Foundations of Physics 42 (2012): 512-523.

DOI: 10.1007/s10701-011-9622-8

The final publication is available at Springer via http://dx.doi.org/10.1007/s10701-011-9622-8.

Reichenbachian Common Cause Systems Revisited

Claudio Mazzola†

Abstract

According to Reichenbach's principle of common cause, positive statistical correlations for which no straightforward causal explanation is available should be explained by invoking the action of a hidden conjunctive common cause. Hofer-Szabó and Rédei's notion of a Reichenbachian common cause system is meant to generalize Reichenbach's conjunctive model to fit those cases in which two or more common causes cooperate in order to produce a positive statistical correlation. Such a generalization is proved to be unsatisfactory in the light of a probabilistic conception of causation. Accordingly, an alternative model for systems of multiple common causes is offered, which is capable of emulating the explanatory efficacy of Reichenbachian common cause systems, while overcoming their major conceptual shortcomings at the same time.

1 Introduction

Reichenbach's principle of common cause is usually understood as a tool for extracting causal knowledge out of statistical data. Its probabilistic formulation [4] claims that positive statistical correlations between two apparently non-causally connected events A and B are due to the action of a hidden common cause C which locally increases their joint probability. Incidentally, Reichenbach claimed that

> if there is more than one possible kind of common cause, C may represent the disjunction of these causes [4, p. 159],

but he never provided a detailed probabilistic model in order to extend his principle to those cases in which two or more common causes cooperate in producing a positive statistical correlation. Hofer-Szabó and Rédei have recently attempted to overcome this lacuna by coining the notion of a Reichenbachian common cause system [1, 2].

The aim of the present paper is twofold. On the one hand, Hofer-Szabó and Rédei's notion of a Reichenbachian common cause system is proved to be unsatisfactory in the light of a probabilistic conception of causation, for it lacks some of the main features displayed by Reichenbach's model for a common cause. On the other hand an alternative definition of a Reichenbachian common system is proposed, which is proved to equal the explanatory efficacy of Hofer-Szabó and Rédei's model while overcoming its major conceptual shortcomings. Finally, systems of any finite number of common

†Centre for Time, Main Quad A14, University of Sydney, NSW 2006, Australia

causes, so reshaped, are proved to exist for any positively correlated pair in a suitable extension of the given classical probability space.

2 Reichenbach's Principle of the Common Cause

Let (Ω, p) be a classical probability space with σ-algebra of random events Ω and probability measure p. For any pair of events $A, B \in \Omega$, the correlation of A and B is the quantity

$$Corr(A, B) \stackrel{def}{=} p(A \wedge B) - p(A)p(B). \tag{2.1}$$

Let us notice that, given the theorem of total probability, any positive correlation between events A and B amounts to a positive correlation between their complements, as well as to a negative correlation between A and $\neg B$ and to a negative correlation between $\neg A$ and B. For this reason, our discussion will focus on positive correlations alone, all other cases being reducible to the ones considered.

Statistical correlations play a crucial role in the probabilistic account of causation, according to which causal connections lead (roughly) to statistical correlations. In other words, if A is a positive cause of B or B is a positive cause of A, then (under suitable circumstances) a positive correlation between A and B must obtain – and symmetrically, if A is a negative cause (or pre-empter) of B or B is a negative cause (or pre-empter of A), then (under suitable circumstances) a negative correlation between A and B must obtain.

The converse, however, may not hold: positive statistical correlations of the form

$$p(A \wedge B) - p(A)p(B) > 0 \tag{2.2}$$

may be satisfied by some A, B, even though neither A is a positive cause of B, nor B is a positive cause of A. Statistical correlations such as (2.2) which denote no direct causal connection can nonetheless be given a causal explanation thanks to Reichenbach's *principle of common cause*, according to which:

> if coincidences of two events A and B occur more frequently than would correspond to their independent occurrence, that is, if the events satisfy relation (2.2), then there exists a common cause C for these events such that the fork ACB is conjunctive [4, p. 163].

What Reichenbach means by a *conjunctive fork* is made clear by the following definition:

Definition 1. *Three arbitrary distinct events $A, B, C \in \Omega$ form a conjunctive fork ACB if and only if the following set of conditions is satisfied:*

$$p\,(A \wedge B\,|C\,) - p\,(A\,|C\,)\,p\,(B\,|C\,) = 0 \tag{2.3}$$

$$p\,(A \wedge B\,|\neg C\,) - p\,(A\,|\neg C\,)\,p\,(B\,|\neg C\,) = 0 \tag{2.4}$$

$$p\,(A\,|C\,) - p\,(A\,|\neg C\,) > 0 \tag{2.5}$$

$$p\,(B\,|C\,) - p\,(B\,|\neg C\,) > 0, \tag{2.6}$$

provided that $p(C) \neq 0$ *and* $p(\neg C) \neq 0$.

Conditions (2.5)-(2.6) characterize C as a positive cause of both A and B and $\neg C$ as a negative cause of both; conditions (2.3)-(2.4) state that, once the existence (or non-existence) of C is taken into account, A and B turn out to be independent of each other.

It is easy to prove that conjunctive forks of the form ACB always give rise to positive correlations of type (2.2).

Lemma 1. *Let* $A, B \in \Omega$ *be two arbitrary events and let* $\{C_i\}_{i \in I}$ *be a partition of* Ω *such that*

$$p(C_i) \neq 0 \qquad \text{for all } i \in I; \tag{2.7}$$

$$p(A \wedge B \,|C_i\,) = p(A \,|C_i\,)\, p(B \,|C_i\,) \qquad \text{for all } i \in I. \tag{2.8}$$

Then the following equality holds [1, p. 1820]:

$$Corr(A, B) = \frac{1}{2} \sum_{i \neq j} p(C_i) p(C_j) [p(A \,|C_i\,) - p(A \,|C_j\,)][p(B \,|C_i\,) - p(B \,|C_j\,)]. \tag{2.9}$$

Proposition 1. *For any* $A, B, C \in \Omega$, *if* A,B *and* C *form a conjunctive fork* ACB, *then* $Corr(A, B) > 0$.

Proof. Let $A, B, C \in \Omega$ form a conjunctive fork ACB. Then, according to Lemma 1 (whose applicability is guaranteed by conditions (2.3)-(2.4)):

$$Corr(A, B) = p(C)p(\neg C)[p(A \,|C\,) - p(A \,|\neg C\,)][p(B \,|C\,) - p(B \,|\neg C\,)].$$

Conditions (2.5) and (2.6) force the term on the right-hand side to be strictly greater than zero, and so $Corr(A, B) > 0$. □

The explanatory power of a conjunctive fork lies precisely in its capability of producing a positive correlation out of a statistical independence [4, pp. 159-160], this way establishing a one-to-one correspondence between (positive) statistical correlations and (positive) causal dependence at the finer-grained level where common causes (or their complements) are taken into account.

3 Reichenbachian Common Cause Systems

Classical probability spaces may be too small to include a conjunctive fork ACB for any positive correlation $Corr(A, B)$ for which no straightforward causal interpretation is available. One may overcome this difficulty either by extending the given probability space [3] or by generalizing the very notion of a conjunctive fork in such a way to put $Corr(A, B)$ down to a set of multiple common causes, rather than a single common cause (and its complement). Hofer-Szabó and Rédei [1, 2] adopt the latter strategy, coining the notion of a *Reichenbachian common cause system*.

Definition 2. *For any $A, B \in \Omega$ satisfying (2.2) a Reichenbachian common cause system for the pair A, B is a partition $\{C_i\}_{i \in I}$ of Ω such that:*

$$p(A \wedge B | C_i) = p(A | C_i)\, p(B | C_i) \text{ for all } i \tag{3.1}$$

and

$$[p(A | C_i) - p(A | C_j)][p(B | C_i) - p(B | C_j)] > 0 \text{ for all } i \neq j, \tag{3.2}$$

provided $p(C_i) \neq 0$ and $p(C_j) \neq 0$ for all $i, j \in I$.

Like Reichenbach's conjunctive forks, Reichenbachian common cause systems for the pair A, B may be similarly proved to give rise to a positive correlation between A and B out of a statistical independence.

Proposition 2. *Let the partition $\{C_i\}_{i \in I}$ of Ω be a Reichenbachian common cause system for the pair A, B. Then $Corr(A, B) > 0$.*

Hofer-Szabó and Rédei introduced the notion of a Reichenbachian common cause system as 'a natural generalization of Reichenbach's original definition of common cause to the case when more than a single factor contributes to a correlation [1, p. 1822]'. Proposition 2 contributes in supporting this claim, providing Reichenbachian common cause systems with as much explanatory power as Reichenbach's conjunctive model.

Nonetheless, to be proper generalization of conjunctive forks, Reichenbachian common cause systems should presumably display all of their relevant causal properties. Unfortunately this is not so: in fact, on the one hand they might fail to be systems of sole common causes, while on the other hand they might happen to exclude arbitrarily some causally relevant factors.

Definition 2 demands that, for any distinct $i, j \in I$ such that C_i and C_j are members of a Reichenbachian common cause system for the pair A, B, the following inequality holds:

$$[p(A | C_i) - p(A | C_j)][p(B | C_i) - p(B | C_j)] > 0. \tag{3.2}$$

However, all (3.2) demands is that, for any two such C_i and C_j

$$p(A | C_i) \neq p(A | C_j) \text{ and } p(B | C_i) \neq p(B | C_j) \tag{3.3}$$

and

$$p(A | C_i) > p(A | C_j) \text{ if and only if } p(B | C_i) > p(B | C_j). \tag{3.4}$$

Clearly, according to (3.3) no two distinct members C_i and C_j of a Reichenbachian common cause system can be simultaneously independent of A, nor they can be simultaneously independent of B, for in that case they would be equal to each other. Still, (3.3) leaves place for exactly one member C_i of a Reichenbachian common cause system to be statistically independent of A, and for exactly one (not necessarily distinct) member C_j to be statistically independent of B; and as a consequence, Reichenbachian common cause systems do not need to be systems of sole (positive and negative) *causes* of A or B. In spite of their name, Reichenbachian common cause systems may include some non-causally relevant factors.

Furthermore, condition (3.2) by itself cannot rule out the possibility that, for some $i \in I$:

$$[p(A|C_i) - p(A)][p(B|C_i) - p(B)] < 0. \tag{3.5}$$

In other words, Reichenbachian common cause systems for the pair A, B may possibly include events which are positive causes for one term of the correlated pair *and* pre-empters for the other; but whether such members can be considered common causes in the Reichenbachian sense is at least disputable. In fact, conditions (2.5)-(2.6) expressly require that common causes should symmetrically increase the probability of their effects, or symmetrically decrease the probability of both; it is then reasonable to expect that a proper generalization of Reichenbach's model should preserve this property.

Condition (3.2) is accordingly too weak a criterion to generalize Reichenbach's conditions (2.5)-(2.6). But on the other hand, it is at the same time too strong. In the first place, there's apparently no need to prevent any two equiprobable members of a system of common causes from exerting the same causal influence on (one of) their effects – hence, there is apparently no need to endorse (3.3). Second, for any two positive (any two negative) common causes C_i and C_j of A and B, there is no reason prohibiting us to suppose C_i to be more causally relevant for A than C_j, while C_j is more causally relevant for B than C_i (and vice-versa). However, this possibility is ruled out by condition (3.4). Under these respects, (3.2) is openly an overstatement.

4 Revisitation

Our discussion suggested that a proper generalization of Reichenbach's conjunctive forks should satisfy two basic requirements – namely, each of its members should be either positively relevant for both A and B or negatively relevant for both, and they all should obey no additional restriction, if not causally justifiable. For both reasons, condition (3.2) appearing in the definition of a Reichenbachian common cause system has to be replaced. The following alternative definition of a Reichenbachian common cause system is thus proposed (to keep it separated from the original notion, we shall indicate it using bold text):

Definition 3. *For any $A, B \in \Omega$ satisfying (2.2) a* ***Reichenbachian common cause system*** *for the pair A, B is a partition $\{C_i\}_{i \in I}$ of Ω such that, for all $i \in I$:*

$$p(A \wedge B|C_i) = p(A|C_i)\,p(B|C_i) \tag{4.1}$$

and

$$[p(A|C_i) - p(A)][p(B|C_i) - p(B)] > 0 \tag{4.2}$$

provided $p(C_i) \neq 0$ for all $i \in I$.

Clearly, (4.2) is a straightforward generalization of Reichenbach's conditions (2.5)-(2.6) to cases in which more than one common cause is taken into account; as such, (4.2) avoids any of the shortcomings affecting (3.2), still preserving its explanatory efficacy. In fact, it is easy to verify that:

Lemma 2. *Let $A, B \in \Omega$ be two arbitrary events and let $\{C_i\}_{i \in I}$ be a partition of Ω such that*

$$p(C_i) \neq 0 \qquad \text{for any } i \in I, \tag{4.3}$$

$$p(A \wedge B \,|C_i\,) = p(A\,|C_i\,)\, p(B\,|C_i\,) \qquad \text{for any } i \in I. \tag{4.4}$$

Then the following equality holds:

$$Corr(A, B) = \sum_i p(C_i)[p(A\,|C_i\,) - p(A)][p(B\,|C_i\,) - p(B)]. \tag{4.5}$$

Proposition 3. *Let a partition $\{C_i\}_{i \in I}$ of Ω be a **Reichenbachian common cause system** for the pair A, B; then $Corr(A, B) > 0$.*

Proposition 3 guarantees that **Reichenbachian common cause systems** offer at least as good causal explanations for non-causally correlated pairs as Reichenbachian common cause systems do. But in addition, (4.2) demands that, for any $i \in I$,

$$p(A\,|C_i\,) \neq p(A) \text{ and } p(B\,|C_i\,) \neq p(B) \tag{4.6}$$

and

$$p(A\,|C_i\,) > p(A) \text{ if and only if } p(B\,|C_i\,) > p(B). \tag{4.7}$$

Condition (4.6) openly requires that, contrary to Reichenbachian common cause systems, **Reichenbachian common cause systems** do not admit any factor which is not causally relevant for either A or B. Condition (4.7), on the other hand, requires that each member of a **Reichenbachian common cause system** is either positively relevant for both A and B or negatively relevant for both, exactly as it was demanded by (2.5)-(2.6). Finally, in no way (4.6) rules out the possibility that equiprobable members of a Reichenbachian common cause system might exert the same causal efficacy on A or B.

5 Existence

[2] prove that, for any non-strictly (i.e. non-perfectly) correlated pair A, B and any positive integer $n \geq 2$, an embedding (Ω', p') of the given classical probability space exists such that a Reichenbachian common cause system of size n exists in that space for A, B. A similar statement is here proved for **Reichenbachian common cause systems**.

Hofer-Szabó and Rédei's proof proceeds by two steps. In Step 1 they prove that, for any two non-strictly positively correlated events A, B in (Ω, p), $3n$ real numbers $\{a_i, b_i, c_i\}_{i=1}^n$ exist such that, if a partition $\{C_i\}_{i=1}^n$ of Ω existed satisfying equations

$$p(A|C_i) = a_i, \tag{5.1}$$

$$p(B|C_i) = b_i, \tag{5.2}$$

$$p(C_i) = c_i, \tag{5.3}$$

then $\{a_i, b_i, c_i\}_{i=1}^n$ would be a Reichenbachian common cause system. Hofer-Szabó and Rédei call such numbers *admissible* for the correlation $Corr(A, B)$. In Step 2, they prove that for any classical probability space (Ω, p) and any non-strict correlation $Corr(A, B)$ with admissible numbers $\{a_i, b_i, c_i\}_{i=1}^n$, there exists an extension (Ω', p') of (Ω, p) such that a n-terms partition $\{C_i\}_{i=1}^n$ of Ω' satisfying equations (5.1)-(5.3) does in fact exist.

Our proof will run along similar tracks, the main difference being that admissible numbers for $Corr(A, B)$ will be demanded to satisfy different conditions than those required by Reichenbachian common cause systems. To mark this difference, numbers playing the role of admissible numbers in the case of **Reichenbachian common cause systems** will be labeled **admissible**.

Definition 4. *Let (Ω, p) be a classical probability space. For any $A, B \in \Omega$ such that $Corr(A, B) > 0$ and any $n \geq 2$, numbers $\{a_i, b_i, c_i\}_{i=1}^n$ are called* **admissible** *if and only if they satisfy the following conditions:*

$$0 < c_i < 1 \text{ for any } i = 1, ..., n \tag{5.4}$$

$$0 \leq a_i, b_i \leq 1 \text{ for any } i = 1, ..., n \tag{5.5}$$

$$1 = \sum_{i=1}^{n} c_i \tag{5.6}$$

$$p(A) = \sum_{i=1}^{n} a_i c_i \tag{5.7}$$

$$p(B) = \sum_{i=1}^{n} b_i c_i \tag{5.8}$$

$$p(A \wedge B) = \sum_{i=1}^{n} a_i b_i c_i \tag{5.9}$$

$$\left[a_i - \sum_{i=1}^{n} a_i c_i\right]\left[b_i - \sum_{i=1}^{n} b_i c_i\right] > 0 \text{ for any } i = 1, ..., n. \tag{5.10}$$

Let us notice that (5.4)-(5.10) are in all identical to the conditions lain down by Hofer-Szabó and Rédei for admissible real numbers, with the sole exception of (5.10): in fact, (5.10) is meant to replace the homologous condition which is imposed on admissible numbers by (3.2) in the way demanded by (4.2).

Lemma 3. *Let (Ω, p) be a classical probability space, let $A, B \in \Omega$ be positively correlated and let $\{C_i\}_{i=1}^n$ be a* ***Reichenbachian common cause system*** *of size $n \geq 2$ for $Corr(A, B)$. Then, for any $m \geq 2$, a set of $3(n + m - 1)$* ***admissible*** *real numbers for $Corr(A, B)$ exists.*

Proof. Let (Ω, p) be a classical probability space, let $A, B \in \Omega$ be positively correlated and let $\{C_i\}_{i=1}^n$ be a **Reichenbachian common cause system** of size $n \geq 2$ for $Corr(A, B)$.

By hypothesis, a set $\{a_i, b_i, c_i\}_{i=1}^n$ of $3n$ **admissible** real numbers for $Corr(A, B)$ exists, such that for any $i = 1, ..., n$, a_i, b_i and c_i satisfy equations (5.1)-(5.3).

Let $c_k \in \{c_i\}_{i=1}^n \subset \{a_i, b_i, c_i\}_{i=1}^n$ be chosen arbitrarily. Since c_k is a real number lying inside the open interval (0,1), it is possible to find a set $\left\{c_k^j\right\}_{j=1}^m$ of $m \geq 2$ identical real numbers, lying inside the same interval, such that

$$\sum_{j=1}^m c_k^j = c_k; \tag{5.11}$$

in fact, this is as much as dividing c_k by m. Furthemore, it is trivially possible to find two sets $\left\{a_k^j\right\}_{j=1}^m$ and $\left\{b_k^j\right\}_{j=1}^m$ of $m \geq 2$ identical real numbers lying inside the closed interval [0,1], such that, for any $j = 1, ..., m$,

$$a_k^j = a_k \tag{5.12}$$

$$b_k^j = b_k. \tag{5.13}$$

It is then easy to see that

$$1 = \sum_{i=1}^n c_i = \sum_{\substack{i=1 \\ i\neq k}}^n c_i + c_k = \sum_{\substack{i=1 \\ i\neq k}}^n a_i c_i + \sum_{j=1}^m a_k^j c_k^j = \sum_{\substack{i=1 \\ i\neq k}}^{n+m} a_i c_i \tag{5.14}$$

$$p(A) = \sum_{i=1}^n a_i c_i = \sum_{\substack{i=1 \\ i\neq k}}^n a_i c_i + a_k c_k = \sum_{\substack{i=1 \\ i\neq k}}^n a_i c_i + a_k \frac{c_k}{m} m = \sum_{\substack{i=1 \\ i\neq k}}^n a_i c_i + \sum_{j=1}^m a_k^j c_k^j = \sum_{\substack{i=1 \\ i\neq k}}^{n+m} a_i c_i \tag{5.15}$$

$$p(B) = \sum_{i=1}^n b_i c_i = \sum_{\substack{i=1 \\ i\neq k}}^n b_i c_i + b_k c_k = \sum_{\substack{i=1 \\ i\neq k}}^n b_i c_i + b_k \frac{c_k}{m} m = \sum_{\substack{i=1 \\ i\neq k}}^n b_i c_i + \sum_{j=1}^m b_k^j c_k^j = \sum_{\substack{i=1 \\ i\neq k}}^{n+m} b_i c_i \tag{5.16}$$

$$p(A \wedge B) = \sum_{i=1}^n a_i b_i c_i = \sum_{\substack{i=1 \\ i\neq k}}^n a_i b_i c_i + a_k b_k c_k = \sum_{\substack{i=1 \\ i\neq k}}^n a_i b_i c_i + a_k b_k \frac{c_k}{m} m = \sum_{\substack{i=1 \\ i\neq k}}^n a_i b_i c_i + \sum_{j=1}^m a_k^j b_k^j c_k^j = \sum_{\substack{i=1 \\ i\neq k}}^{n+m} a_i b_i c_i \tag{5.17}$$

and, for any $j = 1, ..., m$

$$0 < c_k^j = \frac{c_k}{m} < 1 \tag{5.18}$$

$$0 \leq a_k^j = a_k \leq 1, \; 0 \leq b_k^j = b_k \leq 1 \tag{5.19}$$

$$\left[a_k^j - \sum_{i=1}^n a_i c_i\right]\left[b_k^j - \sum_{i=1}^n b_i c_i\right] = \left[a_k - \sum_{i=1}^n a_i c_i\right]\left[b_k - \sum_{i=1}^n b_i c_i\right] > 0, \tag{5.20}$$

so that, for any $i = 1, ..., k-1, k+1, ..., m+n$

$$0 < c_i < 1 \tag{5.21}$$

$$0 \leq a_i, b_i \leq 1 \tag{5.22}$$

$$\left[a_i - \sum_{i=1}^{n} a_i c_i\right]\left[b_i - \sum_{i=1}^{n} b_i c_i\right] > 0. \tag{5.23}$$

By Definition 4, $\{a_i, b_i, c_i\}_{\substack{i=1 \\ i \neq k}}^{m+n}$ is accordingly a set of $3(n+m-1)$ **admissible** numbers for $Corr(A, B)$. □

Let us notice that Lemma 3 is also true for stricly correlated pairs; as a consequence, our existence proof will be more general than Hofer-Szabó and Rédei's – though it will partially rely on it.

Proposition 4. *Let (Ω, p) be a classical probability space and let $A, B \in \Omega$ be a positively correlated pair. Then for any finite $n \geq 2$ an embedding (Ω', p') of (Ω, p) exists such that a partition $\{C_i\}_{i \in I}$ of Ω' is a **Reichenbachian common cause system** of size n for the pair A, B.*

Proof. Let (Ω, p) be a classical probability space and let $A, B \in \Omega$ be positively correlated. Proof will proceed by induction on the dimension of the partition $\{C_i\}_{i=1}^{n}$, assuming $n = 2$ as inductive basis.

For $n = 2$, **Reichenbachian common cause systems** and Reichenbachian common cause systems coincide, both reducing to conjunctive forks. In this case, the existence of an extension of (Ω, p) including a **Reichenbachian common cause system** for the pair A, B is guaranteed by the existence of a similar extension of (Ω, p) including a Reichenbachian common cause system of size $n = 2$ for the same pair.

For $n = m$, the existence of an extension of (Ω, p) including a **Reichenbachian common cause system** for the pair A, B is guaranteed by inductive hypothesis.

For $n = m + 1$, the existence of an extension of (Ω, p) including a **Reichenbachian common cause system** of size m for the pair A, B and Lemma 3 jointly guarantee the existence of $3(m + 1) = 3n$ **admissible** real numbers for $Corr(A, B)$. Given the existence of such numbers, the final step in our inductive proof is essentially identical Hofer-Szabó and Rédei's Step 2 [2, pp. 754-755], except for the fact that admissible numbers must be replaced by **admissible** numbers. □

6 Discussion

While Proposition 3 showed that **Reichenbachian common cause systems** are as explanatory effective as Reichenbachian common cause systems, Proposition 4 showed that the two models cover the same range of cases. In spite of these results, however, there might still be doubts concerning the opportunity of replacing condition (3.2) in Hofer-Szabó and Rédei's model with such a relatively strong condition as (4.2).

First, one may possibly object that both conditions (3.2) and (4.2) are purely *ad hoc* – their sole function being that of guaranteeing inequality (2.2) – and that, as such, they should simply be dropped in favor of the sole screening-off conditions (3.1) and (4.1). There are two different orders of reasons to reject this objection: the one epistemological, the other methodological.

On the epistemological level, Reichenbachian common cause systems and **Reichenbachian common cause systems** are both intended to offer a causal *explanation* for pairs of correlated events which are nonetheless not directly causally connected to each other; as such, they are expected to fulfill two major tasks: removing the correlations to be explained at a deeper level of analysis, and showing how those correlations could have obtained at the surface level. The former task is pursued through the screening-off conditions (3.1) and (4.1); the latter is pursued, *via* Proposition 2 and Proposition 3, by (3.2) and (4.2). Under this respect, conditions such as (3.2) and (4.2) are therefore necessary so that Reichenbachian common cause systems and **Reichenbachian common cause systems** respectively fulfill their explanatory role.

On the methodological level, instead, it is reasonable to expect that a proper generalization of conjunctive forks should not only include more than two screening-off causal factors, but also recover conjunctive forks as the special case in which the numbers of those factors reduces to two. Retaining the sole conditions (3.1) and (4.1), on the other hand, would evidently not do the job.

Similarly, one may wonder why should common causes symmetrically increase or decrease the probability of their effects, as required by condition (4.2): in fact, there is apparently nothing in the very idea of causation imposing such a constraint.

To reply to this objection, let us first notice that Lemma 2 is but a special case of the following statement, obtaining precisely in case $Corr\,(A, B\,|C_i\,)$ reduces to zero for all $i \in I$:

Lemma 4. *Let $A, B \in \Omega$ be two arbitrary events and let $\{C_i\}_{i\in I}$ be a partition of Ω such that, for all $i \in I$, $p(C_i) \neq 0$. Then, the following equality holds:*

$$Corr(A, B) = \sum_i p(C_i)[p\,(A\,|C_i\,) - p(A)][p\,(B\,|C_i\,) - p(B)] + p(C_i)Corr\,(A, B\,|C_i\,). \tag{6.1}$$

Given Lemma 4, it is then possible to prove the following statement:

Proposition 5. *Let $\{C_i\}_{i\in I}$ be a Reichenbachian common cause system of size $n > 2$ for the pair A, B. For any $i \in I$, if*

$$[p\,(A\,|C_i\,) - p(A)][p\,(B\,|C_i\,) - p(B)] \leq 0, \tag{6.2}$$

then

$$Corr\,(A, B\,|\neg C_i\,) - Corr(A, B) > 0. \tag{6.3}$$

Proof. Let $A, B \in \Omega$ and let $\{C_i\}_{i\in I}$ be a Reichenbachian common cause system of size $n > 2$ for the pair A, B, let $C_i \in \{C_i\}_{i \in I}$ and let $\neg C_i = \bigvee_{\substack{j \in I \\ j \neq i}} C_j$. By Lemma 4:

$$Corr(A,B) = p(C_i)[p(A|C_i) - p(A)][p(B|C_i) - p(B)] + p(C_i)Corr(A,B|C_i) + \\ + p(\neg C_i)[p(A|\neg C_i) - p(A)][p(B|\neg C_i) - p(B)] + p(\neg C_i)Corr(A,B|\neg C_i), \quad (6.4)$$

where, by (3.1),

$$Corr(A,B|C_i) = 0. \quad (6.5)$$

If C_i satisfies condition (6.2), then, by the theorem of total probability:

$$p(\neg C_i)[p(A|\neg C_i) - p(A)][p(B|\neg C_i) - p(B)] \leq 0; \quad (6.6)$$

so, in that case, (6.4)-(6.6) lead to

$$p(\neg C_i)Corr(A,B|\neg C_i) - Corr(A,B) \geq 0. \quad (6.7)$$

On the other hand, being $p(C_i) \neq 0$ by hypothesis, $p(\neg C_i) < 1$; and therefore, inequality (6.7) reduces to (6.3), as desired. □ □

To better understand the meaning of Propostion 5 let us recall that, for any classical probability space (Ω, p) and any non-zero probability event $X \in \Omega$, it is possible to define a subspace (Ω_X, p_X) of (Ω, p) such that

$$\Omega_X = \{Y \wedge X | Y \in \Omega\}, \quad (6.8)$$

$$p_X = p(*|X), \quad (6.9)$$

where $p(*|X)$ is the conditional probability measure of p with respect to X. Proposition 5 is thus claiming that, for any classical probability space (Ω, p) and any correlated pair $A, B \in \Omega$, if a Reichenbachian common cause system $\{C_i\}_{i \in I}$ for A, B exists in (Ω, p), then, for any element C_i of that system which do not symmetrically increase the probabilities of A and B, the correlation of A and B in the proper subspace $(\Omega_{\neg C_i}, p_{\neg C_i})$ of (Ω, p) not including C_i is even greater than the correlation they exhibit in (Ω, p). In other words: non-symmetric common causes reduce, rathen than increase, the positive correlation between their effects.

In replying to the previous objection, on the other hand, we remarked that part of the explanatory task of common causes is precisely to show how positive correlations between pairs of causally unrelated events may possibly obtain. This means that common causes are expected to give rise to, or favor, the correlations they are supposed to explain. In this respect, non-symmetric common causes fall short of their explanatory role: in fact they inhibit, rather than favor, those correlations.

7 Conclusion

I showed how the original definition of a Reichenbachian common cause system, in spite of its capability of explaining positive correlations between pairs of causally independent events, is not itself causally satisfactory. I proposed an alternative definition, which proved to be not only causally significant, but also capable of emulating the explanatory efficacy of its original counterpart.

References

[1] Hofer-Szabó, G. and M. Rédei (2004). Reichenbachian Common Cause Systems. *International Journal of Theoretical Physics* 43: 1819-1826. 1826.

[2] Hofer-Szabó, G. and M. Rédei (2006). Reichenbachian Common Cause Systems of Arbitrary Finite Size Exist. *Foundations of Physics* 36: 745-756.

[3] Hofer-Szabó, G., M. Rédei and L. Szabó (1999). *On Reichenbach's Common Cause Principle and Reichenbach's Notion of Common Cause. British Journal for the Philosophy of Science* 50: 377-399.

[4] Reichenbach, H. (1956) *The Direction of Time*. Berkeley: University of California Press.