Network Flow Solution Method for Optimal Evacuation Traffic Routing and Signal Control with Nonuniform Threat

Neema Nassir, Mark Hickman, Hong Zheng, and Yi-Chang Chiu

An efficient two-stage network flow approach is proposed for the determination of optimal scenarios for integrated traffic routing and signal timing in the evacuation of real-sized urban networks with several threat zones, where the threat levels may be nonuniform across zones. The objective is to minimize total exposure to the threat (severity multiplied by duration) for all evacuees during the evacuation. In the problem formulation, traffic flow dynamics are based on the well-known point queue model in a time-expanded network representation. The proposed solution approach is adapted from a general relaxation-based decomposition method in a network flow formulation. The decomposition method is developed on the basis of insights into the optimal flow of traffic at intersections in the solution of the evacuation routing problem. As for efficiency, the computation time associated with the decomposition method for solving the integrated optimal routing and signal control problem is equivalent to the time required for solving the same optimal routing problem (without optimizing the intersection control plan) because the computation time required for determining the optimal signal control is negligible. The proposed solution method proves to be optimal. The method is implemented and applied to a real-sized evacuation scenario in the transportation network of Tucson, Arizona. The method is stress-tested with some inflated demand scenarios, and computation aspects are reported.

In a traffic network, one might wish to explore the means of routing traffic through the network from origins to destinations while controlling the signal systems to enhance this traffic flow. However, combined routing and traffic control problems are complex and computationally expensive. The reason for this complexity is that in mathematical models, the optimal values of the decision variables must be identified for both the routing and the control strategies. Adding the control decision variables (which are usually binary variables) to the routing problem increases the complexity of the problem in a combinatorial manner. The integrated optimal traffic routing and signal timing problem is no exception and is known to be much more complex than the optimal traffic routing problem itself. Despite this complexity, over the past few decades, the significance of intersection delays has motivated a large body of research dedicated to finding a harmonized configuration for integrated optimal traffic routing and signal control.

In this research, an exact analytical solution method based on network flow was developed for generating the optimal integrated routing and signal control plan for real-sized urban evacuation problems—that is, fast enough to apply to no-notice or short-notice urban disaster management scenarios. It was made possible by using a relaxation-based decomposition technique and taking advantage of the efficiency of network flow algorithms in solving linear optimization problems with graph structures.

Another feature of the optimization problem in this research is the incorporation of different threat levels into the optimal routing strategy. Recently, the safety aspects of evacuation routing have been incorporated into evacuation modeling in different contexts. These approaches aim not only to minimize the delay experienced in the process of evacuation but also to optimize some safety measures for the evacuation plan. Opasanon and Miller-Hooks propose SEscape, a pseudo-polynomial network flow algorithm that finds the set of paths and volumes that maximizes the minimum chance of escape for all the evacuees (*1*). Some other approaches also incorporate nonuniform threat levels into the objective functions of their optimization evacuation models to minimize the total threat-weighted time (*2–7*).

The other objective of the optimization problem in this paper is minimizing the total threat exposure of traffic during the evacuation. The traffic exposure to threat is defined as the product of the link threat level and the dynamic traversal time of the link, with consideration of the flow dependency of travel times in the links. With a similar objective function, Nassir et al. formulated and solved an optimal routing problem for a chlorine spill scenario in a real-sized urban network by using CPLEX, a commercial optimization package, to model the problem as a minimum cost flow (MCF) problem and then solving it to optimality (*7*). In this paper, the same evacuation routing problem is integrated with signal timing optimization and solved with the proposed relaxation-based decomposition.

LITERATURE REVIEW

The proposed solution method has two major components that make its application to real-sized networks possible: (*a*) formulation of the evacuation routing problem as a network flow problem, which facilitates the use of efficient network flow algorithms in the evacuation traffic routing, and (*b*) the proposed decomposition method, which efficiently handles the integration between the routing and

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Transportation Research Record: Journal of the Transportation Research Board, No. 2459, Transportation Research Board of the National Academies, Washington, D.C., 2014, pp. 54–62.
DOI: 10.3141/2459-07

the signal control problems. A brief review of the literature on these two aspects of the problem is provided in this section.

Network Flow Algorithms for Optimizing Evacuation

Network flow approaches are powerful, efficient methods for modeling and solving linear optimization problems that have constraints sets with the desired graph structures. However, modeling the details of realistic traffic flow dynamics is not always easy (or even possible) in a graph structure.

Traffic routing problems with constant link travel times during an evacuation can be formulated as general dynamic network flow problems and thus be solved efficiently. A set of dynamic flow problems could favor methods for optimizing evacuation. Specifically, minimizing the network clearance time is one common objective in the evacuation literature. Its dynamic network flow model counterpart is known as the quickest flow problem (QFP) (*8–11*). Another evacuation objective is to minimize the total travel time spent by all evacuees in the evacuation process; such a model is formulated as the minimum cost dynamic flow (MCDF) problem. Whereas solutions to the QFP minimize the time horizon, the earliest arrival flow problem aims to optimize the evacuation process (i.e., maximizing the number of evacuees reaching safety), not only at the ultimate clearance time but also at every intermediate time point (*12, 13*). Therefore, the earliest arrival flow problem is a multidimensional optimization problem on top of the QFP that has been exploited in several evacuation studies (*14, 15*). All of these dynamic flow models are presented in a single destination structure; however, the evacuation problem is not restricted because multiple destinations could be connected to a virtual super sink so that the single destination structure applies. Hamacher and Tjandra present a thorough survey on modeling dynamic network flow for evacuation studies (*16*).

The existing literature on the QFP and earliest arrival flow problems generally assumes constant link travel times. However, when a disaster is placed in a congested urban network, the assumption of constant link travel times may not be realistic; more detailed traffic flow dynamics may need to be incorporated for effective evacuation modeling. Important characteristics of traffic flow (e.g., queue and congestion formation and dissipation) are examined in traffic flow dynamics models. In general, how traffic dynamics are modeled substantially affects the properties of the solution method. Incorporating sophisticated traffic flow dynamics into the constraint set generally destroys the problem's graph structure and makes the problem harder to solve (*17, 18*).

In the literature, a few dynamic traffic flow models with flow-dependent travel times have been proposed and encompassed in single-destination–system-optimal dynamic traffic assignment (SODTA) models, such as those based on exit flow function (*19–22*), delay function (*23*), point queue (PQ) (*24–26*), spatial queue (*27*), and kinematic wave [or the cell transmission model (CTM)] (*28–30*). Among these approaches, the PQ and the spatial queue are the only models that can be easily embedded in a typical network graph structure. In this research, the PQ is adopted as the traffic flow model to take advantage of the efficiency of network flow algorithms.

Integrated Traffic Routing and Signal Control

In the past few decades, the significance of intersection delays has motivated a large body of research dedicated to finding a harmonized configuration for integrated optimal traffic routing and signal control. Abdelfatah and Mahmassani presented a closed-form formulation for the combined SODTA and signal control problem, implemented and applied to the Dallas–Fort Worth, Texas, network; their solution approach is a good example of a simulation-based optimization platform (*31*). Their results indicated improved average travel times in the generated solution versus the SODTA solution without signal optimization. However, the optimality of the generated solution was not guaranteed.

Cova and Johnson presented an analytical model for static lane-based evacuation routing in which evacuees are assigned to nonconflicting paths with a limited number of merging movements (*32*). They added integer constraints to the static traffic assignment and used CPLEX to solve their mixed-integer program to optimality. They applied their method to a midsized subnetwork of Salt Lake City, Utah (20 intersections, 314 nodes, 415 links), and determined the optimal static nonconflicting traffic routing.

Lin and Wang proposed a mixed-integer linear formulation to optimize the combined SODTA with the signal control plan (*33*). Their model benefits from the CTM and explicitly treats the undesired vehicle holding that may appear in analytical SODTA solutions. They tested their method for an illustrative example in a network with one street and two intersections. In 2010, He et al. proposed three heuristics to solve the traffic signal control problem formulated as a 0–1 mixed-integer linear programming problem with the CTM (*34*).

Ukkusuri et al. formulated the combined SODTA and signal control problem as a linear program, which significantly improved the solution method over prior formulations with integer variables (*35*). To model the green time among conflicting intersection movements, they introduced additional signal control variables, intersection cells, and connectors for each intersection that take the form of linear constraints. However, the additional variables in their model increased the problem size. They successfully tested their model for an example network with one intersection.

Xie et al. presented a bilevel simulation-based model for optimizing network evacuation performance subject to lane reversal and crossing elimination (*36*). They developed an integrated Lagrangian relaxation and tabu search method to find the optimal solution. Liu et al. proposed a simulation-based genetic algorithm to solve a mixed-integer model for arterial signal control strategies during an emergency evacuation (*37*). Liu and Luo used a bilevel simulation-based genetic algorithm to solve the problem with crossing elimination and signal optimization (*38*).

Xie et al. proposed a mixed-integer formulation to minimize the number of conflicts among intersection movements for an isolated intersection. They developed a simplex-based heuristic to solve the problem (*39*).

By incorporating nonuniform threat levels into their evacuation model with integrated routing and intersection control, Kimms and Maassen proposed a mixed-integer CTM-based SODTA formulation that minimizes the weighted travel times while prohibiting conflicts between intersection movement (*40*). They proposed a fast heuristic to solve the problem for several real-sized network scenarios. However, the optimality of their heuristic method is not guaranteed.

Bretschneider and Kimms proposed a relaxation-based two-stage heuristic that finds a near-optimal solution for their mixed-integer routing problem for evacuation while prohibiting conflicts at intersections (*41, 42*). Their proposed two-stage relaxation-based method is similar to the relaxation-based decomposition technique proposed in this research; however, their proposed heuristic does not guarantee the optimal solution.

A quick review of the solution approaches mentioned earlier, with attention to the optimality and scalability of the proposed methods for real-sized problems, reveals that the existing solution approaches for integrated single-destination-SODTA and signal control are not capable of solving the problem to optimality with computational guarantees for large networks. At least, no literature indicates that any of the proposed exact solution methods has been successfully tested with real-sized problems. Therefore, the main motivation of this research is to improve the state of the practice in solving the integrated dynamic routing and signal control to optimality in real-sized networks.

METHOD

The analytical optimization model in this research is an integrated traffic routing and signal control strategy intended to minimize the total exposure of traffic to the threat, where the exposure of traffic traversing each link is defined as the product of the link threat level and the dynamic traversal time of the link. Travel times on the links also may be flow dependent. The decision variables to optimize for the evacuation plan are traffic advisory information (consisting of evacuation routes, destinations, and departure times), and signal timing to ensure the most efficient and safest flow of traffic when evacuating the network.

Proposed Decomposition Approach

In the proposed solution platform, the integrated routing and intersection control (IRIC) optimization problem first is decomposed into two major subproblems for optimal traffic routing and optimal signal control.

Subproblem 1 (SP1), the optimal traffic routing subproblem, includes optimizing the departure times, evacuation paths, and destinations of vehicles in the network. When SP1 is solved, the constraints related to traffic control at each intersection are relaxed; the intersections in SP1 are modeled as nodes without constraints on turning movements. After the optimal routing solution is found, Subproblem 2 (SP2), the optimal signal control subproblem, facilitates traffic flow for the generated routing solution.

It is proved in this paper that even though the proposed decomposition technique dramatically improves the computational efficiency of the solution method, the optimality of the solution is guaranteed. The proposed decomposition technique is restricted to single-destination optimal routing problems, with which many evacuation problems can be modeled.

Subproblem 1. Dynamic Traffic Routing

Relaxation of Intersection Constraints

In the routing subproblem, intersection traffic control constraints are relaxed, and intersections are modeled as simple nodes. To imagine better how the intersections in SP1 would look after relaxation, assume that each intersection in the real transportation network is modeled as an interchange with uninterrupted flows and infinite capacity for all movements.

As a result of this relaxation, the dynamic flow feasibility constraints in SP1 would consist of only the traffic flow propagation constraints at links and flow conservation at nodes. After the optimal pattern of evacuation traffic flows is found for SP1, the intersections are transformed back to the real conditions in the network, and the optimal dynamic intersection control plan is generated.

Problem Formulation

The objective of SP1 is to find the optimal routing strategy for evacuating traffic to minimize total exposure risk during the evacuation. The exposure of an evacuee on each link is defined as the product of the travel time and the threat level for each link at the specific time. The decision variables of the optimization problem are the evacuees' combined choices of departure time, route, and destination.

In this research, SP1 is formulated as an MCDF problem, with additional constraints to capture traffic flow dynamics at the links. The PQ traffic flow model is adapted for its simplicity and popularity in reflecting the flow dependency of link travel times. Furthermore, PQ can be incorporated into the constraints without destroying the graph structure of the model (*25, 27*). The PQ model assumes that traffic flow traverses the whole link at the free-flow speed until its end, where a queue may develop (*26*). The flow that exits the queue (or the link) is bounded by the bottleneck capacity. The queue can hold all of the excess flow from one time interval to another. As a result, the link travel times in the PQ model depend on the amount of flow on the link; therefore, PQ travel times are dynamic and flow dependent.

Mathematical Formulation

Consider a network $G(N, A)$ in which N is the set of nodes and A is the set of links. G is divided into a set of mutually exclusive and collectively exhaustive subsets $\{G_1, G_2, \ldots, G_K\}$ (i.e., $G = G_1 \cup G_2 \cup \cdots \cup G_K$ and $G_\alpha \cap G_\beta = \varnothing$, $\forall\alpha = 1, 2, \ldots, K$, $\beta = 1, 2, \ldots, i$, $\alpha \neq \beta$). The subnetwork $G_k\ (N_k, A_k) | k = 1, 2, \ldots, K$ includes the node subset located in the threat Zone k, denoted by N_k, and the link subset including the links whose tail nodes are in the threat Zone k, denoted by A_k. A threat zone k is associated with a hazard level h_k. For simplicity, the set of risk zones $\{G_k\}$ are ordered in decreasing h_k (i.e., $h_1 \geq h_2 \geq \cdots \geq h_k$). The safe area outside the disaster threat zones is zone K, and its hazard level h_K is equal to 0.

The present evacuation problem can be modeled as P_{SP1}:

P_{SP1}:

$$\text{minimize } Z_{\text{SP1}} = \sum_{0\leq\tau\leq T} \sum_{k:G_k\subset G} \sum_{(i,j)\in A_k} h_k \cdot x_{ij}^{\tau} \tag{1}$$

subject to

$$x_{ij}^{\tau+1} = x_{ij}^{\tau} + u_{ij}^{\tau} - v_{ij}^{\tau} \qquad \forall (i, j) \in A; \forall 0 \leq \tau < T \tag{2}$$

$$\sum_{(k,i)\in\Gamma_i^{-1}} v_{ki}^{\tau} + b_i^{\tau} = \sum_{(i,j)\in\Gamma_i} u_{ij}^{\tau} \qquad \forall i \in N; \forall 0 \leq \tau < T \tag{3}$$

$$v_{ij}^{\tau} \leq x_{ij}^{\tau} - \sum_{t=\tau-\theta_{ij}}^{\tau} u_{ij}^{t} \qquad \forall (i, j) \in A; \forall 0 \leq \tau < T \tag{4}$$

$$\sum_{t=0}^{T} b_i^{t} = b_i \qquad \forall i \in N \tag{5}$$

$$u_{ij}^{\tau} \le C_{ij}, v_{ij}^{\tau} \le C_{ij} \qquad \forall (i, j) \in A; \forall 0 \le \tau < T \tag{6}$$

$$x_{ij}^{0} = 0 \qquad \forall i \in N; \forall (i, j) \in A; \forall 0 \le \tau < T \tag{7}$$

$$x_{ij}^{\tau} \ge 0, u_{ij}^{\tau} \ge 0, v_{ij}^{\tau} \ge 0, b_i^{\tau} \ge 0 \qquad \forall i \in N; \forall (i, j) \in A; \forall 0 \le \tau \le T \tag{8}$$

where

τ, t = indexes for discrete time step,
T = time horizon,
h_i = threat level at node i,
x_{ij}^{τ} = number of vehicles in link (i, j),
u_{ij}^{τ} = number of vehicles that flow into link (i, j) during τ,
v_{ij}^{τ} = number of vehicles that flow out of link (i, j) during τ,
Γ_i^{-1} = set of all predecessors of node i,
Γ_i = set of all successors of node i,
θ_{ij} = free-flow travel time of link (i, j),
b_i^{τ} = time-dependent demand in source node i during τ,
b_i = total demand in source node i for entire horizon (i.e., $b_i = \sum_{\tau \in [0,T]} b_i^{\tau}$), and
C_{ij} = bottleneck capacity of link (i, j).

The objective function in P_{SP1} is to minimize the sum of the flow exposed to the defined threat on each link. The sum is over all the links, in all the threat zones, and for all time intervals. The equalities in Equations 2 and 3 are the conservation of flow at links and nodes. The inequality in Equation 4 guarantees the legitimate propagation of flow on the links, with sufficient travel time from entrance to exit on a link. The inequalities in Equations 5 through 8 are demand, queue bottleneck capacity, and nonnegativity constraints, respectively. Equation 7 specifies that the initial flow for all the links at the start time is zero. Even though the links might carry an initial flow (background traffic) when evacuation begins, that flow is assumed to enter the network at the downstream node of the link, along with the evacuation demand that arises directly at the downstream node.

Model P_{SP1} is an MCDF problem because the terms in the objective function of P_{SP1} are directly associated with the flow on the links, and the constraints set in P_{SP1} have a graph structure. To solve P_{SP1}, the problem is transformed into an MCF problem in a time-expanded representation. The primary consideration in such a transformation is that the PQ constraint in Equation 6 must be reflected in the network structure. More detailed discussion about this transformation (link transformation) is provided next.

Solution

A link transformation originally proposed by Zawack and Thompson is adapted to model the PQ traffic flow constraints in the evacuation problem (*25*).

Figure 1 shows the link transformation for a simple example network, with one source node, two sink nodes, and two links. As shown in the transformed network, for all the time intervals and at each time copy of the network, a dummy node is used to represent the queue on the link (black shaded squares in Figure 1). Depending on the congestion state at the link, the flow exiting the source node to the links can either exit and proceed to the sink nodes or stay (hold over) in the queue for another time interval. The exit flow to the sink has a capacity equal to the bottleneck capacity of the link (C_{ij}), and the holdover flow in the queue has a capacity equal to the

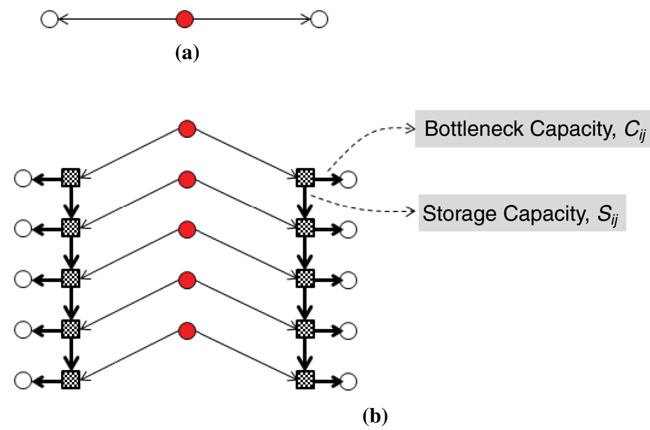


FIGURE 1 Example of transformation from (*a*) base network to (*b*) link-transformed network.

capacity of the storage on the link (S_{ij}). The PQ can be modeled by assigning the appropriate flow on holdover arcs S_{ij}. In this case, the link holdover capacities are infinite

With this transformation, P_{SP1} can be formulated as an MCF problem in the time-dependent network with the embedded PQ traffic flow model. The arc costs for each link are the time elapsed on each link multiplied by the link threat level. This P_{SP1} formulation can be solved with any number of efficient network flow algorithms, such as network simplex, out-of-kilter, and negative cycle canceling (*43*). Standard commercial software packages also can be exploited to solve large-scale MCF problems with a reasonable computation time.

IBM ILOG CPLEX Optimization Studio 12.1 was used in this research to solve the MCF formulation of P_{SP1} (*44*). Experiments with the CPLEX network optimization tool indicate the tool's capability in finding optimal solutions in short computation time. For a P_{SP1} evacuation routing problem with a PQ traffic flow model for a demand of about 150,000 vehicles in an urban network with about 500 nodes, 1,600 links, 70 safe locations, and an evacuation time window of 75 min with a 30-s time discretization, CPLEX generates the optimal solution in about 1 min by using a regular 32-bit desktop computer with an AMD Phenom 8250e 1.90-GHz triple-core processor with 3.25 GB of usable memory. Detailed information about the performance of CPLEX in solving evacuation routing MCF problems with the PQ traffic flow model are available elsewhere (*7*).

Results of these experiments indicate that CPLEX is an effective optimization tool to solve large-scale P_{SP1} problems for no-notice evacuation scenarios.

Subproblem 2. Optimal Intersection Control Plan

Feasibility of SP1 Routing Solution

Up to this point, the solution generated for the evacuation routing problem P_{SP1} is an optimal set of link flows that minimize the total exposure of evacuees to the threat while satisfying constraints about evacuation demand and dynamic traffic flow propagation in the network for all time intervals. These optimal evacuation link flows are used to generate advisory evacuation paths. However, every path flow solution generated from the optimal link flows is not necessarily feasible at the intersections because in the P_{SP1} model, the intersection control constraints were relaxed and intersections were modeled as simple

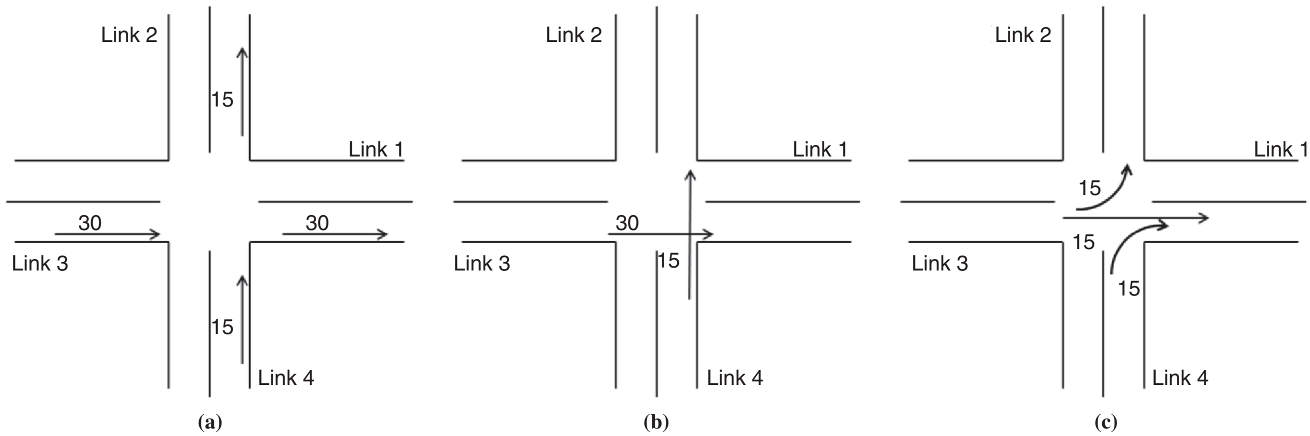


FIGURE 2 Examples of (*a*) link flow solution, (*b*) infeasible path flows, and (*c*) feasible path flows.

nodes. Figure 2 is an example of a feasible solution to P_{SP1} (Figure 2*a*) that can correspond to two sets of path flows: infeasible and feasible movements at the intersection (Figure 2, *b* and *c*, respectively).

As an example, assume a time discretization interval of 30 s, during which the flow for each of the Links 1 through 4 is 30 vehicles. Assume that in the optimal solution to P_{SP1}, during one time interval, that Links 3 and 4 are performing at their capacity of 30, and that Links 1 and 2 have 15 vehicles on each. The reason that movements in Figure 2*b* are infeasible during the 30-s time interval is that this set of movements requires more than one phase because they are conflicting movements. Therefore, given 30 s in each time interval, the green time for each phase in Figure 2*b* must be strictly less than 30 s to accommodate all flows. So the flow of 30 vehicles from Link 3 to Link 4 (at capacity) would be infeasible in a green time less than 30 s. However, Figure 2*c* shows that another set of movements, corresponding to the same set of link flows, can take place in a single phase.

In the next section, it is proved that for every single-destination-SODTA with relaxed intersection control constraints (e.g., P_{SP1}), a feasible solution with nonconflicting path flows exists. The proof to this claim is through a proposed algorithm that is shown to find always a nonconflicting set of intersection movements satisfying the link flows from the optimal solution of SP1. Then, this nonconflicting solution for the optimal link flows in P_{SP1} is proved to be optimal to the integrated problem, IRIC.

Right–Through–Left Algorithm

A right–through–left (RTL) algorithm is developed that finds a feasible set of nonconflicting movements for the optimal solution to P_{SP1}; or, in general, for the optimal solution of every single-destination-SODTA with relaxed intersection control constraints.

Before the algorithm is presented, two essential properties of the SP1-optimal link flow solutions must be mentioned:

Property 1. At each intersection, the flow from any link with positive inflow can be routed to any of the links with positive outflow. This character is exclusive to single-destination optimal routing problems and does not hold for general multidestination problems (*39*).

Property 2 (unidirectionality). At least one of the optimal solutions to P_{SP1} with PQ traffic flow constraints is unidirectional, which means that in the routing solution, each street during each time interval carries vehicles in only one direction at most. It is intuitive because in the optimal routing strategy, routing vehicles in opposing directions of the same street is equivalent to unnecessary traffic circulation in the network. Nassir provides proof for this claim (*5,* Appendix B). In this research, a small movement penalty ($\beta = 10^{-6}$) is added to the costs of moving links in P_{SP1} to guarantee that the optimal solution found for P_{SP1} is necessarily unidirectional.

Given Properties 1 and 2 for the optimal solutions of P_{SP1}, an RTL algorithm is proposed that finds the set of nonconflicting positive-flow movements (NCPFMs) at each intersection, for all time intervals that satisfy the intersection inflows and outflows. This set includes intersection movements for each time interval, with positive flow assigned in such a way that none of the movements pass over (or cross) another; however, this set may include merge movements, diverge movements, or both.

The problem of finding the nonconflicting movements at each intersection is in the form of a static network flow problem, and a flow-augmenting algorithm that generates the solution after a finite number of iterations is used. The proposed RTL algorithm assigns the intersection inflows to the intersection outflows according to a predefined movement prioritization scheme. The priorities in the assignment and the algorithm steps follow.

Step 1. The first priority is to assign the right turns because right turns (Movements 9, 10, 11, and 12 in Figure 3) do not have crossing conflicts with other movements. For all the streets with positive inflows, search for possible right-turn movements; if a right-turn movement to another street with positive flow is possible, then assign the maximum possible flow to this right turn. Complete this step for all possible right-turn movements.

Step 2. The second priority is through movements (Movements 2, 4, 6, and 8). For all positive inflows that remain after Step 1, search for a possible through movement; if a through movement (positive inflow and outflow) is possible, then assign the maximum possible flow to this movement. Repeat this step for all through movements.

Step 3. The last priority is left turns (Movements 1, 3, 5, and 7). For all streets with positive inflows remaining after Steps 1 and 2, assign the remaining flow to possible left-turn movements.

The set of movement flows generated with the RTL algorithm complies with link inflows and outflows, because inflows and outflows

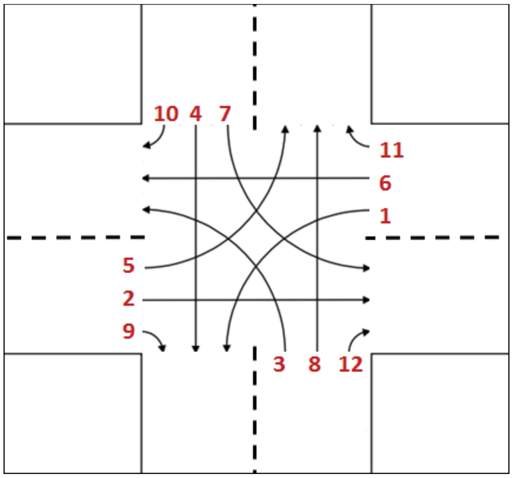


FIGURE 3 Numbering of movements at four-way intersections.

are considered directly. What remains to prove is that the solution the algorithm generates creates no conflicting movements at the intersection.

Proof of correctness. The right-turn movements do not conflict with any other movements. Therefore, only through–through, through–left, and left–left movements have possible crossing conflicts. Each case is explored separately:

- Through–through conflicts (e.g., Movement 2 with Movement 4) cannot exist in the assignment solution. At an intersection with four legs, through movements would cross only after the maximum right turns have been assigned. However, any two crossing movements necessarily create a positive flow for at least one right-turn movement (e.g., Movement 9), which has priority in the assignment process. Therefore, through movements that cross cannot exist (with positive flows) after Step 1.
- Through–left conflicts are either head-on (e.g., Movement 2 with Movement 1) or perpendicular (e.g., Movement 2 with Movement 3). Neither type can exist in the assignment solution because both require at least one street with positive inflow and positive outflow during the same time interval, which contradicts the unidirectionality of the SP1-optimal solution.
- Left–left conflicts also can be characterized as head-on (e.g., Movement 1 with Movement 5) or perpendicular (e.g., Movement 1 with Movement 7). Head-on left–left conflicts cannot exist in the assignment solution because any such movement necessarily creates a positive flow for at least one right-turn movement (e.g., Movement 9 or 11), which takes priority in the assignment process; therefore, head-on left–left conflicts cannot exist with positive flows after Step 1. Perpendicular left–left conflicts cannot exist either, because this type of conflict requires at least one street with positive inflow and positive outflow during the same time interval, which contradicts the unidirectionality of the SP1-optimal solution.

Therefore, the solution to this algorithm has no crossing conflicts of any kind. ■

This proof indicates that for every unidirectional optimal solution to the relaxed routing problem SP1, the RTL algorithm will find a set of NCPFMs for all intersections and during each time interval. Then, a signal control plan based on those NCPFMs is generated. At each time interval, movements will have green time only if they carry positive flow in the solution generated by RTL. As a result, the generated signal plan facilitates uninterrupted intersection flows for the SP1 routing solution.

Optimality Decomposition Method

The main contribution of the proposed optimization platform is solving the IRIC problem to optimality in a short time. The optimality of the solution to the IRIC problem found by the proposed framework is proved in this section.

Proof of optimality. The two problems IRIC and P_{SP1} are identical except for the intersection traffic flow constraints in IRIC that have been relaxed in P_{SP1}. Therefore, the optimal (minimal) objective value to P_{SP1} is a lower bound to the IRIC optimal objective value.

On the basis of the correctness of RTL, for every optimal P_{SP1} solution, the RTL algorithm can generate a feasible set of intersection movements that conserves the P_{SP1}-optimal link flows and the optimal objective P_{SP1} value. Therefore, the solution generated by P_{SP1} and RTL is feasible for IRIC, has the minimum possible objective IRIC value, and is optimal for IRIC. ■

CASE STUDY

A real-sized urban scenario of a chlorine spill was modeled for Tucson, Arizona, with three threat levels: chlorine concentrations of 1,000, 430, and 20 parts per million (ppm) in Zones 1, 2, and 3, respectively. In each zone, the threat level is assumed to be constant over time and equal to these chlorine concentrations.

The generated threat zones were based on a specific scenario in which three railcars carrying liquid chlorine derail, causing a large plume of chlorine gas that is dispersed across the local area according to the Areal Locations of Hazardous Atmospheres (ALOHA) model for gas dispersion (*45*). As shown in Figure 4, gas disperses over a large residential area. Zone 1 (1,000 ppm) covers the whole University of Arizona campus and a large portion of downtown Tucson. Zones 2 (430 ppm) and 3 (20 ppm) also cover several square miles of residential areas in the center and northeast of Tucson.

The network contains 508 nodes and 1,643 links (including centroid connectors) in Zone 3, 211 nodes and 600 links in Zone 2, and 159 nodes and 438 links in Zone 1. All links are major and minor arterials, for which the capacities are calculated according to the actual number of lanes per link (one to three), with a saturation flow rate of 1,800 passenger cars per hour per lane.

Optimization was conducted on the master network that consists of the three threat zones plus 70 sink nodes outside the threat zones. The evacuation time window for all the scenarios was 75 min, and the discretized time interval was 30 s.

The lexicographic P_{SP1} was solved for six demand scenarios. The baseline demand scenario (11,506 vehicles) was generated from background vehicular traffic on the network from a typical weekday traffic assignment. The vehicles on each street and on each centroid connector at the time of the disaster were assigned to the downstream node as the evacuation demand in the baseline scenario. For stress-testing purposes, the same demand pattern was scaled up with factors of 2, 4, 8, 12, and 16 to constitute the other five scenarios.

A small penalty of $\beta = 10^{-6}$ was applied to the P_{SP1} movement links. All of the optimal solutions were verified to be unidirectional; however, an investigation of the optimal flow patterns revealed that

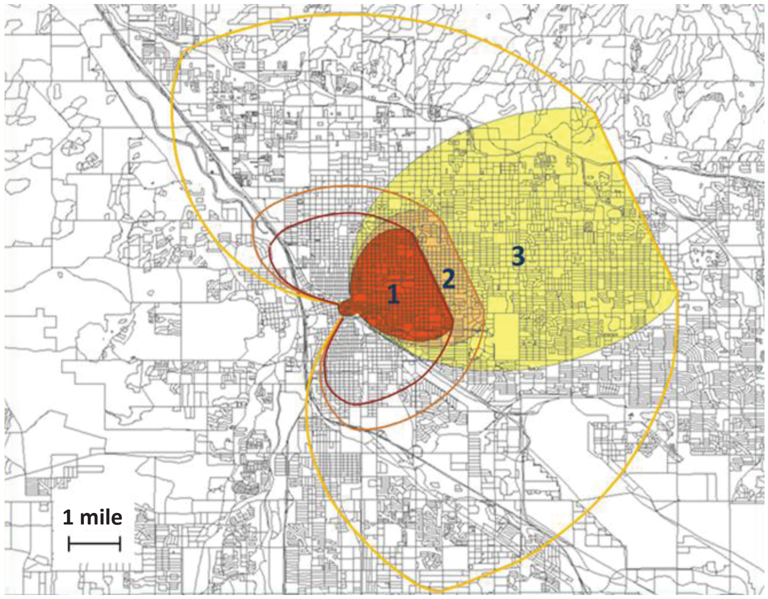


FIGURE 4 Tucson chlorine spill threat zones from dispersion model.

these optimal solutions do not necessarily maintain a constant flow direction over the entire evacuation time window for all the streets. For example, in the optimal routing solution for the baseline scenario, evacuation flows change direction over five streets at least once during the time window, implying that the generated signal timing optimal plan may not be static for all intersections and that some intersections may have dynamic timing over the evacuation time window.

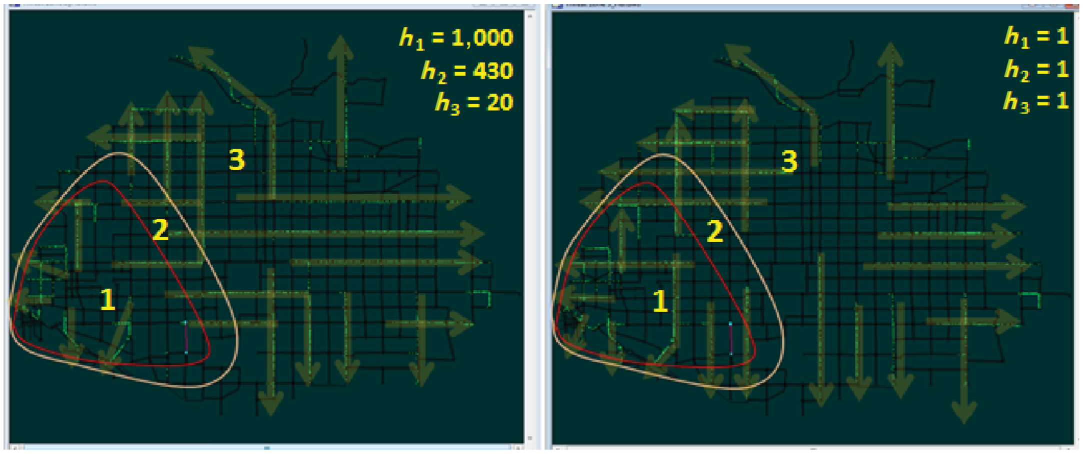
The RTL algorithm was coded in C++ and applied to the P_{SP1}-optimal solutions for all scenarios, and NCPFMs were generated for all intersections in all time intervals. Optimization results for the Tucson evacuation problem with all demand scenarios are presented in Table 1. The most important observation in the results is that computation times are fairly low, which indicates that the proposed model can be applied to large-scale, short-notice or no-notice disaster scenarios because the optimal routing solution can be generated in a reasonably short time. Computation times also increase approximately linearly with increased evacuation demand. A comparison of the computational results of the two stages (CPLEX runs versus RTL runs) confirms the negligible computation time associated with Stage 2, verifying that in practical real-network evacuation scenarios, the proposed decomposition method can reduce IRIC complexity to a relaxed optimal routing problem.

For evacuation clearance time, as evacuation demand increases, the network clearance time increases from 27 time intervals (13.5 min) to 150 time intervals in response to the effect of congestion on traffic flow. Another insight is that in the last scenario with the total demand of 184,096, unlike every other scenario, the network clearance time is equal to the evacuation time window (150 intervals or 75 min), implying that the evacuation time window of 150 intervals might be a binding constraint with a nonzero shadow price. Therefore, an increase in the time horizon could be expected to result in a decrease in the optimal objective value.

Given that the objective function in this research is to minimize total threat exposure, the optimal evacuation flow is expected to be affected by the topological features of the disaster threat zones. Figure 5 shows two screen shots of the simulated optimal IRIC evacuation flows that the proposed method generated for a total demand of 7,866 vehicles. Figure 5*a* illustrates the case in which the threat levels equal the actual chlorine concentration in the Tucson chlorine scenario; Figure 5*b* shows a uniform threat scenario (no difference across Zones 1, 2, and 3). The effects

TABLE 1 Computational Results for Case Study

Total Demand	Evacuation Time Intervals (τ = 30 s)	Network Clearance Time (intervals)	Minimal Total Exposure (ppm × 30 s)	Computation Time (s)	
				CPLEX	RTL
11,506	150	27	21,966,680	2.45	0.22
23,012	150	31	54,867,400	3.9	0.31
46,024	150	45	160,940,480	7.38	0.82
92,048	150	85	547,876,020	15.88	1.29
138,072	150	125	1,168,062,660	31.12	2.12
184,096	150	150	2,036,849,680	60.12	3.63



(a) (b)

FIGURE 5 Optimal evacuation flow pattern across three threat zones for (*a*) nonuniform and (*b*) uniform threat levels (h_x = chlorine concentration in Zone 1, 2, or 3).

of threat zone features are apparent in a comparison of the two evacuation flow patterns. Specifically, flows in the uniform case (Figure 5*b*) illustrate that flows generally take the most direct path out of the evacuation area, in a radial direction away from the hot zone. In the nonuniform case (Figure 5*a*), flows move to areas of lower threat (e.g., from Zone 1 to Zones 2 and 3) before exiting the evacuation area to reduce exposure in the areas of higher threat.

As a result of such a difference in routing strategies, with the same amount of evacuation demand (7,866 vehicles), in Figure 5*a* the network is evacuated in 14.5 min and the average travel time is 3.86 min, whereas in Figure 5*b,* the network is evacuated in 8.8 min and the average travel time is 3.24 min, thus implying that the evacuation could be faster if the routing strategy did not consider the various threat levels. However, results also indicate that even though the network evacuation is slower when threat levels are reflected in the model, the total exposure of the evacuees to the threat decreases by 26% (from 9,315,340 to 6,842,957 min · ppm) when the various threat levels are included.

CONCLUSIONS

The major contribution of this research is a framework developed to solve real-sized optimal IRIC problems in a short time. The framework is made possible by a proposed decomposition method that reduces the computational complexity of the original IRIC problem to an optimal routing problem followed by a fast postprocessing stage. What differentiates this technique from similar relaxation-based methods is that in the proposed method, the optimal solution is generated after only one run of each subproblem. This feature—possible because of the unidirectionality property in the optimal routing solution—significantly decreases computation time.

The proposed solution technique was applied to a hypothetical large-scale chlorine spill scenario in downtown Tucson. The optimal routing and signal control problem for the evacuation of background traffic from the threat area was optimized. The method also was stress tested with some inflated demand scenarios. Results indicate that the proposed method can solve the problem (both original and inflated demand scenarios) to optimality with short computation time.

ACKNOWLEDGMENTS

The authors thank the anonymous reviewers for invaluable comments that improved this paper.

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The Emergency Evacuations Task Force peer-reviewed this paper.