Applied dimensional problems in mathematics courses: how small-scale partnerships across disciplines can improve mathematical problem-solving skills of engineering students

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Structured abstract

BACKGROUND

The underlying problem hampering engineering students' success in problem solving is an inability to apply theoretical mathematical knowledge, rather than a lack of proficiency in basic mathematics. The integration of mathematics with engineering applications is still an issue for large scale programs as well as for universities in the initial stages of curriculum reform. This is because in order to teach integrated thinking, courses need to use problems that effectively combine both mathematical and engineering concepts. However, the creation of such problems imposes a significant cognitive load on a single teacher, or it requires expertise from more than one discipline to be combined, requiring collaborative time and effortful thinking and working through issues between teachers from different disciplines. It is the cognitive effort, the time required and the pitfalls involved in collaborative endeavours that often creates blocks to interdisciplinary integration in teaching.

PURPOSE

A small team of mathematics and engineering teaching staff attempted such collaboration to see if it would be possible to develop maths-engineering integrated problems in five discipline areas that would enhance the students' ability to apply mathematics to engineering problems and to improve their learning experience in a required mathematics course.

DESIGN/METHOD

These problems were used for assignments within a mathematics course, with the aim of helping students to learn how to use mathematics inside of engineering problem solving processes. Students worked in small groups and extra tutorial support was given to help the students through the problem solving processes. Marks were compared for first and second year mathematics and for the related chemical engineering course for the experimental cohort versus a control cohort. In addition, students were surveyed about the benefits of the integrated mathematics course immediately after the course, then were asked about its relevance when doing a later chemical engineering course where the mathematics was being repeated, and then, near graduation, were surveyed again about the effect of these applied problems on improving their understanding and ability to apply theory to real world problems

RESULTS

Results suggested benefits of the integrated mathematics course for learning in the mathematics course itself as well as carry over effects for the related chemical engineering course for the experimental cohort compared to their traditionally taught peers. Survey results showed that in the long term, more than two thirds of engineering and mathematics students agreed that the problems improved their ability to apply mathematics. Most students agreed that the projects helped them to obtain a deeper understanding of mathematics, but that they would have learned more had the problems been more directly/explicitly linked to their later engineering courses.

CONCLUSIONS

The work involved in creating applied dimensional problems that integrated a foundational mathematics course with related engineering courses was worthwhile, resulting in benefits for students in learning mathematics as well as in some longer term benefits for students needing to use mathematics in a related chemical engineering course. Four problems from chemical, civil, mechanical and electrical engineering are appended, which can be adapted and applied in other universities.

KEYWORDS

STEM integration; dimensional Problems, applied mathematics

Introduction

The Accreditation Board for engineering and Technology (ABET) stresses that a [proficient] engineering graduate must have the ability to apply their knowledge of science, mathematics, and engineering (ABET., 1995). In spite of these standards, there is a widespread perception that engineering students frequently lack the ability to apply mathematics to engineering problems both during their course and when they leave university (Kumar & Jalkio, 1999; Lang, Cruse, McVey, & McMasters, 1999). This can in part be attributed to the traditional methods used for teaching mathematics, as these predominantly teach courses in compartmentalised courses (Dugger, 2010), use non-dimensional variables, and they mostly consider idealized models which are only introduced briefly. Associated demonstration problems use simplified versions of real world situations, with most of the fuzzy or difficult issues removed so as to highlight the need for a particular equation or principle. Traditional methods do not explicitly teach problem solving skills (Hyslop, 2010). Together these conditions may foster procedural learning (where students apply the appropriate equations by substituting values into variables and then work through a learned procedure), rather than deep learning (where students need to understand why the specific variables are relevant to the problem as well as the relationships between those variables (Mayer, 1992; Pape & Smith, 2002).

The more abstract method for teaching mathematics is often seen to be the most general and relevant to a broad range of disciplines. At many institutions worldwide, including the site of this study (a large Australian university) mathematics at first and second year level is taught in large classes which include students from many disciplines such as science, engineering, physics, and mathematics (Kumar & Jalkio, 1999). Because of other science and engineering discipline demands, the often substantial mathematical content is compressed into two or three courses (about 80-120 lectures). The teaching of mathematics in a self-contained manner is efficient for covering a broad range of topics, and highlighting the mathematical interconnections amongst these topics. However, it has a number of disadvantages that contribute to student's difficulties in translating problems into mathematical descriptions within applied disciplines where a degree of modelling is required.

One disadvantage (perhaps the most important), is that most first- and second-year mathematics lectures, tutorial problems, and assignments are presented in dimensionless variables. It is easier to formulate a problem using dimensional variables, as the dimensions of each term of an equation inform about magnitudes, and also provide a critical "parity" check on reasoning and calculations. However dimensionless problems are easier to solve, and the distillation of the "physics" into dimensionless constants provides information about the underlying mathematical problem. However, dimensions are generally omitted from "applied" undergraduate problems for a number of reasons. Firstly, for efficiency: it is time consuming to carry many dimensional constants throughout the calculations. Secondly, constants would become tedious when conducting exercises (e.g., on the many techniques of integration or differentiation). The alternative, of teaching students about non-dimensionalisation (and then re-dimensioning an answer for interpretation), is conceptually challenging and hard to teach to early undergraduate students. The topic tends to be taught in 3rd year when students have sufficient mathematical maturity to rescale problems involving integrals and differential equations.

As well as using dimensional variables, there are other ways that mathematics can be more closely linked to applications in order to enhance student learning. Since the early 1990's the NSF in the US has been funding university-wide coalitions attempting to integrate mathematics, engineering and other disciplines via reorganisation of the curriculum (e.g., Froyd, 2001). This integration has occurred in many different forms: within programmes, by coordination of topics, and through assessment (Al-Holou, Bilgutay, Corleto & Demel et al, 1999). There have also been other attempts to create integration of maths and engineering at the course level using scenario-based, problem-based and project-based learning (Bell, Galilea, & Tolouei, 2010; Mills & Treagust, 2003; D.R. Woods, 1997; D. R. Woods et al., 1997). All of these methods are variations' of 'inductive' learning approaches and all have shown significant measures of success (Prince & Felder, 2006).

Many of the integrated curriculum approaches have shown positive outcomes in terms of improved GPA's and reduced retention rates (Al-Holou et al., 1999). In spite of this apparent evidence for improvements in student ability and understanding, few advances have been made towards integration on a wide-scale and 'engineering education remains predominantly dependent upon narrow, discipline-focused undergraduate programs' (Duderstadt, 2008, p iii). Subsequently, national reviews are still highlighting the need for the integration of disciplines (e.g., Universities Australia,

2011) and the application of better teaching methods for teaching mathematics in STEM (e.g., PCAST, 2012).

There are many possible explanations for the lack of uptake of integration between mathematics and engineering, despite the known educational benefits. Firstly, as evidenced by the coalition attempts, integrating mathematics and engineering across the curriculum requires substantial resourcing, leadership and support at high levels. Secondly, when the topics are coordinated by creating real-world projects that apply mathematics to engineering, they often include mastery of other 'real world' skills such as team work or using MATLAB. While these are valuable skills to integrate with mathematical ability, they detract from the core business of applying the maths to the engineering problem and thus reduce the students' chances to gain technical understanding (Kjersdam, 1994; Mills & Treagust, 2003).

There are a number of reasons why teaching in an integrated fashion assists student learning (Everett, Imbrie, & Morgan, 2000), including students' cognitive processes. For example, students are often only able to assimilate information across disciplines when they are given a framework that helps them to see the connections. Integration of applications into mathematics teaching enhances student learning by making these connections and thus attaching new information to existing knowledge (Prince & Felder, 2006). Moreover, once a student has persevered to link two ideas, these ideas are not easily disconnected (Coppola, Ege, & Lawton, 1997).

The theory above (e.g., Prince & Felder, 2006) suggests that there is a need for teachers to actively help students to create the required cognitive connections between concepts in maths and in engineering. Consistent with this theory, a more in-depth analysis of integration programs has shown that there have been varying levels of success in integrating topics which are dependent on the methods that have been used for assimilation (McKenna, McMartin, Terada, Sirivedhin, & Agogino, 2001). The results of this study show that the less successful attempts at coordination of topics occur when one course in a program is merely designed so as to contain the information that is needed by another; e.g. the maths that is needed in an engineering course is taught in a separate maths course in a purely mathematical manner. In such cases, the links across the topics are not made explicit for the students; rather students are left to discover the links for themselves. As one student commented:

> *You don't see the lines between classes and subjects anymore. It takes a while, because around here, you're forced to discover that on your own. Classes are rarely, to never, tied into each other...* (McKenna et al. p. 8).

On the other hand, the more successful programs teach the maths next-to or inside the engineering or visa-versa (Laughlin, Zastavker, & M., 2007; McKenna et al., 2001). For example:

> *..in Math 1B when we were covering differential equations, at the same time in physics 7A we were covering springs and oscillating springs. The two concepts are very closely linked. So we could see essentially the same examples in both classes which made it easy to relate. So I thought that the synchronization helped reinforce the concepts and show us an application for the math that we were learning.* (McKenna et al. p 8-9.)

Teaching mathematics while at the same time teaching solutions to engineering problems is central to the cognitive integration process that needs to occur for students; example, teaching double differentiation while analysing the distance travelled by a piston in an engine, so as to use differentiation to convert distance to velocity and then to acceleration. By teaching in this way, students can 'see' the mathematics in terms of the real world example; in this case, they can see the motion of a piston mapped to differentiation calculations. However, teaching of dual content is the hardest aspect of the subject integration for the teachers. This is because the lesson and assignment preparation requires the creation of authentic example problems that target specific mathematical skills. Consequently, there are few published examples of problems that have been devised to illustrate how maths can be applied to engineering. There are some on the coalition websites (see Al-Holou et al., 1999 for a full list of the coalition websites) and some detailed examples in the literature (e.g., Mourtos, DeJong-Okamoto, & Rhee, 2004; Otung, 2002). However, there are far more examples of design principles being integrated with mathematics or engineering (e.g., Shamel & Al-Atabi, 2003; Sheppard & Jenison) than of mathematics being integrated with engineering applications.

Generating integrated, dimensional problems in mathematics is significantly more difficult than writing tutorial problems for either engineering or mathematics courses. It takes substantial time to translate a meaningful engineering problem into a mathematical framework for effective learning, because it requires knowledge of both the engineering problem and the mathematics. The intellectual process required in devising such problems is complex. It requires recognition of the higher-order relationships that are common to both the mathematics and engineering problems. These relationships need to be extracted and used to map the two different problem domains (mathematics and engineering) onto each other (Gentner, 1983; Halford, 1982). This intellectual process is a “hard” task and takes substantial cognitive effort (Johnson-Laird, 1989). The hard aspect of the mapping task makes it difficult for students to recognise the common relations across the different problem domains on their own (Holyoak, 1986). This explains why students need help from teachers to make these connections, and also, why the task of creating such problems is difficult even for lecturers with expertise in their own knowledge domains. Thus this task cannot be outsourced to tutors or junior teaching assistants.

In spite of the challenges, we believe that the core business of integrating mathematics and engineering can be achieved on a small scale, if individual lecturers are able to successfully collaborate with one or two other teaching staff spanning the mathematics-engineering divide. This paper demonstrates that mathematics and engineering teaching staff can work together to create authentic problems which enable students to apply core mathematical concepts and problem solving skills. We investigated whether the application of these problem sets in a second level compulsory mathematics class had positive effects on student’s problem solving and mathematics skills.

Curricular approach

This study was undertaken in an Australian university in 2008, in a second level mathematics course “Calculus and Linear Algebra II”, a compulsory course for approximately 800 undergraduate engineering and 150 mathematics and physics students.

In order to integrate engineering contexts into a second year mathematics course on a small scale, the decision was made to use engineering problems to teach mathematics, to see whether this type of integration would help students to learn to formulate mathematics when needed in engineering contexts. To this end, five discipline related problems were collaboratively developed by lecturers in mathematics, chemical, civil, mechanical and electrical engineering. (A full description of problems is outlined in the Supplementary material, Appendix A.) Each problem involved developing and evaluating or solving integrals or differential equations, and interpreting the physical meaning of the mathematics results within the context of a real world engineering problem. For example, in the pipe problem, mathematics was used to solve a problem in thermodynamics. The solution required integration of the heat equation, so as to determine the temperature profile in the pipe wall. Then the steady state solution needed to be used so as to calculate heat loss from the pipe under a range of scenarios, and to determine how the thickness of the pipe wall, the material of construction, and dimensions of the pipe would affect heat loss and associated GHG emissions. In the mathematics course, these concepts needed to be developed, not from physical understanding, but from the mathematical equations. In the subsequent year of study, the chemical engineering students then undertook the same problems but from a physical basis, where mathematics was the tool used to analyse the physical problem.

Table 1: Description of the applied mathematics problems used in this study, and the discipline from which the problems originated

Problem	Problem source (discipline)	Problem description
LRC circuit	Electrical engineering	Calculate voltage and current in an LRC circuit, which is a circuit containing a resistor, inductor and capacitor connected in series.
Heat loss from pipe	Chemical engineering	Determine rate heat loss from a pipe, and how this varies with pipe thickness and material properties
Satellite	Mechanical engineering	Calculate the centre of mass and moment of inertia of a satellite
Dam water depth	Civil engineering	Calculate the dynamic level of a reservoir in response to changes in inflows and outflows, through formulation of simple rate models, and answering specific questions relating to the physical meaning of the results
Winding Number	Mathematics	The winding number is a characteristic of a closed curve. The project did not involve a modelling component, or dimensional variables, and was included as an option for non-engineering students in the class who were more interested in abstract mathematics.

Similar mappings between the mathematics and the physical world were used to construct all problems. Two of the problems were drafted by engineering lecturers, and finalised by the mathematics course co-ordinator in consultation with his colleagues in engineering to ensure that both mathematical learning objectives were met. Additional funding from a faculty strategic project ("Supporting at-risk students and increasing engagement in second year mathematics") was available to pay tutors for project development. However extensive teaching experience and familiarity with course objectives and content was required, hence the task was found to be beyond the scope of tutors. About six hours work by the engineering colleague and ten hours by the mathematics course co-ordinator was required to finalise each problem and to write solutions.

For the integrated maths course assignment, students were required to complete their chosen problem, in a small group of two-four chosen members. The assignment accounted for 10% of their final grade. Additional tutoring was provided, with 2 hour drop-in sessions available for each specific project for the 3 weeks of the project duration. Specialist tutors, generally drawn from the engineering disciplines, were used for the project. This was done because the application areas were outside the scope of the regular course syllabus, and we wanted to ensure that the important context of the problems was able to be supported by the tutors.

Data and Methods

The impact of the project on student learning was assessed in two ways, using both quantitative (student marks) qualitative (student surveys) and data, described below

Analysis of Marks The integrated mathematics course being considered for analysis was a second year course that was run in semester1 2008. The related chemical engineering course was run one year later in semester 1 2009. In an attempt to ascertain the benefits of the integrated maths course over a normal maths course, the effects of the course on the students who participated (called the 'experimental group') were compared with the effects on students who did the same courses a semester or a year later, in a traditional non-integrated way (called the 'control group'). We investigated 1. improvements in marks from a first year course to the second year course for the experimental group versus the control group, and 2. marks in the integrated mathematics course and in the related chemical engineering course for the experimental group versus the control group.

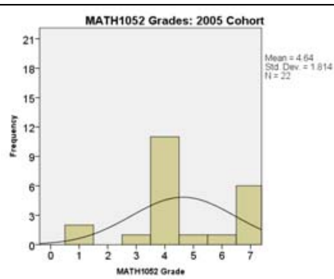
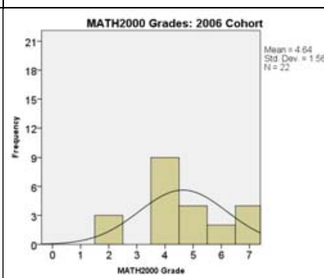
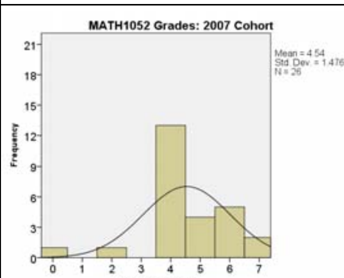
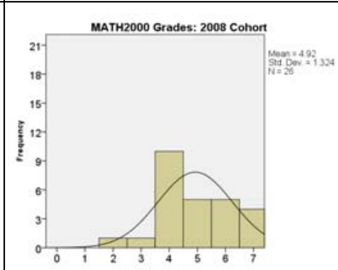
Student surveys Suspecting that students would have different impression of the benefits of a group-based, open-ended applied assignment immediately after it had been completed, than they would have had later on, we took data on student impressions of the assignment at varying intervals:

- *Short-term:* One month after project completion, the entire class was surveyed about whether the project was worthwhile, rewarding, an appropriate level of difficulty, and about the nature of the group work.
- *Medium term:* One year after project completion, 31 chemical engineering students who had completed the "heat loss from a pipe" project were asked whether the project improved their understanding of the material taught in a heat transfer course.
- *Long-term:* 30 months after project completion (immediately prior to graduation), 115 students completed a survey about how the project affected their mathematics and problem-solving skills. Results were analysed according to student discipline (chemical, civil, mechanical engineering, maths, or ITEE: information technology and electrical engineering), and according to the project selected (heat loss from a pipe, water storage in a dam, centre of mass and moment of inertia of a satellite, winding number, and properties of LRC circuits.

Results

Analysis of marks

First to second year mathematics: To investigate the benefits of the integrated course over the students' previous experiences (based on a comment from a student), we investigated whether students who had competed the pre-requisite first year course benefited from the integrated second year course. That is, we compared the first year to second year shift in marks for the experimental group (2007/8) with the corresponding shift for several traditionally run versions of the course. Comparisons were made between all first and second year courses from 2005/6 up until 2012/3. The results for the experimental group and for the 2005/6 and 2008/9 cohorts are summarised in Figure 2 below, in terms of the distributions of marks for the first and second year courses, along with the associated within groups t-tests. The full set of comparisons for all years is shown in Appendix B.

Group	First year math grades	Second year math grades	Mean1 to Mean2 shift & Within subjects t-tests
Control group (Same lecturers as experimental group) First year 2005 Second year 2006			4.64 to 4.64 No change t_21 = 0.0; p = 1.0 NS
Experimental group (Integrated 2nd year course) First year 2007 Second year 2008			4.54 to 4.92 Increase t_25 = -2.18; p = .04 Significant

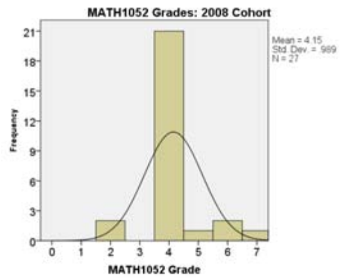
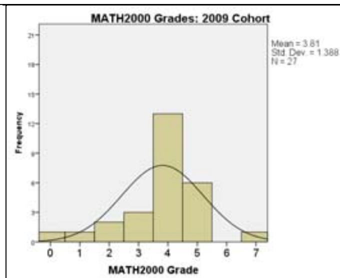
Control group Traditional 2nd year course First year 2008 Second year 2009	 	 	4.15 to 3.81 Decrease $t_{26} = 1.2$; $p = .24$ NS

Figure2: The shifts in grade distributions from first to second year mathematics for the integrated course versus two traditional courses.

The results show that the shift from first year course (M1=4.54) to the integrated second year course (M2=4.94) was in the upward direction, so that as a group, students did significantly better in the second year course than they did in the first year course ($t_{25} = -2.18$; $p = .04$). Furthermore, the mean for the experimental group in second year mathematics was higher than for any other year. For every other first to second year pair there was either no change or a significant downwards shift for the second year group. Across all of the comparison pairs of courses, there were several different lecturers who taught the first year course and the second year course, which add uncontrolled variance to the data. However, the 2005/6 cohort (shown in the first row of table 2) had the same lecturers for both the first and second year courses as did the experimental cohort. Thus the 2005/6 pair could be said to be the best control group in the analysis. However, even in this pair, there was no difference in the marks between the years. On the other hand, with the same lecturers, the marks rose in the second year course when the experimental intervention was used in 2008.

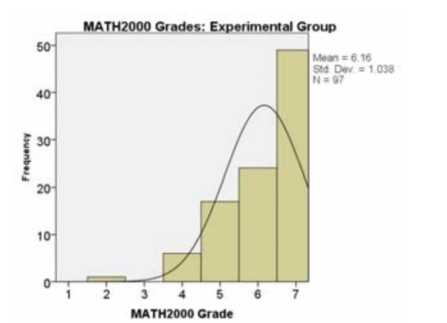
Mathematics and Chemical Engineering To investigate how participating in the integrated course may have benefited the chemical engineering students, we inspected the marks in the integrated course and in the related chemical engineering course (thermodynamics) for the experimental group versus the control group. The results are summarised in Figure 3 below, in terms of the distributions of marks for the mathematics and the thermodynamics courses, for the students in the experimental cohort versus the control cohort.

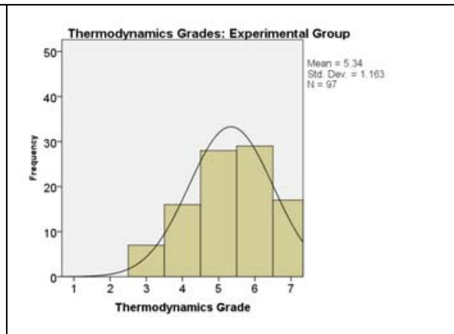
The results show that the integrated mathematics course gave advantages to the thermodynamics students, both in the mathematics course itself while they were doing it (in 2008) and later in thermodynamics (in 2009). That is, the students who did the integrated mathematics course and then later took the related thermodynamics course had their mathematics marks skewed towards the upper grades. This is probably because one of the assignment problems in the integrated course was related to thermodynamics (calculating heat loss through a pipe using integration), and many of the chemical engineering students chose this assignment. Thus the integrated assignment seemed to be a positive experience for the experimental cohort. The results also showed that for these students (the experimental cohort), their later thermodynamics marks had more relative numbers of 5's and 6's than their colleagues in the same thermodynamics course who had completed second mathematics in a different semester (the control cohort). However, while the two-way ANOVA comparing the means for mathematics and thermodynamics gave a significant interaction ($F_{261} = 16$; $p = .001$), the post-hoc tests showed that the thermodynamics mean grades for the control versus experimental group did not differ significantly ($F_{226} = .35$ NS). The interaction was due to the advantage of the experimental group during their mathematics course rather than during their thermodynamics course. However, since ANOVA is a test of differences between means, it does not tell the full story, that there were more 6's and 7's in the experimental group in thermodynamics, suggesting some longer term benefits of the integrated course for at least some of the students in the experimental cohort.

Group	2nd Year Maths grades for students who did Thermodynamics in Sem1 2009	Thermodynamics grades for the same students Sem1 2009

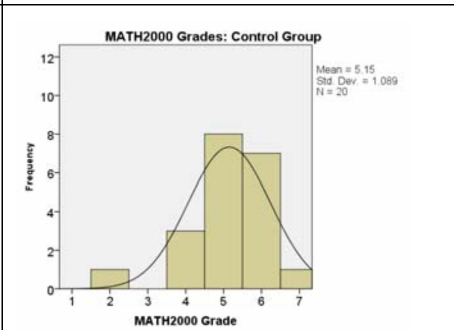
Experimental group

Second year Maths: S1, 2008 then Thermodynamics: S1, 2009





Control group Second year Maths: S2, 2008/ Summer 2008/ S1 2009. then Thermodynamics: S1, 2009



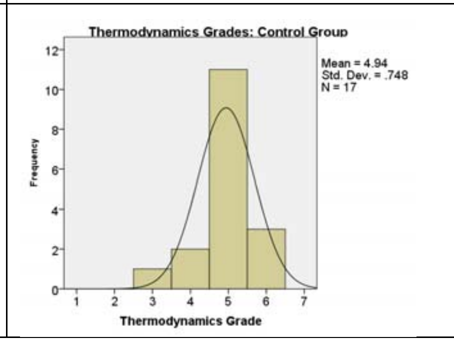


Figure 3: The distributions of marks for the integrated mathematics course and the later thermodynamics course for chemical engineering students in the experimental group versus the control group

The above analysis is based on grades, which are only broad indicators of learning benefits. When comparing marks across semesters, there are many other factors that come into play: different lecturing styles, different emphases on adhering to the normal distribution when allocating final grades, and different cohorts of students. However, by comparing across many years of mathematics and by using students from the same class in thermodynamics, the analysis above has been aimed at reducing any blatant effects of such variables so as to distil some signal from the noise. Together these results gives some indication that the work on integrating the mathematics course with the chemical engineering component was successful, and resulted in some longer term benefits for students comprehension of mathematics as well as their ability to use mathematics in a related engineering course. The following analysis shows that the impressions of the students themselves support these findings.

Student surveys

Student responses were more positive in the long-term survey than in the short and medium term surveys. One month after completing the project, a small but statistically significant majority of the class agreed that completing the problem allowed them greater depth than would normally be possible, and that the level of difficultly was appropriate. However there was no significant agreement that it was a worthwhile part of the course. One year later, 52 % of students surveyed in the third level chemical engineering subject “heat and mass transfer” (i.e. thermodynamics) who recalled doing the pipe project in the maths course agreed or strongly agreed that doing that project improved their understanding of “heat and mass transfer” course content.

Table 2: Discipline based analysis: Percentage of students from each discipline who agreed/strongly agreed (A/SA) or disagreed/strongly disagreed (D/SD) with the survey statements, 30 months after completing the problems.

Discipline	**Chem**	**Civil**	**IT & Elec.**	**Mech. & Min.**	**Eng. overall**	**Math**	**Phys. & Other**
Number of students	16	33	26	18	**93**	12	10

Survey statement 4	**Doing the project helped me learn to apply maths to engineering/physics problems.**						
A/SA	33	73	64	67	**68**	73	44
D/SD	42	7	6	11	**9**	12	22
Survey statement 5	**Doing the project helped me learn to develop a deeper understanding of mathematics.**						
A/SA	56	67	62	61	**62**	50	67
D/SD	13	12	8	17	**12**	33	11
Survey statement 6	**Doing the project helped me remember the maths I learned in MATH2000 longer than I would have if it was just a standard maths course.**						
A/SA	19	36	64	33	**40**	25	44
D/SD	31	30	32	44	**34**	58	44
Survey statement 7	**Doing the project enhanced my mathematical modelling and problem solving skills.**						
A/SA	56	53	62	56	**57**	42	78
D/SD	19	9	19	22	**16**	25	11
Survey statement 8	**The project in MATH2000 was not beneficial to me.**						
A/SA	31	15	23	28	**23**	17	22
D/SD	38	61	65	61	**58**	50	67
Survey statement 10	**I would have learned more if the project problem had been linked more explicitly to courses that I went on to study.**						
A/SA	63	61	58	61	**60**	25	33
D/SD	19	21	12	0	**14**	50	0
Survey statement 12	**Overall, was the project in MATH2000 2008 valuable for your learning?**						
A/SA	63	67	69	94	**72**	33	67
D/SD	19	15	8	6	**12**	33	22

The long term results are the most positive (see Table 2). When surveyed thirty months after completing the integrated mathematics course (MATH2000), almost three-quarters of the engineering students agreed that the applied problems used in MATH2000 were valuable for their learning. At least 60% of engineering students agreed or strongly agreed that the problems helped them to apply and to develop a deeper understanding of mathematics. While the majority of engineers responded positively to all statements about the value of the project, only 40% of engineering students agreed the projects 'helped me to remember the maths taught in MATH2000 longer than I would have if it were just a standard maths course". IT & electrical engineering students were the only discipline surveyed where the majority of students agreed with this statement.

Discipline-based analysis: While there was some variation between different disciplines of engineering in their response to various survey statements, there were no obvious differences between disciplines, except for mathematics students, who responded quite differently to the engineering students. That is, the most positive response in the survey was a 73% agreement by both mathematics and civil engineering students that the problems taught them to apply mathematics (see Table 2).

On the other hand, mathematics students had low numbers of positive responses to all other questions about the value of the project. The difference between mathematics and engineering students may reflect different affinity for abstract mathematics, or different interests. Either way, the results indicate that engineers seem to need the maths to be grounded in the real world in order to grasp the concepts. The key result is that using real world problem solving is not necessary for learning mathematics in an abstract form; rather it has the potential to improve mathematics skills and understanding for engineering students.

The majority of engineering students agreed that they would have learned more had the project been more explicitly linked to other courses they were studying. This was true across all engineering disciplines, but not an issue for mathematics or physics. While the large scale highly funded coalitions (Al-Holou et al., 1999), had the resources and leadership to create linkages across the larger curriculum, this is not yet possible in smaller scale endeavours such as ours, where only two courses are involved. For example, the only direct follow-up from mathematics inside an engineering course occurred in thermodynamics, where students were reminded of the pipe heat loss project when calculating heat loss from a pipe in their in their third year chemical engineering course. However, it is evidence that larger scale curriculum reforms would be valued by students were they to be implemented.

Problem-based analysis: The large majority of students (84%) indicated that they remembered which project they had completed, and those students responded more positively to all survey statements than the students who did not remember which project they completed (see Table 3).

The LRC circuit, followed by the heat loss project (pipe), received the strongest positive feedback from the students across all categories. The winding number project was deemed to be very effective in developing deep understanding of mathematics, but did not receive positive feedback to other questions such as helping to apply mathematics. This is likely because it was the most abstract of the projects, and did not involve a modelling component, or dimensional variables. It was included as an option for non-engineering students in the class who were more interested in abstract mathematics.

When choosing their assignment topics in the integrated mathematics course, the students were not told which projects aligned with their individual disciplines; however, at least 30% of students in most engineering disciplines chose the project most aligned with their field of study. In a comment on the survey, one student suggested that *each project should be labelled with its corresponding engineering discipline, so that students could choose the project most applicable to their future studies*.

Conclusions

Poor mathematics knowledge and ability in students poses a major barrier to student learning in all engineering disciplines (PCAST, 2012). In this report, we have shown that incorporating applied dimensional problems in mathematics courses has the potential to improve the ability of engineering

. Table 3: Problem-based analysis: Percentage of students who agreed/strongly agreed (A/SA) or disagreed/strongly disagreed (D/SD) with the survey statements listed, for the problem which they completed, when surveyed 30 months later. .

Problem	**LRC**	**Pipe**	**Sat.**	**Dam**	**Wind. No.**	**Overall (remem. project)**	**Didn't remem. project**
Number of students	20	16	16	25	21	**97**	18
Survey statement 4	**Doing the project helped me learn to apply maths to engineering/physics problems.**						
A/SA	80	88	64	79	48	**72**	17
D/SD	10	0	21	8	19	**12**	22
Survey statement 5	**Doing the project helped me learn to develop a deeper understanding of mathematics.**						
A/SA	80	81	57	56	71	**69**	22
D/SD	10	6	7	20	10	**11**	29
Survey statement 6	**Doing the project helped me remember the maths I learned in MATH2000 longer than I would have if it was just a standard maths course.**						
A/SA	55	13	50	44	38	**41**	29
D/SD	25	25	36	36	52	**35**	47
Survey statement 7	**Doing the project enhanced my mathematical modelling and problem solving skills.**						
	85	50	79	60	40	**62**	17
	10	13	14	16	10	**13**	39
Survey statement 10	**I would have learned more if the project problem had been linked more explicitly to courses that I went on to study.**						
A/SA	60	63	29	60	52	**54**	56
D/SD	5	19	14	24	29	**19**	6
Survey statement 8	**The project in MATH2000 was not beneficial to me.**						
A/SA	20	25	21	16	14	**19**	39
D/SD	75	44	50	68	62	**61**	39
Survey statement 12	**Overall, was the project in MATH2000 2008 valuable for your learning?**						
A/SA	80	81	64	72	62	**72**	44
D/SD	5	6	14	16	19	**13**	29

students to apply mathematics, and so address a well-discussed and well-documented need in engineering education. One of the key benefits of this approach is that it can be led by informal collaborations between teaching staff from engineering and mathematics, and so does not depend on institutional leadership or curriculum reform.

Applied, dimensionalised problems taught in tandem with mathematical theory can enhance both mathematical and problem-solving skills of engineering students. However, generating such problems in mathematics is non-trivial: it requires time, knowledge of both the engineering problem and the mathematics, and the ability to map the higher order relationships across the two problem domains. In spite of the challenges, it is possible to provide integration for students through small-scale partnerships across disciplines. Once the mappings between problem domains have been realised, and a problem set has been created, these can be reused in tutorials, or modified for use as assessment items again at a later date. Ideally, these resources can be pooled across different universities for maximum benefit. The existing coalition's websites which give examples of integration efforts, would be ideal places for these problems to be shared (e.g. Foundation, Synthesis, etc.; see Al-Holou et al., 1999).

Our analysis of student marks suggest that there are benefits in using applied problems in a mathematics course, both for how students do in the mathematics course as well as for how they fare in related engineering courses.

Engineering students in this study appeared to value application of mathematics to real world problems. A key (although perhaps unsurprising) finding is that the mathematics students in this study did not see the value of the application of mathematics to real world problems as much as the engineering students. This is consistent with the findings of McKenna et al. (2001) who investigated issues that students had when applying integration to real world problems, finding that students perceive mathematics to be abstract and irrelevant to other subjects. This could be due to the way it has always been taught. Our attempts at integration and other attempts like Otung (2002) show that it is possible to teach mathematics in a different way.

Our results also suggest that students may change their perspective on the value of different learning activities over the course of their degree, and that both short and long-term surveys are useful in assessing teaching practices.

The tendency for students to grind through the working out of a mathematical solution and thus not make the essential conceptual links between the mathematics to the context is a real issue with all engineering problems that involve mathematics. The issue that we were trying to address here was that of helping the students to be able to later 'see' the mathematics needed to solve the problem based on seeing it once before when learning the mathematics. The benefits associated with this method could be increased through explicitly linking each specific assignment project to the engineering discipline and courses from which it originated. This could be done with some very simple actions, requiring very little time from teaching staff. For example, the projects should be clearly labelled according with the appropriate engineering discipline. Engineering teaching staff could make a short guest appearance in the mathematics course to explain briefly the origin and significance of their problem, and the engineering course in which students will later encounter the problem. This extension to the method could help to prevent students from tackling the assignments by merely working through the a mathematical equations, and thus not really creating any long lasting conceptual understanding of how the mathematics applies to the problem at hand. We are embarking on further projects to try to address the issue of how to help students to 'see' the mathematics in an engineering problem.

Creating links between previous and current learning is known to benefit student learning (Coppola et al., 1997; Polya, 1971). Thus, addressing the same problem firstly from a mathematical perspective and then in an engineering context should help to address the key issue of engineering students ability to apply mathematics, and so increase their capacity to learn in many engineering courses. Incorporating applications in mathematics courses has a number of benefits for students (Al-Holou et al., 1999) and we have presented a feasible way for individual lecturers to make this happen. Developing authentic, dimensionalised problems for engineering students requires time and genuine collaboration between mathematicians and engineers (on a personal, rather than curriculum review level). The process of developing these problems also develops links and collaborations between engineering and mathematics teaching staff, which can benefit education in multiple ways.

Such an endeavour takes courage and patience and perseverance for all who embark on this journey, and it is not for the faint-hearted. The key barriers to widespread uptake of this process are lack of time and lack of collaborative networks. University management can promote these through encouraging and rewarding collaboration and teaching innovations; thus actions can be taken at department, faculty or organizational scale to address this critical issue in engineering education.

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Copyright statement

Supplementary material Appendix A: four sample problems

We include four problems used in this study, collaboratively designed by engineering and mathematics staff.

Problem 1: Heat loss from a pipe

In almost all process engineering plants, fluids are transported though piping. If the pipe and its contents are hotter than the surrounding environment, heat will be conducted through the pipe wall to the surrounding air. This loss of heat is a waste of energy, which will increase the carbon footprint of the process plant. Furthermore, if enough heat is lost from the pipe, the pipe contents may thicken or solidify, resulting in damage to pipes or pumping equipment, a disaster for any engineering process.

Objects of unequal temperatures in a thermal system tend toward thermal equilibrium. The hotter object transfers some of its heat to the colder object until the objects are the same temperature. Just as water flows down a pressure gradient, heat is conducted down a temperature gradient. Hence for heat conducted through a pipe, the rate of temperature change over time is a function of the temperature gradient:

$$\rho \, C_p \frac{dT}{dt} = k\nabla^2 T \qquad \text{Eqn 1}$$

where: k is thermal conductivity ($Wm^{-1}K^{-1}$), which depends on the pipe wall

T is temperature (°C or K); t is time (s); C_p is specific heat ($J\ kg^{-1}K^{-1}$, not used in this project) and ρ is the density of the pipe wall (kg/m^3).

In cylindrical coordinates, the steady state form of Eqn 1 simplifies to:

$$0 = \frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} \qquad \text{Eqn 2}$$

where r is the radius of the pipe (m).

Fourier's law relates energy transfer to temperature gradient and thermal conductivity:

$$\frac{q}{A} = -k\frac{dT}{dr} \qquad \text{Eqn 3}$$

where q is the rate of heat loss from the pipe ($W=Js^{-1}$), and A= area through which the heat is conducted, i.e. the area of the pipe wall (m^2).

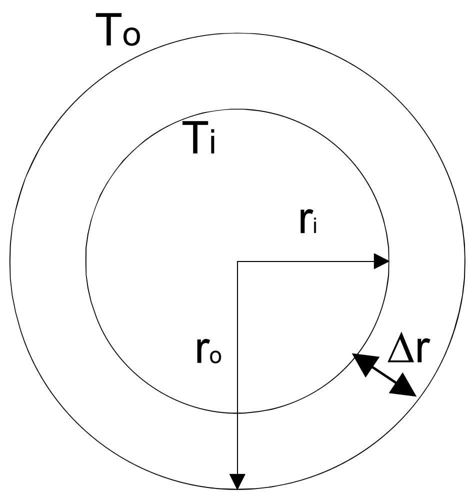


Fig 1: Cross-sectional view of a pipe. r_o and r_i are the other and inner radii and T_o and T_i are the temperatures at the outer wall and inner boundaries of the pipe (prescribed).

(a) Solve Eqn 2 for the temperature throughout the wall of the pipe using the boundary conditions provided on the diagram.

(b) Using your solution to Eqns 2- 3, derive an equation for q, the rate of heat loss from the pipe under steady state conditions, as a function of pipe length h, the temperature of the inside of the pipe wall T_i, and the temperature of the outside wall of the pipe T_o. (The inner radius of the pipe is r_i and the outer radius is r_o, as shown in Fig. 1). Consider ONLY conduction of heat through the pipe wall (ignore heat loss along the pipe length). Does q depend on r in this case? Explain why or why not.

(c) A carbon steel pipe (inner diameter 50 cm, wall thickness of 20 mm) is used to transport high pressure steam around a dairy processing plant. If temperature of the inside wall of the pipe is T_i=150 °C and the outside wall temperature is T_o=30 °C, what is the rate of heat loss per metre of pipe?

(d) If the factory contains 2 km of this steam piping and operates continuously, what will be the total energy lost from steam pipes over a year?

(e) Assuming 0.1 kg of CO_2 are released per MJ of energy consumed, what total annual greenhouse gas emissions correspond to the wasted energy?

(f) Determine the temperature in the pipe wall, midway between the inner and outer walls.

(g) How will the rate of heat loss per unit length of pipe change if the pipe diameter is doubled?

(h) How will the rate of heat loss per unit length of pipe change if the pipe thickness is halved? Compare this with the effect of doubling the diameter.

(i) Find an expression for the mass flow in the pipe F (kg/s)

(j) Find an expression for the mass flow in the pipe F (kg/s), in terms of the steam density ρ_{steam} (kg m^{-3}) and the steam velocity v (ms^{-1}).

(k) Heat loss per unit length may not be the most appropriate basis to consider the heat loss from the pipe if the diameter is changed. Assuming the same velocity and density of steam in both cases, how will doubling the pipe diameter effect Q, the heat loss per unit mass of steam per unit length of pipe? You will need your answer to (i).

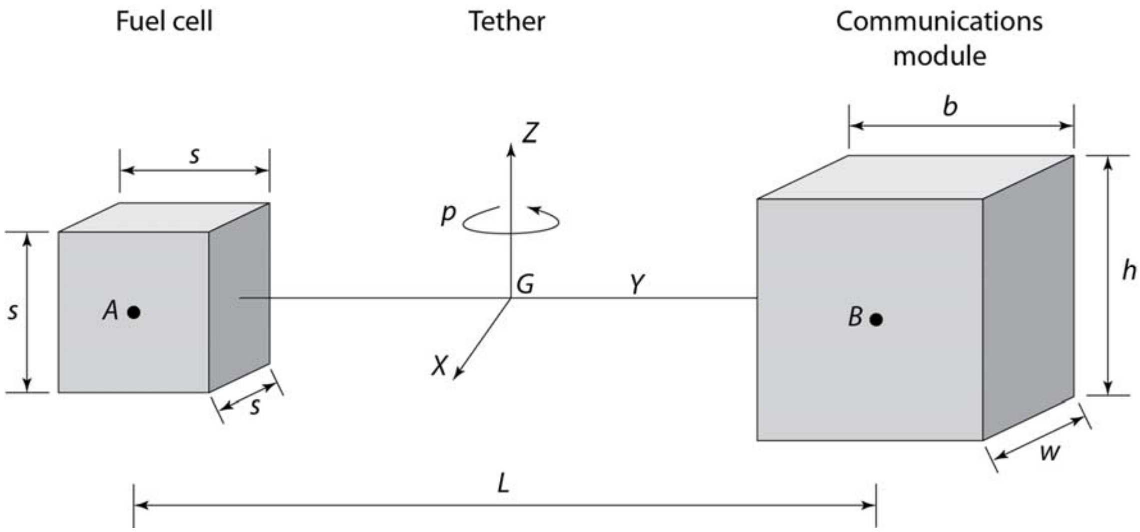
(l) For the carbon steel pipes and temperatures given above, determine whether doubling the pipe wall thickness or doubling the internal diameter of the pipe would be more successful at reducing the energy loss in the plant.

(m) The thermal conductivity and thickness of pipe wall vary for different construction materials (Table 1). For a pipe with inner diameter 50 cm, which of these materials in Table 1 will maximise heat loss from the pipe? Which will minimise heat loss from the pipe?

Table 1 - Common pipe heat transfer characteristics.

Material	Thermal conductivity k ($Wm^{-1}K^{-1}$)	Thickness of pipe wall Δr (mm)
Aluminium	250	17
Brass	109	10
Carbon Steel	54	20
Stainless Steel	16	25
PVC	0.19	30

Problem 2: Satellite problem



A spinning satellite system consists of a communications module and a separate fuel cell module tethered with an adjustable length *L* between two individual centres of masses, A and B. The system spins about the *Z* axis centred at the system centre of mass G and has the following parameters:

- Fuel cell is a cube with side length *s*, mass M_A and uniform mass distribution.
- Communications module is a cuboid of width b, height *h* and width *w* , mass M_B and uniform mass distribution.

(a) Determine the position of the system centre of mass G (X_G, Y_G, Z_G) as a displacement from the fuel cell centre of mass A. Choose A as the origin for your calculations. Explain if and how the geometry of the satellite components affects the system centre of mass position.

(b) Using the definition, determine the mass moment of inertia components I_{BX} of the communications module about its centre of mass B. Use symmetry arguments to write down I_{BY}, I_{BZ}. Write the answers in terms of the mass M_B .

(c) Determine the mass moment of inertia components I_{AX}, I_{AY}, I_{AZ} of the fuel cell about it's centre of mass A (The result can be simply obtained from (b)). Write the answers in terms of the mass M_A

(d) Determine the mass moment of inertia components of the system I_{GX}, I_{GY}, I_{GZ} of the satellite system about it's centre of mass G by calculating the fuel cell and communication module components directly with respect to G. The calculations are simpler if you retain Y_G if you answer *

(e) Repeat (d) by using your answers in (b) and (c) and using the shifting axis theorem (also called the parallel axis theorem).

* In (d) you may wish to make use of the identity $(p+q)^3-(p-q)^3= 6 p^2 q + 2 q^3$. If you use it you should also write down a quick proof.

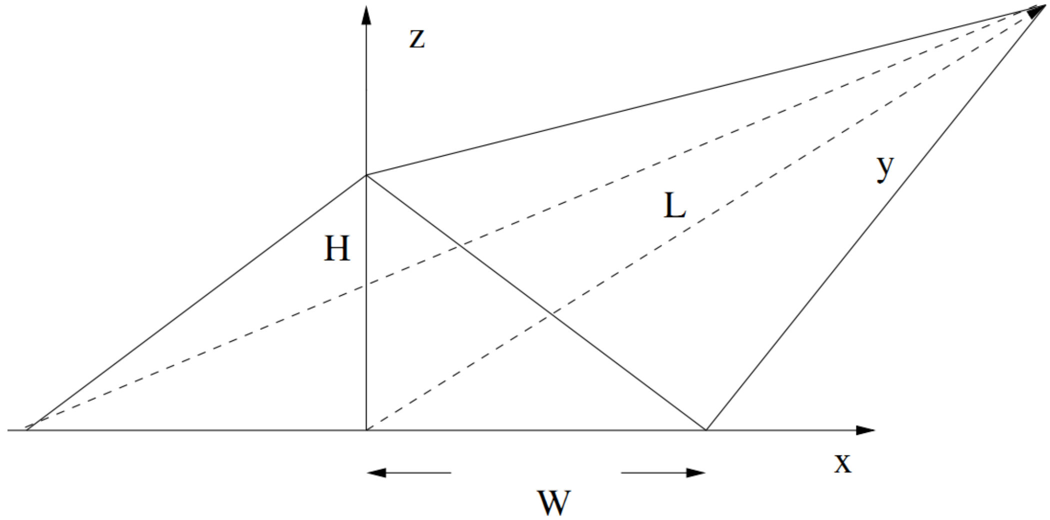
For the following questions you are required to determine numerical answers. Take the fuel cell to have side length *s*=0.4m, mass 45kg. Take the communications module to have dimensions *b*=5m, *h*=4m, *w*=3m, mass 200kg. Assume the initial length *L*=10m and spin rate *p*=1Hz.

(f) Based on your inertia calculations determine the angular momentum components of the satellite system about its centre of mass ($H_i = I_{Gi}\omega_i, i = X, Y, Z$).

(g) The tether is now slowly extended. The external forces on the satellite are negligible so the angular momentum is conserved. Determine what length *L* the tether needs to be extended to in order to reduce the spin rate by 50% from its initial value.

(h) Predict and explain if and how your answers in f) and g) would change if the communications module was in the shape of an ellipsoid with the same mass and mass moment of inertia components.

Problem 3: Dam water depth



Consider a dam which is modelled using the two tetrahedrons shown above. The dam wall has height *H* at its centre and width *2W* . The dam extends a distance *L* up river. The dam is drawn upside down to make calculations easier.

(a) Find the equation of the plane for the dam bottom ($x > 0$).

(b) In order to monitor the water storage in the dam, you are asked to determine at what height you should put markers at the centre of the dam wall (i.e. the *z*-axis to indicate when the dam is fraction *f* full (so $0 \le f \le 1$). Provide a formula for the height as a function of the fraction *f*, i.e. find *f (h)* and hence *h(f).* Also provide the full capacity of the dam.

(c) If the catchment area is *A*, determine a formula for the “height” of rain which must fall in the catchment in order to fill the dam. Assume that the dam is initially empty, there are no outflows, and that all the falling water collects in the dam.

(d) A field rain gauge has the shape of a perfect sphere of radius *R*. It has a small opening of area *α* at the top. Your goal is to determine at what height (relative to the vertical dimension) you need to place a mark to indicate that *r* mm of rain have fallen. First express the fraction that the gauge is filled as a function of height.

In the following questions about the dam and rain gauge you must provide numerical answers for the parameters given.

Dam: *H*=100m, *W*=150m *L*=100km *A*=1500 km2. The numbers are based roughly on Brisbane’s Somerset Dam (Queensland, Australia).

Gauge: R = 40mm, α = 81mm^2.

(e) For the dam provide the actual heights on dam wall to indicate when the dam fraction has reached *f* = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7.0.8, 0.9, 1.

(f) For the specified parameters give the actual required rainfall to fill the otherwise empty dam.

(g) Indicate where you would need to place a mark to indicate that the gauge is 40% full. The actual point must be read off a graph (unless you wish to use the formula for solving cubic equations). How much rainfall (in mm) has fallen when the gauge is 40% full?

Problem 4: LRC circuit

In an LRC circuit the voltage drop across an inductor, capacitor and resistor is given by

$$V_I = L\,\frac{dI}{dt},\ \ V_c = \frac{1}{C}\int_0^t I(s)ds \ \text{ and } V_R = I\,R.$$

If they are arranged in series, Kirchoff 's law states that $V_I + V_I + V_I = V$ or

$$L\,\frac{dI}{dt} + \frac{1}{C}\int_0^t I(s)ds + IR = V(t) \qquad (1)$$

where V (t) is the applied voltage. Below we assume the latter to be of the form V = E_0 sin(ωt). Taking the derivative of (1) gives

$$L\,\ddot{I} + R\,\dot{I} + \frac{1}{C}\,I = \frac{dV}{dt} = E_0\,\omega\cos\omega t. \qquad (2)$$

(a) For L=1,R=2 C=1/3, w=1 and E_0=20, use the method of undetermined coefficients to find the general solution for I . Identify the transient and steady state solutions, and explain what will happen after a long time has elapsed.

(b) For general L, R, C and E_0 find only the steady state solution to (2). Show that your solution can be written as

$$I = \frac{-E_0 S}{R^2 + S^2}\cos\omega t + \frac{E_0 R}{R^2 + S^2}\sin\omega t,$$

where the reactance is $S = \omega L - \frac{1}{\omega C}$.

(c) Write your solution as I_0 sin(ωt − θ). Find I_0 and θ.

(d) Consider the complex ODE:

$$L\,\ddot{J} + R\,\dot{J} + \frac{1}{C}\,J = E_0\,\omega\exp(i\omega t) \quad i = \sqrt{-1}$$

Use the method of undetermined coefficients with the (complex) trial particular solution $J_P = K\exp(i\omega t)$ to find K. Write your answer in terms of R and S.

(e) Find the real part of your solution from part (d). It should agree with your solution from part (b). Why?

(f) The complex impedence Z is defined by Z = R + iS. Show that

$$K = \frac{E_0}{iZ}.$$

The equation is similar to I = V/R for DC circuits.

(g) Use the complex method of part (d) to derive the steady state solution you obtained in part (a). (Re-derive the solution rather than using your formula from part (d).

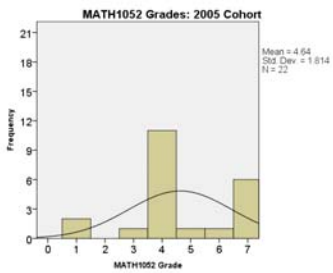
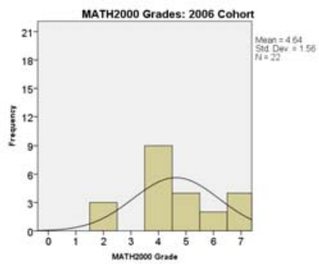
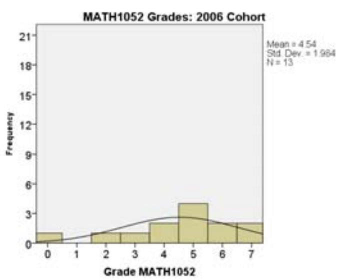
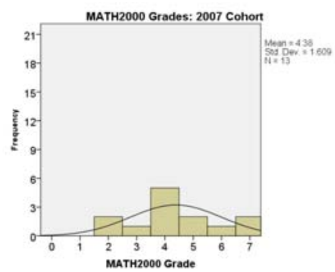
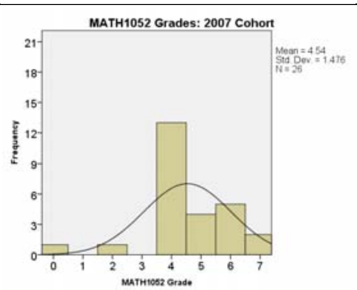
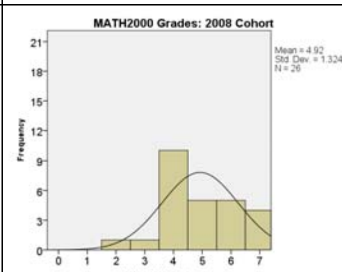
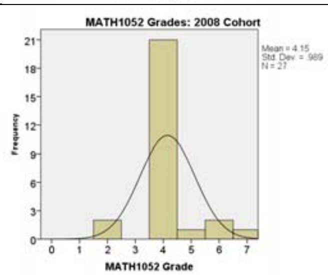
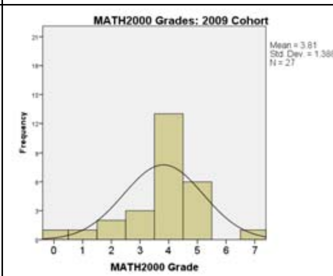
(h) In the steady state the energy supplied to the circuit must be equal to the energy dissipated by the resistor. This is because no energy is consumed by the inductor or capacitor (although energy may be temporarily stored in these components). In one period (T = 2π) the energy loss and input are given by

$$\int_0^{2\pi} I^2 R dt \qquad \int_0^{2\pi} VI dt$$

Show that these two quantities are indeed equal by direct computation. Here V = 20 sin t is the supplied voltage and I is the current in the circuit obtained in (a) and (g).

BONUS QUESTION: Can you prove that the above two integrals must be equal? The proof involves multiplying equation (1) by **I** and integrating.

Appendix B: The shift in grade distributions from first to second year mathematics for the last 8 years (2005/6 to 2012/2013), with corresponding within subjects t-tests.

Cohort Lecturers	First year math grades	Second year math grades	Mean 1 to Mean 2 shift Within subjects t-tests
2005/6 Z/R,I			4.64 to 4.64 No change t_21 = 0.0 p = 1.0 NS
2006/7 H,I/R,I			4.54 to 4.38 Decrease t_12 = .35 p = .73 NS
2007/8 Z/RI			4.54 to 4.92 Increase t_25 = -2.18 p = .04 Significant
2008/9 Z/Z,I			4.15 to 3.81 Decrease t_26 = 1.2 p =.24 NS

<table>
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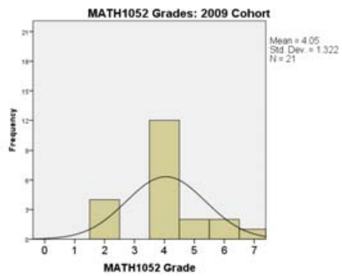
Da,Do/I</td>
<td>
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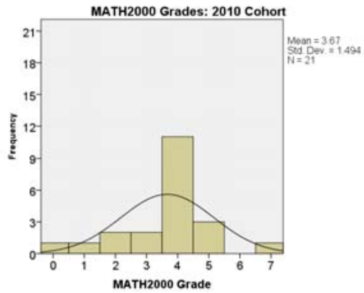
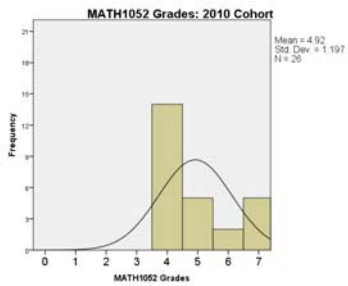
<td>
</td>
<td>4.05 to 3.67
Decrease
t_20 = 1.16
p = .26
NS</td>
</tr>
<tr>
<td>2010/11
Do/I</td>
<td>
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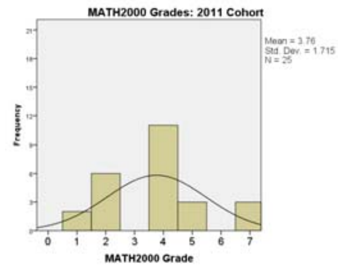
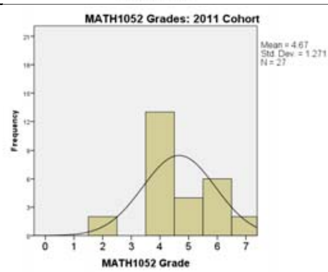
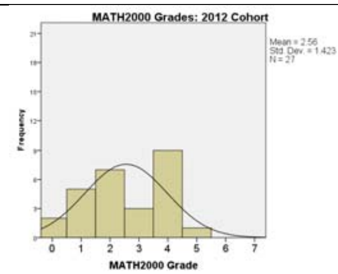
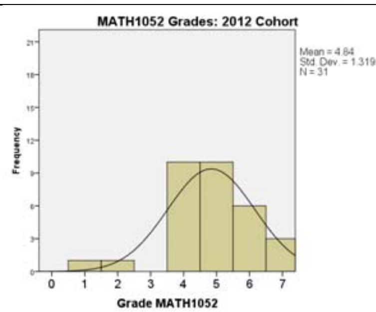
<td>
</td>
<td>4.96 to 3.76
Decrease
t_24 = 4.00
p = .001
Significant</td>
</tr>
<tr>
<td>2011/12
Do/R</td>
<td>
</td>
<td>
</td>
<td>4.67 to 2.56
Decrease
t_26 = 7.43
p = .000
Significant</td>
</tr>
<tr>
<td>2012/13
Do/I</td>
<td>
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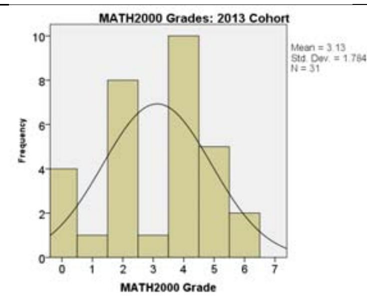
<td>
</td>
<td>4.84 to 3.13
Decrease
t_30 = 4.96
p = .000
Significant</td>
</tr>
</table>

Notes

- The mean was higher in 2008 for second year mathematics than any other year
- 2008 is the only year second year mathematics went up from first year
- The same lecturers taught first year and year second year mathematics in the 2005/2006 years as taught the 2007/8 years (i.e. Z in first year and R,I in second year) which is the best control available in the analysis
- The same lecturer taught second year mathematics in 2008 (the experimental group) and 2102. The mean for the 2012 class was 2.56, as opposed to 4.92 for the experimental group.