A technique for calibration a triple hot-wire probe (DANTEC type 55P91)

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Hot Wire Anemometry has been used as an appropriate technique in the flow measurement process, and particularly in different turbulent flow studies. This technique is, however, rarely used in situations where occasional damage to the probe is to be expected. The basic principle in Hot Wire Anemometry is the convective heat transfer from a heated wire or film element placed in a fluid flow. The changing of any parameters of the flow such as velocity or temperature causes the heat transfer from the heated element to be changed. This changing will be detected by a constant-temperature Hot Wire Anemometry system. In measurements of the velocity component in the mean flow direction, a single hot-wire probe can be used. It consists of a short length of a fine diameter wire attached to two prongs. Wollaston wires with a platinum rod covered by a thick outer diameter are usually used in hot-wire probes. First, the Wollaston wire is soldered or welded to the prongs. Second, the small and fragile platinum wire can be exposed by etching the silver sheath from the wire.

1 Triple hot-wire probes

A triple hot-wire probe is a probe with three wire elements which provides complete information about instantaneous velocity vector at a point. A number of triple hot-wire probes with different configurations have been developed and calibration techniques have been presented. The procedure of calibration and data-reduction would be much simpler for probes with orthogonal sensors. The DANTEC 55P91 triple-wire probe is a probe with orthogonal sensors. It uses gold-plated tungsten wires which are 3.2 mm long with a 1.2 mm sensitive length. The prongs, which are in a claw configuration, are attached perpendicularly to the wire elements to minimise the aerodynamic interference effect (Figure 1).

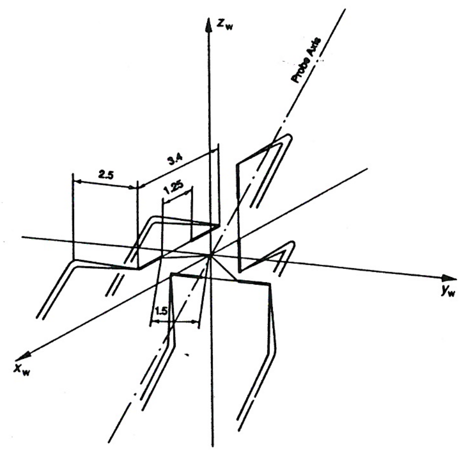

Figure 1: The wire and prong configuration (Bruun, 1995)

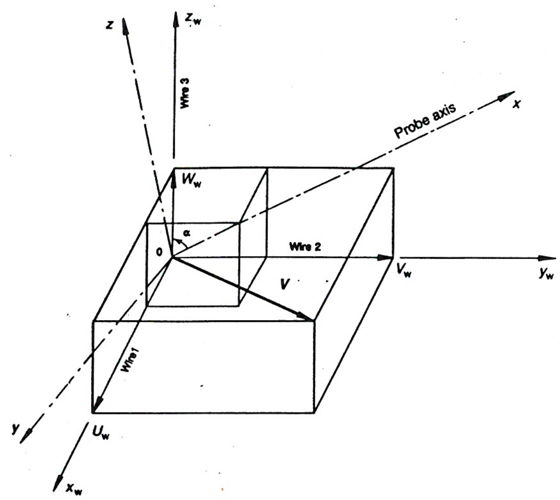

Figure 2: The wire-fixed coordinate system (Bruun, 1995)

The three wires are nominally orthogonal and are inclined at a yaw angle of 54.7° relative to the probe axis. The signal from the probe can be expressed in terms of effective cooling velocity (Equation 1). The three wires form an orthogonal coordinate system, so that the tangential component for one wire is normal for another wire and binormal for the third. The effective cooling speed is a function of velocity components in the wire-fixed coordinate system. Bruun (1995) presented a technique to extract the velocity components in a known coordinate system. In the following a summary of his analysis will be presented. In order to simplify the analysis and data reduction procedure, the same value for sensitivity coefficients (k and h) can be assumed .

$$\left[\frac{E_1^{\,2} - A_1}{B_1}\right]^{\frac{2}{n_1}} = V_{e1}^2 = k^2 U_w^2 + V_w^2 + h^2 W_w^2$$

$$\left[\frac{E_2^{\,2} - A_2}{B_2}\right]^{\frac{2}{n_2}} = V_{e2}^2 = h^2 U_w^2 + k^2 V_w^2 + W_w^2 \quad \Rightarrow \quad \begin{bmatrix} V_{e1}^2 \\ V_{e2}^2 \\ V_{e3}^2 \end{bmatrix} = \begin{bmatrix} k^2 & 1 & h^2 \\ h^2 & k^2 & 1 \\ 1 & h^2 & k^2 \end{bmatrix} \begin{bmatrix} U_w^2 \\ V_w^2 \\ W_w^2 \end{bmatrix} \qquad (1)$$

$$\left[\frac{E_3^{\,2} - A_3}{B_3}\right]^{\frac{2}{n_3}} = V_{e3}^2 = U_w^2 + h^2 V_w^2 + k^2 W_w^2$$

Where E is the output voltage, V_e is the effective cooling velocity, U_w, V_w, W_w are the instantaneous velocity components in the wire-fixed coordinates and k and h are the sensitivity coefficients. The unknown U_w, V_w, W_w can be obtained by a matrix-inversion method.

$$\begin{bmatrix} V_{e1}^2 \\ V_{e2}^2 \\ V_{e3}^2 \end{bmatrix} = D \begin{bmatrix} U_w^2 \\ V_w^2 \\ W_w^2 \end{bmatrix} \Rightarrow \begin{bmatrix} U_w^2 \\ V_w^2 \\ W_w^2 \end{bmatrix} = D^{-1} \begin{bmatrix} V_{e1}^2 \\ V_{e2}^2 \\ V_{e3}^2 \end{bmatrix} \qquad (2)$$

$$D^{-1} = \frac{1}{\Delta} \begin{bmatrix} k^4 - h^2 & h^4 - k^2 & 1 - k^2h^2 \\ 1 - k^2h^2 & k^4 - h^2 & h^4 - k^2 \\ h^4 - k^2 & 1 - k^2h^2 & k^4 - h^2 \end{bmatrix} \qquad (3)$$

where, Δ is the determinate of the matrix D given by

$$\Delta = k^6 + h^6 - 3k^2h^2 + 1 \qquad (4)$$

When all three components of velocity in the wire-fixed coordinate system (see Figure 2) have positive values, a unique solution exists for the above system of three equations. The velocity components, evaluated in the wire-fixed coordinate system (U_w, V_w, W_w), can be transformed to any other orthogonal components. If the x-axis of the new coordinate system is aligned with the probe stem and wire 3 is lying in the (x , z) plane, the transformation relationship for the velocity components U, V and W are :

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = N \begin{bmatrix} U_w \\ V_w \\ W_w \end{bmatrix} \qquad (5)$$

where,

$$N = \begin{bmatrix} \cos 45 \cos 35.3 & \cos 45 \cos 35.3 & \cos 54.7 \\ -\cos 45 & \cos 45 & 0 \\ -\cos 45 \sin 35.3 & -\cos 45 \sin 35.3 & \cos 35.3 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} \end{bmatrix} \qquad (6)$$

In some applications, it may be necessary to position the triple wire probe with a pitch angle, β, in for example the (x , y) plane and rotate it by a roll angle, γ, around its stem. In this case, an additional transformation is required.

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = C\,N \begin{bmatrix} U_w \\ V_w \\ W_w \end{bmatrix} \tag{7}$$

where,

$$C = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix} \tag{8}$$

Bruun (1995) indicates that the above analysis assumes that the three wires are mutually orthogonal and that yaw and pitch coefficients k and h are constants which are independent of the direction of the velocity vector. However, in many investigations it has been found that yaw coefficient is a function of yaw angle but pitch factor is almost constant with variation of pitch angle. Andreopoulos (1983), however, found that pitch factor, h, was also a function of pitch angle. The more common value for h of 1.02, has been used by some researchers such as Jorgensen (1971a) and Chew and Ha (1988).

2 Calibration

The probe should be calibrated before utilising in the measurements. A fast calibration procedure is often needed, so that damage to the wires would not result in lengthy delay of the test program. The calibration procedure can be carried out by placing the probe in a jet of known velocity and known direction. A special arrangement is required such that a range of yaw and pitch angles and speeds could be performed. A number of researchers have calibrated triple hot-wire probes, using different techniques. Jorgensen (1971b) determined the magnitude and dependency of yaw and pitch coefficients on yaw and pitch angles for three different models of single wires. In Jorgensen's method, the effective cooling velocity acting on the sensor was expressed as it was mentioned in

Section 1. The coefficients in the voltage-velocity relationship were obtained by calibrating the wire at zero yaw and pitch angles. By rotating the probe, the anemometer output voltage was read as a function of the angle of inclination. Thus, the variation in the effective cooling velocity was obtained. The results showed that yaw factor decreases with increasing values of yaw angle. However, the assumption of a constant value for yaw factor yields a small error, less than 5%, at angles lower than 45°. The errors from 4% to 10 % was achieved for velocity, when the value of yaw factor was selected for yaw angle of 90°, inside the full range from 0° to 90°. Pitch factor was found nearly constant but tended to increase with increasing values of pitch angle. The errors from 5% to 12 % for the velocity would be predicted, if pitch corrections were neglected. However, using the pitch factor selected for pitch angle of 90°, errors on velocity reduced to between 1% and 2 %. Champagne et al. (1967) showed yaw and pitch factors depend basically on the length-to-diameter ratio, the pitch and yaw angles and the magnitude of velocity. Bruun and Tropea (1985) indicated that the sensitivity coefficients decrease with increasing the velocity magnitude.

Huffmann (1980) performed a precise calibration for the DISA 55P91 triple-wire probe. The automated calibration procedure had two data sets. First, at variable velocity and a fixed angle and second, at fixed velocity and variable angles. Calibration coefficients consisted of the three coefficients of Nusselt number-Reynolds number relationship and the two angularity coefficients. Indeed, if the anemometer is calibrated at ambient conditions but used at a much higher pressure in an environment, where the temperature is usually rising continually, the full Nusselt number-Reynolds number relationship must be used rather than a voltage-velocity correlation. Huffmann (1980) applied the Nusselt number-Reynolds number relationship in the calibration process. The optimisation process was followed until a good accuracy was obtained for the group of coefficients. Zank (1980) performed a running calibration procedure for hot-film anemometer. The calibration was carried out in two steps. The absolute value of the wind speed, measured by the 3D-probe, was first corrected and then, the coordinate system was rotated in order to obtain the corrected direction of the wind velocity vector. Mobarak et al. (1986) investigated the effect of Mach number, yaw and pitch angles on sensitivity factors. The same method, as used by Jorgensen (1971b), was developed to calculate the sensitivity coefficients at different orientations of the probe. Pitch factor was found to increase linearly with pitch angle and to decrease with Mach number. Yaw factor was found to decrease with increasing values of yaw angle and increasing with Mach number. Bergstrom and Hogstrom (1987) showed that yaw factor was a function of yaw angle and pitch factor was approximately constant.

Lakshminarayana and Davino (1988) investigated the sensitivity of a three sensor hot-wire probe to yaw and pitch angle variation. In their study, each sensor of the probe was individually calibrated with the calibration jet normal to the sensor. The total incident velocity sensed by the probe at various positions of yaw and pitch angle was derived using these calibrations. A maximum error of 12 % was found over the yaw angle range of ±40°. This error was less than 4 % at values of yaw and pitch angle below 10°. The sensitivity coefficients were also determined at different yaw and pitch angles. The yaw factor was in the range between 0.8 and 1.3 and the pitch factor was equal to 2.0. Andreopoulos (1983) discussed the problem of nonorthogonality and the directional sensitivity coefficients with the pitch and yaw angles. The DISA type 55P91 triple-wire probe was utilised. It was discovered that the values of yaw and pitch coefficients varied with the yaw and pitch angles respectively. However, it seems impossible to use values of sensitivity coefficients which depend on the unknown velocity to obtain this velocity. Jorgensen suggested to use only one value of sensitivity coefficients, namely the mean value over the expected range of angle variation to evaluate directly the required velocity components. Andreopoulos (1983) found a maximum error of 1.4% on all turbulence quantities when the functional dependence of sensitivity coefficients on the yaw and pitch angles was not considered. However, in flows with relatively high turbulence levels, it was recommended to include the functional dependence, otherwise errors up to 12 % on some normal or shear stresses could be expected. To calculate the sensitivity coefficients at different positions of yaw and pitch angles, a digital technique was used in his study. First, initial values were guessed for yaw and pitch angles and corresponding sensitivity coefficients were evaluated by using the calibration graphs. The velocity components were calculated and new angles were obtained by using these components. By employing this technique, the required computational time was increased by 25% in comparison to the case where the dependency of sensitivity coefficients on yaw and pitch angles was not taken into account.

A three-step procedure was developed by Gieseke and Guezennec (1993) to calibrate a triple hot-wire probe and to extract the velocity. It involved experimental determination of an accurate cooling law for each wire, using these models to develop a set of tables relating anemometer output to flow velocity input, and a table look-up procedure to extract the velocities. The calibration positioning mechanism consisted a rod with one end fixed in space and the other sliding inside a ball joint attached to a traversing mechanism controlled by a computer. This allowed the rod to be oriented at any angular position with respect to the free-stream velocity. The calibration was carried out in two stages; the velocity magnitude sensitivity calibration, and the flow direction calibration. Thus, for one specific orientation of the flow, the anemometer output voltages were recorded at different flow velocities. A curve with cubic polynomial function was fitted to the data using the least square method. A functional dependence was obtained between the anemometer output voltage and the magnitude of effective cooling velocity. In the next stage, the anemometer output voltages were recorded by varying the velocity magnitude and the probe orientation with respect the flow. Thus, at each calibration point, three components of velocity were obtained. The resulting sets of three components of velocity and the output voltages were recorded for processing the data. To extract the velocity, each wire output voltage is an estimate for the effective cooling velocity. This value was corrected using the angular calibration to give the best estimate for the cooling velocity and to extract three components of velocity in the required coordinates.

Gaulier (1977) used a triple hot-wire probe to determine the velocity range in the air stream emerging from domestic fuel burners. Each wire of the probe was calibrated separately by placing the wire perpendicular to an air stream whose velocity was known from a Pitot-tube measurements. The sensitivity coefficients in the effective cooling velocity equations, extracted from Jorgensen, were 0.15 and 1.02 for yaw and pitch factors respectively. Chew and Simpson (1988) also used the sensitivity coefficients extracted from Jorgensen. They mentioned, it was quite sufficient to assume the values of 0.15 and 1.02 for yaw and pitch coefficients respectively, especially for the DISA type 55P91 triple sensors hot-wire probe with gold-plated sensors. Some other researchers such as Lakshminarayana and Poncet (1974) and Frota and Moffat (1983) also assumed constant values for sensitivity coefficients. However, if a good accuracy is required, a precise calibration procedure will be required which includes the functional dependence of sensitivity coefficients to yaw and pitch angles. In the following a calibration procedure for the DISA type 55P91 in the lines of work by Skinner and Rae (1984) will be presented.

3 Calibration procedure

The DISA type 55P91 triple hot-wire probe has been considered and the calibration procedure is presented in individual divisions.

1- The jet is set to a known velocity of Q.

2- The calibration rig is set to give some yaw angle (α) and pitch angle (β).

3- Thus, the velocity components in probe coordinates are given by

$$\begin{aligned} U &= Q\cos\alpha\,\cos\beta \\ V &= Q\sin\alpha \\ W &= Q\cos\alpha\,\sin\beta \end{aligned} \tag{9}$$

4- Now, the velocity components in the wire-fixed coordinate can be calculated.

$$\begin{bmatrix} U_w \\ V_w \\ W_w \end{bmatrix} = N^{-1} \begin{bmatrix} U \\ V \\ W \end{bmatrix} \tag{10}$$

5- The effective cooling velocity components can now be determined.

$$\begin{bmatrix} V_{e1}^2 \\ V_{e2}^2 \\ V_{e3}^2 \end{bmatrix} = D \begin{bmatrix} U_w^2 \\ V_w^2 \\ W_w^2 \end{bmatrix} \tag{11}$$

6- An initial guess for matrix D is necessary since the elements of this matrix are unknown. As the first guess we assume the value of 0 for k and the value of 1 for h.

$$D = D_0 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad (12)$$

7- Thus, the effective cooling velocity components are evaluated at the corresponding output voltages.

$$\begin{bmatrix} V_{e1} \\ V_{e2} \\ V_{e3} \end{bmatrix} \Leftrightarrow \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \qquad (13)$$

8- A range of speeds at constant yaw and pitch angles can be performed. Using King's law for a series of speeds, the coefficients in the voltage-velocity relationship can be determined for each wire. The least square method can be used to fit the best curve and to evaluate the coefficients.

$$\begin{aligned} E_1^2 &= A_1 + B_1 \sqrt{V_{e1}} + C_1 V_{e1} \\ E_2^2 &= A_2 + B_2 \sqrt{V_{e2}} + C_2 V_{e2} \\ E_3^2 &= A_3 + B_3 \sqrt{V_{e3}} + C_3 V_{e3} \end{aligned} \qquad (14)$$

9- Thus, first estimate of the hot wire constants have been obtained. Now, these values can be used to obtain a better value for the D matrix.

10- At a certain point, output voltages can be measured and the above equations can be solved so that the effective cooling velocity for each wire can be determined.

$$\begin{bmatrix} V_{e1} \\ V_{e2} \\ V_{e3} \end{bmatrix} \qquad (15)$$

11- The velocity components in the wire-fixed coordinate system are also known from Equation. 10.

$$\begin{bmatrix} U_w \\ V_w \\ W_w \end{bmatrix} \tag{16}$$

12- Matrix D can then be calculated.

$$\begin{bmatrix} V_{e1}^2 \\ V_{e2}^2 \\ V_{e3}^2 \end{bmatrix} = D \begin{bmatrix} U_w^2 \\ V_w^2 \\ W_w^2 \end{bmatrix} \tag{17}$$

13- Using this new matrix D and performing a range of speeds, steps 1 to 12 will be repeated until the minimum error is obtained. If the matrix D with its elements is defined as follows, the calculated errors will be:

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \qquad \begin{bmatrix} Error_1 = (d_{11} U_w^2 + d_{12} V_w^2 + d_{13} W_w^2) - V_{e1}^2 \\ Error_2 = (d_{21} U_w^2 + d_{22} V_w^2 + d_{23} W_w^2) - V_{e2}^2 \\ Error_3 = (d_{31} U_w^2 + d_{32} V_w^2 + d_{33} W_w^2) - V_{e3}^2 \end{bmatrix} \tag{18}$$

Thus, the calibration procedure can be carried out. First, an arrangement which performs a range of yaw and pitch angles and speeds should be designed and manufactured. Second the above procedure should be followed so that the voltage-velocity relationship coefficients and angular coefficients can be determined.

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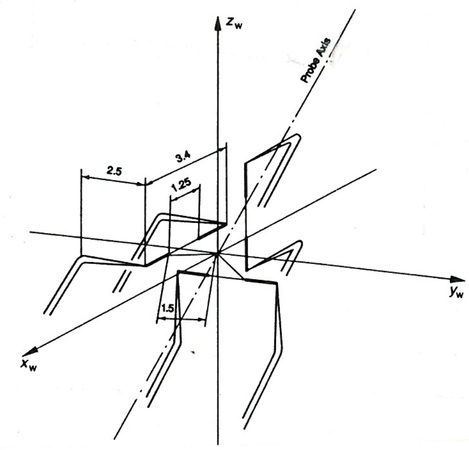

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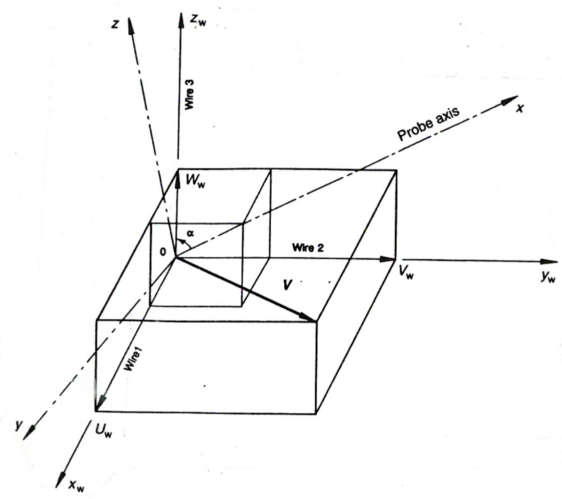

Figure 2: The wire-fixed coordinate system (Bruun, 1995)

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$$\left[\frac{E_2^{\,2}-A_2}{B_2}\right]^{\frac{2}{n_2}} = V_{e2}^2 = h^2U_w^2 + k^2V_w^2 + W_w^2 \Rightarrow \begin{bmatrix} V_{e1}^2 \\ V_{e2}^2 \\ V_{e3}^2 \end{bmatrix} = \begin{bmatrix} k^2 & 1 & h^2 \\ h^2 & k^2 & 1 \\ 1 & h^2 & k^2 \end{bmatrix} \begin{bmatrix} U_w^2 \\ V_w^2 \\ W_w^2 \end{bmatrix} \qquad (1)$$

$$\left[\frac{E_3^{\,2}-A_3}{B_3}\right]^{\frac{2}{n_3}} = V_{e3}^2 = U_w^2 + h^2V_w^2 + k^2W_w^2$$