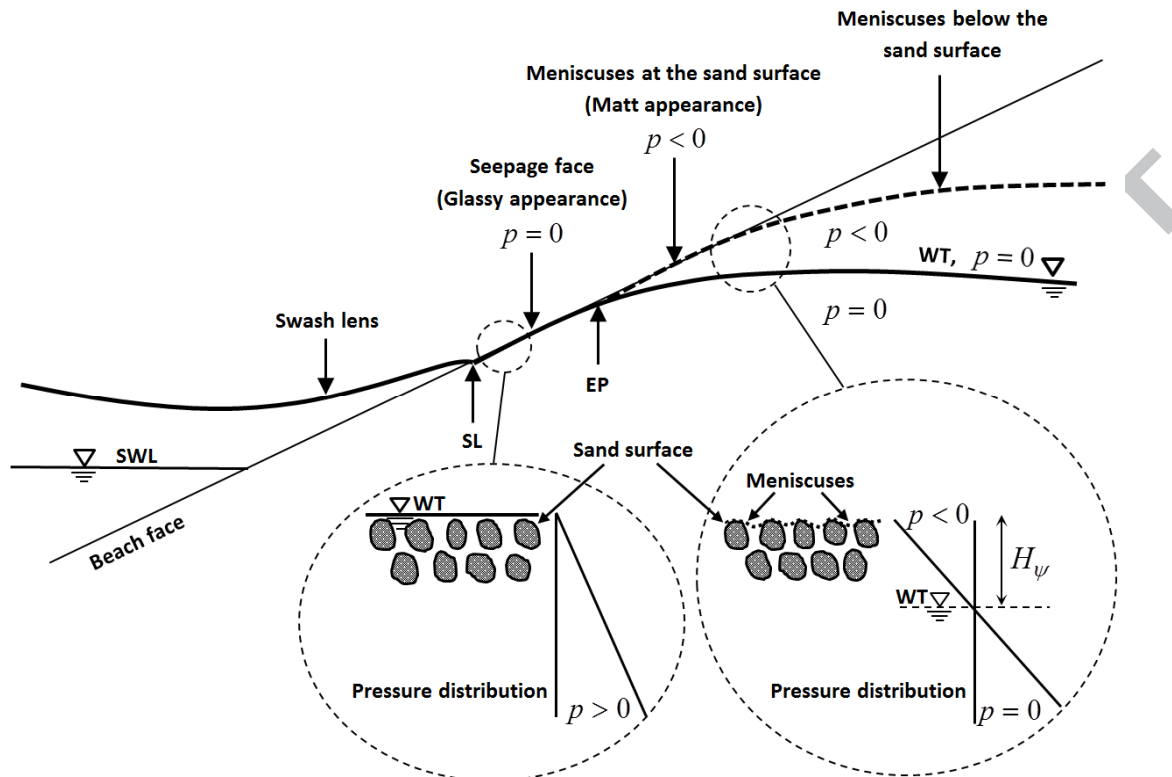
892

$$893 \quad \frac{\bar{p}}{\rho g} = \frac{1}{\pi} \left[ (d - z) \cos^{-1} \left( \frac{z-d}{A} \right) + A \sqrt{1 - \left( \frac{z-d}{A} \right)^2} \right]$$

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A(4)





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Figure 1. Schematic illustration of seepage face and meniscuses formation on the beach face. SL = shoreline (swash front); EP = water table exit point;

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4

WT = water table;  $p$  = pore pressure;  $H_\psi$  = steady capillary fringe thickness.

5

Solid and dashed lines represent the free surface and idealized meniscuses

6

surface, respectively (after Cartwright et al., 2006).

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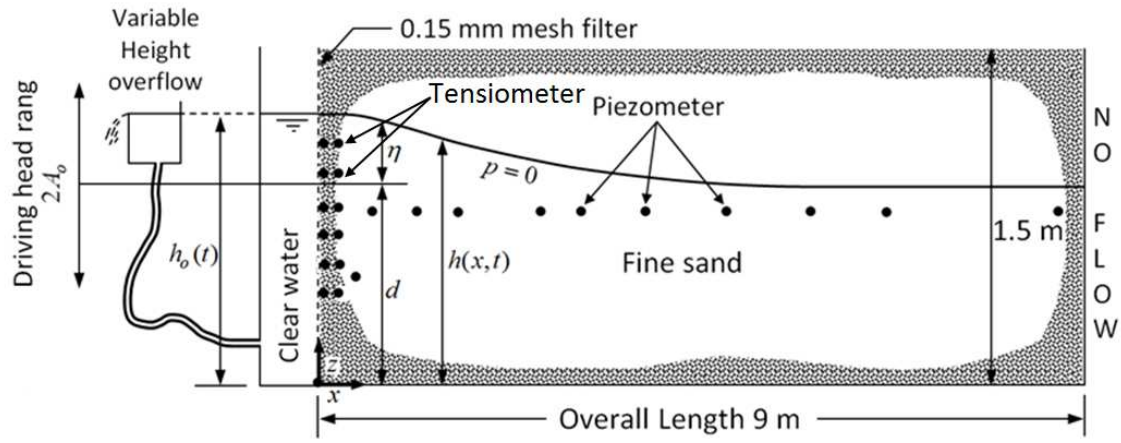
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14 Figure 2. Schematic illustration of the sand flume (after Cartwright et al., 2003)

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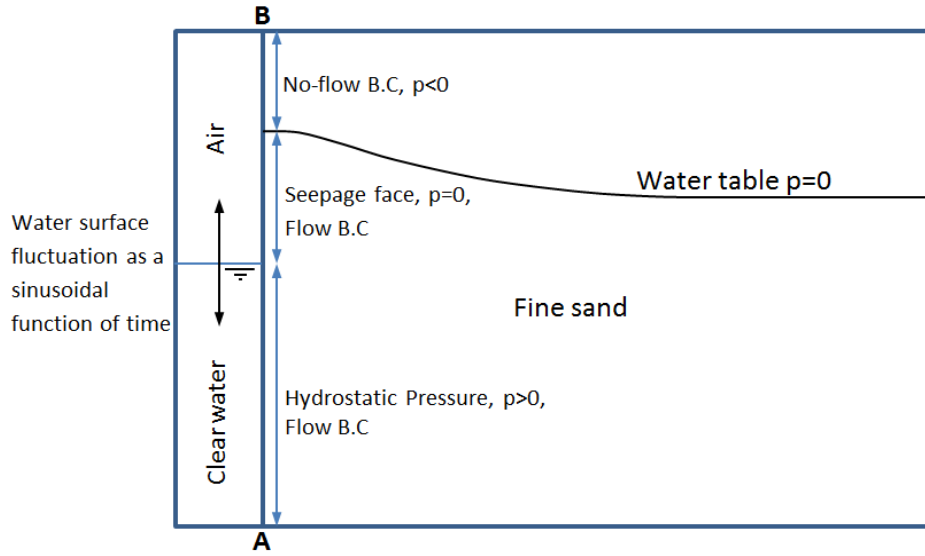
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Figure 3. Periodic Cauchy boundary condition

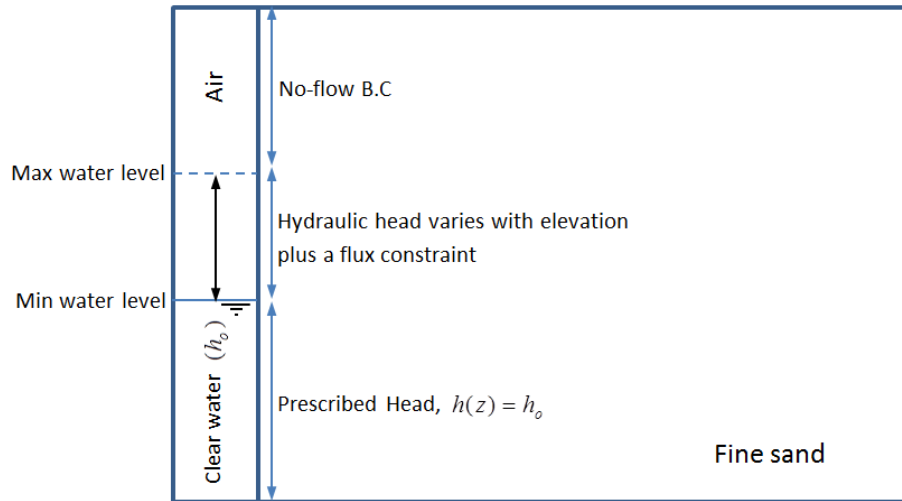
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Figure 4. Hydraulic head combined with flux constraint

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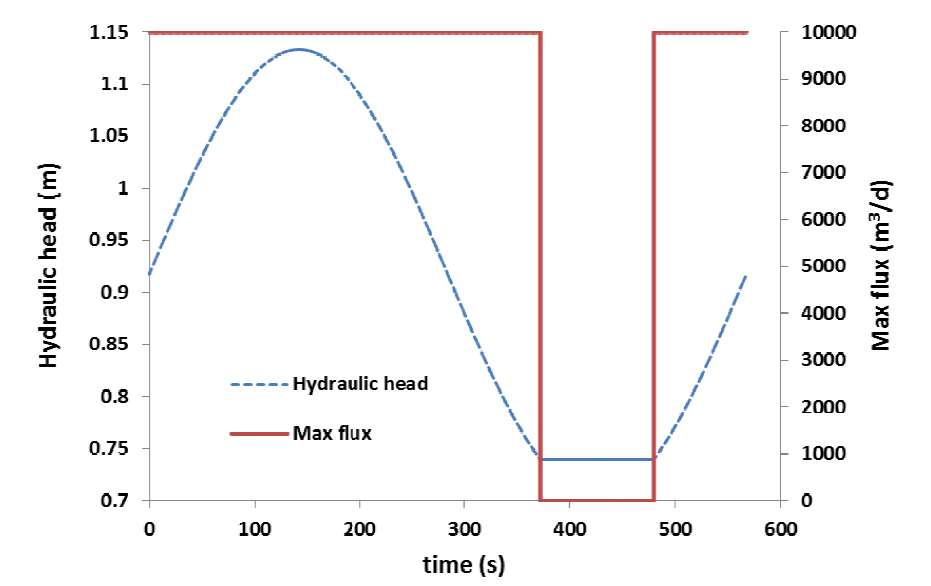
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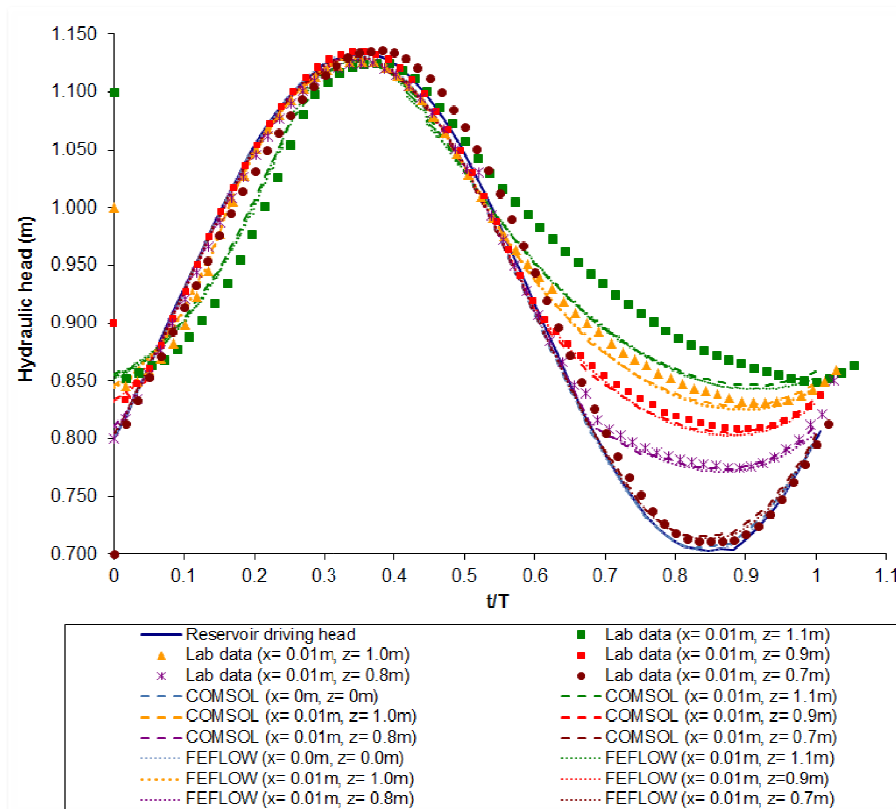
43 Figure 5. A sample of hydraulic head time series and flux constraint ( $z = 0.74 \text{ m}$ )

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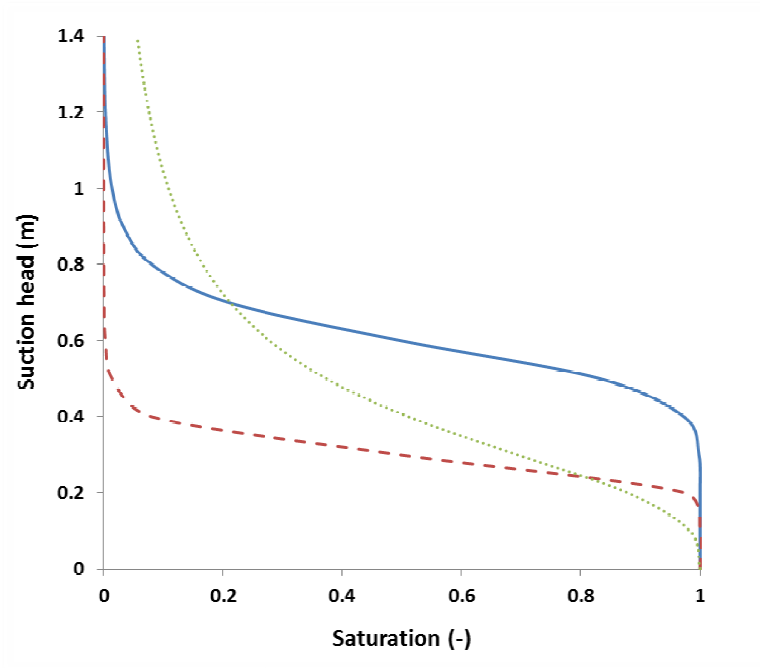
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Figure 6. Comparison of measured and predicted piezometric head time series close to the interface boundary ( $x = 0.01 \text{ m}$ ) at different elevations ( $z = 0.7, 0.8, 0.9, 1.0, 1.1 \text{ m}$ ). The symbols show measured laboratory data and curves show numerical modeling results.



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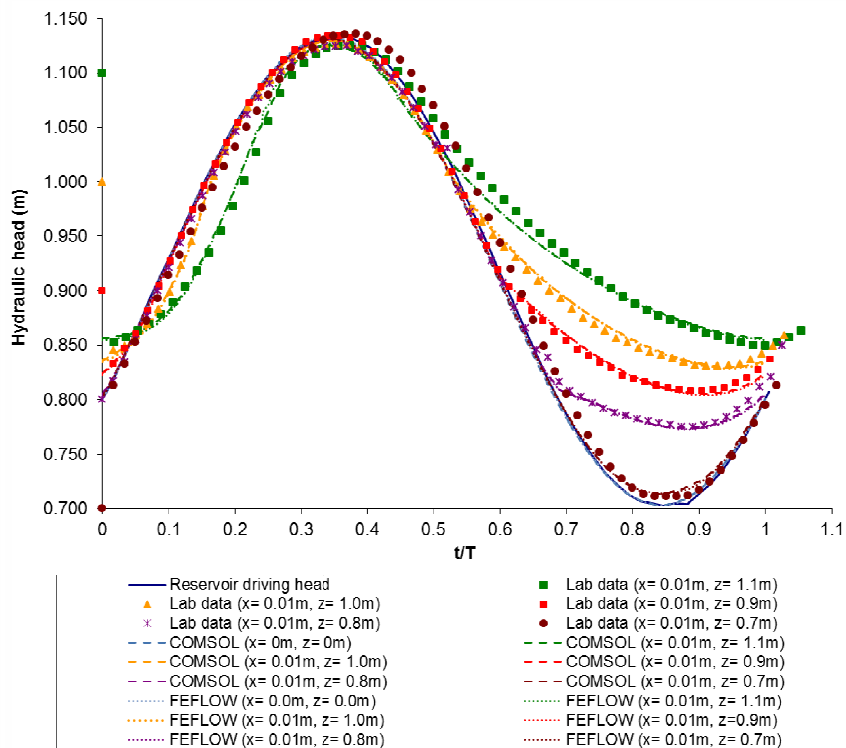
60 Figure 7. Drying ( $\alpha_d = 1.7 \text{ m}^{-1}, \beta = 9$ ) (solid line), wetting ( $\alpha_w = 3.4 \text{ m}^{-1}, \beta = 9$ )61 (dashed line) and modified ( $\alpha = 3 \text{ m}^{-1}, \beta = 3$ ) (dotted line) retention curves

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67 Figure 8. Comparison of measured and predicted piezometric head time series close to the  
 68 interface boundary (using modified retention curve with  $\alpha = 3 \text{ m}^{-1}$  and  $\beta = 3$  for numerical  
 69 models)

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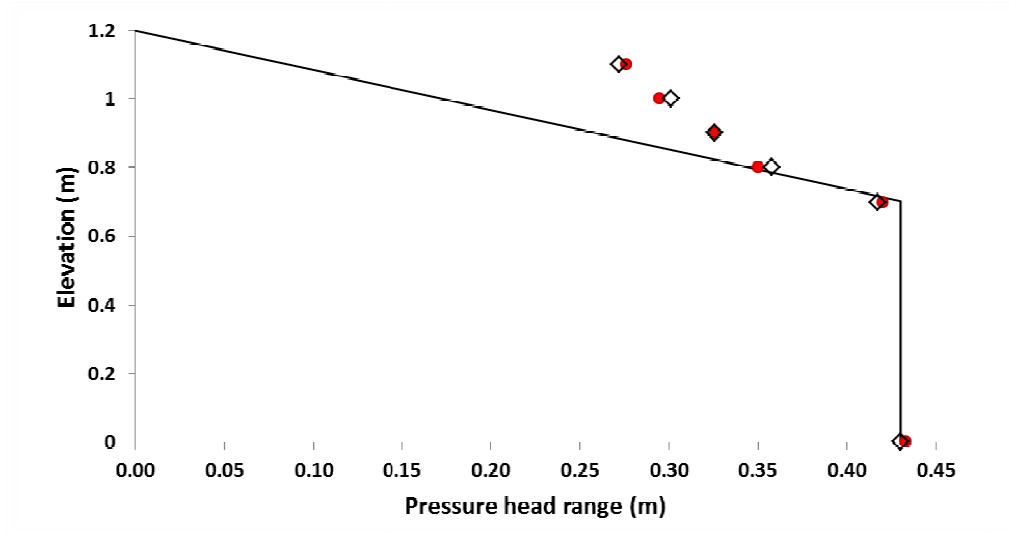
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78 Figure 9. Comparison of measured and predicted pressure head range at  $x = 0.01 \text{ m}$ . Lab  
 79 data (solid circles) and numerical results of FEFLOW using modified retention curve  
 80 (i.e.  $\alpha = 3 \text{ m}^{-1}$  and  $\beta = 3$ ) (open diamonds). Solid line shows the pressure had range in  
 81 the reservoir.

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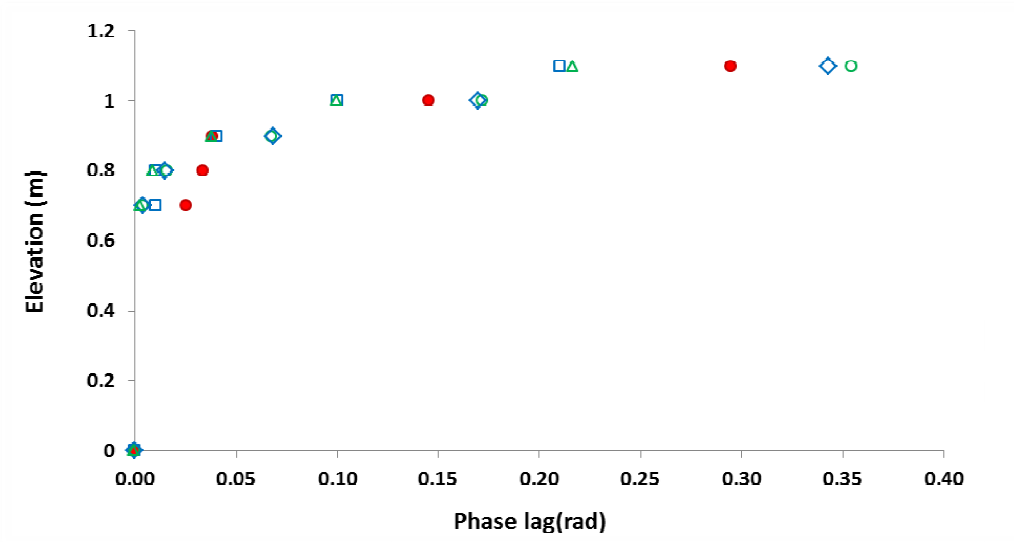
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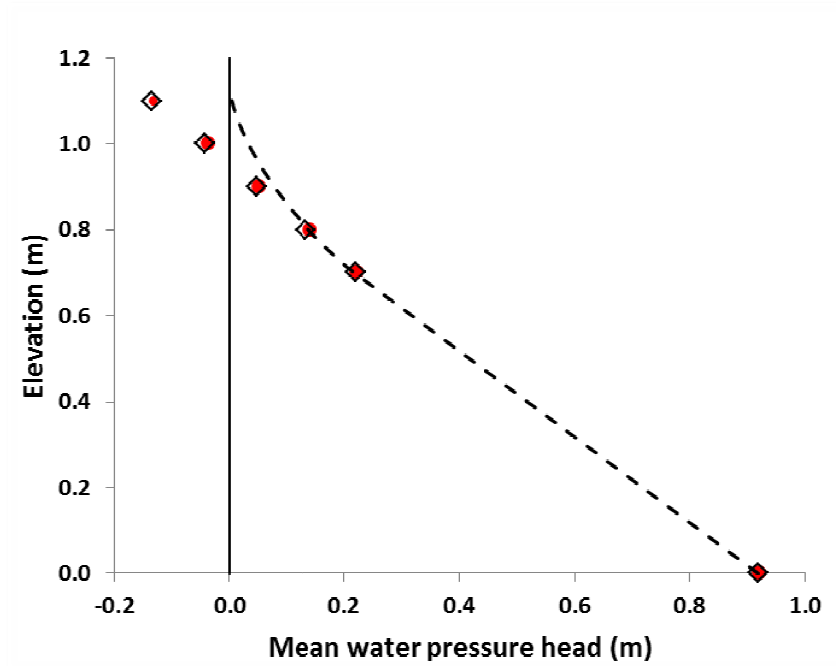
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90 Figure 10. Comparison of measured and predicted phase lag at  $x = 0.01$  m. Lab data (solid  
 91 circles) and FEFLOW and COMSOL results with  $\alpha = 1.7 m^{-1}$  and  $\beta = 9$  open square and  
 92 triangle, respectively. FEFLOW and COMSOL results with  $\alpha = 3 m^{-1}$  and  $\beta = 3$  open  
 93 diamonds and circles, respectively.

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Figure 11. Comparison of measured and predicted mean pressure head profile at  $x = 0.01 \text{ m}$ . Lab data (solid circles) and numerical results of FEFLOW using modified retention curve (i.e.  $\alpha = 3 \text{ m}^{-1}$  and  $\beta = 3$ ) (open diamonds). The dashed line shows the theoretical profile calculated by equation (11). Solid line represents the vertical sand interface.

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Table 1. Hydraulic and moisture properties of the sand

$d_{50}$ (mm)	$K$ (m/s)	$\theta_s$	$\theta_r$	$\alpha$ (m <sup>-1</sup> )	$\beta$	$H_\psi$ (m)
0.260	$4.7 \times 10^{-4}$	0.41	0.09	1.7	9	0.62

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$d_{50}$ , mean grain size;  $K$ , saturated hydraulic conductivity;  $\theta_s$  and  $\theta_r$ , saturated and residual moisture contents, respectively;  $H_\psi$  steady capillary fringe thickness;  $\alpha$  and  $\beta$  are van Genuchten parameters. After Nielsen and Perrochet (2000).

Table 2. Summary of harmonic components.

	$x$ (m)	$z$ (m)	$\bar{h}^*$ (m)	$\bar{p}/\rho g$ (m)	$h_{max}^*$ (m)	$h_{min}^*$ (m)	Range (m)	$R_1$ (m)	$\phi_1$ (rad)	$R_2$ (m)	$\phi_2$ (rad)	$R_3$ (m)	$\phi_3$ (rad)
Lab data	0.00	0.0	0.920	0.920	1.133	0.700	0.433	0.214	2.205	0.005	1.382	0.003	2.175
	0.01	0.7	0.922	0.222	1.125	0.712	0.420	0.214	2.230	0.001	0.587	0.002	3.526
	0.01	0.8	0.941	0.141	1.125	0.775	0.350	0.186	2.238	0.014	4.525	0.007	3.444
	0.01	0.9	0.951	0.051	1.126	0.808	0.326	0.167	2.243	0.026	4.120	0.007	2.968
	0.01	1.0	0.964	-0.036	1.132	0.831	0.295	0.152	2.350	0.031	4.040	0.004	4.498
	0.01	1.1	0.968	-0.132	1.134	0.849	0.276	0.136	2.499	0.036	4.224	0.009	5.877
FEFLOW ( $\alpha=1.7, \beta=9.0$ )	0.00	0.0	0.918	0.918	1.133	0.703	0.430	0.215	1.593	0.000	1.574	0.000	1.563
	0.01	0.7	0.920	0.220	1.131	0.714	0.417	0.211	1.596	0.002	3.048	0.001	1.585
	0.01	0.8	0.933	0.133	1.131	0.772	0.359	0.187	1.603	0.018	3.096	0.009	1.493
	0.01	0.9	0.946	0.046	1.130	0.803	0.328	0.166	1.633	0.024	2.944	0.005	1.269
	0.01	1.0	0.955	-0.045	1.130	0.825	0.305	0.152	1.693	0.028	2.910	0.003	2.806
	0.01	1.1	0.957	-0.143	1.128	0.842	0.285	0.136	1.797	0.031	3.093	0.008	4.332
COMSOL ( $\alpha=1.7, \beta=9.0$ )	0.00	0.0	0.918	0.918	1.133	0.703	0.429	0.215	2.148	0.000	2.180	0.000	1.586
	0.01	0.7	0.919	0.219	1.131	0.712	0.419	0.210	2.151	0.002	4.263	0.001	3.160
	0.01	0.8	0.933	0.133	1.130	0.772	0.359	0.186	2.157	0.018	4.220	0.009	3.149
	0.01	0.9	0.946	0.046	1.130	0.803	0.327	0.166	2.186	0.024	4.071	0.005	2.880
	0.01	1.0	0.955	-0.045	1.129	0.826	0.303	0.152	2.248	0.028	4.038	0.002	4.499
	0.01	1.1	0.957	-0.143	1.125	0.844	0.282	0.135	2.365	0.031	4.297	0.007	0.044
FEFLOW ( $\alpha=3.0, \beta=3.0$ )	0.00	0.0	0.918	0.918	1.133	0.703	0.430	0.215	1.593	0.000	1.574	0.000	1.563
	0.01	0.7	0.920	0.220	1.131	0.714	0.417	0.210	1.597	0.002	2.983	0.001	1.571
	0.01	0.8	0.933	0.133	1.131	0.773	0.358	0.186	1.608	0.019	3.038	0.010	1.408
	0.01	0.9	0.947	0.047	1.130	0.804	0.326	0.164	1.661	0.027	2.802	0.006	1.103
	0.01	1.0	0.958	-0.042	1.130	0.829	0.301	0.148	1.763	0.034	2.771	0.004	2.686
	0.01	1.1	0.965	-0.135	1.127	0.855	0.272	0.126	1.936	0.039	3.029	0.011	4.001
COMSOL ( $\alpha=3.0, \beta=3.0$ )	0.00	0.0	0.918	0.918	1.133	0.703	0.430	0.215	2.148	0.000	2.360	0.000	2.102
	0.01	0.7	0.919	0.219	1.131	0.713	0.418	0.210	2.152	0.002	4.199	0.001	3.242
	0.01	0.8	0.933	0.133	1.131	0.774	0.356	0.185	2.164	0.019	4.151	0.010	3.075
	0.01	0.9	0.947	0.047	1.130	0.805	0.325	0.163	2.215	0.027	3.928	0.006	2.761
	0.01	1.0	0.958	-0.042	1.129	0.830	0.299	0.147	2.320	0.034	3.903	0.004	4.444
	0.01	1.1	0.965	-0.135	1.125	0.856	0.269	0.124	2.502	0.039	4.174	0.011	5.753

41 Mean water head ( $\bar{h}^*$ ), Mean pressure head  
 42 ( $\bar{p}/\rho g$ ), Maximum water elevation ( $h_{max}^*$ ), Minimum water elevation ( $h_{min}^*$ ), Pressure head  
 43 range (*Range*), First harmonic amplitude ( $R_1$ ), First harmonic phase ( $\phi_1$ ), Second harmonic  
 44 amplitude ( $R_2$ ), Second harmonic phase ( $\phi_2$ ), Third harmonic amplitude ( $R_3$ ), Third harmonic  
 45 phase ( $\phi_3$ ).

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**Highlights**

- Pore pressure response to tide close to the boundary was measured in a sand flume
- The data show the influence of seepage face and meniscus formation at the boundary
- The data was used to assess capability of a numerical solution of Richards equation
- Two different methods were used to simulate seepage face formation at the boundary
- Model-data comparison shows a good agreement but sensitive to retention parameters