

# Philoponus and the Subtraction Argument

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The subtraction argument for the existence of an empty world has been challenged in the case of where the world has an infinite number of objects. Drawing on the reconstruction of Philoponus' traversal argument by George Couvalis (2013). I argue that a subtraction argument based around time units can adequately deal with worlds that are inhabited by an infinite number of objects.

## Introduction

An *empty world* is a possible world devoid of any concrete particulars. Concrete particulars are objects that have a spatio-temporal location. Abstract particulars by contrast have no spatio-temporal location. In that sense, an empty world may contain abstract particulars such as numbers, sets, or even perhaps, laws of nature. But it will not have any concrete particulars such as tables, chairs and teapots. The philosophical view that maintains that there is a possible world that is the empty world is called *metaphysical nihilism*.

Why should we be interested in the empty world? Philosophical interest in the empty world relates to the fundamental metaphysical question of *why is there something rather than nothing?* The word "nothing" in this context is ambiguous between *why are there any concrete particulars at all?* and *why is there anything at all?* (including abstract objects). In this paper I shall take the meaning of "nothing" to refer to "no concrete particulars". Thus, if we are interested in the question of *why there is something rather than nothing*, we are in fact asking *why are there any concrete particulars at all?* If one maintains that there is no empty world then this is to say that in all possible worlds there are concrete particulars. That is, it is impossible that there "be nothing". If that is the case, then the fundamental metaphysical question is answered by maintaining that *the reason* why there is something rather than nothing is because it is impossible for there to be an empty world.

Baldwin (1996) presents what he calls *the subtraction argument* to establish the possibility of there being a world with no concrete particulars. The argument proceeds by establishing a set of premises which allow for the possibility that 1) There is

a world with a finite number of concrete particulars. 2) No concrete particular in this world necessarily exists. And 3) the non-existence of any one of these objects does not necessitate the existence of any other. The argument proceeds by moving from world to world “subtracting” objects until one arrives at an empty world.

Baldwin’s subtraction argument has taken heavy fire on each of its three fronts, in particular, the claim that there is a finite world. If it is a metaphysical truth that each world has an infinite number of objects then the “subtraction process” will fail, since, for each object that is subtracted, the totality will not be diminished.

In this paper I will discuss Baldwin’s subtraction argument for the existence of an empty world. I will consider one of the key objections against the subtraction argument, namely, that the world *must* contain an infinite number of objects thus rendering subtraction impossible. Following this I will present Couvalis’ reconstruction of Philoponus’ traversal argument against the infinite past time of the world. Next, I will show that if we relate the creation of objects to the creation of time units, then a subtraction argument along Philoponan lines in a world with an infinite number of objects still makes subtraction possible. I will then consider some possible sources of objections. I will conclude that a subtraction argument based on time shares similar weaknesses to Baldwin’s subtraction argument. Namely, if time is infinite then the subtraction process will fail.

## The Subtraction Argument

Baldwin (1996:232) maintains that there are three fundamental metaphysical assumptions for the subtraction argument. These are listed as A1 to A3 respectively:

- (A1) There might be a world with a finite domain of concrete objects.
- (A2) These concrete objects are, each of them, things which might not exist.
- (A3) The non-existence of any one of these things does not necessitate the existence of any other such thing.

I will render the argument in an informal version of a modal logic called S5. The idea here is that there is an infinite number of possible worlds inhabiting a logical space. Each world in an S5 logical space can “see” or have access to, each other. In S5, if cats are objects in one world then they are possible in all worlds. This is not necessarily the case in modal logics weaker than S5. The subtraction argument now runs as follows.<sup>1</sup>

Starting from the actual world  $w$ , by A1 there is a world  $w^1$  that consists of a finite domain of concrete objects. Let  $c_1$  be a member of this domain, then by A2 there is a world  $w^2$  that has all the members of  $w^1$  except for  $c_1$  and any other things whose existence is implied by  $c_1$ . Since by A3 the non existence of  $c_1$  in  $w^2$  does not necessitate the

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<sup>1</sup> Baldwin appears to assume S4. I assume S5 to simplify the argument without loss of generality.

existence of any new thing in the domain of  $w^2$ , it follows that the domain of  $w^2$  has less members than  $w^1$ . The process is repeated until we reach a world  $w^{\min}$  that has one or more concrete objects such that the non existence of any one of these objects implies the non existence of all. By A2 the non existence of any of these objects is possible, and since by A3 the non existence of any of these objects would not bring anything new into existence, there is a world  $W^{\text{nil}}$  which contains none of these objects.  $W^{\text{nil}}$  is an empty world consisting of no concrete particulars.

I will now discuss objections to A1 of the subtraction argument. My purpose here is not to examine the objections in detail, but rather show how they might motivate the view that it must be a metaphysical truth that each world has an infinite number of objects, to the conclusion that A1 is false.

Baldwin (1996:233) draws one source of objections to A1 of the subtraction argument from Lewis (1986:81–86). Lewis maintains that if the members of a set are concrete then the set must be concrete too. Let us call this “the Lewis assumption”. Unit sets are sets with just a single object, so if the object happens to be a concrete particular, like those imagined in the subtraction argument, then a set which has a concrete particular as its sole member, must, according to the Lewis assumption, be a concrete object as well. Thus, since Socrates is a concrete particular the unit set whose sole member is Socrates must be concrete as well. In this sense, “concreteness” is a hereditary property of unit sets. And since it is possible to apply the unit set operation *ad infinitum*, if the progenitor of an infinite unit set chain is concrete, then every unit set descendant will be concrete too. Thus, for any world that has a single concrete particular, by the Lewis assumption, that world will have an infinite number of concrete particulars in virtue of the unit set operation.

Even if one did have doubts about the Lewis assumption and a universe with an infinite number of objects built upon a foundation of sets, Rodriguez-Pereyra (1997:163) have contributed a second source of objections. They argue that since the parts of concrete objects are themselves concrete, and if each space-time region is infinitely divisible, then if a concrete object exists in some space-time region, then it must have infinitely many concrete parts. Thus if one concrete object exists, then infinitely many will exist.

I have listed just two ways in which the world could have an infinite number of objects, though no doubt many more could be envisioned. If the subtraction argument is to meet this threat then it cannot do so by dealing with each counter-example to its finiteness assumption (A1) one objection at a time. Rather we must modify the subtraction argument such that it can still be successful even if the universe has an infinite number of objects. But if infinite collections are immune to subtraction, then how can subtraction work at all? The way forward for the friend of subtraction lies with Couvalis’ reconstruction of Philoponus’ traversal argument. I will now discuss the traversal argument, following this, I will show how it can be motivated as an intuition pump for a reconstructed subtraction argument.

## The Traversal Argument

According to a report by Simplicius (Wildberg, 2007), Philoponus' traversal argument against Aristotle's belief that the past is infinite, consists of three assumptions: (1) If the existence of something requires the pre-existence of something else then the first thing will not come to be without the prior existence of the second. (2) An infinite number cannot exist in actuality nor be traversed in counting nor be increased.<sup>2</sup> (3) Something cannot come into being if its existence requires the pre-existence of an infinite number of other things, one arising out of the other.

Intuitively the traversal argument can be captured by the following analogy. Suppose Socrates has an infinite number of ancestors arranged as a (non dense) descending chain. Suppose further from (1) that Socrates could only have existed if the previous member existed first, and for each member of the chain its existence is causally determined by its previous member and so on. But since the chain is infinite and each member requires the existence of a previous member to come into being, then by (2) no matter how far we go back *to begin* the causal process which eventually brings Socrates into being, we must always go back one more and so on *ad infinitum*. Thus given these assumptions, by (3) Socrates cannot have an infinite number of ancestors.

Couvalis (2013:69–70) argues that the traversal argument is not a purely logical argument, that is, a piece of formal apparatus which provides a model based on purely logical or mathematical principles. Rather, it is a metaphysical argument which draws upon assumptions about what we think some feature of the world is like, in order to make its case. Specifically, Couvalis draws attention to the role that the A-theory of time is playing in Philoponus's argument in order to make the case for the finite past time of the world. My purpose here is to show how the traversal argument can be used as a basis to reconstruct the subtraction argument. To that end I will also need to draw attention to the role that *time* is playing in the reconstruction that I am offering. I will now set the groundwork for that task with a discussion of the distinction between the A-theory of time and the B-theory of time.

The A-theory of time is the view that there is an ontological difference between the past and the future. Informally it is the idea that the past exists or has existed but the future does not yet exist. It is often expressed as the view that time flows, giving an objective difference between the past and the future. The B-theory by contrast is the view that there is no ontological difference between the past and the future. Events in time are merely earlier than or later than other events. The B-theory is most commonly thought of as the “block universe” view of time and space. Couvalis in his reconstruction of the traversal argument, makes explicit the A-theory assumptions

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<sup>2</sup> This assumption appears question begging. It is important to remember that Aristotle like Philoponus doesn't think that actual infinities can exist. The traversal argument shows that regardless of whether the past is considered to be an actual or potential infinite, it cannot be traversed. On this point see Couvalis, 2013:70–71.

that Philoponus tacitly invoked in his traversal argument. Couvalis' reconstructed traversal argument is given below (2013:72).

- 1) The past and the present consist of time units.
- 2) If the A-theory of time is true, time has come into existence successively as a series of units.
- 3) If the A-theory of time is true, every unit of time (except a first unit) can only come into existence when its predecessor has come into existence before it.
- 4) The A-theory of time is true.
- 5) There cannot exist a present member of an infinite series of units in which each member, past and present, could only come into existence after its predecessor has come into existence.
- 6) No unit of time can come into existence that has an infinite series of units preceding it coming into existence in succession before it (inferred from premises 1–5).
- 7) All units preceding the present unit are past units that came into existence in succession before it.
- 8) A present unit exists.
- 9) So, there cannot be an infinite number of past units.

So, Couvalis' reconstructed traversal argument seeks to make explicit the role that the A-theory of time is playing in Philoponus' traversal argument. Or, to draw on the Socrates' ancestor analogy above, it could be seen as making explicit the role that the A-theory of time is playing in that analogy. To demonstrate, consider what the Socrates' ancestor analogy world be like if time were B-theory like. In the B-theory there is no requirement that time units come into being in succession one unit at a time. It is possible in a B-theory like world for the whole temporal chain to come into existence *holus bolus*. So if the ordering relation of time were like the integers with Socrates at 0, and if we suppose that the past *is not* ontologically different from the future then, Socrates could have an infinite number of ancestors. As such what Couvalis' treatment of Philoponus' traversal argument reveals, is that the A-theory of time or, A-theory like assumptions, are required for Philoponus' conclusion that past time must be finite.

So how does Couvalis' reconstructed traversal argument help us with the subtraction argument? With Couvalis' traversal argument we gain a new metaphysical fact. If a given world is A-theory like then it must have a finite past time. Philoponus' traversal argument however, *does not* preclude the world from having an infinite number of objects in it. Given the finite past time of an A-theory world, then it would seem possible to "count-down" the time units until we reach an empty world. It thus opens up the possibility for a reconstructed subtraction argument based around time. I will now motivate a reconstructed subtraction argument based around time that allows for a universe with an infinite number of objects in it.

## The Reconstructed Subtraction Argument

To demonstrate this, consider a chessboard universe, which has a finite number of squares along the x-axis or “time axis” such that each square along the x-axis is a time unit. Suppose further that there is an infinite number of squares up the y-axis and the y-axis represents “objects”. Thus the relation between the two is such that for each time-unit on the x-axis there are an infinite number of objects on the y-axis. Suppose further that instead of subtracting objects as we move from world to world, we subtract time units. Thus at each world, as we remove a time unit, we remove an infinite number of objects. Though the number of objects remains constant as we move from world to world, since one can never subtract from an infinite collection, nevertheless, *the duration* of the world is decreased. Finally we reach a world with a single time unit along the x-axis and an infinite number of objects, up the y-axis. After this time unit is removed the world will cease to have any objects in it, it will be an empty world.

I now will discuss each premise or underlying metaphysical assumption of the reconstructed subtraction argument in turn, highlighting any modifications or redundancies to the original subtraction argument.

The reconstructed subtraction argument will make explicit the claim that it is possible that there is an A-theory like world with a finite number of past time units. The claim that all A-theory like worlds must have a finite number of past time units is given by the conclusion Couvalis’ reconstructed traversal argument. The first metaphysical assumption of the reconstructed subtraction argument will also allow for the fact that even though these A-theory worlds have a finite number of past time units, they can nevertheless have an infinite number of concrete particulars. Thus we have RSA1:

(RSA1) There might be an A-theory like world with a finite number of past time units and an infinite domain of concrete objects.

Premise A2, however, from the original subtraction argument is still required. If any of these concrete objects necessarily exists, then there could not be a time unit where it did not exist. There is a further point to make about A2. For the reconstructed subtraction argument to work, it is required that no *time unit* necessarily exists. If the first time unit is considered to be concrete on account of a supposed causal efficacy and if it is also considered to necessarily exist, then it could not be shown that there is an empty world. For a reconstructed subtraction argument for the existence of an empty world, we would need to make explicit the assumption that no object (including time units if they are considered concrete) necessarily exists. By contrast, if it can be shown that the first time unit is concrete and necessary then the original subtraction argument and *a-fortiori* the reconstructed subtraction argument are unsound in virtue of their second assumption being false. It is beyond the scope of this paper however, to consider whether the first unit of time necessarily exists. Setting that issue aside then, we shall keep A2 but rename it as “RSA2”.

(RSA2) These concrete objects are, each of them, things which might not exist.

A3 from the original subtraction argument maintained that “The non-existence of any one of these things does not necessitate the existence of any other such thing”. This was required so that subtraction of objects did not automatically spawn new objects to replace them. Given, however, that we are now dealing with a world in which there is an infinite number of objects, this premise is no longer required. Instead we will substitute A3 with a new assumption. That is, that concrete particulars are connected to time units. This allows for the claim that when a time unit is “subtracted” then any object that came into existence at that time unit is subtracted too. More explicitly:

(RSA3) For each world  $w \in W$ , for each concrete particular  $c$ , and for any time  $t$ , if  $c$  came into existence at time  $t$  in  $w$ , then if there is a world  $w^*$ , such that,  $t$  does not exist at  $w^*$ , then  $c$  does not exist at  $w^*$ .

RSA3 is motivated by the thought that whilst Socrates might have been born at a slightly different time, (or a slightly different place) that is, maybe he was born premature, or could have been conceived a few months prior to his actual conception, it does not seem possible that he could have been born at a totally different time (and a totally different place). So if there is a world whose history extends only so far as, say, 10,000 years prior to the birth of Socrates, then we would want to say that Socrates is not a member of that world. That is, that Socrates has been “subtracted” from that world.

I will now summarise, the three key metaphysical assumptions of the RSA:

(RSA1) There might be an A-theory like world with a finite number of past time units and an infinite domain of concrete objects.

(RSA2) These concrete objects are, each of them, things which might not exist.

(RSA3) For each world  $w \in W$ , for each concrete particular  $c$ , and for any time  $t$ , if  $c$  came into existence at time  $t$  in  $w$ , then if there is a world  $w^*$ , such that,  $t$  does not exist at  $w^*$ , then  $c$  does not exist at  $w^*$ .

Assuming S5, RSA now runs as follows:

Starting from the actual world  $w$ , suppose  $w$  is an A-theory-like world with an infinite number of concrete objects but only a finite number of past time units. Let  $t_n$  be a member of this domain and the present temporal unit of this domain. Then by RSA2 there is a world  $w^2$  which has all the temporal units of  $w^1$  except for  $t_n$ . Let  $t_{n-1}$  denote the present temporal unit of  $w^2$ . Then by RSA2 and RSA3 any object  $c$  that came into existence at  $t_n$  in  $w^1$  no longer exists at  $t_{n-1}$  in  $w^2$ . The process is repeated until we reach a world  $w^{\min}$  which has only one temporal unit. By RSA2 the non existence of this temporal unit is possible and so, by RSA3, the non existence of any concrete particular which accompanies this temporal unit. Finally, there is a world  $W^{\text{nil}}$  that contains no time units, since  $W^{\text{nil}}$  has no time units, by RSA3 it contains no concrete particulars,  $W^{\text{nil}}$  is an empty world.



I will now consider a possible source of objections, namely, that the reconstructed subtraction argument is too dependent on the A-theory of time, as such, if our best science tells us that the A-theory is false then the reconstructed subtraction argument is unsound. I will also within the context of this discussion, further explore the role that the A-theory and B-theory of time play within the reconstructed subtraction argument. I will show that the reconstructed subtraction argument is not dependent on the A-theory of time, but it is dependent on time being finite.

## Is the Subtraction Argument Too Dependent on the A-theory?

Is the A-theory of time true? If there is good reason to think that it is not then the reconstructed subtraction argument is at risk of being unsound. Such an argument against the A-theory of time might be persuasive if we consider that our best science tells us that the B-theory is true. I will now show that the reconstructed subtraction argument is not dependent on whether time is A-theory like or B-theory like but only on whether the world has a finite number of time units.

Recall that B-theory like time maintains that there is no ontological difference between the past and the future. That is, all the time units of a B-theory like universe could have come into existence *holus-bolus*. Time units in this sense are to be treated like spatial coordinates. Though we occupy just one position on the time line, other positions on the time line are not ontologically different from the present one. If the entirety of space-time came into existence *holus-bolus* then, contrary to the traversal argument considered above, it is possible to have an infinite number of past time units prior to the present time unit. Consider again the case of Socrates and his ancestors, if Socrates and his ancestors came into existence in a B-theory like *holus-bolus* fashion then it is possible that there could be an infinite number of them. In such a universe the reconstructed subtraction argument would not work. The original subtraction argument failed in a world where there is an infinite number of objects. The reconstructed subtraction argument likewise fails in a world where there is an infinite number of time units.

But if our best science tells us that the B-theory is true, it also tells us that past time or “space-time” has a finite duration starting from the big bang to the present time unit. Thus even if one does concede the B-theory to the best science argument, one does not need to concede the point that past time, at least, is infinite. That is, there is a sense in which the universe is approximately 13 billion years old, *and that* is the amount of time it took us to get from the big bang to the present moment. Suppose then that the world is B-theory like and has a finite number of time units in its entirety. Under such a scenario, the “B-theoryness” of the world does not prevent the subtraction argument from working. For, if the world is B-theory like and has  $n$  time units, we can then imagine a possible world  $w^*$  that is B-theory like but has  $n-1$  time units in its entirety and so on. Eventually we reach a world with only one time unit.



So, the reconstructed subtraction argument is not so much dependent on whether the world is A-theory like or B-theory like, but rather, whether there is a finite number of time units. For example, consider now a B-theory like world with a finite number of *past time* units from its creation to the present time unit, but an infinite number of *future time* units. If we consider this world in *its entirety* then there is an infinite number of time units, as such subtraction will not work. In this sense it shares the same weakness as the original subtraction argument.

An anonymous reviewer has highlighted the following objection,<sup>3</sup> If the “best science” argument is right, then space-time and matter, as general relativity suggest, are to be considered as somewhat intimately related and not as distinct as the reconstructed subtraction argument suggests. I do not think there is an incompatibility here. The intended thought with the RSA3 is that if we view subtraction from the standpoint of “subtracting time”, then to say that “time is subtracted, is to say that space-time and matter at that “time unit” is subtracted too. It is not the case that we subtract a time unit and are left with “something”. More generally, worlds that are infinite in one respect may be finite in another. Whether subtraction is possible sometimes depends on the perspective that one takes on a given world.

## Conclusion

A necessary condition for the success of a reconstructed subtraction argument, that is, a subtraction argument based on time, be it A-theory time or B-theory time, is that there be a finite number of time units in its entirety. We can conclude from Couvalis’ reconstructed traversal argument that if the world is A-theory like, then it *must* have a finite number of past time units. Assuming then that no time unit necessarily exists, we can then conclude via a subtraction argument that there is an empty world. If the world is B-theory like however, and it can be established through perhaps a separate chain of argument that the temporal duration of the world is finite in its entirety, then regardless of the cardinality of objects that occupy it, a subtraction argument based on time to the conclusion that there is an empty world, is possible. Establishing through a subtraction argument that an empty world is possible if certain conditions are met, is however, only one aspect of the problem of *why there is something rather than nothing?* The problem of how something could come from nothing still leers ominously from the metaphysical darkness, ready to pounce.

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<sup>3</sup> A second objection was raised by this reviewer as to the nature of empty worlds. It is beyond the scope (and word limit) of this paper to give that topic the attention it deserves. See Coggins, 2010 for an analysis of that problem.

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