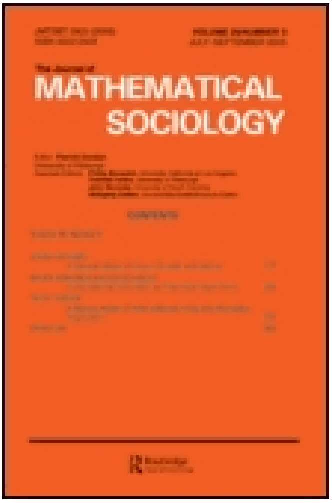

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An equilibrium-correction model for dynamic network data

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AN EQUILIBRIUM-CORRECTION MODEL FOR DYNAMIC NETWORK DATA

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We propose a two-stage MRQAP to analyze dynamic network data within the framework of an equilibrium-correction (EC) model. Extensive simulation results indicate practical relevance of our method and its improvement over standard OLS. In addition, empirical illustration shows that the EC-model yields interpretable parameters, in contrast to an unrestricted dynamic model.

Keywords: Network Analysis, Dynamics, Quadratic Assignment Procedure, Cognitive Social Structure

1. INTRODUCTION

In network analysis interest in longitudinal investigations increases (see for example Burt, 2000; Doreian & Stokman, 1996; Feld, 1997). Current models for these analyses are often based on Markov Chain methods (see Leenders, 1996, for overview). Although these models have proven to be useful (Snijders, 2001; van de Bunt, 1999), they do have some potential limitations. One such limitation is that these models do not make a distinction between "change" effects and "level" effects of explanatory variables. As we believe that this distinction is useful in network studies, we propose a model that explicitly incorporates "change" and "level" effects.

The model specification we propose to use is the equilibrium-correction model (EC-model), which is often used in time-series econometrics (see

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Greene, 2000). This model describes effects on temporal changes in a dependent variable, which can for example be relationship strength. An advantage of the EC-model is that it explicitly specifies effects of changes in explanatory variables over time (short-term effects) and effects of a variable that describes an equilibrium relation (long-term effects). Especially, as we will show later, when we consider cognitive variables such as those based on perceived networks the distinction between short-term and long term effects may be relevant.

In the dependent variable the EC-model mirrors models like a longitudinal p*-model (Robins & Pattison, 2001) and the actor-oriented SIENA-model (Snijders, 2001), which address the probability of relationship change. However, a difference is that the actor-oriented approach is defined in continuous time, while the EC-model we propose is defined in discrete time. Furthermore, we use an adjusted "multiple regression quadratic assignment procedure" (MRQAP) approach for statistical inference. This a non-parametric approach in contrast to SIENA, which is based on an explicit probabilistic network evolution model.

As is well known, inference on network data based on ordinary least squares (OLS) or non-linear least squares (NLS) can lead to spurious results. Autocorrelation (serial as well as structural) may lead to underestimation of standard errors, which makes correct inference based on these estimates impossible (see Johnston & DiNardo, 1996). Although the equilibrium-correction model handles serial autocorrelation, when it is considered for network data it seems wise to rely on a MRQAP approach for parameter testing (Hubert & Schultz, 1976; Krackhardt, 1988). MRQAP, which builds on the bivariate QAP work of Hubert (1987; Hubert & Schhultz, 1976), is a non-parametric method and therefore makes no a-priori distributional assumptions.

We should emphasize that the MRQAP approach that we use here tests the null hypothesis that all independent variables have a zero coefficient (Krackhardt, 1987b, 1988). In contrast, the actor-oriented approach tests the effect of a single independent variable, controlling for the effects of the other independent variables. Thus, although we use OLS in the estimation of the multiple-regression coefficients, the statistical inference we use is different from what is most commonly used.

The outline of the paper is as follows. In section 2 we first briefly discuss the equilibrium-correction model and the MRQAP approach. In Section 3 we report on the extensive simulations to check if the model works in practice. In section 4 we discuss an empirical illustration. In the final section we present our conclusions.

2. QAP-ING AN EQUILIBRIUM-CORRECTION MODEL

In econometric time series analysis the equilibrium-correction model is often used due to some nice features of this model. Most important, the model handles serial autocorrelation (which occurs when observations are dependent over time), while it also gives interpretable parameters. In the following we first discuss the advantages of the EC-model. Second, we discuss the MRQAP approach, which is practically relevant as network data are prone to structural autocorrelation because of the inherent row and/or column dependency between observed relations (Lincoln, 1984).

2.1 An Equilibrium-Correction Model

There are several ways to deal with serial autocorrelation in network data. Serial autocorrelation implies that the error terms ($\varepsilon_{ij,t}$) are correlated over time, for example like $\varepsilon_{ij,t} = \rho\varepsilon_{ij,t-1} + v_t$, with $0 < \rho < 1$, and where v_t might be distributed as $N(0, \sigma^2_v)$. In such data there is a correlation between observations in subsequent periods. In this exemplary case we can say that the data have a first-order dynamic structure. A general model to handle first-order dynamics is the so-called autoregressive distributed lag model, ADL(1,1) model, which is given by

$$y_{ij,t} = \beta_0 + \rho y_{ij,t-1} + \beta_1 x_{ij,t} + \beta_2 x_{ij,t-1} + e_{ij,t}. \quad (1)$$

In this model it is assumed that $y_{ij,t}$ depends on its own past, and also on current and past explanatory variables $x_{ij,t}$. Of course, (1) can be extended to include more than one explanatory variable, in which case $x_{ij,t}$ denotes a vector.

A potential drawback of (1) is that it may not always be easy to interpret the estimated parameters. For example, there is the possibility that β_1 and β_2 get opposite signs. One way to facilitate parameter interpretation amounts to rewrite (1) into the equilibrium-correction model, that is

$$y_{ij,t} - y_{ij,t-1} = \gamma_0 + \gamma_1(x_{ij,t} - x_{ij,t-1}) + \gamma_2(y_{ij,t-1} - \gamma_3 x_{ij,t-1}) + e_{ij,t}. \quad (2)$$

It is easy to see that the parameters in (2) are uniquely related with those in (1) by

$$\gamma_0 = \beta_0, \quad \gamma_1 = \beta_1, \quad \gamma_2 = (\rho - 1), \quad \text{and} \quad \gamma_3 = -(\beta_1 + \beta_2)/(\rho - 1). \quad (3)$$

The EC specification enables a sensible interpretation of the parameters. In the EC-model, γ_1 can be interpreted as the short term effect of x

on y as it captures the effect of changes in x on those in y. Furthermore, γ_3 can be interpreted as indicating the long-term equilibrium relation between y and x, while γ_2 measures the speed of adjustment of y to that long-term equilibrium.

For time series data, OLS (or NLS) yields consistent estimates of γ_1, γ_2, γ_3 (Greene, 2000, p.118–120). However, for network data, with potential structural autocorrelation it may not. To solve this issue, Krackhardt (1988) proposed a method for parameter inference that is robust against structural autocorrelation. This method we discuss next.

2.2 MRQAP To Handle Structural Autocorrelation

A major problem with network data is that it is sensitive to structural autocorrelation, and hence a straightforward application of OLS might result in spurious findings (see Greene, 2000; Jonston & DiNardo, 1996). Structural autocorrelation may occur because row and/or column entries in a socio-matrix are dependent. Krackhardt (1988) proposes the MRQAP as an inference procedure that is robust against structural autocorrelation. The MRQAP entails a non-parametric test for the significance of parameter estimates. It compares OLS parameter estimates based on the original data with a reference distribution of OLS estimates that are estimated using random data.

There are different approaches to generate MRQAP reference distributions (Krackhardt, 1987b,1988), which give similar results. An often-used approach permutes simultaneously the rows and columns of the dependent network data matrix to generate random data, which then become the basis for the distribution of the coefficients under a null hypothesis. The advantage of this particular form of permutation (simultaneous row and column permutation) is that any structural autocorrelation is preserved under each permutation. Thus, all coefficients derived to produce the null hypothesis distribution share this amount and form of structural autocorrelation (Krackhardt, 1988).

For example, assume

$$y_{ij} = \begin{bmatrix} y_{11} & \cdots & y_{1j} & \cdots & y_{1k} \\ \vdots & \ddots & & \cdot^{\cdot^{\cdot}} & \vdots \\ y_{j1} & & y_{jj} & & y_{jk} \\ \vdots & \cdot^{\cdot^{\cdot}} & & \ddots & \vdots \\ y_{k1} & \cdots & y_{kj} & \cdots & y_{kk} \end{bmatrix},$$

then one permutation of y_{ij} that keeps the possible structural auto-correlation intact is[1]:

$$y_{ij}^* = \begin{bmatrix} y_{kk} & \cdots & y_{k1} & \cdots & y_{kj} \\ \vdots & \ddots & & \iddots & \vdots \\ y_{1k} & & y_{11} & & y_{1j} \\ \vdots & \iddots & & \ddots & \vdots \\ y_{jk} & \cdots & y_{j1} & \cdots & y_{jj} \end{bmatrix}$$

With each randomly permuted matrix, the parameters are re-estimated, and these re-estimated parameters comprise the reference distribution against which the observed estimates are compared. A particular observed coefficient is deemed significantly different from random if it lies in the tails of this reference distribution.

2.3 Two Stage MRQAP

The method described above gives valid results for the null hypothesis that all regression coefficients are equal to zero. However, problems may arise if we want to test this hypothesis, while there is serial auto-correlation. We cannot straightforwardly use this approach for the EC-model nor to the ADL(1,1) model. Let us focus on the ADL(1,1) model since we may derive the EC-model from it. In the instances where the ρ parameter is not equal to zero the random permutation of $y_{ij,t}$ has consequences for the estimation of ρ, β_2 and β_3 in (1) during the QAP-procedure. In the following discussion of these problems with MRQAP, we will indicate a randomized $y_{ij,t}$ in the MRQAP as $y_{ij,t}^*$ and also will identify parameter estimates that are generated by the MRQAP with an asterisk, for example, ρ^*.

Consider again the ADL(1,1) model in (1). MRQAP offers a basis to test whether ρ is a spurious result due to structural autocorrelation. Under the null hypothesis of MRQAP, the expected value of ρ^* is zero, that is, there is no relation between $y_{ij,t}^*$ and $y_{ij,t-1}^*$. If this null hypothesis is true, but the OLS estimate of ρ is different from zero, then this presumably is due to neglected structural autocorrelation. In that case we should consider that the OLS value of ρ is due to neglected structural autocorrelation or is just zero indeed.

[1]In this instance of a permutation, row i replaced row j, row j replaced row k, row k replaced row i, and hence column l replaced column j, column j replaced column k, column k replaced column l. For a more detailed example, see Krackhardt (1987b).

Similarly, we could analyze the β_2 and β_3 parameters in the ADL(1,1) model, but here a problem could arise. Note again that there is no linear relation between $y^*_{ij,t}$ and $y_{ij,t-1}$ (the expected value of ρ^* is zero). However, there may be a linear relation between $y^*_{ij,t}$ and $\pi(y_{ij,t-1})$, where $\pi(.)$ represents the randomization function that describes the permutation of rows and columns that created $y^*_{ij,t}$. This relation implies that possible serial autocorrelation did not disappear; however, it does not have a first-order structure anymore. Actually, the serial autocorrelation in the data has taken a form that can best be interpreted as a form of structural autocorrelation. In the MRQAP the serial autocorrelation that the ADL(1,1) specification controls for in the initial (not randomized) data, becomes autocorrelation that is not explicitly controlled for in the ADL(1,1) model. Hence, the permutations in an MRQAP change the level of serial autocorrelation (ρ^*), which affects the estimation of the other parameter estimates, β^*_2, and β^*_3. This has consequences for the usefulness of the reference distributions, generated by MRQAP to assess the sizes of β_2, and β_3.

A consequence of this increase in the level of structural autocorrelation is that the variation in the size of the estimates of the parameters increases. As ρ does not correct for serial autocorrelation anymore, the estimates of the other parameters would increasingly differ from zero for increasing levels of serial autocorrelation. This would make the MRQAP a conservative test.

To deal with the above problem, we have to control for serial correlation during the MRQAP procedure. A two-stage MRQAP (TS MRQAP) does this. In the first stage we test the null hypothesis that all regression coefficients, including ρ, are zero. If we can not reject the null hypothesis that ρ is larger than zero, we conclude that the ADL(1,1) model, and hence the EC-model are both inappropriate. If we can reject the null hypothesis that ρ is larger than zero we use the second stage to asses the hypothesis that all regression coefficients except ρ are equal to zero. In this second stage, we not only randomize $y_{ij,t}$, but also in a similar way randomize $y_{ij,t-1}$.This keeps intact the relation between $y^*_{ij,t}$ and $y^*_{ij,t-1}$, and hence $\rho^* = \rho$. When applying MRQAP, we then explicitly control for serial autocorrelation, which allows the assessment of whether the other parameter estimates are spurious due to neglected structural autocorrelation.

An additional remark needs to be made with regard to γ_3 in the EC-model, (2). For $\rho < 1$, when ρ becomes larger (and $|\rho-1|$ thus becomes smaller), $\gamma_3 = -(\beta_1 + \beta_2)/(\rho - 1)$, would go to infinity when ρ approaches 1. The TS MRQAP may then lead to overestimation of γ_3, especially when ρ is large. To counter this outcome we need to control for ρ when testing the null hypotheses that $\gamma_3 = 0$. As γ_3 is zero when $\beta_1 + \beta_2 = -\gamma_2\gamma_3 = 0$, it suffices to test whether this condition holds, given that ρ is less than 1.

3. SIMULATIONS

In this section we present some simulations to see whether TS MRQAP, as we described in the previous section, works in practice. These simulations would indicate whether a TS MRQAP analysis of the ADL(1,1) and the EC-model is robust against structural autocorrelation.

3.1 Data Generating Process

As is done in Krackhardt (1988), we generate random data with varying levels of structural and serial autocorrelation on a dependent variable ($y_{ij,t}$) and a single independent variable ($x_{ij,t}$). This data-generating process (DGP) implies that there is neither a short-term nor a long-term relation between x and y. We estimate the parameters for the two period ADL(1,1) model in (1) and the associated EC-model in (2), with the following data:

$$y_{ij,t} = K_R\zeta_{yi,t} + K_C\zeta_{yj,t} + K_B\zeta_{yij,t} + \rho y_{ij,t-1} \tag{4}$$

$$x_{ij,t} = K_R\zeta_{xi,t} + K_C\zeta_{xj,t} + K_B\zeta_{xij,t} \tag{5}$$

where K_R and K_C represent the levels of structural autocorrelation in respectively the rows and columns of the matrix and ρ is the serial autocorrelation parameter. The $\zeta_{xi,t}, \zeta_{xj,t}, \zeta_{xij,t}, \zeta_{yi,t}, \zeta_{yj,t}$, and $\zeta_{yij,t}$ are randomly distributed gaussian variables ($N(0,1)$). The autocorrelations take values between $0 < K_B \leq 1$, $K_R = 1 - K_B$, $K_R = K_C$ and $0 < \rho < 1$, with steps of .05. Thus, 441 combinations of structural and serial autocorrelation values have been evaluated.

3.2 Tests

In the simulations we record the percentage of rejections (based on 1000 runs) of the (true) null hypotheses, that is, that there are no short-term and long-term relations between dependent and explanatory variables. As both the dependent and independent variables are random, we would expect to find no relations between them. On the other hand, we would expect the relation between the dependent (y_t) and lagged dependent (y_{t-1}) to be as large as ρ. Therefore, we only test the null hypothesis ($\rho = 0$).

All inference of the parameters in the EC-model can be done on the basis of the ADL(1,1) model. An advantage of this model is that it is linear in the parameters. From the ADL(1,1) parameter estimates we derive the parameter values and (asymptotic) standard errors of the EC-model parameters (see Greene 2000, pp. 118–120). We determine the robustness against autocorrelation as the degree to which the t-test and TS

MRQAP-test reject the null hypotheses of no significant effects at the $\alpha = .10$ level. We expect for TS MRQAP that the rejection rate of the null hypotheses to be α on average (see Krackhardt, 1988).

3.3 Simulation Results

Figures 1a to 3c and Tables 1a and 1b summarize our simulation results. First, Figure 1a shows us that the TS MRQAP analysis of ρ is robust against structural autocorrelation, but slightly liberal (not to a disturbing extent though). With increasing levels of structural autocorrelation, the number of rejections based on the MRQAP-test remains 10% when indeed there is no serial autocorrelation. As expected we see that the t-test is not robust against structural autocorrelation (see Figure 1b). This graph indicates that the t-test based rejection rate of the null-hypothesis that $\rho = 0$ increases as structural autocorrelation increases.

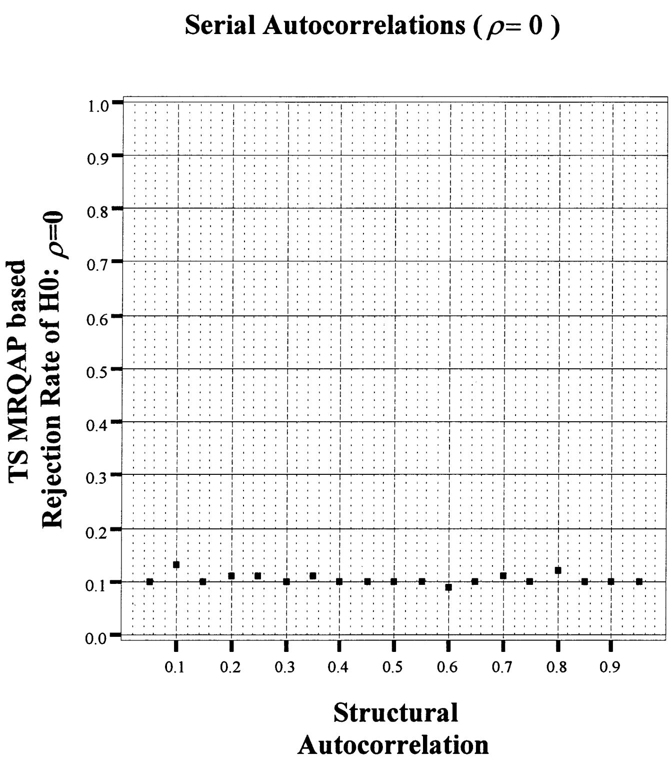

FIGURE 1a TS MRQAP based Rejection Rates of H0: $\rho = 0$.

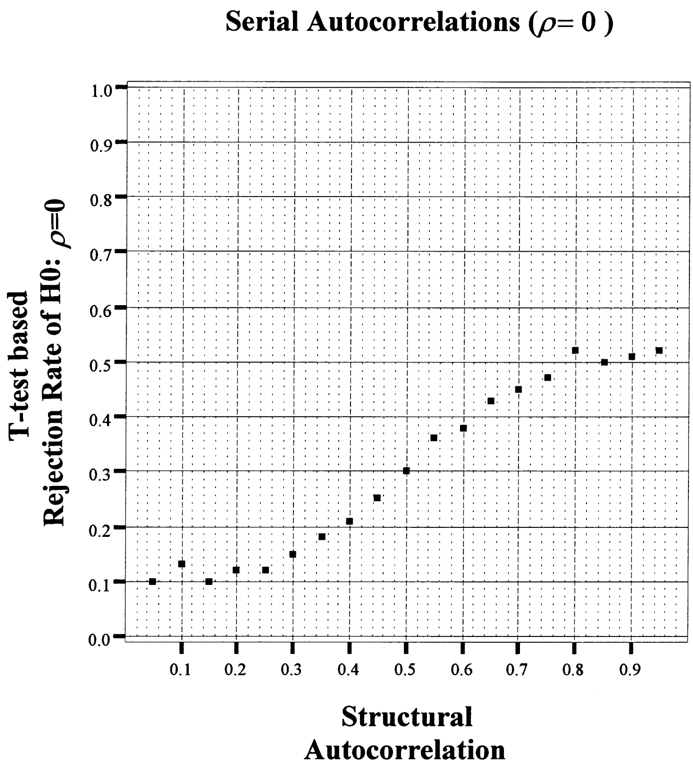

FIGURE 1b T-test based Rejection Rates of H0: $\rho = 0$.

Second, Table 1a shows that regular MRQAP is conservative, because the rejection rate goes to zero in the analysis of β_2. These results are similar for γ_2 and β_3 and we therefore do not report those results. When we control for serial autocorrelation, as we do in the TS MRQAP analysis, results are satisfactory (see Table 1b). Furthermore, Figure 2a shows us that TS MRQAP analysis of β_2 (and γ_2 and β_3) is robust against structural autocorrelation, without becoming a test that is conservative. And, as expected, Figure 2b shows that the t-test of β_2 (and γ_2 and β_3) is not robust against structural autocorrelation.

Figure 3a shows that when we do not control for ρ, the TS MRQAP-analysis of γ_3 ($= -(\beta_2 + \beta_3)/(\rho - 1)$) is not robust against increasing levels of serial autocorrelation. When the structural autocorrelation is indeed zero, the TS MRQAP-analysis rejects the null-hypothesis that $\gamma_3 = 0$ more often with increasing ρ. However, as discussed above, to test whether $\gamma_3 = 0$ it is sufficient to test that $\beta_2 + \beta_3 = 0$. From Figure 3b it becomes clear that TS MRQAP-analysis of this condition is robust against structural

TABLE 1a MRQAP Based Rejection Rates for β_2

Structural Autocorrelation (K_R)	Serial Autocorrelation (ρ)																				
	.00	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	1.00
.00	.09	.12	.08	.10	.08	.07	.10	.09	.08	.08	.05	.06	.05	.04	.04	.02	.02	.02	.01	.01	.01
.05	.11	.11	.08	.09	.09	.10	.09	.09	.08	.06	.06	.05	.05	.04	.03	.02	.03	.01	.01	.01	.01
.10	.11	.10	.10	.10	.08	.09	.08	.09	.07	.06	.07	.06	.04	.05	.03	.03	.02	.02	.01	.01	.01
.15	.12	.10	.11	.08	.10	.10	.10	.07	.08	.07	.07	.04	.06	.04	.03	.03	.02	.02	.01	.01	.00
.20	.10	.09	.09	.09	.08	.08	.07	.08	.08	.08	.07	.05	.04	.03	.03	.03	.01	.01	.02	.01	.00
.25	.10	.11	.10	.10	.09	.09	.08	.07	.08	.06	.07	.07	.04	.04	.02	.03	.02	.01	.01	.01	.00
.30	.10	.11	.11	.11	.10	.08	.11	.08	.08	.06	.06	.06	.04	.04	.03	.03	.03	.02	.01	.01	.01
.35	.11	.10	.08	.09	.09	.07	.08	.07	.08	.07	.06	.06	.04	.04	.04	.03	.02	.02	.01	.01	.01
.40	.09	.12	.12	.10	.10	.09	.08	.07	.07	.05	.06	.05	.05	.04	.04	.03	.02	.01	.01	.01	.00
.45	.09	.09	.10	.11	.12	.08	.07	.08	.06	.06	.07	.05	.05	.04	.03	.03	.02	.02	.01	.01	.01
.50	.09	.10	.09	.10	.11	.07	.09	.08	.10	.07	.06	.05	.04	.03	.04	.02	.03	.01	.02	.01	.01
.55	.09	.10	.09	.11	.08	.10	.08	.09	.08	.06	.05	.04	.04	.04	.03	.02	.01	.01	.01	.01	.01
.60	.10	.09	.10	.09	.07	.08	.09	.08	.08	.07	.06	.06	.04	.04	.03	.02	.01	.02	.01	.01	.01
.65	.09	.10	.11	.10	.08	.09	.08	.08	.07	.06	.05	.06	.04	.03	.03	.03	.03	.02	.01	.01	.01
.70	.10	.11	.10	.10	.09	.07	.08	.06	.06	.08	.06	.05	.04	.04	.03	.02	.02	.02	.02	.01	.01
.75	.10	.11	.10	.10	.09	.09	.09	.08	.07	.07	.04	.05	.04	.04	.03	.03	.02	.01	.01	.01	.01
.80	.10	.09	.10	.09	.09	.09	.08	.06	.06	.07	.06	.05	.04	.03	.03	.03	.02	.02	.01	.01	.00
.85	.10	.11	.10	.10	.09	.09	.07	.08	.07	.05	.06	.04	.04	.03	.04	.03	.02	.01	.02	.01	.01
.90	.10	.09	.08	.08	.09	.09	.09	.10	.07	.06	.05	.05	.04	.05	.02	.02	.02	.01	.02	.01	.00
.95	.10	.09	.09	.09	.10	.07	.09	.08	.07	.07	.05	.05	.05	.03	.03	.02	.02	.02	.01	.01	.01
1.00	.11	.09	.12	.10	.10	.09	.08	.07	.06	.07	.05	.06	.04	.04	.03	.02	.02	.02	.01	.01	.01

Under the DGP the expected rejection rate is .10

TABLE 1b MRQAP Based Rejection Rates for β_2

Structural Autocorrelation (K_R)	Serial Autocorrelation (ρ) .00	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	1.00
.00	.08	.09	.10	.10	.10	.11	.10	.09	.11	.09	.10	.08	.09	.09	.08	.10	.09	.13	.10	.09	.09
.05	.11	.09	.11	.11	.11	.10	.10	.11	.09	.11	.10	.09	.10	.10	.12	.12	.10	.09	.10	.09	.11
.10	.09	.10	.10	.10	.11	.10	.08	.10	.09	.09	.10	.09	.11	.10	.11	.10	.09	.10	.09	.09	.12
.15	.10	.11	.11	.10	.10	.12	.11	.11	.08	.10	.12	.11	.08	.11	.11	.11	.10	.11	.11	.10	.10
.20	.10	.12	.10	.10	.09	.10	.11	.09	.09	.10	.11	.11	.09	.08	.09	.11	.10	.10	.11	.10	.10
.25	.11	.11	.10	.10	.10	.11	.09	.09	.09	.12	.12	.12	.10	.09	.10	.10	.09	.11	.09	.11	.11
.30	.09	.10	.09	.10	.11	.11	.11	.09	.10	.09	.08	.12	.10	.10	.11	.10	.11	.11	.10	.10	.10
.35	.12	.10	.11	.12	.10	.10	.12	.10	.09	.10	.08	.08	.09	.10	.11	.09	.11	.11	.10	.11	.10
.40	.09	.10	.11	.09	.08	.09	.09	.09	.10	.10	.09	.10	.10	.11	.11	.09	.10	.10	.10	.10	.10
.45	.10	.10	.09	.10	.10	.12	.09	.11	.09	.10	.10	.10	.10	.09	.10	.11	.08	.11	.11	.09	.10
.50	.12	.10	.08	.10	.11	.12	.10	.10	.09	.09	.09	.10	.11	.12	.10	.11	.09	.12	.10	.10	.10
.55	.10	.11	.11	.11	.11	.09	.09	.11	.10	.12	.11	.11	.10	.09	.10	.09	.12	.10	.10	.08	.11
.60	.10	.10	.10	.10	.09	.09	.11	.12	.11	.08	.10	.10	.10	.10	.10	.10	.11	.09	.11	.10	.10
.65	.10	.09	.10	.12	.11	.12	.10	.10	.10	.09	.10	.11	.09	.09	.09	.10	.10	.11	.09	.08	.10
.70	.09	.11	.10	.10	.11	.10	.10	.12	.11	.12	.10	.11	.10	.10	.10	.09	.10	.09	.10	.08	.11
.75	.11	.11	.11	.09	.13	.10	.09	.09	.13	.11	.09	.11	.10	.10	.10	.09	.10	.10	.10	.10	.10
.80	.11	.08	.10	.11	.08	.08	.08	.10	.11	.08	.09	.10	.09	.09	.09	.10	.09	.09	.12	.10	.08
.85	.11	.10	.11	.10	.10	.09	.10	.09	.09	.10	.11	.12	.09	.08	.10	.09	.10	.10	.12	.09	.10
.90	.10	.11	.10	.10	.11	.10	.10	.11	.10	.10	.10	.11	.11	.10	.11	.11	.10	.09	.10	.09	.09
.95	.10	.09	.09	.09	.11	.12	.11	.11	.10	.10	.13	.11	.09	.10	.10	.09	.09	.09	.10	.08	.10
1.00	.10	.09	.09	.10	.09	.11	.10	.09	.10	.10	.10	.10	.11	.10	.10	.10	.11	.12	.09	.10	.10

Under the DGP the expected rejection rate is .10

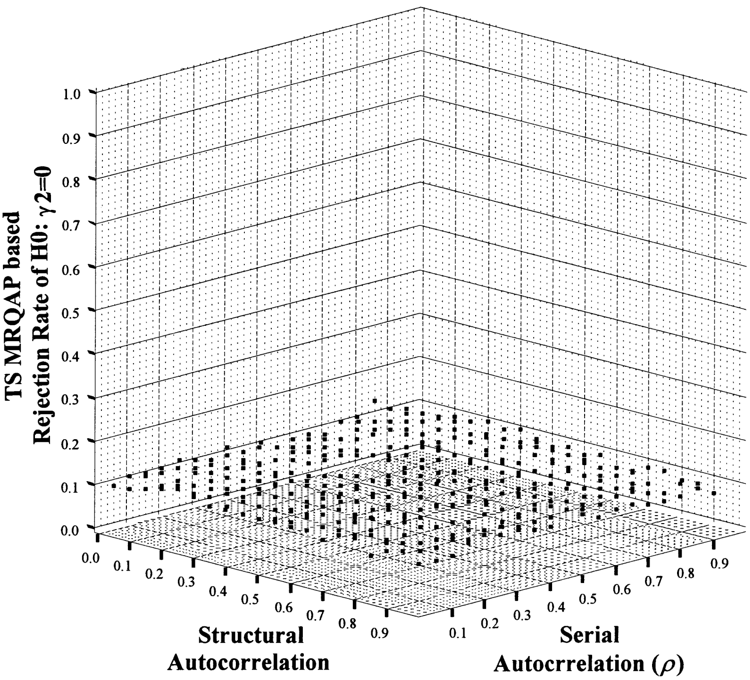

FIGURE 2a TS MRQAP based Rejection Rates of H0: $\gamma_2 = 0$ ($\beta_2 = \gamma_2$).

autocorrelation. Figure 3c again shows that the t-test of $\gamma_3 = 0$ is not robust against structural autocorrelation.

To summarize our simulation results, it seems that TS MRQAP has excellence performance, and it is more reliable than the OLS-based t-statistics.

4. AN EMPIRICAL ILLUSTRATION: CHANGES IN ACCURACY

To illustrate the usefulness of EC-models we present an example in which we analyze both ADL(1,1) and EC-models. In this example, we focus on accuracy of social structural perception. In the example we show that indeed the ADL(1,1)-model may give results that have a difficult interpretation, while the interpretation of the EC-model is much more straightforward. First, we will give a short background on the importance of accuracy studies and we discuss the value of a longitudinal study on accuracy. Subsequently, we discuss the data and show some results.

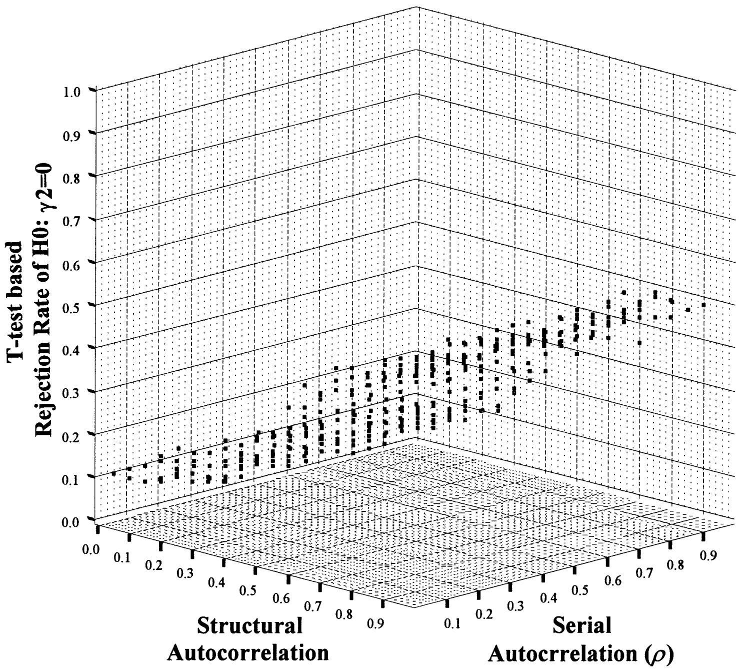

FIGURE 2b T-test based rejection rates of H0:$\gamma_2 = 0$ ($\beta_2 = \gamma_2$).

4.1 Accuracy of Perceptions

Krackhardt (1990) shows that individuals that accurately perceive the network structure of relationships have greater power in that network. Casciaro (1998) suggests that accurate perceptions may not only affect the individual's ability to get what he or she wants, but also have consequences for groups and organizations. Those individuals who perceive the social structure, which defines the access to resources, more accurately are better able to obtain the resources that are needed for groups and organizations (Burt, 1992).

Several studies have shown that degree centrality in networks enhances individuals accuracy of perceived networks (Bondonio, 1998; Casciaro, 1998). Degree centrality is measured as the number of people that have a direct relationship with a focal individual. In this illustration we focus on the effects of indegree centrality and outdegree centrality. The indegree is the number of relationships that a focal individual receives, while the outdegree is the number of relationships that originate from that focal individual.

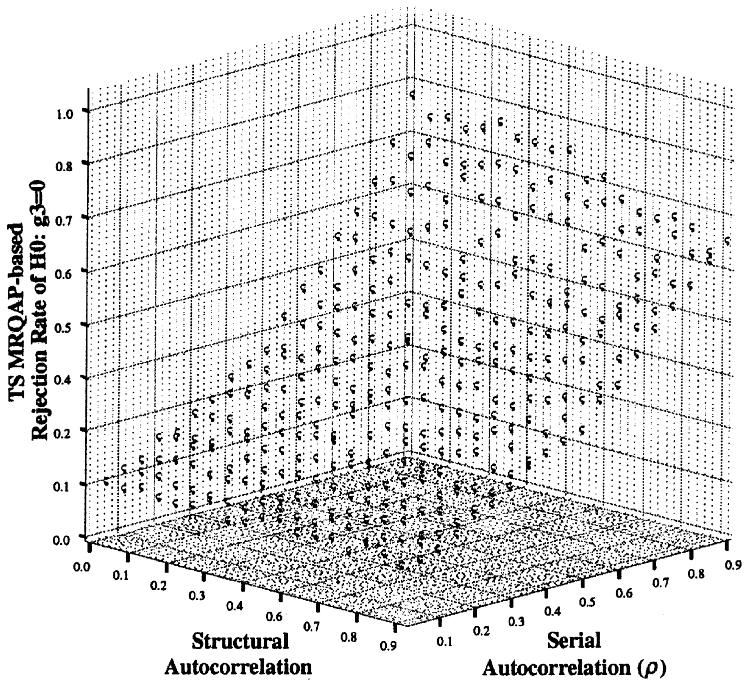

FIGURE 3a TS MRQAP based Rejection Rates of H0:$\gamma_3 = 0$.

Centrality indicates the potential for communication in which an actor could be involved (Freeman, 1979). More involvement in the total communication that occurs in the network could have two effects on perception accuracy. First, a central individual receives more information about the structure of the network. Or better, such an individual receives information on the perceptions about the network structure of more other individuals in the network. To the extent that such a central person actively seeks out others (as opposed to being the passive recipient of others inquests), this effect is especially captured by outdegree. Second, the perceptions of an individual who is frequently sought out by others tend to carry more sway. That is, more individuals will take notice of the perceptions of such a central individual, and therefore his or her perceptions are more likely to become dominant. This effect of centrality would be especially captured by the indegree.

If centrality indeed enhances perceptual accuracy, it should do so over time. For example, changes of centrality should be reflected in enhanced or diminished accuracy. In our illustration, we study whether centrality influences the accuracy of social structural perceptions over time. In other

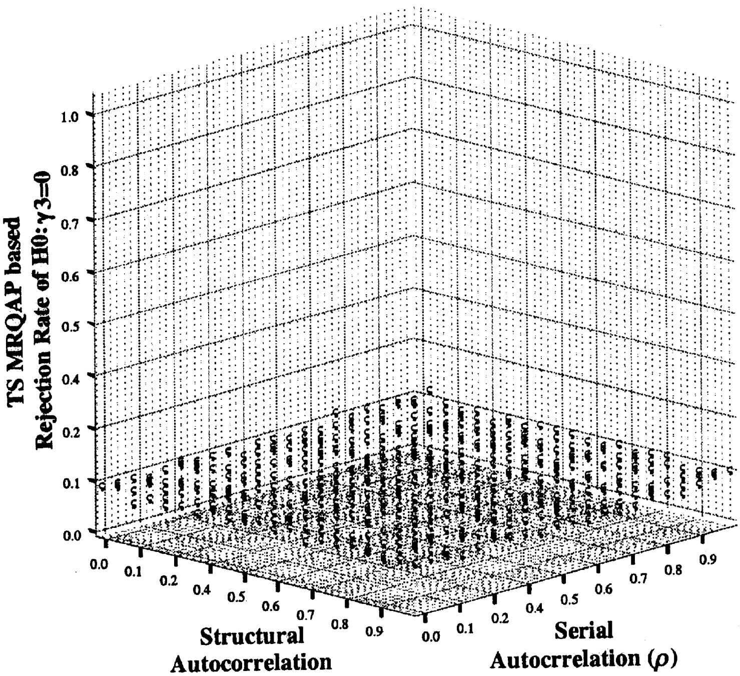

FIGURE 3b TS MRQAP based Rejection Rates of H0:$\gamma_3 = 0$ based on $\beta_2 + \beta_3 = 0$.

words, here we study whether centrality affects the consistency of perception accuracy. Especially, we try to determine what centrality measures affect accuracy.

4.2 Dependent Variable

In this illustration, accuracy implies a minimum deviation from a certain reference or benchmark. Krackhardt (1987) defines the locally aggregated structure as such a reference for perceived social structure. In the locally aggregated structure (LAS) a relationship exists when both individuals that are involved in the relationship claim it is present. For example, if individual i claims to have a relationship with j (R_{ij}) and j confirms this, then this (directed) relationship is present in the locally aggregated structure. We measure the accuracy of individual k's perceptions as the absolute deviation of individual k's perceptions from this reference.

However, still different accuracies may be determined. Examples are the accuracy of individual k concerning the entire network (Krackhardt 1987) or the accuracy of individual k concerning the relationships of each

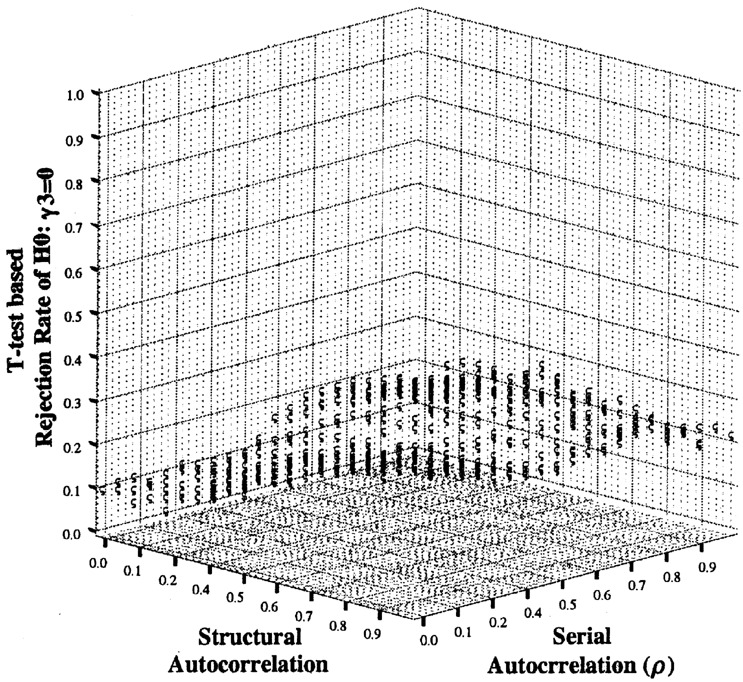

FIGURE 3c T-test based Rejection Rates of H0: $\gamma_3 = 0$.

individual in the network (Bondonio 1998). To keep things simple in our illustration, we only look at the perceptions individual *k* has of his/her direct relationships. That is, accuracy hereafter will refer to the accuracy (as defined by comparing perceptions to the LAS benchmark) of the perceiver *k* in assessing his/her own direct ties to a set of alters, *j*. Thus, the dependent variable is a matrix M(*k,j*) where cell (*k,j*) captures the extent to which *k* is accurate in assessing *k*'s tie to *j*.

TABLE 2a Values Dependent Variable in ADL(1,1) Model

		"Actual" R_{kj} $({}_aR_{kj})$	
$\lvert {}_kR_{kj} - {}_aR_{kj} \rvert$		No Relationship (0)	Relationship (1)
k's perception of R_{kj} $({}_kR_{kj})$	No Relationship (0)	Accurate (0)	Inaccurate (1)
	Relationship (1)	Inaccurate (1)	Accurate (0)

TABLE 2b Values Dependent Variable in EC-Model

		Period t-1	
		Accurate (0)	Inaccurate (1)
Period t	Accurate (0)	Consistently accurate (0)	More accurate (–1)
	Inaccurate (1)	More inaccurate (1)	Consistently inaccurate (0)

The ADL(1,1) model and the EC-model both have different dependent variable matrices. In our illustration the dependent variable in the ADL(1,1) model is the accuracy of individual k on R_{kj} in period t. Given that our data is dichotomous, the value of this variable is always one or zero as can be seen in Table 2a.

The ADL(1,1) models in our example specify the effects of previous accuracy, current centrality and previous centrality on future accuracy. A problem with the ADL(1,1) specification could be that current centrality and previous centrality have opposite effects. It would then be difficult to understand the effects of centrality.

We therefore rely on the EC-model. In our illustration the EC-model assumes an effect induced by the levels of centrality and an effect of change in the level of centrality. These are different effects, with substantively different meanings. The level effect of centrality would explain the equilibrium level of accuracy. The change in the level of centrality would explain the deviation from the equilibrium level of accuracy.

As mentioned, a consequence of using the EC-model is that the dependent variable in the EC-model differs from that of the ADL(1,1) model. In the EC-model the dependent variable is the change in accuracy or the instability of accuracy. Table 2b shows that there are three possible values for change in accuracy when data are dichotomous. The value is zero if no change occurs either because individual k remains accurate or inaccurate. The value becomes positive when an individual becomes more inaccurate and the value becomes negative when an individual becomes more accurate. Consequently, a negative parameter estimate indicates better accuracy, while a positive parameter estimate means lower accuracy.

4.3 Explanatory Variables

In our models we consider the effects of indegree and outdegree centrality in three types of networks. First, we consider the centrality of individual k in the consensus structure (CS). In this structure a relationship exists if a majority of individuals (more than 50%) perceive the relationship to exist. The centrality measures based on this structure reflect whether the group as a whole considers an individual to be central. Second, we consider centrality in the LAS. The centrality measures based on the LAS reflect whether the group of direct contacts agrees with individual k on his/her centrality. Third, we also consider the structure as perceived by each individual personally (the slices of the cognitive social structure). In these structures we measure the centrality individual k perceives individual j to have. The network we study is an advice request network.

Note that the measures based on the first two structures are group-based measures that reflect the importance of the perception of k for others (indegree) and the access others think that k has to resources. The measures in the third structure reflect how central k perceives j to be. The indegree measure shows how important j is to others according to k, while the outdegree measure shows how much access j has to social resources according to k.

TABLE 3a Results of the ADL(1,1)-Model with as Dependent Variable "Accuracy of Advice Relationships" (LAS) in Period 2 ($t = 2$) and Different Degree Measures as Explanatory Variables

			Two stage MRQAP	Standard OLS statistic
		Estimates	*P-value*	*P-value*
Constant		.74	.24	.10
Serial autocorrelation parameter	ρ	**.39**	**.03**	**.00**
Indegree CS (*t*)	β_{11}	−.20	.14	.05
Indegree CS (*t-1*)	β_{21}	.19	.29	.15
Indegree LAS (*t*)	β_{12}	.08	.24	.11
Indegree LAS (*t-1*)	β_{22}	−.09	.44	.27
Indegree SLICE (*t*)	β_{13}	**−.03**	**.06**	**.04**
Indegree SLICE (*t-1*)	β_{23}	**.03**	**.03**	**.02**
Outdegree CS (*t*)	β_{14}	**−.27**	**.08**	**.03**
Outdegree CS (*t-1*)	β_{24}	.09	.22	.14
Outdegree LAS (*t*)	β_{15}	−.04	.32	.16
Outdegree LAS (*t-1*)	β_{25}	*.08*	*.13*	*.05*
Outdegree SLICE (*t*)	β_{16}	.01	.57	.53
Outdegree SLICE (*t-1*)	β_{26}	.00	.81	.81
			Adj.$R^2 = .16$	

Boldface and Italic numbers represent significant results $\alpha \leq .10$. P-values are two-sided.
TS MRQAP is based on 10000 simulations
Dependent Variable: Accurate = 0, Inaccurate = 1

TABLE 3b Results of the Equilibrium-Correction Model With as Dependent Variable "Change in Accuracy: Advice Relationships" (LAS) and Different Degree Measures as Explanatory Variables, Where *Δ* Denotes the Change Variable

			Two stage MRQAP statistics	Standard OLS Statistics
		Estimates	*P-value*	*P-value*
Constant		.74	.24	.10
Short-term Adjustment Parameter	γ_2	**−.61**	**.03**	**.00**
Indegree CS (Δ)	γ_{11}	−.20	.14	.05
Indegree CS *(t-1)*	γ_{31}	−.02	.84	.77
Indegree LAS (Δ)	γ_{12}	.08	.24	.11
Indegree LAS (*t-1*)	γ_{32}	−.02	.90	.86
Indegree SLICE (Δ)	γ_{13}	**−.03**	**.06**	**.04**
Indegree SLICE (*t-1*)	γ_{33}	.00	.89	.66
Outdegree CS (Δ)	γ_{14}	**−.27**	**.08**	**.03**
Outdegree CS (*t-1*)	γ_{34}	−.29	.20	.10
Outdegree LAS (Δ)	γ_{15}	−.04	.32	.16
Outdegree LAS (*t-1*)	γ_{35}	.06	.62	.54
Outdegree SLICE (Δ)	γ_{16}	.01	.57	.53
Outdegree SLICE (*t-1*)	γ_{36}	.03	.51	.43
			Adj.R^2 = .42	

Boldface and Italic numbers represent significant results at $\alpha \leq .10$. P-values are two-sided.
Italic numbers represent significant results for standard OLS based t-test at $\alpha \leq .10$.
TS MRQAP is based on 10000 simulations
Dependent Variable: Consistent = 0, More Inaccurate = 1; More Accurate = −1

4.4 Data

We collected data on a group of 13 individuals on perceived advice request relationships over two periods. Hence we study 156 changes in accuracy. The data setting is similar to that described in Krackhardt & Porter (1985, 1986). The individuals in the network are employees of a big fast food chain. Employees are subject to standard rules that apply throughout the chain. For example, they have to wear prescribed uniforms. Most of the employees are high school students who work to earn some spending money. Furthermore, working at that specific restaurant comes with social status, because it is a popular hangout place for students. These data were collected in the beginning of the 1980s and have not been presented before in Krackhardt & Porter (1985, 1986). The reason was that those papers focused on turnover as a dependent variable, and in this branch there was no turnover between the two periods. This is beneficial to this study, because it means that the number of observations is higher than if there had been mutations in the group.

4.5 Empirical Results

Tables 3a and 3b show the results of our empirical analysis where the dependent variables are respectively, LAS-based accuracy and change in LAS-based accuracy. Equation 3 shows the algebraic relation between the coefficients in these tables. Furthermore, the first index of the parameter symbols (e.g. β_{11}, β_{21}, and γ_{31}) indicate the similar parameters in equations 1 and 2.

Table 3a immediately shows a difficulty with the interpretation of the ADL(1,1) model. It shows that *k*'s accuracy at time *t*, based on LAS, is positively and negatively related to the indegrees that *k* perceives *j* to have now and in a previous period, respectively (Indegree Slice at time $t = -.03$, $p = .06$ and Indegree Slice at time $t{-}1 = .03$, $p = .03$). Since accuracy in this model is defined by *j* agreeing with *k*'s perception of the tie from *k* to *j*, this would mean that *j*'s and *k*'s current perceptions are more similar when *k* perceives *j* to have a high current indegree. However, this result also implies that the current perceptions of *k* and *j* are less similar, when *k* perceived *j* to have a high indegree in the previous period. These results are not intuitive. On the other hand, in Table 3b (the EC-model), we see that the change in accuracy is affected by the change in the perceived indegree (Indegree Slice $\Delta = -.03$, $p = .06$) and not the level of perceived indegree (Indegree Slice $t{-}1 = .00$, $p = .89$). This result allows for a more meaningful substantive interpretation. As *k* perceives *j* to become a more popular source of information, *k* becomes more accurate in his/her assessment of his/her tie to *j*.

In Table 3a, we also see that current outdegree as perceived by the majority of individuals in the network enhance accurate perception (Outdegree CS $t = -.27$, $p = .08$), which is (by definition) similar to the change effects in the EC-model (see table 3b, Outdegree CS Δ). As no other effects where found for the EC-model, it seems that change in LAS-based accuracy is mainly driven by CS-based centrality and individual's perceptions about the centrality of partners.

Some other results are worth emphasizing in this illustration. In Table 3a, for example, LAS-based outdegree of previous periods is not significant (Outdegree LAS $t{-}1 = .08$, $p = .13$). However, based on the standard OLS t-test we would falsely infer that the accompanying parameter significantly differs from zero ($\beta_{25} = .08$, $p = .05$). We emphasize again that the TS MRQAP approach doesn't test such hypotheses. However, this approach does allow us to select a set of relevant explanatory variables, while not making inferences on spurious results due to autocorrelation.

5. CONCLUSIONS

In this paper we proposed to use TS MRQAP for analyzing dynamic network data, captured by an equilibrium-correction model. Our simulation results emphasize that under conditions of serial and structural autocorrelation it is relevant to follow the TS MRQAP. Especially, the two-stage procedure is needed to control for disturbing effects of serial autocorrelation. Although estimation of the ADL(1,1) model is needed to make inferences on the long-term effect parameter (γ_3) in the EC-model, the latter model has more interpretable coefficients, that is the "level"-effect and the "change"-effect. Our empirical analysis illustrates this.

The empirical results suggest that the effect of a change in *k*'s perception of *j*'s indegree centrality is larger than the level effect on accuracy of perceptions. Accuracy of perceptions about relations increases when those relations are with someone that is deemed to be more central. Note that this is LAS-based accuracy, i.e., accuracy with reality defined as a confirmation between two individuals. As has been demonstrated empirically (Krackhardt, 1987a; Kumbasar, Romney & Batchelder, 1994), individuals (here *j*) overestimate there own centrality. Hence, as *k* deems *j* to be more central this will on average imply a higher confirmation between *j* and *k*.

As suggested above, one alternative explanation could be that more central individuals have a more dominant perception. Especially, as *k* perceives *j* to have a higher indegree the effect could show that *k* adopts the perceptions *j* hold about the network. Further study is needed to give more definite answers. This illustration makes clear changes in perceptions of partners' centrality are associated with changes in accuracy over time.

Another finding is the effect of CS-based outdegree on changes in accuracy. Interestingly, the EC-model suggest that mainly the change in information sources enhances accuracy, while we could not find an effect of the amount of information sources (i.e., outdegree level). It suggests that a given level of centrality accuracy doesn't change, but changes in level of outdegree result in immediate change of accuracy.

More substantive, as accuracy enhances performance (Krackhardt, 1990) we may hypothesize that because increasing outdegree centrality enhances accuracy, centrality enhances performance. Higher outdegree centrality implies a broader range of information sources that leads to greater accuracy. Hence, higher outdegree may contribute fundamentally different to performance than increasing indegree. The latter would increase deference, while the former enhances perception of the social environment.

Specifically, the EC-model could be of great help in further research to shed some light on these hypotheses. Finally, we want to conclude with the remark that the equilibrium-correction model can easily be extended to incorporate more change effects, like for example changes between period $t=1$ and $t=2$, $t=2$ and $t=3$, and so on. This could provide additional insights in the structure of dynamic effects in network data.

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