AGU PUBLICATIONS

Water Resources Research

RESEARCH ARTICLE

10.1002/2014WR016084

Key Points:
- Analytical solution of Boussinesq equation
- Understanding of recession of sloping aquifer
- Determination of aquifer property using Brutsaert's method

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Citation:
Hogarth, W. L., L. Li, D. A. Lockington, F. Stagnitti, M. B. Parlange, D. A. Barry, T. S. Steenhuis, and J.-Y. Parlange (2014), Analytical approximation for the recession of a sloping aquifer, *Water Resour. Res.*, *50*, 8564–8570, doi:10.1002/2014WR016084.

Received 1 JUL 2014
Accepted 13 OCT 2014
Accepted article online 16 OCT 2014
Published online 7 NOV 2014

Analytical approximation for the recession of a sloping aquifer

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Abstract An approximation is obtained for the recession of a sloping aquifer. The analytical approximation can provide a useful tool to analyze data and obtain physical properties of the aquifer. In contrast to the case of a horizontal aquifer, when plotting the time derivative of the flux versus the flux on a log scale, the result shows that the flux derivative reaches a minimum value and that the curve can have a slope of unity as often observed. Illustration of the application of the analytical results to the Mahantango Creek data is also discussed.

1. Introduction

Recession flow analysis of an aquifer is a powerful tool to estimate catchment hydraulic parameters. Starting with the seminal work of *Brutsaert and Nieber* [1977], the method has been used and refined for nearly 40 years. Conceptually based on the solution of the Boussinesq equation for horizontal aquifer, plotting $-dQ/dt$ vs. Q on a log scale gives straight lines of slope 3 for short times and 3/2 for long times. Here Q is the drainage flux and t is the time. The long time slope has been the subject of much research. *Brutsaert and Lopez* [1998] pointed out that a slope of 1 rather than 3/2 is frequently observed. Theoretically, such a slope is predicted after linearization of Boussinesq equation or using Laplace's equation [*van de Giesen et al.*, 2005]. Recently, *Bogaart et al.* [2013] noted that a zero slope, i.e., dQ/dt constant, could be obtained if the aquifer is not horizontal.

The present work derives an analytical expression for $Q(t)$ for a sloping aquifer making curve fitting data easier. Existing analytical solutions are based on kinematic approximations [*Beven*, 1981, 1982; *Harman and Sivapalan*, 2009] or the assumption that the aquifer is infinite vertically [*Daly and Porporato*, 2004]. Our result applies to the Boussinesq equation, and thus is not applicable for the case when catchment drainage is largely controlled by a river network [*Mutzner et al.*, 2013]. Thorough discussions of sloping aquifers, including recession based on the linearized Boussinesq equation as well as related models and experiments are provided by *Basha and Maalouf* [2005] and *Rupp and Selker* [2006a]. A recent general review of catchment hydrology can be found in *Troch et al.* [2013] and covers most aspects of that problem. An earlier review by *Tallaksen* [1995] on recession analysis covers many aspects of Brutsaert's method.

The advantage of having an analytical expression to curvefit field data is a practical one rather than a theoretical one. As soon as there is more than one parameter to be obtained using numerical solutions of the Boussinesq equation, for instance, it would require a huge number of calculations. As pointed out by *Bogaart et al.* [2013]: "Application of the method of *Brutsaert and Nieber* [1977], i.e., linking (two) recession parameters to aquifer properties, to sloping aquifers requires analytical solutions to (the Boussinesq) equation." This is our aim.

2. Formulation

Our approach is an extension of our earlier analytical approximation for a horizontal aquifer adding gravity corrections. For convenience, the notation used by *Stagnitti et al.* [2004] for a sloping aquifer is kept. *Bogaart et al.* [2013] point out that the formulation of *Harman and Sivapalan* [2009] has some advantages for a sloping aquifer but not for a horizontal aquifer, which is required in our derivation. Dimensionless variables are denoted without bars, as

$$x=\bar{x}/L \tag{1}$$

$$h=\bar{h}/D \tag{2}$$

$$t=\bar{t}\,Dk\cos i/fL^2 \tag{3}$$

where i is the slope of the aquifer, L is the length of the aquifer measured along bedrock, h is the watertable height measured perpendicular to bedrock, f is the drainable porosity, t is time, and x is distance measured along bedrock; the spatial domain extends from $x=0$ (aquifer exit) to $x=1$; D is a characteristic depth of the aquifer and k is the saturated conductivity. The Boussinesq model, given by *Brutsaert* [1994, equation (2)], is

$$\partial h/\partial t=\partial[h\partial h/\partial x]/\partial x+\varepsilon\,\partial h/\partial x \tag{4}$$

where

$$\varepsilon=L\tan i/D. \tag{5}$$

Discharge takes place at $x=0$ and the drainage flux is given by

$$Q=dI/dt=(h\partial h/\partial x+\varepsilon h)_{x=0} \tag{6}$$

where I is the total drainage from the aquifer at $x = 0$, i.e.,

$$I=1-\int_0^1 hdx. \tag{7}$$

Equation (7) assumes that the initial depth is constant equal to D, or $h=1$. This, of course, will rarely be possible in the field. However, the recession analysis of Brutsaert applies to "low flow," "base flow," and drought flow" [*Brutsaert and Nieber*, 1977]. Since the initial conditions will rarely be known in the field, the analysis implies that when base flow is obtained, the actual initial conditions have low impact. *Bogaart et al.* [2013] also use a steady state initial condition as well as the present one. It is interesting that their Figure 3 shows that, as the aquifer dries, the watertables for both initial conditions become rather similar. Thus, we take the initial condition

$$h=1;\ \ 0\le x\le 1\ \text{ at }\ t\ =\ 0. \tag{8}$$

We also take a sudden drawdown at $x=0$, or

$$h=0\ \text{ at }\ x=0\ \text{ for }\ t>0. \tag{9}$$

Equation (9) ensures that the solution captures the full recession curve. Finally, as long as the watertable reaches $x=1$, there is no flux there or

$$\partial h/\partial x=-\varepsilon\ \text{ at }\ x=1. \tag{10}$$

The numerical solution for equation (4) and conditions (8), (9), and (10) is obtained as in *Stagnitti et al.* [2004].

3. Analytical Approximation

The aim is to produce an approximation for I, with the same degree of accuracy as the approximation obtained for $\varepsilon=0$ [*Parlange et al.*, 2001]. We designate the zero-slope approximation $I(\varepsilon=0)$ by I_0. *Mendoza et al.* [2003] also give the analytical form for $Q_0=dI_0/dt$ and *Rupp and Selker* [2006b] give dQ_0/dt as well.

To extend I_0 for $\varepsilon\neq 0$, we have to estimate the interaction of the two physical processes, one due to the Darcy flux is often called "diffusion," and the other due to gravity, in equation (4). If they did not interact, we would have simply the result

$$I=I_0+\varepsilon t. \tag{11}$$

Interaction between the two processes will reduce the drainage as given in equation (11). We call t_D the drying time of the aquifer, which is infinite when $\varepsilon=0$. We then can replace I_0 in equation (11) by $I_0(1-t/t_D)$ so

Table 1. Values of t_D Based on Analytical and Numerical Results

	Numerical	Analytical
$t_D(\varepsilon=10)$	0.147	0.146
$t_D(\varepsilon=1)$	2.361	2.358
$t_D(\varepsilon=0.1)$	28.34	28.99
$t_D(\varepsilon=0.01)$	290.87	298.33

that at $t=t_D$, the diffusion term disappears. Making the correction of order t is also intuitive since gravity introduces terms of order t. By symmetry, it would also be tempting to replace εt by $\varepsilon t(1-\alpha I_0/I_{0D})$, where α is an unknown coefficient and $I_{0D}=I_0(t_D)$. However, as ε increases and t_D decreases, gravity dominates so that the correction $(1-\alpha I_0/I_{0D})$ must go to 1. This behavior is obtained by replacing εt by $\varepsilon t(1-\alpha I_0/I_{0D})^{t_D}$ in equation (11). Altogether, then, we try the simple approximation

$$I=I_0(1-t/t_D)+\varepsilon t(1-\alpha I_0/I_{0D})^{t_D}. \tag{12}$$

It is important to emphasize that equation (12) is an intuitive interpolation between the two limiting terms, I_0 and εt, entering equation (11). We tried other interpolations and settled on equation (12) because it is simple, as we want for practical applications, and quite accurate as we shall see later.

Instead of using α, it will be convenient later on to replace it with

$$\alpha=(a-1)/a; \quad a^{t_D}=\varepsilon t_D \tag{13}$$

and equation (12) becomes

$$I=I_0(1-t/t_D)+(t/t_D)[a+(1-a)I_0/I_{0D}]^{t_D}. \tag{14}$$

For a given ε, only one parameter, t_D, has to be obtained, with $a(t_D)$ given by equation (13). At $t=t_D$, we have $I=1$ and $Q=0$, the first condition is automatically satisfied by equation (14) and $Q=dI/dt=0$ gives

$$t_D(a-1)=(1-I_{0D})I_{0D}/(t_D dI_0/dt)_{t_D}. \tag{15}$$

If ε is large, t_D is small, then $I_0 \sim t^{1/2}$ and equation (15) gives

$$\varepsilon t_D=(1+2/t_D)^{t_D} \tag{16}$$

or, since t_D is small

$$\varepsilon t_D \simeq 1+t_D ln(1+2/t_D). \tag{17}$$

Hence, in the limit of $t_D \to 0$, equation (17) yields $\varepsilon t_D=1$ as expected.

For the other limit, when $\varepsilon \to 0$, $t_D \to \infty$, then $(1-I_0) \sim t^{-1}$ [*Parlange et al.*, 2001] and equation (15) gives $t_D(a-1) \simeq 1$ and from equation (13), $\varepsilon t_D=e$. It is interesting that equation (17), obtained for a small t_D, predicts $\varepsilon t_D=3$ for a large t_D, which is close to $\varepsilon t_D=e$. Further, when ε is small, Q remains small around t_D so that taking $\varepsilon t_D=3$ instead of e will still result in a very small Q. For simplicity then, we are going to keep equation (17) for all values of ε and to simplify the equation further, replace t_D in the RHS of the equation by $(1+e)/2\varepsilon$, which is the arithmetic average of e/ε (for low ε) and $1/\varepsilon$ (for large ε). This results in a very simple expression for t_D given by

$$\varepsilon t_D=1+[(1+e)/2\varepsilon]ln[1+4\varepsilon/(1+e)]. \tag{18}$$

Table 1 compares results for t_D obtained by the numerical method of *Stagnitti et al.* [2004] and from equation (18). Considering the

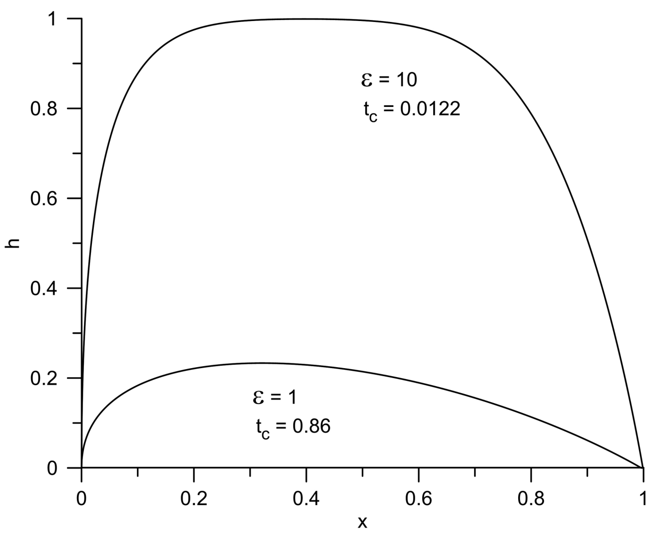

Figure 1. Numerical watertable level, h, at $t=t_c$ when the top of the aquifer starts drying; for $\varepsilon=10$ and $\varepsilon=1$.

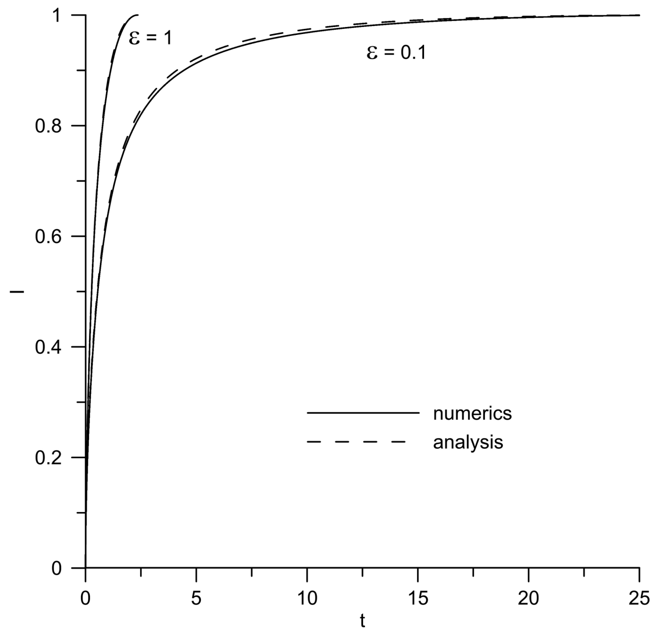

Figure 2. Analytical and numerical values of the cumulative drainage, I, as a function of time; for $\varepsilon=1$ and for $\varepsilon=0.1$.

simplifications used to obtain the equation, the results are surprisingly good.

4. Discussion

Brutsaert's method requires curve fitting predictions of Q and dQ/dt to the data. Using an analytical expression makes the curve fitting very easy. To obtain the dimensionless expressions of I, $Q=dI/dt$ and $dQ/dt=d^2I/dt^2$, only ε is required; t_D is then obtained from equation (18), and a from equation (13), with equation (14) yielding I.

We believe that the value of ε taken a priori cannot be too large for Brutsaert's approach to be reliable for reasons to be discussed below. Figure 1 gives the numerical watertables for $\varepsilon=10$ and 1, i.e., large and moderate values, at $t=t_c$, which is the first time when $h=0$ at $x=1$, whereas t_D is the time when $h=0$ for $0 \le x \le 1$.We did not plot watertables for lower values of ε, e.g., 0.1, as they look very similar to the case $\varepsilon=1$, but with smaller maxima for h. Note that *Stagnitti et al*. [2004] and *Bogaart et al*. [2013] used $\varepsilon=10$ and 20, respectively, in their examples, which are large values. Their results will prove useful in the following discussion.

As mentioned earlier, the method applies at low flow when the impact of initial conditions does not affect the shape of the watertable extensively. Thus, if we look at *Bogaart et al*. [2013, Figure 3], this happens when more than half of the aquifer is dry and the discharge is quite low (see their Figure 2a). At that stage, the recession flow becomes representative of a small portion of the aquifer only. *Stagnitti et al*. [2004] give similar watertable shapes for $\varepsilon=10$. On the other hand, if we look at Figure 1 here when $t=t_c$, i.e., when the aquifer is still nearly completely saturated, for $\varepsilon=10$, the watertable is near its initial position, i.e., hardly dry, whereas for $\varepsilon=1$, and a fortiori for lower values of ε, the aquifer has drained considerably. Thus, we assess that the approximation equation (18) can be applied with confidence with ε of order 1 or less. For example, for $\varepsilon=1$ and 0.1, Figure 2 compares the numerical and analytical (equations (13), (14), and (18)) values of I, with the good accuracy shown. As it is difficult to read the error

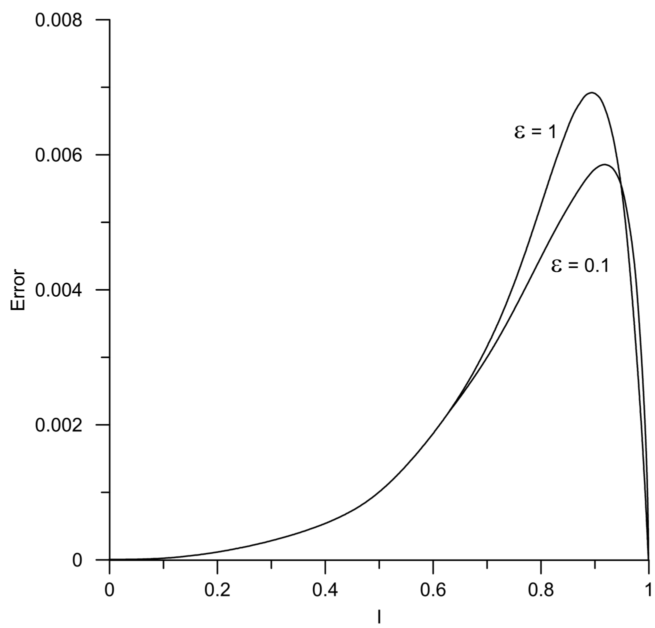

Figure 3. Error (difference ΔI between analytical and numerical I values for $\varepsilon=0.1$ and 1), plotted as a function of the numerical I.

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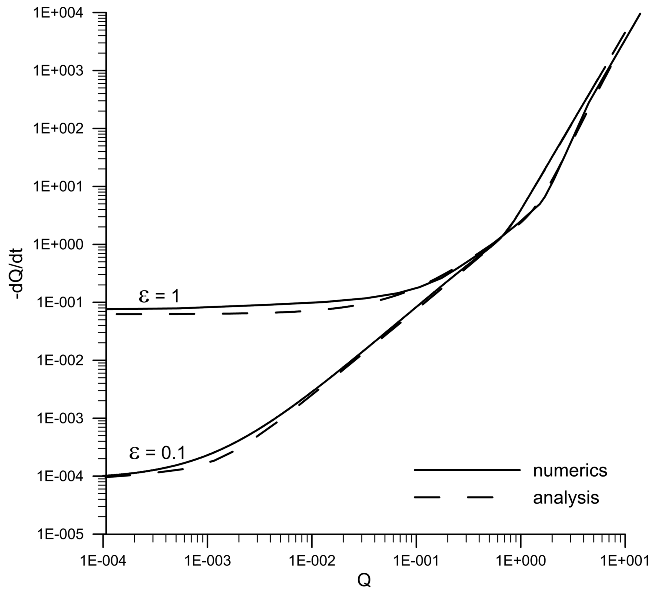

Figure 4. Analytical and numerical plots of $-dQ/dt$ versus Q on a log scale; for $\varepsilon=1$ and for $\varepsilon=0.1$.

of the analysis on Figure 2, Figure 3 gives the error ΔI of the approximation, i.e., the difference between the analytical and the numerical results given in Figure 2, as a function of the numerical I. We observe that the errors for $\varepsilon=0.1$ and 1 are almost the same (they are in between for $0.1 < \varepsilon < 1$), illustrating that between 0.1 and 1 the error is largely independent of ε. It might be tempting to try to correct empirically I, on the LHS of equation (14) to reduce the error even further. However, this complicates equation (14) and would remove its practical attraction. In addition, as will be illustrated later, the error of the present approximation is irrelevant for curvefitting field data.

However, a more important criterion is the comparison of $-dQ/dt$ vs. Q as a log scale as it forms the basis of Brutsaert's method. As shown in Figure 4, the analytical results remain very accurate. Figure 5 shows more details with ε going from 0.2 by increments of 0.4 for the analytical results. As usual, the early times give a slope of 3 and the change of slopes as Q decreases occurs fairly abruptly at a "corner." For the low value of 0.2, as in the case for $\varepsilon=0.1$ in the previous figure, the slope below the corner is equal to 1.5. This is, of course, what happens when $\varepsilon=0$, except that in that case, the slope does not change as Q decreases further. Here on the contrary, the slope decreases until it reaches a minimum value. This feature is present for positive ε and is often observed in the field. For the high value of $\varepsilon=1.4$, the curve below the corner is well represented by a line of slope 1 until it reduces further. For the intermediary values $\varepsilon=0.6$ and 1, the curve below the corner has a mixture of those two features, primarily a slope of 1.5 for $\varepsilon=0.6$ and 1 for $\varepsilon=1$. This feature might be important in practice as a slope of 1 below the corner has often been reported, e.g., see *Brutsaert and Lopez* [1998] and might be the result of the aquifer slope (see also the discussion by *Bogaart et al.* [2013]).

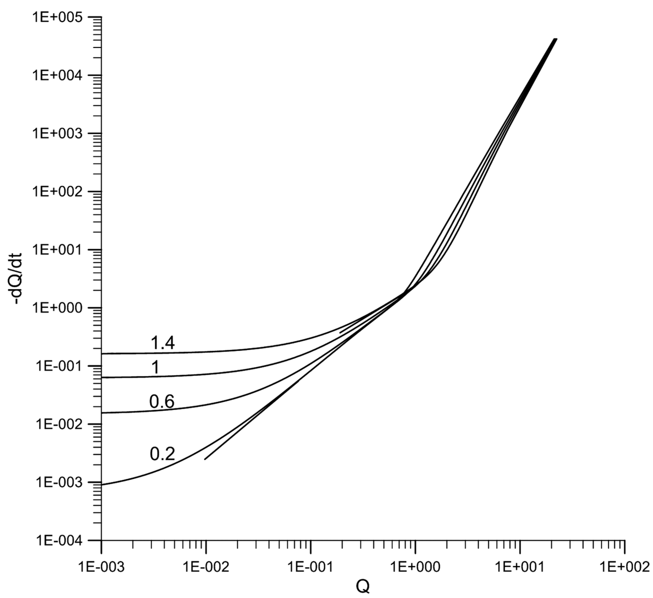

Figure 5. Analytical plots of $-dQ/dt$ vs. Q on a log scale for different values of ε (1.4; 1; 0.6; 0.2). Tangents of slope 1 and 3/2 are shown for $\varepsilon=1.4$ and 0.2, respectively.

5. Illustration

We consider the Mahantango Creek data obtained from the USGS National Water Information Systems. Data from this catchment have been widely used and will allow us to illustrate several features of the present approximation. Details of the catchment can be found in *Szilagyi and Parlange* [1998] and *Parlange et al.* [2001] and will not be repeated here. In *Parlange et al.* [2001], the

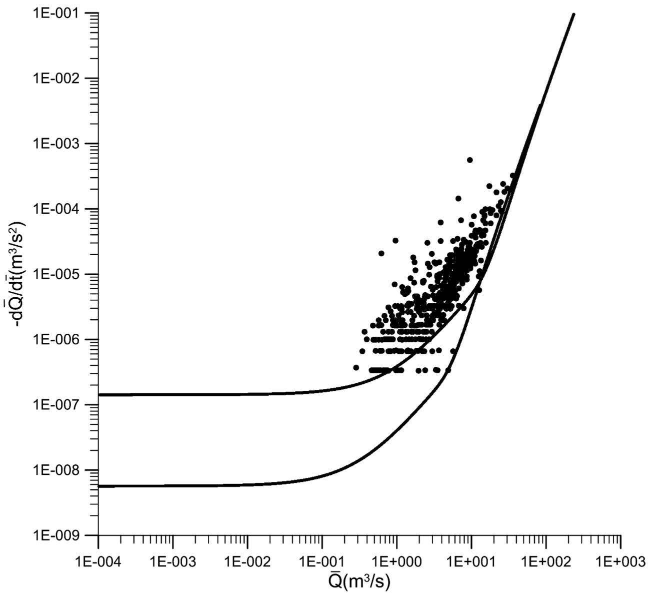

Figure 6. Data obtained from the Mahantango Creek and analytical curves obtained for $\varepsilon=1$. Those curves are translated to fit either the extreme right points (lower curve) or ignoring half the points for the two lowest values of $-d\overline{Q}/d\bar{t}$ (upper curve).

solution for $\varepsilon=0$ was fitted to the average of the observations. Here instead, we shall consider a lower envelope of the data, as suggested by *Brutsaert and Nieber* [1977], to minimize the impact of precipitation and thus initial conditions. Also, it is more convenient to use if curve fitting involves corners between different slopes [*Mendoza et al.*, 2003] as they become more apparent with a lower envelope [*Brutsaert and Lopez*, 1998]. Finally, *Shaw and Riha* [2012] showed that individual recession curves may not fit well the average observations, but those that start near the lower envelope remain near the boundary as expected (see their Figure 2). Data in Figure 5, $\overline{Q}$ and $d\overline{Q}/d\bar{t}$ have the dimensions indicated and

$$\overline{Q}=d\bar{I}/d\bar{t};\, d\overline{Q}/d\bar{t}=d^2\bar{I}/d\bar{t}^2;\;\; \bar{I}=IAfD, \tag{19}$$

where A is the drainage area. The dimensionless results from equation (14) are translated to make them dimensional in a log-log plot. The shifts in the horizontal (H) and vertical (V) directions are given by

$$H=k\cos iAD^2/L^2 \tag{20}$$

$$V=Ak^2\cos^2 iD^3f^{-1}/L^4. \tag{21}$$

In Figure 6, we fit two curves with $\varepsilon=1$ simply translating vertically and horizontally the curve given in Figure 4 or 5. One curve fits all the rightmost points, the other ignores half the points for the two lowest values of $-d\overline{Q}/d\bar{t}$. Those choices are arbitrary; for instance, those outer points could be dismissed as unreliable. However, our interest is not to assess the accuracy of measured data but rather to demonstrate the flexibility of the analytical approximation. We took $\varepsilon=1$ because it is about the largest value one can take realistically. If we had instead considered the envelope of the points further left in the continuous cloud of points, where a 3/2 slope is observed and a lower ε would be required since, with $\varepsilon=1$, the slope 3/2 is practically eliminated. In fact, even $\varepsilon=0$ could be used, as was done in *Parlange et al.* [2001, Figure 5]. Of course, if we assume that the observed lowest values of $-d\overline{Q}/d\bar{t}$ have physical meaning, then the upper curve is preferable and the curve with $\varepsilon=0$ (not shown here) is a poor representation of the data. Using an average curve with $\varepsilon=0$, *Parlange et al.* [2001] chose translations $H=1/0.036$ and $V=1/22000$. With $\varepsilon=1$, the two envelopes of Figure 6 give $H=1/0.036\times2.6$, $V=1/22000\times20$ for the upper curve and $H=1/0.036\times7.3$, $V=1/22000\times500$ for the lower curve, which differ rather significantly from the values of *Parlange et al.* [2001].

6. Conclusion

Using the Boussinesq equation and extending the approximation obtained for a horizontal aquifer, we obtained a very accurate approximation for a sloping aquifer. The introduction of a slope ε adds great flexibility to interpret the relationship between $-d\overline{Q}/d\bar{t}$ and $\overline{Q}$ when a log scale is used. When the aquifer is horizontal, $-d\overline{Q}/d\bar{t}$ is largely proportional to $\overline{Q}^b$, with $b=3$ for short times and 3/2 for long time. With a non-zero slope, b goes to 0 for long times and between 0 and 3, an additional value of 1 is possible. This value of 1 is important as it is often observed in the field. The lower the aquifer slope, the less the value of 1 is observed and the higher the slope, the more the value of 1 is observed. As the recession analysis of Brutsaert is for low flow to minimize the impact of initial water distribution, the solution is most likely restricted to ε not much above 1. If ε is too large, much of the aquifer is totally dry by the time the method is usable so that the data are not representative of the whole aquifer. Another important feature of the solution of a sloping aquifer is that $-d\overline{Q}/d\overline{t}$ reaches a minimum after a long time and the greater the slope, the higher the minimum is. The existence of a minimum is also observed in the field, although, of course, one has to be cautious in giving too much importance to this feature as it could also be partly caused by the difficulty in obtaining data when the aquifer is very dry.

Acknowledgments
The authors are grateful to Olivier Rey-Lescure for his continuing technical help in setting up the figures. The Mahantango Creek data are obtained from the USGS National Water Information Systems: http://waterdata.usgs.gov/nwis/nwisman/?site_no=01555500.

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