Efficient Negative Cycle–Canceling Algorithm for Finding the Optimal Traffic Routing for Network Evacuation with Nonuniform Threats

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A new network flow solution method is designed to determine optimal traffic routing efficiently for the evacuation of networks with several threat zones and with nonuniform threat levels across zones. The objective is to minimize total exposure (as duration and severity) to the threat for all evacuees during the evacuation. The problem is formulated as a minimum cost dynamic flow problem coupled with traffic dynamic constraints. The traffic flow dynamic constraints are enforced by the well-known point queue and spatial queue models in a time-expanded network presentation. The key to the efficiency of the proposed method is that, for any feasible solution, the algorithm can find and can cancel multiple negative cycles (including the cycle with the largest negative cost) with a single shortest path calculation made possible by applying a proposed transformation to the original problem. A cost transformation function and a multisource shortest path algorithm are proposed to facilitate the efficient detection and cancelation of negative cycles. Zone by zone, negative cycles are canceled at the border links of the zones. The solution method is proved to be optimal. The algorithm is implemented, tested, and verified to be optimal for a midsized example problem.

Over the past few decades, many human-caused and natural disasters (e.g., hurricanes, building fires, bomb threats, and chemical spills) have occurred. The increasing number and intensity of emergencies raise interest in the optimal preparation of an evacuation plan before an emergency arises. Evacuation problems have been studied extensively by many researchers in various modeling paradigms, with different selections of decision variables, objective functions, and constraint sets; moreover, the solution methods have generated a wide spectrum of approaches suitable for problem-specific purposes, contexts, capabilities, and performances.

Evacuation management policies can be modeled, enhanced, and optimized with mathematical optimization techniques. Tactical decisions that provide the scope and context of the evacuation plan are made in various ways depending on the evacuation control tools available to the decision makers. Common decisions selected for optimization focus on generating evacuation advisory information, including departure times, evacuation routes, and destination choices for evacuees (*1–5*). Additional tactical decisions may entail optimizing traffic control and reconfiguring the network to take advantage of flexibilities in the network and create a more efficient evacuation plan (*6–9*).

Without considering the fine details of traffic flow dynamics on roadways, an evacuation can be formulated as a general dynamic network flow problem, which optimizes the evacuation objective as a network flow problem on a dynamic network. Minimizing the network clearance time is one of the common objectives in the evacuation literature; its dynamic flow model counterpart is known as the quickest flow problem (*10–13*). Another evacuation optimization objective is to minimize the total travel time spent by all evacuees in the evacuation process; such a model is formulated as the minimum cost dynamic flow (MCDF) problem. Whereas solutions to the quickest flow problem minimize the time horizon, the earliest arrival flow problem aims to optimize the evacuation process (i.e., maximizing the number of evacuees reaching safety), not only at the ultimate moment of clearance time but also at every intermediate time point (*14, 15*). Therefore, the earliest arrival flow problem is a multidimensional optimization problem on top of the quickest flow problem, exploited in several evacuation studies over the past decade (*16, 17*). All of these dynamic flow models are presented in a single-destination structure; however, this structure does not restrict the evacuation problem because multiple destinations could be connected to a virtual supersink so that a single destination network structure applies. A thorough survey on modeling dynamic network flow for evacuation studies is available elsewhere (*18*).

Network flow algorithms are powerful, efficient methods for modeling and solving evacuation problems. However, the details of real-world traffic flow dynamics are not easy (or even possible) to model in a network structure. Modeling traffic flow dynamics captures important traffic flow characteristics, such as the queuing discipline and the formation and spread of congestion. In the literature, several traffic dynamic models have been proposed and encompassed in single-destination, system-optimal, dynamic traffic assignment (SODTA) models, such as models based on exit flow function (*19–22*) or delay function (*23*), the point queue (PQ) model (*24–26*), the spatial queue (SQ) model (*27, 28*), and the kinematic wave and cell transmission models (*29–31*).

In general, the method used to model traffic dynamics substantially affects the properties of the solution and the solution algorithms. Incorporating sophisticated traffic flow dynamics into the constraint

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set generally destroys the problem's graph structure and makes the problem harder to solve (*32, 33*). As a result, most analytical evacuation models that are solved by network flow algorithms either lack detailed dynamic flow features or use simple ones such as queue models.

In this research, two simple traffic flow models (PQ and SQ) that can be embedded in a network structure are adapted as the traffic flow propagation models to take advantage of the efficiency of network flow algorithms. The PQ and SQ models can be modeled in a time-expanded network structure with simple network transformations (*26, 27*).

Another feature of the optimization problem in this research is the incorporation of threat levels into the optimal routing strategy. Researchers in different contexts recently have incorporated safety aspects in evacuation routing problems. These approaches aim to optimize not only the experienced delay but also some safety measures for the evacuation plan. Opasanon and Miller-Hooks proposed a pseudo polynomial network flow algorithm (SEscape) that finds the set of paths and volumes that maximizes the minimum chance of escape for all the evacuees (*34*). Liu et al. applied a SODTA model to minimize the weighted travel time among all evacuees in the network (*35*). Yao et al. developed a robust linear programming model to minimize the weighted travel times (*36*). Kimms and Maassen incorporated nonuniform threat levels into their integrated routing and intersection control evacuation model (*37*). They proposed a mixed-integer SODTA formulation based on a cell transmission model that minimizes the weighted travel times while prohibiting intersection movement conflicts.

With an objective function similar to that of this research, Nassir et al. formulated and solved an optimal routing problem that minimizes the traffic exposure to threat in a real-sized network chlorine spill scenario by modeling the problem as a minimum cost flow (MCF) problem and solving it to optimality with CPLEX (*28*). In this paper, the same problem formulation is tackled by developing a new network flow solution algorithm that benefits from the special pattern of link costs in the problem.

The remainder of the paper is organized as follows. First, the problem statement is presented. Then, the proposed cost transformation and cycle-canceling algorithm are illustrated. Numerical studies are described to verify the results for a midsized network example. Finally, the conclusions from this work are provided.

PROBLEM SPECIFICATION

Overview

The objective of this research is to design an optimal routing for traffic evacuation that minimizes total risk exposure during an evacuation. The exposure of an evacuee on each link in a certain threat zone is defined as the product of the dynamic travel time and the threat level of that zone. The decision variables for the optimization problem are the departure times, routes, and destinations of the evacuees.

What differentiates this optimization problem from typical evacuation models, which usually optimize the clearance time or system travel time, is the consideration of threat levels (or exposure severity) on different links in the network. This feature not only introduces new parameters to the model but also changes the objective function and the solution method required to solve the problem. The single-destination-SODTA approaches, which are the most commonly used for evacuation problems, are no longer applicable to the specified problem because of the difference in the objective function.

The formulation of the minimum exposure evacuation problem in this paper is similar to that of the MCDF problem, with additional constraints to include traffic flow dynamics. The two traffic flow models selected for this research, PQ and SQ, were chosen for their popularity in measuring queuing effects (formation and dissipation) in the evolution of traffic dynamics. The PQ and SQ models also can be embedded in the constraints without destroying the model's graph structure.

The PQ model, first proposed by Vickrey, assumes that traffic flow traverses at the free-flow speed on the entire link until its end, at which point a queue may develop (*25*). The flow can exit the queue (or the link) with a limited capacity called bottleneck capacity, which is equal to the maximum number of vehicles that can traverse a link during one time interval. The queue can hold the excess flow with a finite or infinite capacity. The SQ model is similar to the PQ model, except that queue length is bounded by the maximum number of vehicles that the link can physically accommodate under jam conditions. In the PQ model, flow entering a link is not bounded, which means that the queue never spills over to its upstream links. However, in the SQ model, queue spillover may occur; when the queue length reaches its capacity, no vehicles are allowed to enter the link.

The link travel times in the PQ and SQ models depend on the amount of flow on the link, and therefore, travel times are dynamic and flow dependent.

Mathematical Formulation

Consider a network $G(N, A)$, where N is the set of nodes and A is the set of links. Network G is divided into a set of mutually exclusive and collectively exhaustive subsets $\{G_1, G_2, \ldots, G_K\}$ (i.e., $G = G_1 \cup G_2 \cup \cdots \cup G_K$ and $G_\alpha \cap G_\beta = \varnothing$, $\forall \alpha = 1, 2, \ldots, K$, $\beta = 1, 2, \ldots, K$, $\alpha \neq \beta$). Subnetwork $G_k\,(N_k, A_k)$ for $k = 1, 2, \ldots, K$ is a subnetwork that includes the node subset located in the threat zone k, denoted by N_k, and the link subset including the links whose tail nodes are in the threat zone k, denoted by A_k. A threat zone k is associated with a hazard level h_k. For simplicity, the set of risk zones $\{G_k\}$ are ordered in decreasing h_k (i.e., $h_1 \geq h_2 \geq \cdots \geq h_K$). The safe area outside the disaster threat zones is zone K, and its hazard level h_K is equal to 0.

Notation

τ, t = indexes for discrete time step,
T = time horizon,
h_i = threat level at node i,
$x_{i,j}^{\tau}$ = number of vehicles in link (i, j) during τ,
$u_{i,j}^{\tau}$ = number of vehicles that flow into link (i, j) during τ,
$v_{i,j}^{\tau}$ = number of vehicles that flow out of link (i, j) during τ,
Γ_i^{-1} = set of all predecessors to node i,
Γ_i = set of all successors from node i,
b_i^{τ} = time-dependent demand in source node i during τ,
$\theta_{i,j}$ = free-flow travel time of link (i, j),
b_i = total demand in source node i for entire horizon (i.e., $b_i = \sum_{\tau\in[0,T]} b_i^{\tau}$), and
$C_{i,j}$ = bottleneck capacity of link (i, j).

The evacuation problem in this study can be modeled as P:

P:

$$\min Z = \sum_{0\leq\tau\leq T} \sum_{k:G_k\subset G} \sum_{(i,j)\in A_k} h_k \cdot x_{i,j}^{\tau} \tag{1}$$

subject to

$$x_{i,j}^{\tau+1} = x_{i,j}^{\tau} + u_{i,j}^{\tau} - v_{i,j}^{\tau} \qquad \forall (i,j) \in A; \forall 0 \le \tau < T \tag{2}$$

$$\sum_{(k,i)\in\Gamma_i^{-1}} v_{k,i}^{\tau} + b_i^{\tau} = \sum_{(i,j)\in\Gamma_i} u_{i,j}^{\tau} \qquad \forall i \in N; \forall 0 \le \tau < T \tag{3}$$

$$v_{i,j}^{\tau} \le x_{i,j}^{\tau} - \sum_{t=\tau-\theta_{i,j}}^{\tau} u_{i,j}^{t} \qquad \forall (i,j) \in A; \forall 0 \le \tau < T \tag{4}$$

$$\sum_{t=0}^{T} b_i^t = b_i \qquad \forall i \in N \tag{5}$$

$$u_{i,j}^{\tau} \le C_{i,j},\, v_{i,j}^{\tau} \le C_{i,j} \qquad \forall (i,j) \in A; \forall 0 \le \tau < T \tag{6}$$

$$x_{i,j}^{0} = 0 \qquad \forall i \in N; \forall (i,j) \in A; \forall 0 \le \tau \le T \tag{7}$$

$$x_{i,j}^{\tau} \ge 0, u_{i,j}^{\tau} \ge 0, v_{i,j}^{\tau} \ge 0, b_i^{\tau} \ge 0 \qquad \forall i \in N; \forall (i,j) \in A; \forall 0 \le \tau \le T \tag{8}$$

The objective function in model P is to minimize the sum of the flow being exposed to the defined threat on each link. The sum is over all links, in all threat zones, and for all time intervals. The equalities in Equations 2 and 3 are the conservation of flow at links and nodes, respectively. Equation 4 guarantees the legitimate propagation of flow on the links, with sufficient travel time from entrance to exit on a link; specifically, this inequality states that the number of vehicles leaving a link at a particular time cannot exceed the total number of vehicles on that link at that time, minus the number of vehicles on that link that have not been on the link long enough to reach the downstream node, under free-flow conditions.

The inequalities in Equations 5, 6, and 8 are demand, bottleneck capacity, and nonnegativity constraints, respectively. Equation 7 specifies that the evacuation flow for all the links at the start time is zero.

Whereas the traffic flow constraints in model P are written to comply with the PQ model, adding the inequality in Equation 9 to the constraint set adjusts P to the SQ model. The storage capacity constraint is

$$x_{i,j}^{\tau} \le S_{i,j} \qquad \forall (i,j) \in A; \forall 0 \le \tau \le T \tag{9}$$

where $S_{i,j}$ is capacity of the storage on the link (i, j).

In both the PQ and SQ models, model P is an MCDF problem because the terms in the objective function of P are directly associated with the flow on the links and the constraint set in P has a graph structure. To solve P, the problem is transformed into an MCF problem in a time-expanded representation. The primary factor to consider is that in such a transformation, the PQ and SQ constraints (Equations 6 and 9) must be reflected in the network structure. More detailed discussion about this transformation (link transformation) is provided in the next section.

METHOD

In the method proposed in this paper, instead of solving P as a linear optimization problem, the problem is transformed into a special MCF problem in a time-expanded network that has a set of specially defined turnstile costs at the borders of the threat zones. A link transformation (LT) and a cost transformation (CT) are applied to the base roadway network in problem P to do this.

Link Transformation

To model the SQ (or PQ) traffic flow constraints in the evacuation problem, an LT originally proposed by Drissi-Kaaitouni and Hameda-Benchekroun is adopted (*27*).

Figure 1 shows the LT transformation for a simple example network with one source node, two sink nodes, and two links. In the transformed network, for all time intervals and at each time copy of the network, a dummy node (shaded squares in Figure 1) is generated for each link in the base network, which represents the queue on the link. The flow exiting the source node to the links can either proceed to the sink nodes or stay in the queue (hold over) for another time interval, depending on the congestion state on the link. The flow moving to the sink has a capacity equal to $C_{i,j}$, and the holdover flow in the queue has the capacity equal to $S_{i,j}$.

The PQ and SQ models can be modeled by assigning the appropriate flow capacity on holdover arcs ($S_{i,j}$). In the former case, link holdover capacities are infinite; in the latter case, holdover capacities are set to the physical capacity of storage at the link. Therefore, the rest of the solution procedures in this paper are on the LT-transformed network and are the same for the PQ and SQ models.

Cost Transformation

The CT function proposed in this paper is inspired by the turnstile cost first introduced by Hamacher and Tufekci (*38*). The proposed turnstile cost function for link (i, j) at time τ ($f_{i,j}^{\tau}$) is presented in this section, and the problem of finding the minimum turnstile cost flow P' is proved equivalent to the original problem P.

Denote the sets of boundary (interzonal) links (that cross between two different threat zones) as $B_{\alpha,\beta} = \{(i, j) | i \in G_\alpha, j \in G_\beta, \alpha \ne \beta\}$, where node i belongs to risk zone α, node j belongs to risk zone β, and $\alpha \ne \beta$. Let h_α and h_β be the severity of threat associated with zones α and β, respectively. The proposed turnstile cost function for link (i, j) at time τ is

$$f_{i,j}^{\tau} = \tau \times (h_\alpha - h_\beta) \qquad \forall (i,j) \in A; 0 \le \tau < T \tag{10}$$

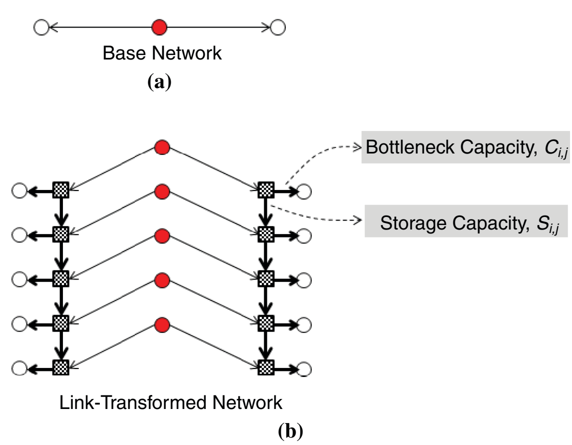

FIGURE 1 Examples of link transformation: (*a*) base network and (*b*) link-transformed network.

where node i belongs to risk zone $i \in N_\alpha$ and node j belongs to risk zone $j \in N_\beta$. The resulting $f_{i,j}^\tau$ is equal to the differential threat level at the two ends of the link, multiplied by the time index τ. Therefore, $f_{i,j}^\tau$ is equal to 0 for all the links that have equal threat levels at the two ends or simply the links lying inside the zones. Such links may be called intrazonal links as opposed to interzonal (or boundary) links, which have nonzero transformed costs.

The mathematical formulation of the transformed cost (min-turnstile-cost) problem is

P':

$$\min Z = \sum_{0 \le \tau \le T} \sum_{(i,j) \in A} f_{i,j}^\tau \cdot x_{i,j}^\tau \tag{11}$$

subject to Equations 2 through 9.

Theorem 1. The problem P is equivalent to the min-turnstile-cost problem P'.

Proof. The only difference between the two problems is in the objective functions, and the objective functions are proved equivalent.

Rewrite the objective function of P as

$$\sum_{0 \le \tau \le T} \sum_{(i,j) \in A} h_i \cdot x_{i,j}^\tau = \sum_{0 \le \tau \le T} \sum_{a: G_a \subset G} h_a \cdot \left(\sum_{(i,j) \in A_a} x_{i,j}^\tau \right) \tag{12}$$

The last term on the right-hand side of Equation 12 is the total time spent in threat zone a, where A_a is the set of links whose tail nodes are located in threat zone a. It can be rewritten as

$$\sum_{(i,j) \in A_a} x_{i,j}^\tau = \sum_{b: G_b \subset G} \sum_{(i,j) \in B_{a,b}} \left(\tau \cdot x_{i,j}^\tau \right) - \sum_{b: G_b \subset G} \sum_{(k,i) \in B_{b,a}} \left(\tau \cdot x_{k,i}^\tau \right) - \sum_{i \in N_a} \left(\tau \cdot b_i^\tau \right) \tag{13}$$

The last term on the right-hand side of Equation 13 is related to the evacuation demand b_i^τ at zone a. Therefore, it is constant and can be eliminated. Using Equation 13, the right-hand side of Equation 12 can be rewritten as

$$\sum_{0 \le \tau \le T} \sum_{a: G_a \subset G} \left(\sum_{b: G_b \subset G} \sum_{(i,j) \in B_{a,b}} \left(h_a \cdot \tau \cdot x_{i,j}^\tau \right) - \sum_{b: G_b \subset G} \sum_{(k,i) \in B_{b,a}} \left(h_a \cdot \tau \cdot x_{k,i}^\tau \right) \right) \tag{14}$$

which can be rewritten again as

$$\begin{aligned} \sum_{0 \le \tau \le T} \sum_{a: G_a \subset G} \sum_{b: G_b \subset G} \sum_{(i,j) \in B_{a,b}} (h_a - h_b) \cdot \tau \cdot x_{i,j}^\tau &= \sum_{0 \le \tau \le T} \sum_{a: G_a \subset G} (h_a - h_b) \cdot \tau \cdot x_{i,j}^\tau \\ &= \sum_{0 \le \tau \le T} \sum_{(i,j) \in A} f_{i,j}^\tau \cdot x_{i,j}^\tau \end{aligned} \tag{15}$$

Because the right-hand side of Equation 15 is the objective function of P', the proof is concluded.

Figure 2 shows how the CT transformation is applied. What makes P' easier to solve is that in the network representation of P' (the CT-transformed network), the links with nonzero costs (bold arrows in Figure 2) are restricted to only interzonal links, whereas in the original case, all the links with positive travel times had positive arc costs.

Solution Method

Overview

The solution method in this paper is a type of negative cycle–canceling algorithm for solving the MCF problem. The typical negative cycle–canceling algorithm starts with a feasible solution of flow from the origins to the destinations and continuously improves the solution by canceling the negative cycles of flow in the residual

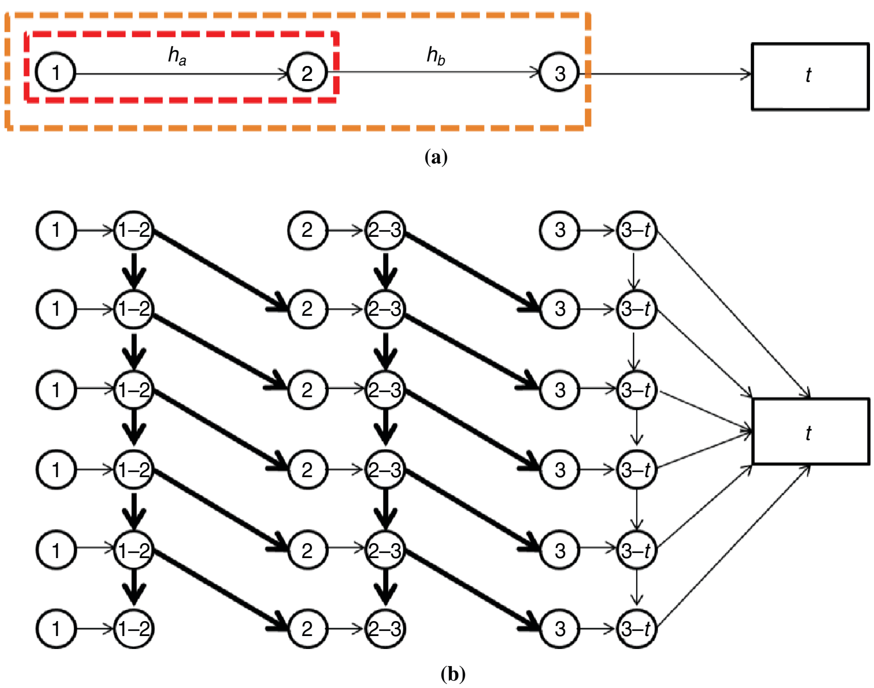

FIGURE 2 Examples of CT transformation: (*a*) original network with two threat zones, and (*b*) LT-transformed network.

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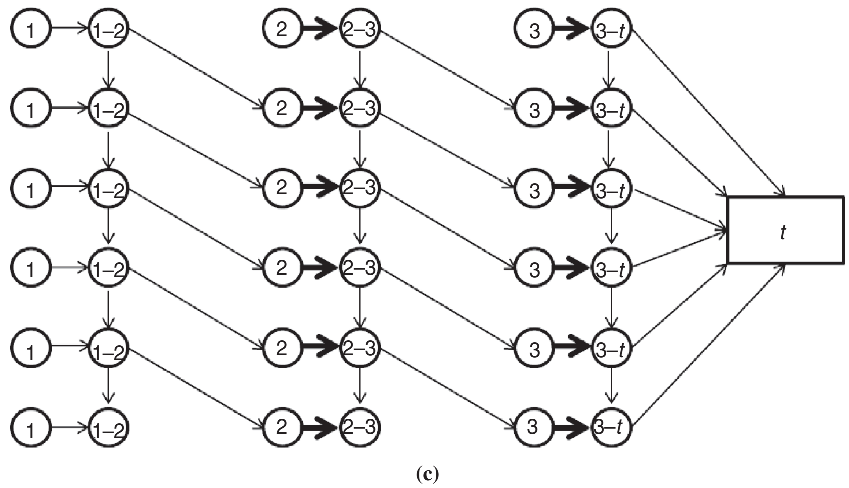

FIGURE 2 *(continued)* Examples of CT transformation: (*c*) LT- and CT-transformed network (bold arrows denote nonzero arc costs).

network. The optimal solution is generated when no negative cycles remain in the residual network. More information about the residual network transformation, the negative cycle–canceling algorithm, and other network flow algorithms to solve the MCF problem are available elsewhere (*39*).

In this paper, an efficient way to identify the greatest negative cost cycle on the transformed network is proposed in taking advantage of the special structure of the turnstile cost.

The solution method has multiple stages, the number of which is equal to the number of threat zones in the evacuation. The solution process starts with the innermost threat zone (which contains no other threat zones), sets it as the study subnetwork, and isolates it from the rest of the network (denoted by the red dotted line in Figure 3*b*). Isolating a subnetwork means temporarily removing the links that connect the subnetwork to the rest of the network.

Then, for the first study subnetwork (SS_1), a feasible flow is generated to evacuate all the demand from SS_1. Next, a proposed negative cycle detection (NCD) algorithm iteratively finds the negative cycles and cancels them until no negative cycles remain in SS_1. Then, the study subset moves one threat zone out to the second study

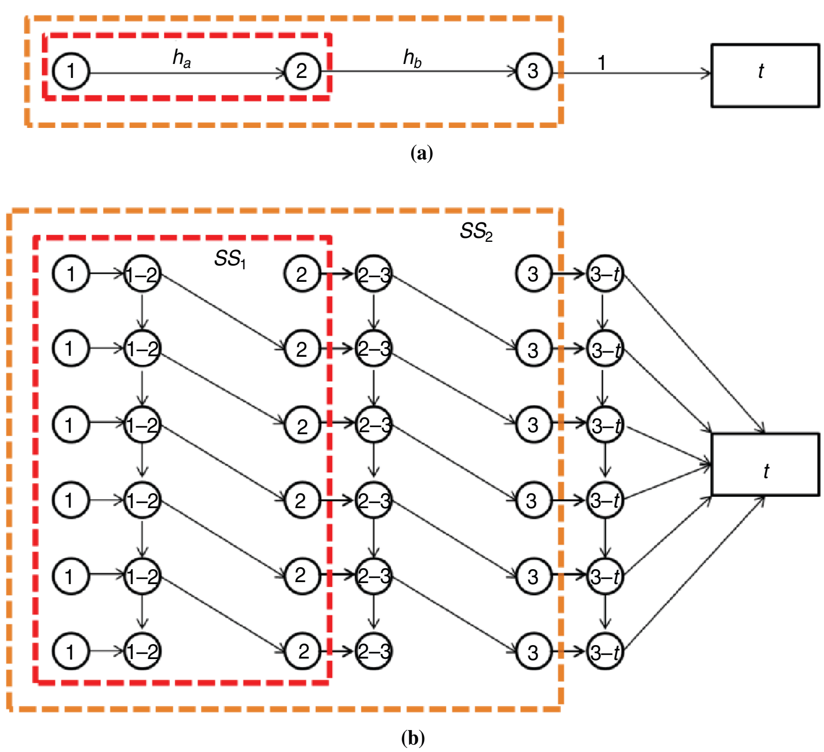

FIGURE 3 Examples of (*a*) original network with two threat zones and (*b*) SS_1 and SS_2 in LT- and CT-transformed network.

subnetwork (SS_2). The same steps are repeated for SS_2, except that the feasible flow to begin this stage evacuates all the demand in SS_2 plus all the flow from SS_1 at Stage 1. The algorithm continues until the last stage (k), in which SS_k is the whole evacuation area. Figure 3 illustrates an example CT-transformed network and the associated study subnetworks in the solution algorithm.

NCD and Cancellation

The NCD algorithm proposed in this research is based on a multi-source shortest path (MSP) algorithm running on the residual network to find the negative cycles. The MSP algorithm is an efficient tool for solving optimal tour or walk problems in intermodal networks (*40*). The MSP algorithm is similar to typical label-setting shortest path algorithms except that the labeling process in MSP starts with more than one node: the sources or the roots of the shortest path tree. This way, when an MSP algorithm terminates, the final calculated label at each node is the distance (or shortest travel time, or other parameter) of that node measured from the whole set of sources (i.e., whichever source node is closest to the labeled node).

At each iteration, the NCD algorithm finds a label and a path for all the border nodes i of the subnetwork, where border nodes of the SS are the nodes outside the SS that are connected to at least one node in the SS. The associated path to node i is the optimal path that has the largest negative flow cost to i from another node j (which is the optimal node among all candidate nodes on that border). The optimal path to i determines the optimal upstream node j, too. The label of i equals the total cost associated with the path from j to i.

In NCD, at each iteration, the labels and paths for all the border nodes are calculated in a single MSP run by setting the initial labels of all the border nodes to zero and running the MSP algorithm with all those nodes as sources. After all the nodes are labeled by MSP, the NCD algorithm searches for the node with the largest negative label and augments the maximum possible flow to the path that is associated with that largest negative label or node. In this way, the largest negative cycle among all possible cycles from all the border nodes is detected and canceled.

The NCD algorithm detects not exactly negative cycles but, more precisely, negative paths. This process is called cycle cancellation in this paper so readers can relate it to the well-known cycle-cancellation network flow optimization method that has similar algorithmic steps.

The main advantage of the NCD algorithm over existing cycle-canceling algorithms is that at each iteration, the NCD algorithm detects and cancels the cycle that has the largest negative cycle cost. This feature avoids redundant flow cancellations with incremental improvements, decreases the number of NCD iterations, and leads to faster optimization. Another advantage of the NCD algorithm is that at least one cycle is detected in each iteration. Experiments reveal that, in practical applications, in each MSP run at every NCD iteration, multiple cycles can be detected and canceled, including the cycle with the largest negative cost.

The NCD algorithm for the SS_i in the residual network is as follows.

Step 0. Start with a feasible flow to evacuate the SS_i in the LT-transformed subnetwork and update the residual network accordingly.

Step 1. Label all the SS_i border nodes as zero.

Step 2. Run the MSP algorithm with all SS_i border node copies, at all the time intervals, as the set of sources.

Step 3. The border node with the smallest label is the downstream node of an augmenting path, representing the cycle with the greatest negative cost. Find the augmenting path and the upstream node by backtracking the sequence of predecessors.

Step 4. Augment all of the border nodes with negative labels (because they correspond to negative cycles) to the largest possible flow along the associated path, in an increasing order of labels.

Step 5. $i := i + 1$; go back to Step 0.

Negative cycles never exist in the study subnetworks, even at the beginning of stage i because all possible negative cycles that could have existed in SS_i are canceled in stage $(i - 1)$, on the borders of the SS_{i-1}. Otherwise, if a negative cycle remained in the study subnetwork, the NCD algorithm would fall into an infinite loop during the shortest path run.

Again, all the flows that are augmented in the NCD algorithm are augmented along negative cost paths in the study subnetwork and not along negative cost cycles.

Figure 4 illustrates a simple NCD iteration for an example subnetwork with three nodes (Nodes 1 through 3) and a single border node (Node 4). Figure 4*a* shows a feasible flow in SS_i, and Figure 4*b* shows the residual network and the NCD iteration. The iteration starts by setting the labels (L) to zero for the copies of Node 4 in all time intervals; then, after an MSP calculation, the labels are updated. The smallest label is –4, which belongs to the node that is selected as the downstream node of the augmented flow. Figure 4*c* is the augmented flow path, and Figure 4*d* is the optimal solution—that is, the result after augmentation.

The solution algorithm is effective only under the condition that all arc costs in the threat zones are zero; this condition is achieved by the proposed CT. By isolating the study subsets from the rest of network, the solution algorithm traces the negative cost cycles at the borders. After all the cycles at the first border are removed, the algorithm moves to the outer subnetwork. Therefore, negative cycles never exist in study subnetworks, and all the cancellations take place at the borders of the threat zone.

After one NCD run on study subnetwork i (SS_i) in the LT-transformed network, the number of detected cycles is, at most, equal to the product of the number of time intervals in the horizon and the number of SS_i border nodes. The negative cycle with the greatest negative cost is among the detected negative cycles. The negative cycles detected by NCD are augmented to their capacity limit, one by one, starting with the greatest negative cycle and proceeding in decreasing order. However, the residual network must be updated after each augmentation, and some or all of the remaining negative cycles (with zero remaining capacity) may disappear after an update because of possible overlaps between the canceled cycle and the remaining cycles. Cancellation continues until all of the detected cycles either are canceled or disappear; then, another NCD iteration is run. The same canceling and NCD iterations continue until the NCD algorithm detects no negative cost cycles. The resulting flow is optimal for the SS_i evacuation. The next step moves to SS_{i+1}.

Proof of Optimality

Theorem 2 (negative cycle optimality condition). A feasible solution x^* is an optimal solution to an MCF problem if and only if the residual network $G_r(x^*)$ has no negative cycles.

Proof. Ahuja et al. provide the proof (*41,* p. 98, Theorem 5.1). ■

The problem P' is an MCF problem. If it is proved that the proposed algorithm generates a solution in a residual network with no negative

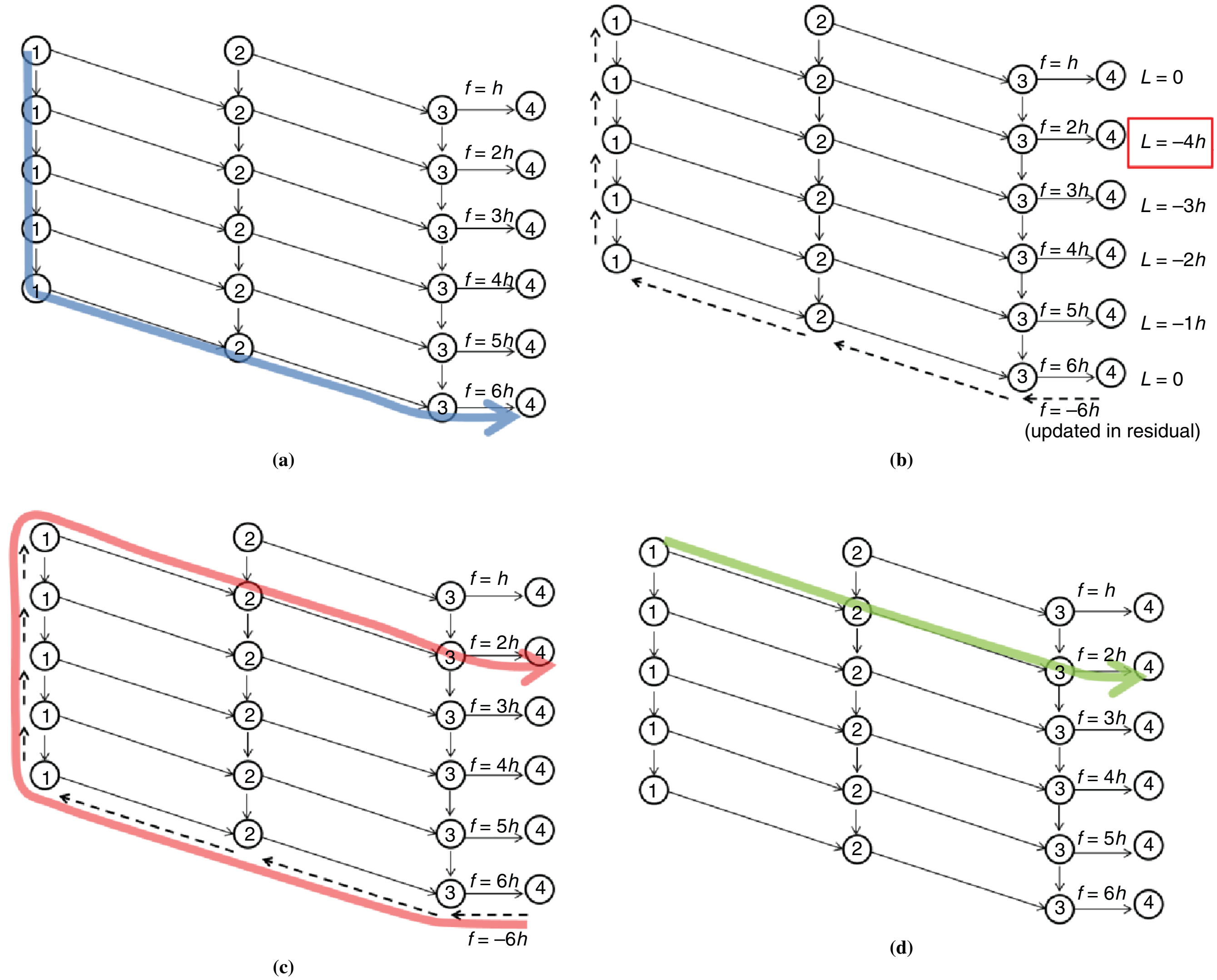

FIGURE 4 Examples of NCD iteration: (*a*) one feasible flow, (*b*) multisource residual network shortest path run (MSP run), (*c*) augmented flow, and (*d*) optimal solution (f = transformed cost, as defined in Equation 10).

cost cycles, then the optimality of the solution is proved according to Theorem 2.

Because this proof is based on mathematical induction, two propositions must be proved:

Proposition 1. No negative cost cycles exist in the residual network of SS_1.

Proof. According to the definition of turnstile cost in the CT transformation, no arcs with a negative (or nonzero) arc cost exist in SS_1; therefore, no negative cycles exist in the residual network of SS_1. ■

Proposition 2. If no negative cycles exist in the residual network of SS_i, then no negative cycles exist in the residual network of SS_{i+1} at the end of stage *i*.

Proof. Because no negative cycles exist in the residual network of SS_i, if a negative cycle exists in the residual network of SS_{i+1}, then it must cross the SS_i border on at least two arcs (because nonzero arcs can appear only at the borders) and generates a negative path in SS_i from one arc to another. However, it is a contradiction, because all of the possible negative paths inside SS_i are removed at the end of stage *i* of the algorithm. Therefore, at the end of stage *i,* no negative cycles exist in the residual network. ■

Given Propositions 1 and 2 and by using mathematical induction, one can infer that at the end of the last stage of the algorithm, no negative cycles exist in the residual network of the entire evacuation area. Thus concludes the proof.

IMPLEMENTATION AND VERIFICATION

The solution algorithm in this paper was coded in C++ and applied to an example network, and the solution quality was verified for several scenarios. Because of the computational limitations of the proposed algorithm, the network chosen for this test is a midsized example network. For a realistic scenario with the same model characteristics as in this paper, see elsewhere (*28*). When implementing the solution algorithm, to generate a feasible solution to start each

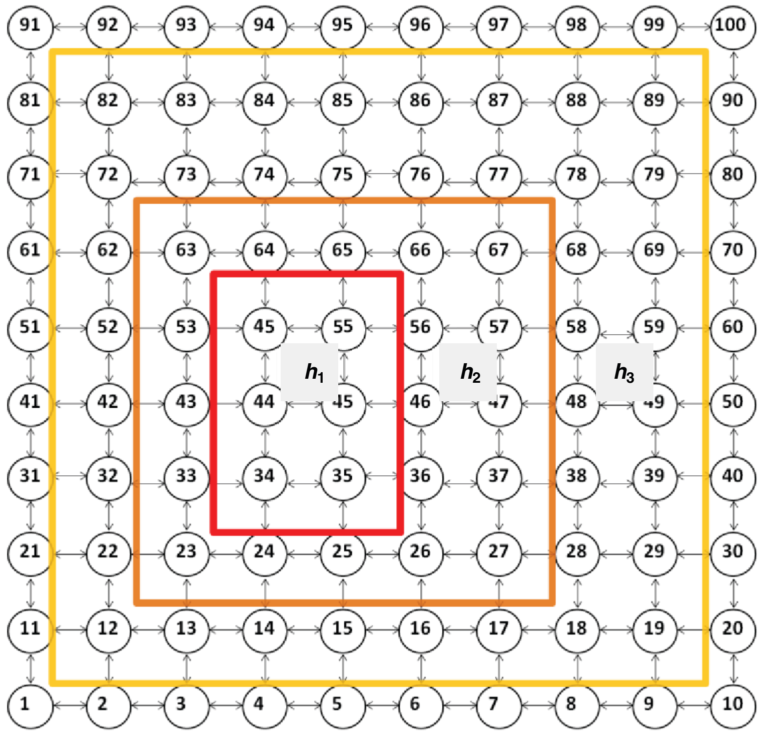

FIGURE 5 Test network with three threat zones.

stage, an all-or-nothing traffic assignment is performed on the roadway network for evacuation demand in the study subnetwork and inflow from the previous stage.

The test network is a grid with 100 nodes and 360 links. Three threat zones with threat levels h_1, h_2, and h_3 are defined for the test network, which have threat values equal to 3, 2, and 1, respectively (Figure 5). In the test network, the threat level outside Zone 3 (which represents the safe area) is set to zero.

All links in the network are two-way streets with two lanes in each direction and a free-flow travel time equal to 1 min. The storage capacity for the links is calculated on the basis of 30 ft of space occupied by each vehicle in the queue. The time resolution is 1 min, and the number of time intervals is set to 50. Ten scenarios were designed with randomly generated patterns for total demand, which ranged from 7,127 to 32,927 vehicles.

For verification purposes, the original MCF problem for each scenario was coded and solved with the CPLEX commercial optimization software package (IBM ILOG CPLEX Optimization) (*42*). For all of the generated scenarios, the optimality of solutions was confirmed: the objective values found by the proposed solution method were equal to those found by CPLEX when solving the original MCF, for all of the generated scenarios. The results for the 10 scenarios are listed in Table 1.

Results indicate that as evacuation demand increases from 7,127 to 32,927, the total number of MSP runs required to detect and cancel the negative cycles increases from 27 to 290. As a result, computation time also increases from 7 s to 282 s. From the reported total number of MSP runs and total number of canceled cycles (Table 1), the average number of cycles detected in one MSP run can be inferred as 5.1 to 6.5 cycles across different scenarios.

TABLE 1 Test Results for PQ and SQ Models

Scenario	Total Demand	Total Number of MSP Runs	Total Number of Canceled Cycles	Optimal Objective Value	Computational Time (s)	Optimal Objective Value of SQ Model
1	7,127	27	161	26,979	7	26,979
2	7,927	33	184	35,504	7	35,504
3	10,327	41	267	53,402	15	53,402
4	13,927	72	429	85,131	32	85,131
5	14,427	82	478	83,723	39	83,723
6	18,027	131	702	117,940	62	117,940
7	18,927	119	744	138,723	49	138,723
8	20,827	154	792	170,450	110	170,450
9	21,927	195	1,006	223,631	172	223,631
10	32,927	290	1,772	456,868	282	456,868

An intuitive interpretation of the results demonstrates the increase (degradation) of the optimal objective value as demand increases, which can be justified with two main reasons. First, when the network is more crowded, the average delay in the threat area increases, which increases the total exposure to the threat and consequently degrades the objective value. Second, when the number of evacuees is higher, total exposure increases, which is again equivalent to degradation of the objective value.

For all 10 scenarios in the test application, the problem formulation based on the PQ model had the same optimal objective value as that of the problem formulations based on the SQ model. This observation may not always hold true, because the feasible set in the SQ formulation is a subset of that for the PQ formulation. However, in this example with high demand levels, the explicit representation of physical queues on the links does not lead to an increase in total exposure. The representation provides greater confidence in the evacuation plan because it performs optimally with a more realistic traffic flow model.

CONCLUSIONS

The solution method proposed in this research finds the optimal routing of traffic to evacuate a network with several threat zones, where the threat level may depend on the exposure or risk in each zone. However, this method can be applied to all zone-based optimal routing problems in which the arc costs are proportional to arc travel times.

The CT proposed to exploit the special arc cost pattern in this study does not require any assumption about the shapes, locations, and amounts of threat associated with the threat zones. The solution algorithm is presented and tested in the context of concentric threat zones but easily can be extended to threat zones with arbitrary shapes and desired threat levels.

The proposed algorithm always finds the optimal solution and is implemented and tested for a midsized grid test network. The solution quality is verified to be optimal. The authors have demonstrated the use of this model under reasonable traffic flow dynamics by using constraints that reflect the PQ and SQ traffic flow models. The PQ and SQ models produce two similar optimal flow solutions with equal objective function values for all scenarios tested; however, this result is not necessarily true for arbitrary scenarios in general.

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