Modeling Transit and Intermodal Tours in a Dynamic Multimodal Network

Alireza Khani, Brenda Bustillos, Hyunsoo Noh, Yi-Chang Chiu, and Mark Hickman

A fixed-point formulation and a simulation-based solution method were developed for modeling intermodal passenger tours in a dynamic transportation network. The model proposed in this paper is a combined model of a dynamic traffic assignment, a schedule-based transit assignment, and a park-and-ride choice model, which assigns intermodal demand (i.e., passengers with drive-to-transit mode) to the optimal park-and-ride station. The proposed model accounts for all segments of passenger tours in the passengers' daily travel, incorporates the constraint on returning to the same park-and-ride location in a tour, and models individual passengers at a disaggregate level. The model has been applied in an integrated travel demand model in Sacramento, California, and feedback to the activity-based demand model is provided through separate time-dependent skim tables for auto, transit, and intermodal trips.

Modeling drive-to-transit trips has been a challenging problem in transportation network modeling for decades. As a part of multimodal modeling, intermodal travel is more complicated not only because two modes of transportation are involved in a trip but also because the choice of park-and-ride is included in the travel. The combination of mode choice, route choice, and mode-transfer choice is involved in modeling intermodal travel. In the literature, intermodal trips (one-way travel from an origin to a destination) have been frequently addressed, and many approaches have been proposed for modeling user behavior (*1–5*). However, additional complexity is added to the problem when the model deals with intermodal tours (round-trip travel between an origin and multiple destinations). The intermodal tour problem is more complex because of the constraint on the park-and-ride choice, meaning that travelers have to return to the same park-and-ride location they originally parked their cars. This constraint changes the problem from a general shortest path to a shortest-tour problem.

In the context of dynamic network modeling, the concept of time has an important role. Its importance relates to the congestion levels in the network, transit schedules, and departure and arrival time considerations and preferences. In other words, travelers in the drive-to-transit mode may consider the time of travel to avoid major congestion in the auto network, the possibility of missing the transit vehicle at the park-and-ride (or being too early at the park-and-ride), and preferable arrival times at their final destinations. Therefore, although modeling a dynamic intermodal tour is more realistic compared with traditional static approaches, it is not a trivial problem. In this study, that problem is addressed, so that a more realistic model is developed for planning and operational purposes. In addition, one objective of the model is to integrate an activity-based travel demand model with a dynamic traffic assignment (DTA) model. Previous studies of such integration include CEMDAP-VISTA (*6*), TASHA-MATSim (*7*), and OpenAMOS-MALTA (*8*). Those models focused on how to integrate more temporally rich models of demand and supply. In this integration framework, there is more interest in the supply side, or in capturing dynamic traffic and transit network assignment and loading. For that purpose, it is assumed that the activities of each traveler are given.

A. Khani, Center for Transportation Research, University of Texas at Austin, 1616 Guadalupe Street, Suite 4.202, Austin, TX 78701. B. Bustillos and Y.-C. Chiu, Civil Engineering and Engineering Mechanics, University of Arizona, 1209 East 2nd Street, P.O. Box 210072, Tucson, AZ 85721-0072. H. Noh, Pima Association of Governments, 1 East Broadway Boulevard, Suite 401, Tucson, AZ 85701. M. Hickman, School of Civil Engineering, University of Queensland, Saint Lucia, Queensland 4072, Australia. Corresponding author: A. Khani, akhani@email.arizona.edu.

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LITERATURE REVIEW

As one of the first studies in the area of multimodal assignment, Abdulaal and LeBlanc introduced two ways to combine mode choice and route choice models (*9*). In one approach, mode choice and route choice are performed sequentially, and in the other, they are performed simultaneously. Fernandez et al. further studied two major issues in multimodal transportation modeling: (*a*) how users choose their mode of the trip and then, depending on the answer, how the best route is chosen and (*b*) how the transfer point from the private to the public mode is selected (*10*). Fernandez et al. proposed the following three approaches to model intermodal trips in a static network:

1. Using the generalized cost of the combined mode, people choose their path to minimize the cost of their trip.
2. People choose their mode of travel while the combined mode is considered as a separate mode, and then the shortest path is found in the selected mode.
3. As an extension of the second model, people may also include the choice of transfer point as the submodel.

Modesti and Sciomachen proposed an algorithm for finding a multi-objective shortest path in a multimodal transportation network (*1*). They introduced a utility function for weighting the links according to their cost and time and used the classical Dijkstra's shortest-path algorithm to find the path with maximum utility. Ziliaskopoulos and Wardell developed an algorithm for finding the intermodal least-time path in a multimodal network with time-dependent link travel times and turning delays (*2*). Their label-correcting algorithm is designed for all time intervals, and its complexity is independent of the number of modes. Abdelghany proposed a dynamic assignment and simulation framework for different modes of transportation, incorporating a multiobjective time-dependent shortest path in the DYNASMART traffic assignment and simulation model (*3*).

Lozano and Storchi also applied a label-correcting algorithm to find the shortest viable hyperpath with a predefined maximum number of modal transfers (*11*). The approach is useful when there is no exact schedule for the transit system (i.e., the transit network is frequency based). Because their algorithm considers more than one criterion, the result of the algorithm is not necessarily optimal, and the user can choose the best hyperpath(s) from among the output, according to their preferences.

A multimodal assignment formulation was proposed by Garcia and Marin in the form of a variational inequality considering the combined modes (*12*). They used a nested logit model, solved by simplicial decomposition, in capturing the choice of mode, the transfer point between modes, and the route. Compared with Fernandez et al., they formulated the problem in a hyperpath space and performed stochastic assignment with elastic demand. Zhou et al. developed an integrated framework to model choices of departure time, mode, and path in a multimodal transportation system (*4*). As a part of the model, a time-dependent least-cost path algorithm based on Ziliaskopoulos and Wardell was used to generate intermodal paths. Khani et al. (*5*) and Nassir et al. (*13*) proposed algorithms for the intermodal path and tour problems, taking into account the scheduled service in the transit network.

From this review, there may be a more efficient and flexible algorithm for intermodal path generation that can be used in the assignment model. Moreover, incorporating a dynamic multimodal system, including the interaction of the auto and transit networks, a transit schedule in the transit path choice model, and a park-and-ride constraint in the intermodal tours, provides the motivation to pursue a more advanced multimodal assignment and simulation model. Therefore, algorithms with more efficient computation time that are appropriate for modeling intermodal trips and tours are developed in this study. The combination of the path models with a dynamic multimodal simulation and assignment tool including a DTA model is proposed to facilitate a comprehensive multimodal transportation network model.

The paper is divided into four sections beginning with the literature review. The proposed methodology is described in the next section, which contains four subsections dedicated to the mathematical model, intermodal shortest path algorithm, park-and-ride model, and multimodal simulation model. The application of the model in a real network is then described, and concluding remarks appear in the final section.

DYNAMIC INTERMODAL TOUR ASSIGNMENT MODEL

Fixed-Point Formulation for Dynamic Multimodal Assignment Problem

The intermodal tour assignment must be modeled as part of an integrated dynamic traffic and transit assignment model. The intermodal demand is defined as a separate class of travelers, different from auto and transit users, but the interaction of the intermodal demand with other travelers is captured in the integrated model. The proposed model is formulated as a fixed-point problem, while the equilibrium for each group of passengers is modeled by variational inequalities. The notations follow.

Notations in the DTA Model

Q_a = auto demand (dynamic trip table),
$\boldsymbol{D}$ = vector of transit dwell times,
$\boldsymbol{P}_a$ = vector of auto paths (DTA solution),
$\boldsymbol{C}_a$ = vector of auto path costs resulting from $\boldsymbol{P}_a$,
$c_a(\boldsymbol{P}_a, \boldsymbol{D})$ = auto path costs as result of simulating auto paths $\boldsymbol{P}_a$ and transit vehicles with dwell times $\boldsymbol{D}$, and
$s(\boldsymbol{P}_a, \boldsymbol{D})$ = transit vehicle trajectories as result of simulating auto paths $\boldsymbol{P}_a$ and transit vehicles with dwell times $\boldsymbol{D}$.

Notations in the Schedule-Based Transit Assignment Model

Q_t = transit demand (dynamic passenger table),
$\boldsymbol{S}$ = vector of transit vehicle trajectories (transit schedule),
$\boldsymbol{P}_t$ = vector of transit paths (transit assignment solution),
$\boldsymbol{C}_t$ = vector of transit path costs resulting from $\boldsymbol{P}_t$,
$c_t(\boldsymbol{P}_t, \boldsymbol{S})$ = transit path costs as result of simulating transit paths $\boldsymbol{P}_t$ on transit network with vehicle schedules $\boldsymbol{S}$, and
$d(\boldsymbol{P}_t, \boldsymbol{S})$ = transit dwell times as a result of simulating transit paths $\boldsymbol{P}_t$ on transit network with schedule $\boldsymbol{S}$.

The integrated auto and transit assignment model (without intermodal tours) can be modeled as the following system of equations:

Dynamic traffic equilibrium:

$$\left[c_a(\boldsymbol{P}_a^*, \boldsymbol{D})\right]^T\left[\boldsymbol{P}_a - \boldsymbol{P}_a^*\right] \geq 0 \qquad \forall Q_a$$

Dynamic transit equilibrium:

$$\left[c_t(\boldsymbol{P}_t^*, \boldsymbol{S})\right]^T\left[\boldsymbol{P}_t - \boldsymbol{P}_t^*\right] \geq 0 \qquad \forall Q_t$$

Integration model:

$$\boldsymbol{D} = d\left(\boldsymbol{P}_t, s\left(\boldsymbol{P}_a, \boldsymbol{D}\right)\right)$$

The first set of equations above explains the dynamic equilibrium in the auto network with the variational inequalities (VIs). In the equilibrium state, travelers with the same origin, destination, and departure time have equal travel time. The second set of equations shows a similar VI formulation for equilibrium in the schedule-based transit network. In the transit equilibrium solution, each passenger takes the available path with minimum travel time. The third equation is a fixed-point formulation to model the interaction between the auto and the transit networks. In the fixed-point problem, given the transit dwell times and the current auto and transit assignment solution from the equilibrium models, new dwell times are calculated by simulation. In this model, the effect of auto and transit passengers' decisions is taken into account when a multimodal equilibrium solution is found.

With the inclusion of the intermodal tours, the multimodal assignment model is extended to the following:

Dynamic traffic equilibrium:

$$[c_a(\boldsymbol{P}_a^*, \boldsymbol{D}, \boldsymbol{I})]^T[\boldsymbol{P}_a - \boldsymbol{P}_a^*] \geq 0 \qquad \forall Q_a \cup Q_i$$

Dynamic transit equilibrium:

$$[c_t(\boldsymbol{P}_t^*, \boldsymbol{S}, \boldsymbol{I})]^T[\boldsymbol{P}_t - \boldsymbol{P}_t^*] \geq 0 \qquad \forall Q_t \cup Q_i$$

Intermodal tour equilibrium:

$$[c_i(\boldsymbol{I}^*, \boldsymbol{C}_a, \boldsymbol{C}_t)]^T[\boldsymbol{I} - \boldsymbol{I}^*] \geq 0 \qquad \forall Q_i$$

Integration model:

$$\boldsymbol{D} = d(\boldsymbol{P}_t, s(\boldsymbol{P}_a, \boldsymbol{D}))$$

Notations in the Dynamic Intermodal Tour Assignment Model

- Q_i = intermodal demand (dynamic tours),
- $\boldsymbol{I}$ = vector of optimal park-and-ride locations (intermodal assignment solution),
- $\boldsymbol{P}_a$ and $\boldsymbol{C}_a$ = extended to include auto part of intermodal paths and costs,
- $\boldsymbol{P}_t$ and $\boldsymbol{C}_t$ = extended to include transit part of intermodal paths and costs, and
- $c_i(\boldsymbol{I}, \boldsymbol{C}_a, \boldsymbol{C}_t)$ = intermodal path costs resulting from park-and-ride solution $\boldsymbol{I}$ given auto path cost $\boldsymbol{C}_a$ and transit path costs $\boldsymbol{C}_t$.

In the extended set of equations, the first two VI formulations use $\boldsymbol{I}$ as an input, meaning that the unimodal (i.e., auto-only or transit-only) trips of the intermodal tours are modeled in the equilibrium models. The unimodal trips are optimized for the best paths, given the trips' origins and destinations, fixed in the intermodal equilibrium model (i.e., in the vector $\boldsymbol{I}$). The third equation is the one in which the trips' origins and destinations are determined for the intermodal tours by assigning the optimal park-and-ride location to each tour. This equilibrium is also modeled by a VI, taking auto and transit path costs as input. Finally, the integration of the three models is established by using the fixed-point formulation in the fourth equation. Similar to the previous integrated models for auto and transit, the equilibrium in the multimodal system is measured by transit dwell time.

Intermodal Dynamic Shortest Path Algorithm

The intermodal shortest path model with a time-dependent auto network and a schedule-based transit network was developed by Khani et al. (*5*). To model a multimodal transportation network and complete a trip chain in a tour, a high-resolution network representation was adapted, moving toward a more realistic behavioral representation of the path in the vicinity of each park-and-ride location. Accessing the park-and-ride location by car, searching for a parking spot, parking the car, and walking to the transit stop or station are modeled explicitly to estimate the total mode transfer delay, excluding the waiting time for a transit vehicle.

The intermodal path algorithm finds the optimal path between an origin and a destination with a preferred arrival time (PAT) at the destination while all available transportation modes are considered. The best path through a park-and-ride location is defined by meeting the traveler's desire to arrive at the destination at or before the PAT with the minimum total travel time. The algorithm developed in Khani et al. (*5*) is the sequential run of a trip-based shortest path (TBSP) algorithm in the transit network and a multisource time-dependent shortest path (MTDSP) algorithm in the auto network (*14*). Proper settings in the algorithm are required for incorporating the mode transfer and for ensuring a multimodal path. As a result, a transit shortest path tree is found connecting the park-and-ride locations to the destination, and a set of auto shortest path trees are found from the origin to the park-and-ride locations (Figure 1). The algorithm's main input data are general transit feed specification (GTFS) data for the transit network, a network with time-dependent link travel times for the auto network (a typical DTA network), and the set of access, egress, and mode transfer links connecting the auto network to the transit network (*15*).

The multisource shortest path algorithm (i.e., adding multiple nodes to the scan-eligible list at the beginning of the algorithm), introduced by Klein, can be used in reverse from the possible park-and-ride locations back to the origin, for a given traveler (*16*). Initially, the TBSP algorithm is applied from the ultimate destination back to the set of possible park-and-ride lots. The multisource approach then finds the best source node (park-and-ride location) to the target node (the origin) by automatically comparing the travel time from the destination to each source node. Because the initial labels of the source nodes (the park-and-ride locations) are set by the TBSP algorithm, the MTDSP also takes into account the transit travel time to the destination to select the best park-and-ride location. The complexity of the intermodal path algorithm is $O(S^2 + P + N^2)$, where $O(\odot)$ represents the worst-case complexity of the model in the O notation, S is the number of transit stops, P is the number of park-and-ride locations, and N is the number of nodes in the auto network. In general, if S and N are comparable, searching for the

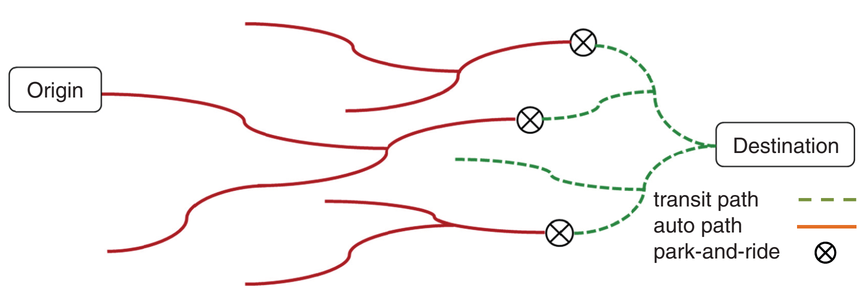

FIGURE 1 Set of intermodal paths to destination through park-and-ride locations.

optimal transit path dominates the complexity of the algorithm. The complexity of the typical schedule-based transit shortest path algorithm is $O(S^2)$, but using a hierarchical trip-based network in the TBSP algorithm decreases the average run time of the algorithm notably (*5*).

Modeling Park-and-Ride Choice in Intermodal Tours

The intermodal shortest path algorithm introduced in the previous subsection is appropriate for one-way trips and results in the optimal path for the initial auto-to-transit trip only. However, if daily travel is modeled in the form of complete tours (departing from and returning to home), it is important to consider the park-and-ride location and the constraint of returning to the same park-and-ride later in the tour, for the transit-to-auto trip. To deal with this problem, the intermodal shortest path algorithm can be extended to an intermodal shortest tour algorithm and to the dynamic assignment and simulation of the intermodal tours.

The solution to the intermodal optimal tour problem is based on the same model elements introduced before (TBSP algorithm, MTDSP algorithm, and park-and-ride choice model). However, instead the origin and destination with PAT being used as the inputs to the algorithm, a complete tour (origin–home, the set of destinations visited in the given time window and with a given sequence, and the return home) forms the input. Then, the optimal park-and-ride location is found with the minimum travel cost for the entire tour. In general, if a park-and-ride location is optimal in a multimodal trip, there is no guarantee that it is also optimal for a subsequent multimodal trip, especially in a dynamic network. Therefore, an algorithm is developed to enumerate all the possible park-and-ride locations and to find the location with minimum tour cost. Obviously, this enumeration will be a computationally expensive procedure, with linearly higher computational time with a higher number of park-and-ride locations. The complexity of the problem will be even higher when there are multiple destinations in the tour. There have been some efforts in the past to model this problem [e.g., Nassir et al. (*13*)]. The proposed algorithm in this study uses some simplifying but logical assumptions that make the problem much less computationally demanding while maintaining optimal results. These assumptions are as follows:

1. There is one primary destination in each tour considered as an anchor activity. This anchor activity, with a given start time and duration, along with the home activity, defines the primary locations visited in the intermodal tour.
2. The intermodal trips are used to travel to and from the primary destination, and the secondary destinations are visited by either auto or transit.

The input destinations and time windows are reviewed in the preprocessing stage, and the potential intermodal trips for the tour are selected to make new trip chains for the park-and-ride assignment. The resulting (simplified) tour contains the preferred departure time (PDT) from the origin and the PAT to the primary destination in the first half-tour, and the PDT from the primary destination and the PAT to the next destination in the second half-tour. A simplified tour representation, forming the input to the algorithm, is shown in Figure 2.

The park-and-ride assignment model is a combination of two intermodal shortest path algorithms at the trip level, with some modifications to account for the constraints on the park-and-ride location introduced in the tour. The algorithm finds the travel cost associated with each park-and-ride location in the first half-tour while removing the infeasible park-and-ride facilities from the choice set according to the available time windows. That is, if a path through a specific park-and-ride is so long that its travel time exceeds the available time window, the park-and-ride is removed from the choice set (e.g., Park-and-Ride Location 1 in Figure 2). From a similar procedure for the second half-tour, and combining the two half-tours, the feasible park-and-ride locations are sorted by total tour cost, and the optimal park-and-ride location can be assigned to the tour. The steps of the algorithm are as follows:

Step 1. The forward auto shortest path tree is found from the PDT at the origin, T_1, and the label l_1 is set for each park-and-ride showing the auto travel cost.

Step 2. The backward transit shortest path tree is found at the PAT at the anchor activity, T_2, and the label l_2 is set for each park-and-ride showing the transit travel cost.

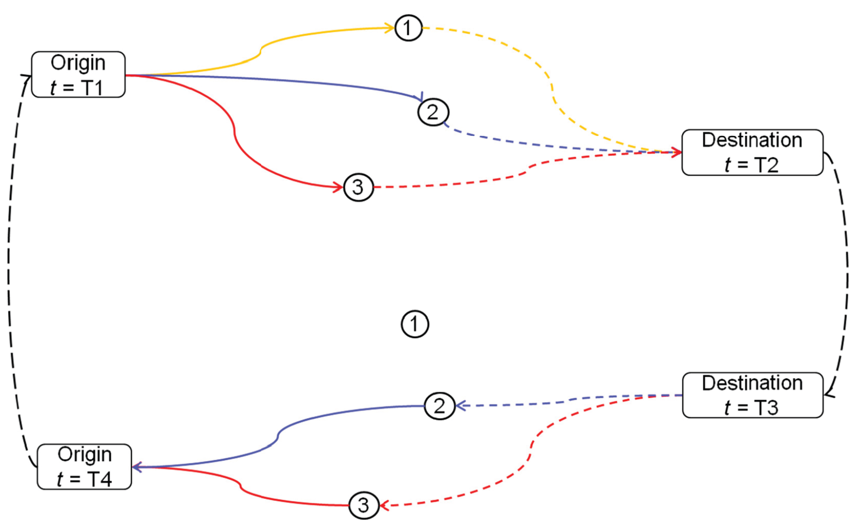

FIGURE 2 Typical intermodal tour with feasible paths through park-and-rides.

Step 3. $C_o = l_1 + l_2$ is calculated as the travel cost from the origin to the primary destination through each park-and-ride location if the intermodal trip is feasible; the total travel time is less than or equal to the time window $T_2 - T_1$.

Step 4. The forward transit shortest path tree is found, beginning at the PDT from the primary destination, T_3, and the label l_3 is set for each park-and-ride location showing the transit travel cost.

Step 5. The backward auto shortest path tree is found at PAT to the next destination, T_4, and the label l_4 is set for each park-and-ride location showing the auto travel cost.

Step 6. $C_i = l_3 + l_4$ is the travel cost from the primary destination to the next destination through each park-and-ride location if the intermodal travel is feasible; the total travel time is less than or equal to the time window $T_4 - T_3$.

Step 7. The sum of the intermodal travel costs, $C_t = C_o + C_i$, is the total tour travel cost through each park-and-ride. Therefore, the optimal park-and-ride location with minimum value of C_t is assigned to the tour.

The result of this algorithm is the optimal park-and-ride location. Moreover, the auto and transit parts of the tour are determined. The proposed model is a combination of four shortest path algorithms, and its complexity is $2 \times O(N^2) + 2 \times O(S^2) + O(P)$ where P is the number of park-and-ride locations. In fact, because of the greater complexity of the transit networks, the overall complexity of the algorithm is $O(S^2)$, where S is the number of stops in the transit network. Moreover, the average run time is significantly lower with the transit network hierarchy and trip-based structure (*14*). Considering that the intermodal tour problem is a challenging problem, the proposed model is computationally efficient.

Dynamic Simulation of Transit and Intermodal Tours

The results of the park-and-ride choice model are used in a simulation-assignment framework to evaluate the experience of passengers in a dynamic multimodal network. The transit vehicle simulation is part of the DynusT (*17*) DTA model, and the transit passenger simulation is part of the dynamic transit simulation and assignment model, FAST-TrIPs (flexible assignment and simulation tool for transit and intermodal passengers) (*18*). In fact, the auto trips of the intermodal tours are given to the DTA model, the transit trips are given to the transit passenger simulation model, and a software interface is used to integrate the two models.

After the park-and-ride assignment is run, the next step of the model is to simulate the auto trips in the DTA model, so that all auto trips (single-occupancy vehicle, high-occupancy vehicle, trucks, etc.) and the auto parts of the intermodal tours are assigned in the congested network. This process is traditionally in the scope of the DTA models, and more information can be found in the related references. The part that is important for the intermodal model is the simulation of the drive-to-transit trips, so that the actual arrival time at the park-and-ride location is estimated under appropriate roadway congestion. This information is critical in the passenger assignment since the transit path assignment is sensitive to the passenger arrival time at the park-and-ride location.

The third step of the model is to reassign a proper transit path to the intermodal passengers according to their actual arrival times at the park-and-ride locations. A transit path algorithm similar to that for the intermodal assignment is used for this purpose, with slight modifications in regard to the departure time; that is, the TBSP algorithm is used starting with the actual arrival time at the park-and-ride (*14*). After this path reassignment step for the intermodal passengers, the whole transit network is simulated, including the simulation of transit-only and intermodal passengers. In the simulation model, many factors are considered, such as passenger arrival time at the transit stop, transit vehicle arrival time at the stop, and capacity constraints in the transit vehicles. Therefore, any inconvenience during the trip, such as missing a transit vehicle trip, is captured, and the path can be adjusted in an iterative process (described in the next section).

One important property of the transit model is the simulation of the transit vehicles along with the other vehicles in a congested traffic network in the DTA model. This approach allows consideration of possible delays resulting from transit vehicles' interaction with other vehicles. To make this mechanism possible, a new set of arrival and departure times to and from each stop is estimated in the DTA model. These arrival and departure times in turn are given to the transit model (FAST-TrIPs) for more accurate assignment and simulation of transit passenger movements. This adjustment takes into account the fact that the transit schedule does not necessarily represent the actual transit vehicle arrival at the stops, and informed users' behavior is based on their actual experience with the transit operations, not just on the schedule. The simulation model also produces high-resolution outputs, such as the trajectory of the passengers, boarding and alighting activities for every transit vehicle at each stop, and other measures by market segment such as transit and intermodal or bus and rail transit ridership. It also produces time-dependent transit and intermodal skims for feedback to the activity-based travel demand model.

APPLICATION IN INTEGRATED DYNAMIC TRAFFIC AND TRANSIT ASSIGNMENT MODEL

Integrated Model Framework

The transit and intermodal tour assignment and simulation model has been implemented in three modules and was applied in a real case study as part of the SHRP 2 C10-B project. The project goal was to develop an integrated dynamic travel model in a high-resolution multimodal transportation network. As part of this project, an activity-based travel demand model (DaySim) was used to model daily travel activities and produce tour-based demand data for the network model. The multimodal network consists of DynusT as the DTA model and FAST-TrIPs as the transit and intermodal assignment model. The demand and network models are integrated, meaning that auto, transit, and intermodal skim tables from DynusT and FAST-TrIPs are fed back to the demand model (DaySim). However, only the network models are explained in this paper.

In the dynamic multimodal network, there is an iterative process of running DynusT and FAST-TrIPs with feedback to each other, until the multimodal network conditions converge. The structure of the model is shown in Figure 3, in which the tours are given to the intermodal assignment model, and after the tours are split into auto and transit trips, these parts of the tours are given to the appropriate model (DynusT or FAST-TrIPs) for further processing. After each model is run, required information is provided for the next iteration; the equilibrium travel times ("dwell times" in Figure 3) for transit vehicles are passed from the DTA model into FAST-TrIPs, and updated dwell times for every transit vehicle at each stop are fed

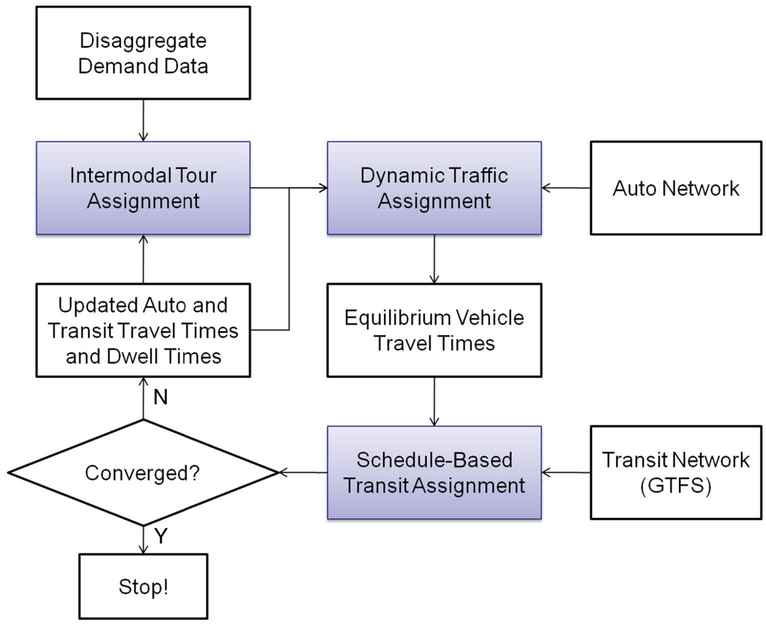

FIGURE 3 High-level structure of dynamic multimodal network model.

from FAST-TrIPs back to DynusT for the next round of simulating transit vehicles. In addition, new time-dependent link travel times and transit vehicle trajectories ("updated auto and transit travel times" in Figure 3) are fed back to the intermodal assignment model for (re)-assigning tours into park-and-ride locations. Thus, if the chosen park-and-ride does not result in a satisfactory experience (minimum time or cost), the passenger may decide to choose a new park-and-ride location.

The convergence of the multimodal assignment model is measured by the relative gap (G_i) on the dwell time of transit vehicles, defined by

$$G_i = \frac{\sum_{r,s} \left(d_i^{r,s} - d_{i-1}^{r,s} \right)}{d_{i-1}^{r,s}}$$

where $d_i^{r,s}$ is the dwell time of transit vehicle r at stop s in iteration i. The gap is defined as being consistent with the fixed-point problem proposed previously. When the transit solution does not change during an iteration, meaning that transit passengers find their best paths and continue using them, the dwell times will not change significantly. Therefore, the impact of the transit vehicles on the auto network is essentially fixed, and the next set of transit vehicle trajectories will be very similar to the existing ones. That is how the system reaches a converged solution and the effect of the auto and transit networks on each other becomes negligible.

Results of Case Study

The proposed model was applied to the Sacramento, California, regional transportation network. For the network preparation, GTFS (*15*) files were used to build the schedule-based transit network in the hierarchical trip-based format (*14*). In fact, for this application, more than one set of GTFS files were used because the transit routes of five different agencies were combined to produce a regional transit network. The regional network contains 3,797 stops, 110 routes, and 2,954 vehicle trips. In addition to the transit network, the intermodal network containing 23 park-and-ride locations was modeled and the mode transfer delays were estimated. Walking links were also generated by using the estimated distance to a stop, considering the size of each traffic analysis zone (TAZ); transit stops that were either in the TAZ or within 0.5 mi of the TAZ centroid were selected as accessible stops, and a walking time was estimated for those stops. The transfer links were generated by using a 0.25-mi distance criterion between pairs of stops.

Two iterations of the multimodal assignment model were run for the 24-h period. Each iteration included a 30-iteration DynusT run to achieve equilibrium in the auto network. In total, the transit and intermodal assignment took about 4 h for each of the two iterations on a computer with Intel Core i5 CPU and 16 GB of RAM. After two iterations the model was terminated (by reaching the maximum number of set iterations), and the dwell time gap reached the value of 0.16.

The main transit measures of effectiveness (MOEs) produced by the model are shown in Tables 1 and 2. These results were obtained by using a model without calibration, and therefore one should not expect very realistic numbers. However, with the default model parameters,

TABLE 1 Transit Vehicle Simulation Results in Sacramento Case Study

MOE	Total Value	Average Value
Number of transit vehicle trips	2,955	na
Travel distance (vehicle miles, per vehicle trip)	30,893	10.46
Operating time (vehicle hours, per vehicle trip)	115,797	0.65
Speed (mph)	na	15.25
Ridership (unlinked passenger trips, per vehicle trip)	88,667	30.02
Average load along the route (persons)	na	7.35
Average dwell time at each stop (s)	na	6.81

NOTE: na = not applicable.

TABLE 2 Transit Passenger Simulation Results in Sacramento Case Study

MOE	Total Value	Average Value
Number of passenger-linked trips	42,803	na
In-vehicle distance (passenger miles)	257,795	6.02
In-vehicle time (passenger minutes)	954,119	22.29
Waiting time (passenger minutes)	371,823	8.69
Walking time (passenger minutes)	355,330	8.30
Number of transfers	58,872	1.38
Transfer time (passenger minutes)	218,593	5.11
Total travel time (passenger minutes)	$1.66 \times 10^{+6}$	38.79

TABLE 3 Intermodal Passenger Measures in Sacramento Case Study

MOE	Average Value
Drive access time (min)	37.4
Transit in-vehicle distance (mi)	4.6
Number of transfers within transit network	0.5
Mode change delay at park-and-ride locations (min)	11.1
Transit in-vehicle time (min)	11.3
Transit transfer time (min)	1.5
Walk egress time (min)	1.4
Transit travel time (min)	25.3
Intermodal travel time (min)	62.7

the results were satisfactory in most cases. For example, as shown in Table 1, the transit vehicle simulation indicates the average operational speed of 15.25 mph and an average ridership of 30 passengers per vehicle trip. These numbers are intuitive considering that all transit vehicle trips, including those in peak and off-peak, are simulated. However, as shown in Table 2, the average travel time in the transit network is 38.79 min, with 22.29 min of in-vehicle time.

Table 3 shows the intermodal trip characteristics, including the travel time in different parts of these trips. According to the table, the average auto travel time to access park-and-ride locations is 37.4 min in the network, and the average transit travel time is 25.3 min for trips from the park-and-ride locations to the destination. The higher drive access time implies that travelers preferred to use auto in a larger proportion of their trips and to access park-and-rides closer to their destinations (or with better transit service to their destinations). Results also show that people spent about 11 min to park their cars, walk to transit stops, wait for a transit vehicle, and board a transit vehicle at the park-and-ride locations.

While the aggregate-level transit results are good enough for the initial tests of the model, some disaggregate sets of transit outputs are provided by FAST-TrIPs and investigated to show how the proposed model can be used as a planning tool for scenario analyses. Figure 4 compares the transit vehicles' travel time in the simulation model and in the GTFS schedule. The simulated travel time is in fact the result of the DTA model and includes the effect of traffic delay. However, transit dwell time as a part of the total travel time is estimated in the transit assignment on the basis of the passengers' boarding and alighting activities. The graph shows that travel times are in general consistent with the GTFS schedule.

Figure 5 depicts the average number of onboard passengers along the route for the inbound and outbound trips of the light rail [light rail transit (LRT)] lines. For the Gold Line, results are consistent with the expectations: a higher morning peak is seen in the inbound trips while a moderate afternoon peak is seen in the outbound trips. For the Blue Line, because it passes through the downtown area, there are two peaks for both inbound and outbound travel, each representing either the a.m. or the p.m. peak period. Figure 6 shows the number of onboard passengers along the route for the Blue Line. In the inbound direction toward the downtown area, the vehicle load increases and then decreases quickly. This pattern is consistent with

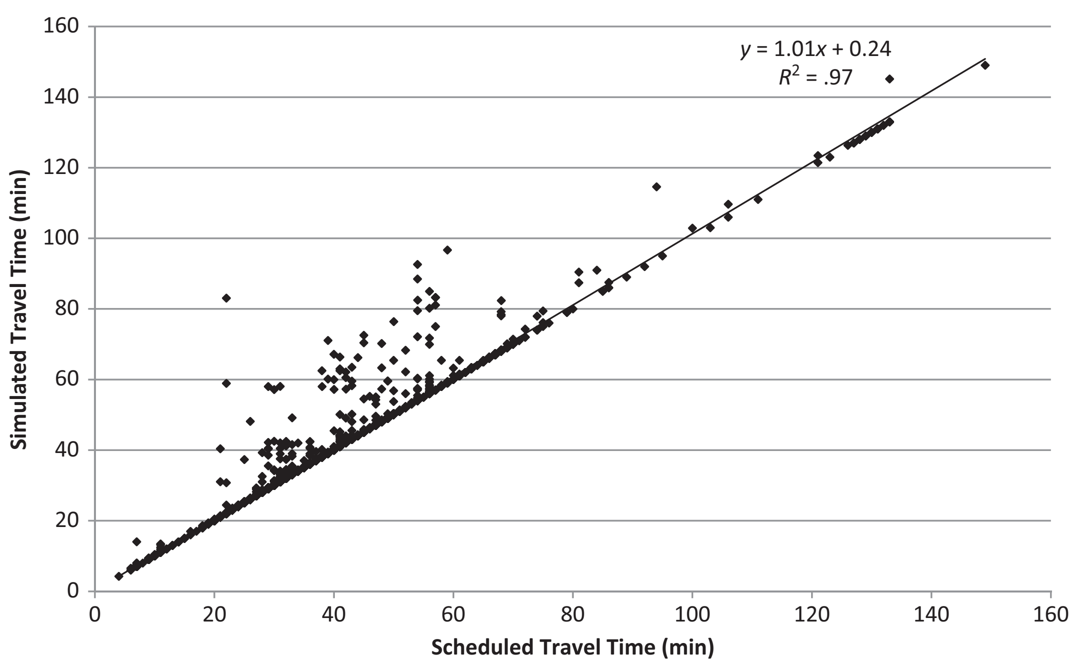

FIGURE 4 Transit vehicle travel times.

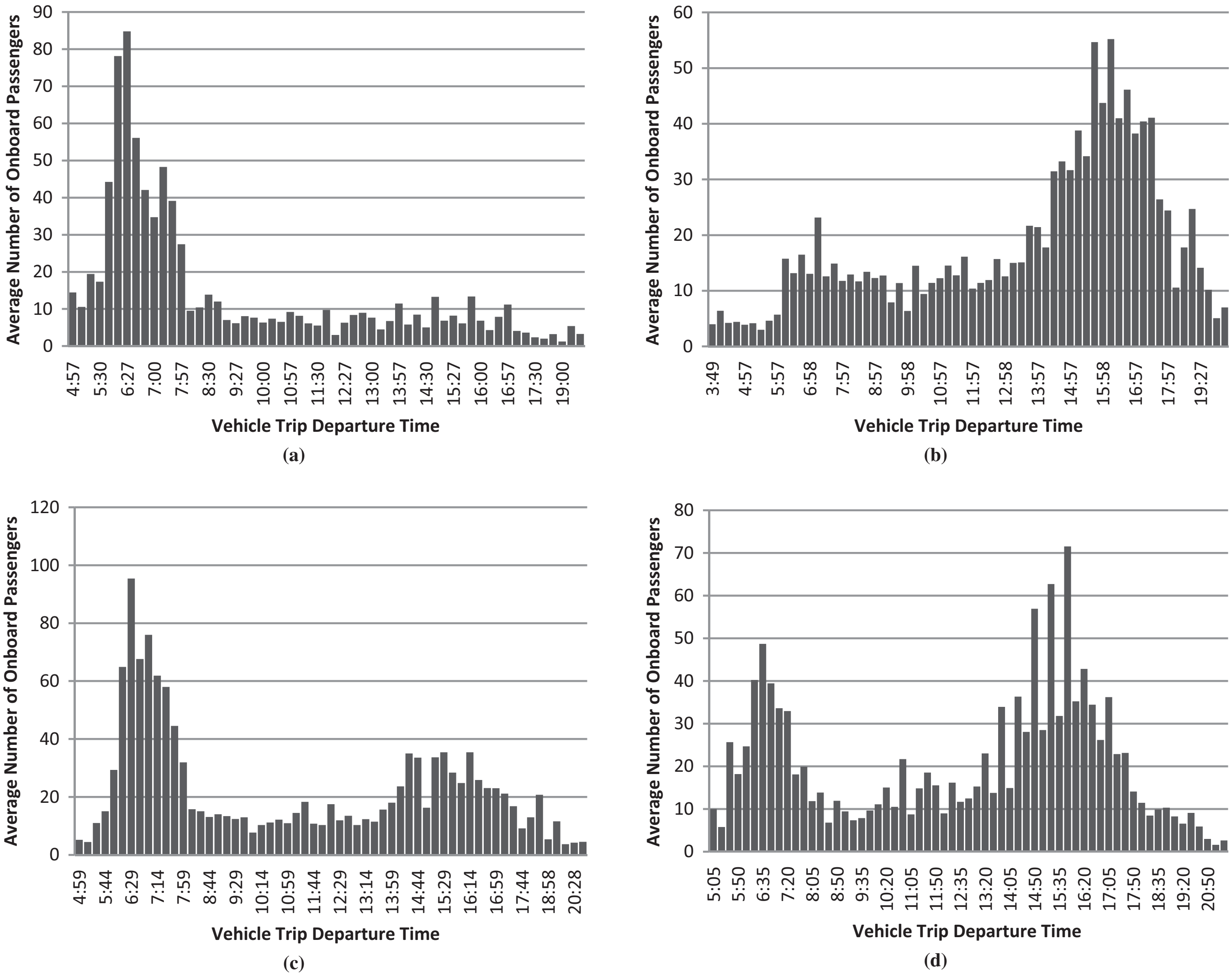

FIGURE 5 Average number of onboard passengers on LRT lines: (*a*) 507 Gold, inbound; (*b*) 507 Gold, outbound; (*c*) 533 Blue, inbound; and (*d*) 533 Blue, outbound.

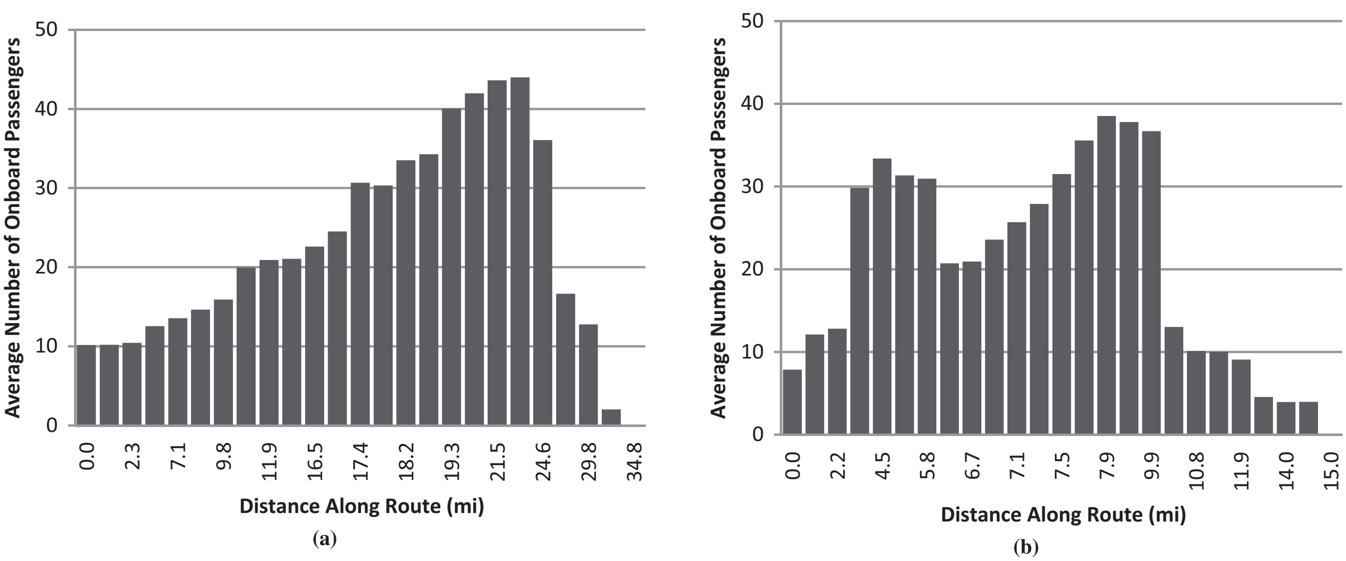

FIGURE 6 Average number of onboard passengers on LRT Blue Line: (*a*) inbound and (*b*) outbound.

the fact that people use the LRT system to access the downtown area for work trips. In the outbound direction, the vehicle load is high at locations closer to the downtown area and starts to decrease, but there is another peak farther down along the route. The two peaks are still lower than that for the inbound trips, representing temporal dispersion in the transit use in the afternoon.

With the results shown in this section, it can be concluded that the proposed model is an appropriate tool for modeling transit and intermodal tours, taking into account the impact of traffic congestion on the transit network. Evidence showed that the model without calibration produced results that are close to expectations. However, further validation of the results is required before the models are used for decision-making purposes.

CONCLUSIONS

A practical model was developed in this study for the comprehensive modeling of transit and intermodal tours as well as auto trips in a dynamic multimodal network. The model has the advantage of representing user behavior more realistically through (*a*) modeling a schedule-based transit network, (*b*) integrating a dynamic congested auto network, and (*c*) including preferred arrival and departure times of passengers in the outbound and inbound half-tours. The proposed model also takes into account the park-and-ride location constraint in the intermodal tours. The computational efficiency of the model is suitable for planning purposes although further improvements in the model efficiency remain an ongoing point of research. The model was applied in the Sacramento regional model, and the general results are promising. The outputs are provided in a disaggregate form, giving the opportunity for planners to analyze different aspects of the system in more detail.

The transit and intermodal assignment models are parts of a model called FAST-TrIPs, developed at the University of Arizona (*18*). The model has undergone several improvements by the authors as a result of its application in the past 3 years. Among such improvements and considered as future work are applying a logit route choice model for improved behavioral modeling (*18–21*), incorporating capacity constraints in the transit assignment model (*18, 20, 21*), and modeling transit system reliability. In addition, validation of the results is an essential future work before the model is used for decision making or scenario analyses.

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