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Geometric parameters influence on Piano Key Weir hydraulic performances

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Abstract: *The Piano Key Weir is a recent evolution of the traditional labyrinth weir. Thanks to a reduced foot print, this nonlinear weir can be placed on the top of gravity dams. The Piano Key Weir geometry involves a large number of geometric parameters. Several experimental studies have been carried out to investigate the main geometric parameters influencing the weir hydraulic efficiency and to define their optimal value. In this paper, the experimental data gathered at the University of Liege are re-examined to show how the weir height, the keys widths and the overhangs positions influence, for a given crest length magnification ratio, the weir discharge capacity. The theoretical rating curve of a standard linear weir is considered for comparison. The analysis highlights that the keys widths and overhangs lengths ratios influence significantly the Piano Key Weir efficiency, but less than the weir height. Considering the above mentioned results, a cost efficient design proposed in the literature is also proved to be close to the hydraulic optimum.*

Keywords: *physical modelling, parametric study, discharge increase ratio, rating curve.*

1. INTRODUCTION

The Piano Key Weir (PKW) is a recent evolution of the traditional labyrinth weir, firstly devised to circumvent the drawbacks of the latter (Lempérière & Ouamane, 2003). Using a rectangular layout and inclined bottoms creating overhangs, the PKW is structurally simple and efficient, can be placed on existing or new gravity dam crest sections and multiplies significantly the discharge capacity compared to a standard linear weir of same width. Compared to a traditional labyrinth weir with the same cycle shape in plan view (same crest print), a PKW is around 10% more efficient for an upstream head equal to its height (Anderson & Tullis, 2012). Several prototypes have already been built in France and Vietnam for instance (Fig. 1).

Figure 1 - PKW on the Etroit dam in France (left) and Van Phong PKW in Vietnam (right)

The PKW geometry involves a large number of parameters. A nomenclature has been specifically developed by several laboratories to unify the notations (Pralong *et al*, 2011). The basic element of a PKW is the PKW-unit, made of one inlet and two adjacent half-outlets (Fig. 2). The main geometric parameters are the height of the inlet and outlet keys, denoted P_i and P_o respectively, their widths W_i and W_o, the unit width W_u, the number of PKW-units N_u, the lateral crest length B_h, the lengths B_o and

B_i of the up- and downstream overhangs, the base length B_b and the wall thickness T_s. *i*, *o* and *s* indexes refer respectively to the inlet key, the outlet key and the side wall. W_u is equal to $W_i+W_o+2T_s$ and the total width *W* of the weir is equal to N_uW_u. The developed crest length L_u of a PKW-unit is equal to W_u+2B_h and the total developed crest length *L* of the weir is equal to N_uL_u. Parapets (vertical extensions of the crest) may be added to the weir. Their height is referred to as P_p. Depending on the existence of zero, one or two overhangs, PKW have been classified in 4 types (Lempérière *et a*l, 2011): type-A with two overhangs, type-B with a single upstream overhang, type-C with a single downstream overhang and type-D without overhang. Though the classical crest layout of a PKW is rectangular, Cicéro *et a*l (2013) recently considered trapezoidal PKW.

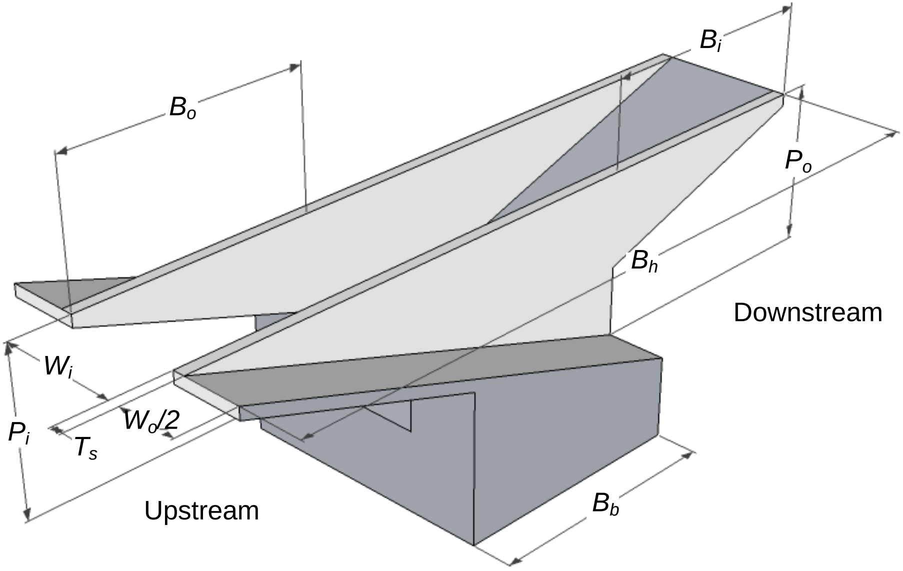


Figure 2 – PKW unit and main geometric notations

Since the invention of the PKW, several researches have been carried out all over the world to understand its hydraulic behaviour, optimize its design and objectify its advantages and drawbacks. In particular, a lot of experimental investigations focused on the geometric parameters influencing the PKW hydraulic efficiency. Their results enabled the development of general design equations to predict the head-discharge relation of a PKW, such as those by Leite Ribeiro *et a*l (2012b) and by Machiels *et a*l (accepted).

Regarding the PKW geometric parameters, Ouamane & Lempérière (2006) and Leite Ribeiro *et a*l (2012a) show that the crest length magnification ratio L/W is the main parameter controlling the discharge capacity. A value of 5 seems to be a reasonable compromise between weir efficiency and structure complexity (Lempérière, 2009; Lempérière *et a*l, 2011), while L/W ratio of existing PKW ranges from 4 to 8 (Pfister *et a*l, 2012). From a detailed investigation of the flow features over a PKW, Machiels *et a*l (2011 and 2012) identify the keys height *P*, the keys widths ratio W_i/W_o and the overhangs positions ratio B_o/B_i as the main geometric parameters influencing the PKW hydraulic efficiency for a given L/W ratio.

To maximize the hydraulic efficiency of a PKW, Machiels *et a*l (accepted) show that the height to unit width ratio P/W_u should be around 1.33; Leite Ribeiro *et a*l (2012a), Anderson & Tullis (2013) and Machiels *et a*l (accepted) show that the optimal range of the ratio W_i/W_o is in the range 1.25-1.6; finally, Lempérière *et a*l (2011) and Machiels *et a*l (accepted) show that a ratio B_o/B_i higher than one with B_i different from zero is more efficient.

On another hand, because of building constraints on the keys width, PKW designed with a high value of the P/W_u ratio are relatively high (several meters). They are thus not necessarily adapted to specific design criteria. For instance, in dam rehabilitation, the PKW height defines the height of the demolition works to be done in the dam body and also the decrease in the reservoir level needed during the construction. Both these criteria are of major concern regarding the costs of a project as well as the dam safety during the construction. For instance, French prototypes, built on the crest of existing gravity dam, show P/W_u ratio ranging from 0.65 to 1.1 (Vermeulen *et a*l, 2011). In addition, assuming that the global cost of a PKW is directly proportional to the volume of the structure, the optimal geometry, from an economic point of view, corresponds to the highest released discharge per cubic meter of concrete. In such a case, Machiels (2012) suggests an optimal value of the P/W_u ratio close to 0.5. For such low height PKW, Machiels *et a*l (accepted) show that W_i/W_o and B_o/B_i ratios equal to 1 are relevant. In the same framework, Lempérière (2009) proposed a cost efficient design of PKW with the following ratios: L/W=5, P/W_u =0.83, W_i/W_o=1.25 and B_i/B_o=1.

In this paper, the experimental data gathered by Machiels (2012) are re-examined to show how the height to unit width ratio P/W_u, the keys widths ratio W_i/W_o and the overhangs positions ratio B_o/B_i influence, for a given crest length magnification ratio L/W, the PKW discharge capacity. The theoretical discharge capacity of a standard linear weir is considered for comparison. In parallel, the pertinence of the Lempérière (2009) design is analysed from a hydraulic efficiency point of view.

2. EXPERIMENTAL DATA

Machiels (2012) extensively tested a large number of physical models representing 2.5 units of a PKW with a unit width W_u equal to 0.3 m, equal inlet and outlet key heights (P_i=P_o=P) and flat topped crests. Considering the L/W value of 5 suggested by Lempérière *et a*l (2011), all the models considered a 0.6 m long lateral crest, a 0.2 m long base length and were designed to investigate P/W_u ratios ranging from 0.33 to 2.00, W_i/W_o ratios ranging from 0.46 to 2.18 and B_o/B_i ratios ranging from 0 to ∞.

The models have been built using PVC plates whose thickness was 0.015 m (equal to wall thickness T_s). They have been placed on a 0.20 m high support to avoid tailwater effects and have been tested in a 7.2 m long horizontal flume equipped with a pumping system whose maximum capacity was 300 l/s. All the models have thus been tested for specific discharges q ranging from 0 to 0.4 m³/s/m. The flume width has been adjusted to the model width using specific convergent structures (channel application). Measurements with an electromagnetic flow meter (accuracy of 1l/s) and an ultrasonic probe (accuracy of 0.5 mm) enabled to derive the head/discharge relation of each model.

More details on the experimental facility and models characteristics can be found in Machiels (2012) or Machiels *et a*l (accepted). In this paper, respecting the criterion defined by Erpicum *et a*l (2013) to avoid scale effects on PKW discharge capacity evaluation, only the results with an upstream head higher than 30 mm have been considered.

3. METHOD

The PKW is a free surface weir and its discharge Q_P is thus proportional to the upstream head H to the power 3/2 as

$$Q_P \propto \sqrt{2gH^3} \tag{1}$$

As summarized by Leite Ribeiro et al (2012a), two approaches may be chosen to derive the proportion factor, which represents the effect of the crest length and shape. On one hand, referring to the developed crest length L, the discharge coefficient $C_{P,L}$ is closely related to the crest shape. Eq. 1 writes as (Leite Ribeiro et al., 2012b)

$$Q_P = C_{P,L} L \sqrt{2gH^3} \tag{2}$$

In this approach, L varies with the head as the effective crest length decreases with increasing heads because of local submergence on the upstream apex for instance. $C_{P,L}$ also varies with the head as it includes both frontal and side weirs effects. On the other hand, referring to the width of the weir W, Eq. 1 writes (Ouamane & Lempérière, 2006; Machiels *et a*l., 2011)

$$Q_P = C_{P,W} W \sqrt{2gH^3} \tag{3}$$

with a discharge coefficient $C_{P,W}$ accounting for both crest shape and developed crest length effects.

Whatever the approach to model the PKW discharge, it is common to look at its discharge capacity by comparison with a standard linear weir of same width, considering the discharge increase ratio *r* (Leite Ribeiro *et a*l, 2012a & 2012b). In this paper, using Eq. (3), *r* is defined as

$$r = \frac{Q_P}{Q_S} = \frac{C_{P,W} W \sqrt{2gH^3}}{C_S W \sqrt{2gH^3}} = \frac{C_{P,W}}{C_S} \tag{4}$$

with Q_S and C_S the discharge and the discharge coefficient of a standard ogee crested weir (OCW), respectively. In this study, the discharge coefficient of the OCW has been computed as follows, with a design head H_d equal to the PKW unit width W_u (0.30 m). This value has been chosen as a crude evaluation of the maximum head upstream of a PKW.

$$C_S = 0.494 \left(\frac{H}{H_d} \right)^{0.12} \tag{5}$$

The minimum head value considered in this study is 0.03 m. From Eq. (5), the corresponding C_S value for the OCW is 0.375. The PKWs considered in this research have 0.015 long flat topped crests. For the minimum head, H/T_s equals 2 and the weir is almost sharp crested (Hager and Schleiss, 2009), with a corresponding $C_{P,L}$ value equal to 0.42. For lower heads, considering Eq. (2), *r* value should thus tend to the crest length magnification ratio multiplied by the ratio of the minimum discharge coefficients, i.e. to a value of 5.6.

4. DISCUSSION

Figure 3 presents the evolution of the discharge increase ratio *r* as a function of the non-dimensional upstream head *H/P* for the models with varied height. All the tested PKWs are more efficient than the OCW on the wide tested range of upstream head. For very low heads, *r* values are the most important and close to the maximal value computed in the previous section. They decrease rapidly with increasing upstream head ratio and seem to tend to one for *H/P* ratios higher than 2.5.

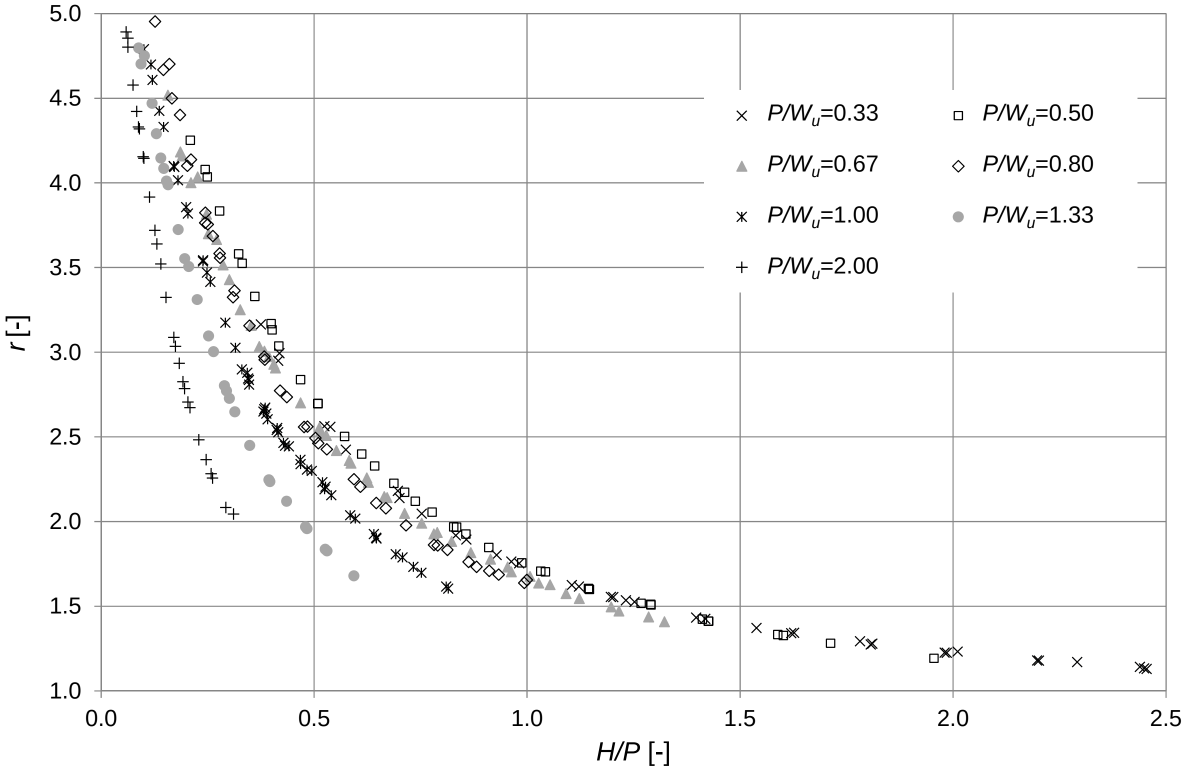


Figure 3 – Variation of the discharge increase ratio regarding the non-dimensional upstream head

At constant non dimensional upstream heads, the lowest PKWs (P/W_u=0.33 and P/W_u=0.50) are the most efficient and the highest ones the less efficient. However, the same comparison in relation to the absolute values of the upstream head (Fig. 4) shows that the overall efficiency of PKWs increases with increasing keys height for P/W_u ratio lower or equal to 1.33. Above this value, the discharge capacity is similar whatever the keys height. As shown by the results of Machiels *et a*l (accepted), once a sufficient weir height is reached, it becomes useless to increase it further since the weir efficiency would remain unaffected. This apparent difference between Figures 3 and 4 is easily understood as, for a given range of upstream head, high PKWs operate under smaller non dimensional head ratios than low PKWs. In Figure 3, the PKW with a P/W_u ratio equal to 1.33 operates for H/P between 0.08 and 0.60 while the PKW with a P/W_u ratio equal to 0.50 operates for H/P between 0.21 and 1.95. To discharge the same discharge per unit length, high PKWs require lower upstream heads than low geometries.

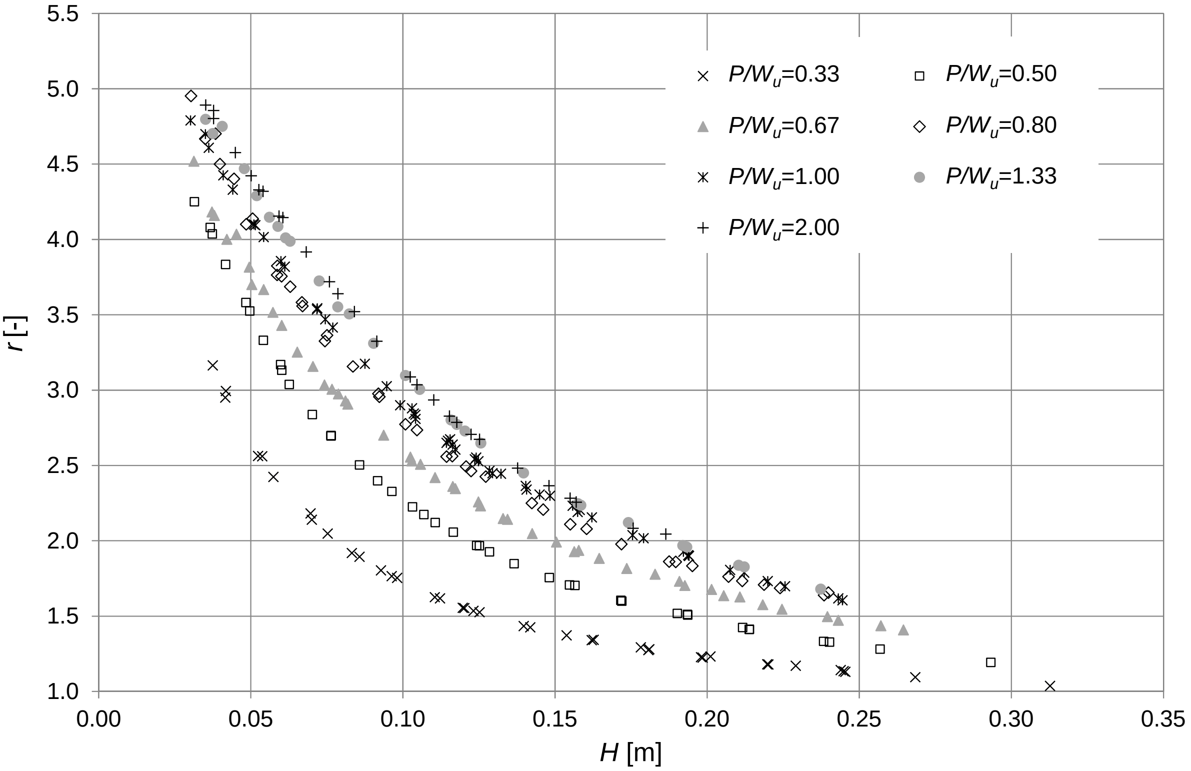
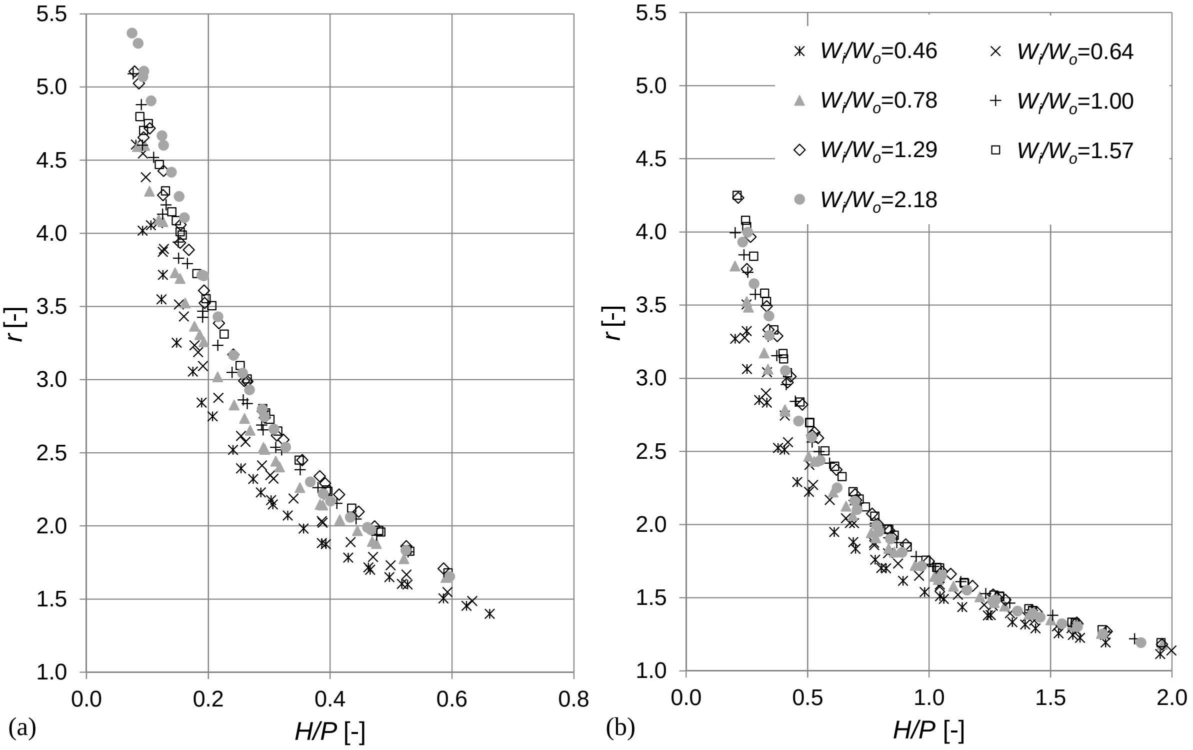


Figure 4 – Effect of the height to unit width ratio on the PKW rating curve and comparison to OCW

Increasing by a factor 4 the weir height (P/W_u=0.33 to P/W_u=1.33) increases by a factor 2 to 4 the gain in efficiency of a PKW compared to an OCW at low and high heads, respectively. The weir height is thus a key factor governing PKW hydraulic efficiency. The cost efficient design proposed by Lempérière (2009), with a low P/W_u ratio equal to 0.83, shows a high discharge capacity. It is indeed only about 10% less than the discharge capacity measured with P/W_u equal to 1.33.

In the following paragraphs, the effect of the keys widths and overhangs lengths ratios is analysed considering two different PKW heights, i.e. P/W_u equal 0.5 and 1.33. These models correspond to, respectively, the technical and economic optimum and the hydraulic optimum identified by Machiels (2012).

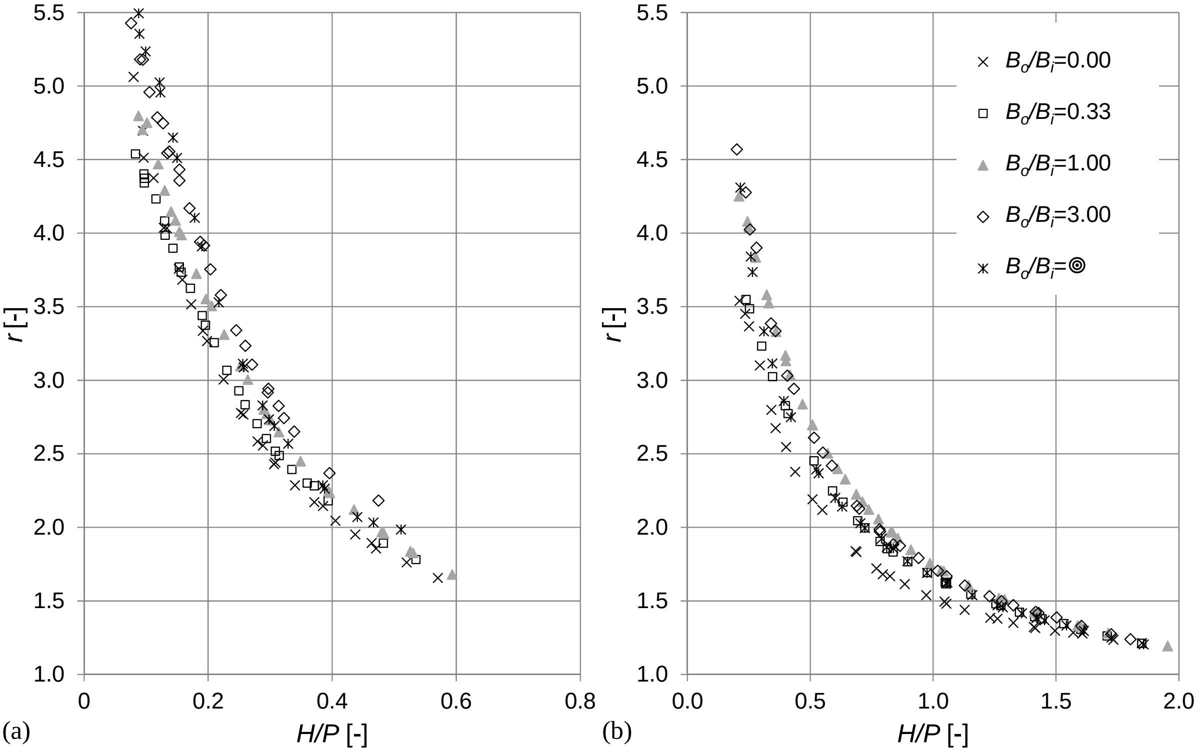
As already shown by Machiels *et a*l (accepted), the highest hydraulic efficiency of a “high” PKW is reached for W_i/W_o ratio between 1.29 and 1.57, except for very low heads where a value of 2.18 gives better results (Fig. 5(a)). For a “low” PKW (Fig. 5(b)), W_i/W_o ratio equal to 1 is as efficient as a ratio equal to 1.29 or 1.57. The model with W_i/W_o equal to 2.18 is less efficient in the whole range of tested heads. Whatever the PKW height, optimization of the W_i/W_o ratio enables to increase the weir efficiency by about 30%, which remains far below the effect of the weir height optimization. With a keys widths ratio equal to 1.25, the Lempérière (2009) design is optimal.



(a) (b)

Figure 5 – Effect of the keys widths ratio on the discharge increase ratio. P/W_u=1.33 (a) and P/W_u=0.50 (b)

Regarding the overhangs lengths ratio, the highest hydraulic efficiency of a “high” PKW is reached for a value of 3, except for very low heads where a type-B PKW (B_i=0) is more efficient (Fig. 6(a)). For a “low” PKW (Fig. 6(b)), B_i/B_o ratio equal to 1 is as efficient as a ratio equal to 3. The type-B model is less efficient whatever the head.



(a) (b)

Figure 6 – Effect of the overhangs lengths ratio on the discharge increase ratio. P/W_u=1.33 (a) and P/W_u=0.50 (b)

Again, whatever the PKW height, the optimization of the B_i/B_o ratio enables a much lower increase in the weir efficiency than the optimization of the weir height. The range of increase in efficiency is around 20%. With an overhangs lengths ratio equal to 1, the Lempérière (2009) design is one again close to the most efficient geometry.

5. CONCLUSION

Experimental data gathered at the University of Liege (Machiels, 2012) to define the optimal value of 3 main geometric parameters ratios governing the PKW discharge capacity have been re-examined with the aim to show how these ratios influence, for a given crest length magnification ratio, the weir rating curve. The analysis has been performed by comparison with the theoretical discharge capacity of a standard linear ogee crested weir.

The analysis shows the importance to study the effect of the PKW height as a function of absolute values of the upstream head. Indeed, analysis considering the non-dimensional upstream head ratio H/P may lead to wrong conclusions on the most efficient geometry maximising the discharge released under a given range of upstream head.

The analysis also highlights the primary importance of the weir height. Its optimization can increase by a factor 2 to 4 the gain in efficiency of a PKW compared to an OCW at low and high heads, respectively. The keys widths and overhangs lengths ratios are secondary parameters as their optimization leads to gains in efficiency of around 30 and 20%, respectively. This relative effect of these 3 geometric parameters is in agreement with the findings of Leite Ribeiro *et a*l (2012b)

Finally, the simple and cost efficient PKW design proposed by Lempérière (2009) is proved to be close to a hydraulic optimum. An increase in efficiency of about 15% may be gained by optimizing the weir height and overhangs position.

6. ACKNOWLEDGMENTS

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