Supply Chain Contracting for
After-sales Service and Product Support

Supply Chain Contracting for After-sales Service and Product Support

Het contracteren van after-sales service en garantie in logistieke ketens

Thesis

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation

The growth of the service sector and economy has been a remarkable trend during the past 50 years in the development of the world's economy (Buera and Joseph, 2012). Nowadays, according to the World Bank, service business contributes more than 70% of the GDP in many countries[1]. For manufacturing companies, especially for producers of durable goods, providing more and more product-related services has been a growing revenue stream. For example, as shown in Table 1.1, service generates much more revenue for Rolls-Royce than the original equipment (OM) during the past five years. The process of adding services into products is widely referred as *servitization* (Vandermerwe and Rada, 1988; Tukker and Tischner, 2006; Kastalli and Looy, 2013; Smith et al., 2014). Under servitization, the importance of service is highlighted and manufacturing companies have to constantly innovate their business model and operational strategy for providing value-added services for customers (Fischer et al., 2014).

Servitization can be represented by several different business models (Cohen et al., 2006; Wang et al., 2011).

- A product-oriented business model (item (1)(2)(4) in Table 1.2). The consumers buy products and the manufacturers provide after-sales services which include repair, maintenance, product refurbishing, and recycle, etc.

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[1]http://data.worldbank.org/indicator/NV.SRV.TETC.ZS/

Key financial data

	2009	2010	2011	2012	2013
Order book £m*	47,102	48,490	51,942	49,608	60,296
	+8%	+3%	+7%	-4%	+22%
Engine deliveries*	844	846	962	668	753
Underlying revenue £m	4,481	4,919	5,572	6,437	6,655
	0%	+10%	+13%	+16%	+3%
Underlying OE revenue £m	1,855	1,892	2,232	2,934	3,035
Underlying service revenue £m	2,626	3,027	3,340	3,503	3,620
Underlying profit before financing £m	493	392	499	743	844
	-13%	-20%	+27%	+49%	+14%

TABLE 1.1: Key financial data of Rolls-Royce, 2013 (Rolls-Royce, 2013).

- A use-oriented business model (item (3) in Table 1.2). The manufacturers have the ownership of products and provides the consumers with usage and function. Examples include product leasing or sharing.

- An outcome-oriented business model (item (5)(6) in Table 1.2). The manufacturers deliver solutions and results, and the consumers pay for performance or outcomes. "Power-by-the-Hour"[2] usage of the product can be an example of this case.

	Service priority	Business model	Terms	Example	Product owner
(0)	None	Disposal	Dispose of products when they fail or need to be upgraded	Razor blades	Consumer
(1)	Low	Ad hoc	Pay for support as needed	TVs	Consumer
(2)	Medium-high	Warranty	Pay fixed price as needed	PCs	Consumer
(3)	Medium-high	Lease	Pay fixed price for a fixed time; option to buy product	Vehicles	Manufacturer; leasing company
(4)	High	Cost-plus	Pay fixed price based on cost and prenegotiated margin	Construction	Customer
(5)	Very high	Performance based	Pay based on product's performance	Aircraft	Customer
(6)	Very high	Power by the hour	Pay for services used	Aircraft engines	Manufacturer; service provider

TABLE 1.2: Models of after-sales services (Cohen et al., 2006).

[2]http://www.rolls-royce.com/news/press_releases/2012/121030_the_Hour.jsp

In response to the different business models for providing services, companies have different service contracts with customers. In service management practice, such contracts typically have one of the following structures.

Warranty. A warranty is a contractual agreement between the buyer and the manufacturer upon the sale of the product. During the warranty period, the manufacturer has to guarantee the availability of the product and failure recovery is free of charge for the customer. Warranty contracts are broadly used in the sales of consumer durables (B2C, e.g. white goods, as shown in item (2), Table 1.2) and industrial equipment (B2B, e.g., airplanes, production lines) (Murthy and Djamaludin, 2002). For example, HP provides a one-year warranty for their laptops. During warranty, parts and labor for repair are fully covered[3]. Another example is ASML that offers a 12 month warranty on their lithography systems for semiconductor manufacturing (Helleputte, 2014).

Transaction-based contracts. Under transaction-based contracts, customers have to pay for service transactions, such as spare parts and labor, to the service providers. Time & Material contracts are one example of the transaction-based contracts, under which customers pay for the direct labor hours at specified fixed hourly rates and actual cost for materials[4]. An example is the US Federal Transit Administration that uses Time & Material contracts for construction of highways[5]. This type of contracts can be related to the business models given in item (1) and item (4) in Table 1.2.

Performance-based contracts. Performance-based contracts (PBC) are becoming more popular in recent years with the growth of the outcome-oriented business model. Under performance-based contracts, suppliers conduct repair and maintenance for products, but the service activities are not compensated by customers. Customers only pay for what they use, or the predefined product performance, rather than the ownership of the physical assets[6] (Hypko et al., 2010). Currently, performance-based contracts are widely adopted in public service acquisition (Hensher and Stanley, 2003; Fearnley et al., 2004), defence (DoD, 2003), airlines (Smith, 2013), industrial equipment[7], advertisement (Dellarocas, 2012), and healthcare (Jiang et al.,

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[3]http://www8.hp.com/us/en/support-drivers/total-care/totalcare-for-laptops.html

[4]http://www.niaid.nih.gov/researchfunding/sop/pages/contracttype.aspx

[5]http://construction.about.com/od/Cost-Control/a/Time-And-Material-Time-And-Materials-Contract.htm

[6]PBC can also be applied when a customer buys and owns the product, like item (5) in Table 1.2. Here the "physical assets" mostly refer to the service related materials, such as spare parts. Under PBC, the customer usually does not buy the parts but pays for performance.

[7]http://www.gtspower.com/Services/Power-as-a-Service

2012), etc. PBC can be related to the business models given in item (3), item (5), and item (6) in Table 1.2.

Successful implementation of service contracts, especially for performance-based contracts, requires managers to solve many challenging tasks such as performance management, service resource management and supply chain coordination.

Performance measurement. Service performance can be measured by different indicators, such as operational availability, operational reliability, and cost per unit usage, etc (Defense Acquisition University, 2005). Although product availability is usually a key metric, different ways to deal with the "unavailability" may result in various impact on the suppliers' or the customers' benefits. For example, performance-based contracts can be executed based on penalizing under-performance. In such a case, suppliers have to pay penalties to the customers if product downtime occurs, or have to compensate the customers' revenue loss due to product unavailability. On the other hand, performance-based contracts can also be conducted based on uptime payment. The customers only pay for product uptime, and if the products are not working, the suppliers do not receive payment during the time period of failure restoration. Different payment schemes may have different impact on the suppliers' decision-making for managing service and product support.

Resource management. The resources of a company, such as spare parts and repair capacity, are critical to the provision of after-sales service and product support. Product failures usually occur randomly, and require either repair or replacement by spare parts. If suppliers can not deliver the right parts at the right time due to stockouts or insufficient repair capacity, it may not be possible to quickly restore products from system failures, and customers may suffer a loss. On the other hand, if suppliers maintain high levels of spare parts and repair capacity, the total operational cost increases, although a high product availability can be realized. Service suppliers have to make the optimal decisions in trading off between cost and efficiency, in the presence of different service contracts.

Supply chain coordination. For capital goods, the final product usually consists of many subsystems and components, which are produced by part suppliers. As a consequence, the performance of the final product is not purely affected by the final assembler, but also depends on the part suppliers' effort. However, in many cases, the part suppliers' production and quality control process can not be fully dictated by the final assembler who is responsible for supplying service to the end customers. Therefore, appropriate incentive alignment methods need to be

developed to coordinate the supply chain. Usually, the final assembler only buys parts from the suppliers, and the suppliers are not involved in the after-sales service, except for selling spare parts. In such a case, the part suppliers can not be incentivized to exert efforts on improving the reliability of their products or on maintaining a high level of service capacity. In order to improve the performance of the final product, managers need to design mechanisms to coordinate the suppliers' activity of product quality improvement and repair capacity management.

In this thesis, we study the aforementioned problems in the presence of different business models in after-sales and product support management. The results derived from our models provide managerial insights for managers to design service contracts and operate their service networks.

1.2 Contribution

By applying methods from economics (game theory and contract theory) to analyzing OM (Operations Management) issues (service contracting), we make the following contributions to literature.

- In Chapter 2, we incorporate the customer's initial purchase decision of the number of products into the model with after-sales service, where in most literature, product and after-sales service are separated. We formulate a Stackelberg game model and analyze the comparisons between Warranty + Transaction-based contracts with Performance-based contracts with two different penalty terms. Furthermore, we incorporate the impact of part obsolescence and Lifetime-buy planning into the decision model, which is, to our knowledge, the first work to discuss this issue in such a setting.

- In Chapter 3, we study the supply chain coordination problem under Performance-based contracts and come up with a penalty-sharing mechanism which can achieve the First Best solution in coordinating the supply chain members' activity. We contribute to the quality management research in OM literature, where people normally focus on quality control by inspection under a transaction-based framework. Our work is different as it studies quality management along supply chains in the context of a Performance-based contracting framework and proposes the channel coordination contract. Moreover, we also discuss the efficiency loss under the more traditional price-only and cost sharing contracts.

- In Chapter 4, we systematically compare service capacity setting under Time & Material contracts and Performance-based contracts, where in literature, discussions are usually given only under PBC. Moreover, we also incorporate the customer's effort on product failure prevention, which is usually discussed at the supplier's end. In addition, we study the uptime payment PBC, whereas in literature, PBC are usually formulated with a lump sum + penalty structure.

1.3   Structure

The structure of this manuscript is as follows.

Chapter 2. Contracting for Products and After-sales Service When Components Expire. In Chapter 2, we study joint contracting for durable products and the after-sales service between a customer and a supplier, while the supplier simultaneously has to plan the spare part inventory, as the part may expire. The customer makes decisions on how many products to buy to satisfy the external demand. At the same time, the customer must take the after-sales service into account since only working products can generate revenue. On the supplier's side, when selling the product, spare parts need to be well planned to service the product, because those parts may expire and become difficult to reacquire once stockouts occur. We establish game-theoretic models for the cases of Warranty + Transaction-based contracts (W+T) and Performance-based contracts (PBC) with two different penalty terms and come up with the equilibrium solutions. We find that a penalty on the customer's lost revenue can result in a higher product availability but a lower profit. W+T with longer warranty can be more profitable if spare parts stockouts can be replenished at a low cost.

Chapter 3. Coordinating Product Support Supply Chains under Outcome-based Compensations. In Chapter 3, we study the supply chain coordination problem under an outcome-based contract for product support. In our setting, the final product is assembled from parts which are manufactured by the Original Equipment Manufacturer (OEM) and the supplier. As a result, the quality of the final product is affected by the effort of the supplier and the OEM, who is responsible for servicing the end customer but cannot directly control the supplier's quality management activity. We formulate Principle-Agent models to fit the moral hazard setting and capture the supply chain members' decisions under price-only contracts, repair cost sharing contracts, and penalty sharing contracts. We show that penalty sharing

contracts can lead to the First Best solutions which coordinate the supply chain, while price-only contracts and repair cost sharing contracts result in efficiency loss due to over-investment in service capacity and under-investment in part failure rate reduction. The results hold even when repair capacity is decentralized or the final product is nonseparable.

Chapter 4. Contract Choice for Product Support. In Chapter 4, we compare Time & Material contracts (T&M) and Performance-based contracts with downtime penalty (PBC-P) and Performance-based contracts with uptime payment (PBC-U). Under PBC-P, the customer pays the supplier a lump sum and the supplier pays a penalty to the customer based on the product downtime; under PBC-U, the customer pays the supplier based on the product uptime. We show that if the supply chain members are risk neutral, equal profits can be realized across contracts for the supplier. If the supplier becomes risk averse to the profit variance caused by uncertainty of product failures, PBC-P are the best contract for the supplier while T&M are better than PBC-U if the product failure rate is high. PBC-P also dominate when the customer exerts effort on reducing the product failure rate, and T&M lead to the lowest profit in this case.

Chapter 5. Conclusion and Future Research. In this chapter, we draw conclusions of the thesis and discuss potential directions for future research.

1.4 Co-authorship

The main body of this thesis (Chapter 2, Chapter 3, and Chapter 4) is written based on the output of the author's research work done during his Ph.D. study at Rotterdam School of Management, Erasmus University. The author's contribution mainly includes idea generation, problem identification, model formulation, result analysis and discussion, and thesis (all chapters) writing. Dr. Nishant Mishra (Erasmus University) acts as the daily supervisor and is fully involved in the whole research process. Besides, Professor Serguei Netessine (INSEAD) contributes to idea development and model evaluation for the research in Chapter 4. Professor Rene de Koster (Erasmus University) acts as the promoter and gives valuable suggestions on the general orientation of the research and polishes all the chapters throughout this thesis. Furthermore, Professor Geert-Jan van Houtum (Eindhoven University of Technology), Dr. Vladimir Karamychev (Erasmus University), and Professor Serguei Netessine (INSEAD) acting as the members of the author's Ph.D. committee, review the entire manuscript and come up with

many insightful comments. Finally, the Chapter *Samenvatting (Summary in Dutch)* is written by Tim Lamballais Tessensohn (Erasmus University) by translating the Chapter *Summary*.

Chapter 2

Contracting for Products and After-sales Service When Components Expire

2.1 Introduction

Since the early 1990s, after-sales support activities have been acknowledged as a relevant source of revenue, profit and competitive advantage in most manufacturing industries (Cohen et al., 2006). The after-sales service market for durables has been found to be up to four or five times larger than the market for new products (Bundschuh and Dezvane, 2003), and is responsible for 40%-50% of profits of the companies (Dennis and Kambil, 2003). For the customers, after-sales support for durables is significant because product downtime can result in huge loss in revenue, productivity, and even reputation[1]. As such, acquisition of products is usually accompanied by considering the after-sales services in the customers' purchasing plan. On the other hand, the suppliers have been changing their business models, from selling products and services separately to combining products and services as a package to better satisfy customers' requirements (Sawhney et al., 2004; Cook et al., 2006). With the advent of *servitization*, which is defined as the process of adding service to products to add value (Vandermerwe and Rada, 1988), both the suppliers and the customers no longer view products and services apart, but

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[1]e.g., see http://www.dba-oracle.com/art_dbazine_high_avail.htm

focus on the "solutions" which are delivered by integrating products and services (Cohen et al., 2006).

Spare parts are critical to operating product support because whether the defective components can be replaced timely determines the overall product availability. For spare parts management, life cycle mismatch brings about many challenging jobs for managers (Bradley and Guerrero, 2008). Components may become obsolete quickly due to rapid technological changes, the market, or new regulations, while the products are intended to be in service for decades. For example, it is estimated that 60% of the integrated circuits on aerospace products become obsolete within five years[2]. Shortage of parts may result in significant challenges to product support and increase maintenance costs for the supply chain partners. Mitigating part obsolescence may include redesigning a system or reverse engineering, sourcing parts from the third party aftermarket, and *Lifetime-buy*. A Lifetime-buy is a purchase decision which is made by the part buyers (the supplier in our model) to purchase a sufficient volume of parts to sustain the product for its life (Bradley and Guerrero, 2009). In practice, Lifetime-buy is a strategy adopted in "nearly every electronic part obsolescence management program no matter what other reactive or pro-active strategies are being followed (Feng et al., 2007)".

The after-sales service contracts for durables usually consists of warranty and an extended service period. Under warranty, the supplier has to guarantee full availability of products with free repair service. After that, the supplier offers services based on per-transaction activities, i.e., the supplier charges a service fee for each repair. We call this type of service contracts "Warranty + Transaction-based contracts" (hereafter W+T). More recently, Performance-based contracts (hereafter PBC), which allow the customers to only pay for outcomes, have been increasingly adopted in both public and private sectors. For example, PBL (Performance-based Logistics) now becoming an almost mandatory acquisition model for service and maintenance in the defense industry in several countries (DoD, 2003; Phillips, 2005; Aerospace Systems Division, 2007). In commercial sectors, KLM equipment services offers a "power by the hour" arrangement, which allows customers to pay only for the number of hours they actually use[3]. Another example is Honeywell's mobile power solution, which "allows customers to pay for their battery power as it is used, effectively turning battery power into a fixed recurring expense instead of a periodic (and often unplanned for) capital expenditure[4]". The essence of PBC is buying performance outcomes, not individual parts and repair actions (Kim et al., 2007). Under

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[2]see http://www.boeing.com/commercial/aeromagazine/aero_10/elect_textonly.html
[3]see http://www.afiklmem.com/AFIKLMEM/en/g_page_news/300414AIRCHINA_GE90.html
[4]see http://www.honeywellbatteries.com/powerbythehour.html

PBC, suppliers are usually penalized if product unavailability occurs. Furthermore, managing part obsolescence is also an important issue under PBC. For example, the US government is planning to use more PBL contracts to deal with electronics obsolescence[5].

In the context of the aforementioned business setting, we build a dynamic game-theoretic model to study the problem of joint acquisition of products and the after-sales service under W+T or PBC in a supply chain, where the customer decides on product order quantity; the supplier plans Lifetime-buy quantity of critical parts which are tend to be obsolete, to guarantee operation of the products, and set the contract terms. By analyzing the equilibrium solutions, we find that different penalty terms under PBC may create different incentives for the supplier. Given the same number of spare parts, the supplier compensating the customer's lost revenue can lead to lower profits, compared to paying a penalty based on the number of spare parts stockouts. If the supplier can voluntarily set the Lifetime-buy quantity, compensating the customer's lost revenue may lead to an overstock on spare parts and lower efficiency, although product availability is higher. In addition, we also compare PBC with W+T. We find that PBC are not always dominate W+T. Warranty can improve the overall product availability, but the benefit is also affected by the part reacquisition cost. We show that if it is easy and cheap to reacquire the part, W+T can result in higher profits than PBC, even with few spare parts and long warranty. However, if the reacquisition cost is high, profits under long-warranty W+T may be lower than PBC.

The remainder of this chapter is organized as follows. After a brief literature review in §2.2, We describe the problem setting in §2.3. In §2.4, we formulate the contracting decision models under W+T and PBC. Subsequently, in §2.5 and §2.6, we investigate the problem when the quantity of spare parts are exogenously and endogenously specified, respectively. We then give some extensions of the model in §2.7 by discussing a special case of a full warranty contract, checking the robustness of the results given other type of revenue functions, and introducing risk aversion. In §2.8, we conclude the results of this chapter.

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[5]see http://www.kmimediagroup.com/military-logistics-forum/172-mlf-2009-volume-3-issue-5/1607-obsolescence-management.html

2.2 Literature Review

This chapter relates to the literature of supply chain contracting, after-sales service, and part obsolescence management. First, there are many papers in OM (Operations Management) literature studying production/capacity decision and contracting problems in decentralized supply chains. For example, Anupindi et al. (2001) develop a general framework for the analysis of decentralized distribution systems on inventory allocation problems. Bernstein and Federgruen (2005) investigate the equilibrium behavior of decentralized supply chains with competing retailers under demand uncertainty. Bassamboo et al. (2010) examine the classical problem of capacity and flexible technology selection with a newsvendor network model of resource portfolio investment. In addition, Lariviere and Porteus (2001) analyze a wholesale price-only contract of coordinating supply chains with newsvendor retailers. Krishnan et al. (2004) discuss buy back contracts in supply chain with retailer promotional effort. Cachon and Lariviere (2005) summarize revenue sharing contracts in general supply chain models. Plambeck and Taylor (2005) study quantity flexibility contracts with more than one downstream firm and ex-post renegotiation. Cachon (2004) gives a comprehensive review in supply chain contracts research. Examples of more recent works are (Ha et al., 2011; Krishnan and Winter, 2010; Shin and Tunca, 2010), etc. Although following similar modelling and analyzing methods, our work differentiates previous papers by introducing spare parts and product availability into the decisions for product quantity, which is oftentimes ignored in previous works. In this chapter, we study the setting where the customer has to consider the after-sales service when making decisions on product order quantities for durable goods, because only working products can generate revenue for the customer.

Secondly, there are also related articles studying after-sales service. Cohen and Whang (1997) develop a product life-cycle model that studies a set of strategic choices facing manufacturers as they design the joint product/service bundle for a product which may require maintenance and repair support after its sale. Iyer (1998) analyzes how manufacturers should coordinate distribution channels when retailers compete in price as well as important nonprice factors such as the provision of product information, free repair, faster check-out, or after-sales service. In particular, some recent papers look at PBC issues. For example, Kim et al. (2007) analyze two practically important issues of contracting in service supply chains – performance requirement allocation and risk sharing. Kim et al. (2010) study how the low rate of system disruption influences the efficiency of PBL contracts with two different measurements. Bakshi et al. (2015)

examine reliability signaling under Performance-based contracts and Resource-based contract. What is new in this chapter is as follows. In (Kim et al., 2007) and (Bakshi et al., 2015), parts are assumed to be repairable, thus replenishment is not necessarily needed. However, in this chapter, we focus on the components like electronics which are normally scrapped once failed, and furthermore, due to the obsolescence risk, replenishing those parts may become problematic. Another difference is that in those papers, product order quantity is not considered or assumed to be an exogenous parameter. In this chapter, we incorporate the customer's order quantity decisions in the contracting model, rather than only consider after-sales service. Besides, we also look at different penalty terms under PBC and find out the different impact on product availability and contract efficiency.

Finally, the model in this chapter also relates to spare parts inventory and part obsolescence management, especially with Lifetime-buy decisions. For spare parts inventory management, Muckstadt (2005) gives comprehensive reviews on the models and algorithms of inventory planning. On the other hand, Bradley and Guerrero (2009) investigate optimal Lifetime-buy decisions in a setting with multiple obsolete parts. Unlike those papers, the main focus in this chapter is not giving optimal spare parts inventory control policies, but is exploring how expired spare parts impact product availability and the supply chain members' profits, when the product and service are provided under different type of contracts.

In summary, we make the following contributions to literature. First of all, we capture the decision for joint purchasing of the products and the after-sales services in the model, where revenue is impacted by product availability. Secondly, we investigate the impact of part obsolescence on contracting decisions. To the best of our knowledge, this is the first work that studies the impact of Lifetime-buy planning on the comparison of service contracts. Finally, within PBC, we compare two forms of penalty schemes: penalty on lost revenue and penalty on spare parts stockouts and show how each type of penalty can affect inventory decision and profits.

2.3 Model Setting

2.3.1 Problem Description

Consider a two-player supply chain consisting of one customer (she) and one supplier (he), who are both risk neural, as shown in Figure 2.1. The customer needs to buy some identical durable products to generate revenue by satisfying the external demand that she faces. The product typically consists of subsystems or parts. The supplier is the final assembler of the product and the only after-sales service provider (which is a reasonable assumption with regards to particularly critical equipments and services, due to the complex and proprietary nature of the products). During the long life cycle of the product, some critical components may fail, which leads to product downtime. As the after-sales service provider, the supplier has to plan and stock spare parts so that failed components can be replaced. The problem becomes more challenging when those components (especially like the electronic components) become obsolete due to various factors such as rapid technological or the market changes. Component obsolescence implies that its manufacturer discontinues the production or supply, and reacquiring those components usually incurs much higher cost as this is generally done via remanufacturing or via sourcing from other spare parts sellers.

To mitigate the risk of component obsolescence, a Lifetime-buy strategy is adopted by the supplier. The supplier buys a certain number of components as spare parts before they become obsolete to cover the total maintenance and service demand for the entire life of the product. In the model, we assume that when contracting with the customer to sell the product, the supplier considers the obsolescence issue and incorporates the Lifetime-buy cost into his planning; however, at that time, the components may have not been obsolete yet. We make such an assumption based on the following reasons. First, the time period that the electronic components become obsolete is usually much shorter compared to the life cycle of the product. For example, the life cycles of OTS (off-the-shelf) electronic parts, such as memory and microprocessors, oftentimes last on the order of 2 years. But the product, such as communication equipment and large Internet routers, can be used for 20 years or more (Bradley and Guerrero, 2009). Moreover, we also observe that "designers frequently find that components and technologies have been made obsolete before the newly designed equipment is ready for production (Trenchard, 2005)". Second, the time when the components expire is sometimes unknown. Although some part manufacturers may give End-of-life notice to the buyers, "electronic parts can also be

made obsolete with little or no warning" (Bradley and Guerrero, 2008). So the supplier has to plan in advance to avoid such unexpected situation. Since reacquiring the obsolete components is usually very expensive, obsolescence issue has become one of the main costs in the life cycle of long-field life systems. Due to the above reasons, before signing the contract, it is necessary for the supplier to include the obsolescence cost within his budget planning (Rojo et al., 2010).

Because only working products can generate revenue for the customer, she also takes product availability and the after-sales service into account when buying the product, i.e., contracts are designed with regards to both the product and the after-sales support. In this chapter, the product and service are delivered based on two type of contracts - W+T and PBC. We assume the service contract period (i.e., the life cycle of the product) is $[0, T]$, where at time 0, the products start to be used by the customer. Under W+T, the supplier charges price p for each product and offers a certain period of warranty. Let $t, t < T$ be the warranty duration. Under warranty, i.e., during $[0, t]$, the supplier repairs all the product failures free of charge. After warranty, i.e., $(t, T]$ is the period of the transaction-based contract. The supplier charges p_s for each repair which includes replacement of failed parts. Under PBC, theoretically, there are no specific prices for products or services. The customer pays the supplier for the products' real performance during the contract period. In this chapter, we use the "initial payment + penalty" as the structure of the PBC, which is common in the literature (e.g., Kim et al., 2007). The supplier first charges an initial payment w for each product. During the contract period, if product failures can not be repaired due to spare parts stockout, the supplier has to pay corresponding penalties to the customer. Penalties can be based on the number of spare parts stockouts $\mathbf{B}$ or the customer's revenue loss $\mathbf{R}_L$ (if all product failures are repaired, there will be no loss for the customer). Let κ be the penalty rate, then the supplier's penalty is $\kappa\mathbf{B}$ or $\kappa\mathbf{R}_L$. In the former, $\kappa > 0$ and in the latter, $\kappa \in [0, 1]$. In this chapter, we assume κ can be set by the supplier or exogenously given (e.g., by negotiation).

The business event sequence is as depicted in Figure 2.1. First, both of the supplier and the customer observe the external demand. Then the supplier gives the contract terms (p, p_s under W+T or w, κ under PBC). After that, the customer gives the order quantity N. We assume that N takes continuous values, which is a common approximation in game theoretic models (Cachon and Netessine, 2004). Then the supplier delivers the product to the customer and starts to provide after-sales service. Meanwhile, he also plans for the amount of Lifetime-buy for the spare parts. Although there can be multiple components in one product, we assume there is one critical component which can lead to product failure once it is defective. Furthermore,
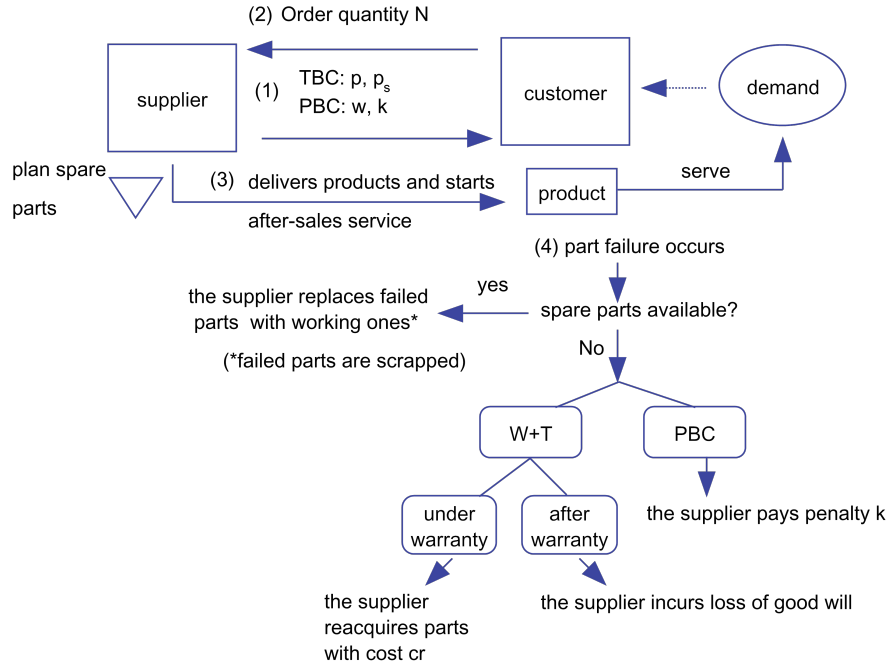
FIGURE 2.1: Sequence of events.

the component may become obsolete[6]. The one-component assumption is also used in OM literature (e.g., Kim and Netessine, 2013). Let s be the Lifetime-buy quantity of the spare parts and $\delta \equiv s/N$. We call δ the Lifetime-buy ratio in this chapter. We allow for δ to be exogenously or endogenously specified. Exogenous δ would represent the case when critical components are mandated to be purchased by the supplier for servicing the product and the quantity of the components is specified exogenously, whereas endogenous δ would capture the case when the supplier can set the number of spare parts in stock based on his service plan. During the after-sales contract period, once part failure occurs, the supplier will replace the failed parts with the working units stocked in the spare parts inventory. We assume that failed parts are scrapped, which is often the case for electronic components (such as processors and memory chips) that can no longer be economically repaired. However, if spare parts are run out when a part is failed, a stockout occurs. The supplier will make different reactions depending on the types of contacts.

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[6]For the components without the obsolescence issues, replenishment is normally not a problem. Then, the shortage of those components does not have big impact on product availability since they can be replenished quickly. In this chapter, we focus on the components which can become obsolete and thus have a much bigger impact on product availability and the supply chain members' profit. However, if considering multiple components with heterogeneous failure rates and all those parts may become obsolete, the model becomes highly complex and intractable for game-theoretic analysis.

(i) Under W+T, if spare parts stockout occurs within the period of warranty, the supplier has to reacquire the parts since he has to guarantee a full product availability. If stockout occurs after warranty, the supplier is not obliged to repair all the failures, although repairing failed parts can be a revenue stream for the supplier[7].

(ii) Under PBC, if spare parts stockout occurs, the supplier pays a penalty to the customer for product unavailability. As stated above, the penalty can be based on the number of stockouts or a proportion of the customer's lost revenue. The supplier maintains the products and at the end of the contract period, performance is measured based on the overall product availability and payment is made according to the contract.

When planning for after-sales service and setting the contract terms, the supplier's cost structure is as follows. The unit production cost of the whole product for is c_p. The unit production cost of the critical component, i.e, the unit Lifetime-buy price is c_s. Once the component becomes obsolete, the unit reacquisition cost is $c_r, c_r \geq c_s$. The supplier also incurs the holding cost of leftover inventory c_h if not all the parts are used.

Particularly, we assume commitment is not an issue in the analysis. In other words, We do not consider default or incomplete fulfil of contracts. Specifically, for the proposed number of spare parts, the commitment can be ensured in some ways. For example, under W+T, the customer can ask the supplier to show the payment for the predefined number of spare parts before paying for the products to the supplier. Under PBC, as long as spare parts shortage occurs, the supplier has to pay a penalty. So the supplier has no incentive to deviate from the equilibrium solution. On the other hand, if the supplier refuses to replenish the spare parts and repair the failed product during warranty, court action may be initiated by the customer. Then, the supplier may have to pay a huge fine because of the default. Also, loss of reputation may make the supplier lose more customers in the long run. Such default is not commonly seen in manufacturing industry.

Finally, we first assume that both the customer and the supplier are risk neutral and maximize their expected profit. In §2.7.3, we introduce risk aversion for both players.

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[7]The supplier may also incur a cost related to the loss of goodwill. In our model, this cost is ignored.

2.3.2 Product Availability

The critical components are subject to random failures during the process of running. Normally a Poisson process is formulated to model the failure process. However, the discrete distribution will cause tractability issues for our game-theoretic analysis. Following literature such as (Kim et al., 2007; Zipkin, 2000), we approximate the failure occurrence by a continuous process. Let $\mathbf{x}$ denote the average number of failures of the N components in unit time. We assume that $\mathbf{x} \in [0, \infty)$ is a continuous random variable with density function $f(x)$, and distribution $F(x)$. $F(0) = 0$, $\lim_{x \to \infty} F(x) = 1$, and $F(x)$ is continuously differentiable with respect to (w.r.t.) x in $[0, \infty)$. Then, the number of failed components, i.e., the demand for spare parts in a time duration $\Delta\tau$ is $N\mathbf{x}\Delta\tau$.

If there are no spare parts available when a failure occurs and no replenishment is made, the product then becomes unavailable. The number of working products at time τ is denoted by $\mathbf{N}_v(\tau)$. We define product availability as $\mathbf{A} = \int_0^T \mathbf{N}_v(\tau)d\tau$. Let $A_P(\delta, T)$ and $A_{W+T}(\delta, t, T)$ be the expected product availability under PBC and W+T (we use the subscript $W+T$ and P to represent W+T and PBC throughout this chapter). We have the properties summarized in Proposition 2.1.

Proposition 2.1. *(1) $A_P(\delta, T)$ is increasing and concave in δ. $A_{W+T}(\delta, t, T)$ is increasing in δ and t, for $t \in (0, T)$.*

(2) Given the same value of δ, $A_{W+T}(\delta, t, T) \geq A_P(\delta, T)$.

The proof can be found in Appendix C.

The results in Proposition 2.1 can be explained as follows. Since downtime only occurs when no spare parts are available, product availability increases in stock quantity δ. Under W+T, warranty implies 100% uptime. Thus product availability increases in the duration of warranty t. Finally, product availability under W+T is higher than under PBC, because warranty in W+T increases the overall uptime.

2.3.3 Revenue Function

We model the customer's revenue $\mathbf{R}$, as a quadratic function in the number of working products. Specifically, $\mathbf{R} = \int_0^T \mathbf{r}(\tau)\mathbf{N}_v(\tau)d\tau$, where $\mathbf{r}(\tau) = \boldsymbol{\xi} - \mathbf{N}_v(\tau)$. $\mathbf{r}(\tau)$ is the revenue generated per

working product. $\boldsymbol{\xi}$ represents the stochastic demand and has mean D and variance $Var(\boldsymbol{\xi})$. We assume that $\boldsymbol{\xi}$ is independent of $\mathbf{x}$. The form of the revenue rate $\mathbf{r}(\tau)$ implies that it is increasing in the external demand $\boldsymbol{\xi}$ and and decreasing in the number of working products (as more working products would result in a lower per-unit revenue if demand is fixed). To rule out triviality, we also assume that $\boldsymbol{\xi}$ takes values no less than N so that $\mathbf{r}(\tau)$ cannot be negative. Additive or multiplicative shocks with linear inverse demand function are often used in the literature, e.g., (Anupindi and Jiang, 2008). Then we have the expected revenue functions under PBC and W+T as $\mathbb{E}[\mathbf{R}_P] = DNA_1(\delta, T) - N^2 A_2(\delta, T)$ and $\mathbb{E}[\mathbf{R}_{W+T}] = DNA_3(\delta, t, T) - N^2 A_4(\delta, t, T)$, where $A_1(\delta, T) = A_P(\delta, T)$ $A_2(\delta, T) = \mathbb{E}[\mathbf{A}_P{}^2]$, $A_3(\delta, t, T) = A_{W+T}(\delta, t, T)$ and $A_4(\delta, t, T) = \mathbb{E}[\mathbf{A}_{W+T}{}^2]$[8].

Our insights do not change when we consider other forms of the revenue function, such as the Cobb-Douglas function or a newsvendor-type framework discussed in §2.7.2.1 and §2.7.2.2. For a list of notations, please refer to Table 2.1.

TABLE 2.1: List of notations.

p	Unit price of the product	$\mathbf{x}$	Average number of failures in unit time
p_s	Price for each repair service	$\mathbf{C}, C$	Total costs, normalized expected total costs
w	Initial payment under PBC	$\mathbf{B}, B$	Spare parts stockouts, normalized expected stockouts
κ	Penalty rate under PBC	$\mathbf{L}$	Spare parts leftover
N	Product order quantity	$\mathbf{A}$	Product availability (PA)
c_p	Unit production cost of the product	A_1, A_2	Expected PA , $\mathbb{E}[\mathbf{A}^2]$ under W+T
c_s	Unit Lifetime-buy cost of spare parts	A_3, A_4	Expected PA , $\mathbb{E}[\mathbf{A}^2]$ under PBC
c_r	Unit reacquisition cost of spare parts	$\boldsymbol{\xi}, D$	External demand, $\mathbb{E}[\boldsymbol{\xi}]$
s	Lifetime-buy spare parts quantity	$\mathbf{R}$	The customer's revenue
δ	s/N, Lifetime-buy ratio	$\mathbf{r}$	Unit revenue rate of working product
T	Contract period	Π, π	Profit of the supply chain, profit of the supplier/customer
t	Duration of warranty	E	Efficiency

---

[8]Expectations are w.r.t. all the random parameters throughout this chapter.

2.4 Contracting

As described in §2.3, the Lifetime-buy ratio δ, can be decided exogenously or endogenously. In practice, the former case represents the setting when some critical parts are mandated to be stocked with the product, or the supplier has various service levels at different prices. In the latter case, δ can be set by the supplier himself. We start by looking at the First Best solution as the benchmark, i.e., the case of an integrated supply chain. After that, we discuss the decision problems of the decentralized setting, where the customer and the supplier maximize their own profits.

2.4.1 First Best

In the centralized decision-making setting, the integrated firm solves for the optimal order quantity of the products and Lifetime-buy quantity of spare parts (in case of exogenous δ) that maximize the expected profit, which is the difference between the expected total revenue and the expected total costs. The firm's decision problem can be written as

$$\max_{N,\,(\delta)} \mathbb{E}[\Pi^{SC}] = \mathbb{E}(\mathbf{R}) - \mathbb{E}[Nc_p + N\delta c_s + c_h\mathbf{L}], \tag{2.1}$$

where $\mathbf{L} = N(\delta - \mathbf{x}T)^+$, is the leftover inventory after the contract period. The total costs consist of production cost of the products, spare parts (Lifetime-buy) inventory cost, and inventory leftover holding cost. (δ) means δ can be exogenous or endogenous. This also applies to the decentralized cases. In addition, let N^0 be the optimal order quantity, and δ^0 be the optimal Lifetime-buy ratio for the supply chain. By solving the optimization problem in Equation (2.1), we have the optimal solutions as given in Table 2.2, where $C(\delta, T) = \mathbb{E}[Nc_p + N\delta c_s + c_h\mathbf{L}]/N$, and $A'(\cdot)$ is the function of the first derivative w.r.t. δ.

We assume D is large enough to ensure a positive order quantity N^0. We use the First Best solution as the benchmark for the analysis of contract efficiency in the decentralized setting. We define the efficiency E as the ratio of the supply chain's profit at equilibrium in the decentralized setting with respect to the maximum profit in the centralized setting, i.e., $E_{W+T/P} = \Pi^*_{W+T/P}/\Pi^0$. Next, we formulate the decision problems in the decentralized setting under W+T and PBC.

TABLE 2.2: Optimal solution of the First Best case.

	Exogenous δ	Endogenous δ
δ^0	-	$\frac{DA_1'(\delta^0,T)-C'(\delta^0,T)}{DA_1(\delta^0,T)-C(\delta^0,T)} = \frac{A_2'(\delta^0,T)}{2A_2(\delta^0,T)}$
N^0	$\frac{DA_1(\delta,T)-C(\delta,T)}{2A_2(\delta,T)}$	$\frac{DA_1(\delta^0,T)-C(\delta^0,T)}{2A_2(\delta^0,T)}$
Π^0	$\frac{[DA_1(\delta,T)-C(\delta,T)]^2}{4A_2(\delta,T)}$	$\frac{[DA_1(\delta^0,T)-C(\delta^0,T)]^2}{4A_2(\delta^0,T)}$

2.4.2   Warranty + Transaction-based Contract

Under W+T, the supplier sells the products with a certain warranty period t. We assume t is an exogenous parameter. Once the warranty period expires, the service is operated under TBC, where the supplier can repair the product on a per-transaction basis. The supplier can thus generate additional profit by servicing the product. From the sequence of events in Figure 2.1, we know that the supplier makes decisions on the price of the product (including warranty) p and the price of service p_s in TBC (and δ if endogenous). The customer decides on N, i.e., how many products to buy given the supplier's prices. The supplier's problem under W+T can be written as

$$\max_{p,p_s,(\delta)} \mathbb{E}(\pi_{W+T}^s) = N_{W+T}^* p + \mathbb{E}[\mathbf{S}(N_{W+T}^*,\delta,t,T)]p_s - \mathbb{E}[\mathbf{C}_{W+T}(N_{W+T}^*,\delta,t,T)]$$

$$s.t.$$

$$N_{W+T}^* = \arg\max \mathbb{E}[\pi_{W+T}^c(N)],$$

$$\mathbb{E}[\pi_{W+T}^c(N)] = \mathbb{E}(\mathbf{R}_{W+T}) - Np - p_s\mathbb{E}[\mathbf{S}(N,\delta,t,T)].$$

$$(2.2)$$

where $\mathbf{S}(N,\delta,t,T) = \min\{N(\delta - \mathbf{x}t)^+, N\mathbf{x}(T-t)\}$, which is the number spare parts used during TBC (the minimum function guarantees the supplier cannot sell more parts than needed); and $\mathbf{C}_{W+T}(N_{W+T}^*,\delta,t,T)$ is the supplier's total costs under W+T. Since the supplier has to repair any failures during the warranty period, he might incur the cost of reacquisition. The spare parts stockouts during warranty can be calculated as $\mathbf{B}_W = N(\mathbf{x}t - \delta)^+$. Then total costs for the supplier can be expressed as $\mathbf{C}_{W+T}(N,\delta,t,T) = Nc_p + N\delta c_s + c_h\mathbf{L} + c_r\mathbf{B}_W$, i.e., the sum of production cost of the products, Lifetime-buy cost of spare parts, leftover inventory holding cost, and the reacquisition cost. In addition, we use superscripts c and s to denote the customer and the supplier respectively throughout this chapter.

2.4.3 Performance-based Contract

Under PBC, we model the payments between the customer and the supplier via an "initial payment + penalty" type of mechanism. Here the supplier charges the customer a fee w for delivering the product, and is penalized by the customer if under-performance (measured by product unavailability) occurs. We capture the performance-based characteristic by considering two forms of penalty: the supplier can compensate a proportion of the customer's lost revenue (PBC-L), or be penalized on the number of stockouts of spare parts (PBC-B). The decision variables for the supplier are the initial payment w and the penalty rate κ (and δ if endogenous). And the customer still decides on the order quantity N.

(i) Under PBC-L, the supplier's problem can be written as

$$
\begin{aligned}
\max_{w,(\kappa),(\delta)} \quad & \mathbb{E}(\pi_{PL}^s) = N_{PL}^* p - \kappa \mathbb{E}[\mathbf{R}_L] - \mathbb{E}[\mathbf{C}_P(N_{PL}^*, \delta, T)] \\
& s.t. \\
& N_{PL}^* = \arg\max \mathbb{E}[\pi_{PL}^c(N)], \\
& \mathbb{E}[\pi_{PL}^c(N)] = \mathbb{E}[\mathbf{R}_P] + \kappa \mathbb{E}[\mathbf{R}_L] - Nw, \\
& (0 \le \kappa \le 1).
\end{aligned}
\tag{2.3}
$$

where the customer's lost revenue $\mathbf{R}_L = \int_0^T (\xi - N) N d\tau - \int_0^T [\xi - \mathbf{N}_v(\tau)]\mathbf{N}_v(\tau)d\tau$, i.e., the difference between the revenue when all N products are working and when product unavailability possibly occurs ($N_v(\tau) \le N(\tau)$ for $\tau \in [0, T]$).

(ii) Under PBC-B, the supplier's problem can be written as

$$
\begin{aligned}
\max_{w,(\kappa),(\delta)} \quad & \mathbb{E}(\pi_{PB}^s) = N_{PB}^* p - \kappa \mathbb{E}[\mathbf{B}(N_{PB}^*, \delta, T)] - \mathbb{E}[\mathbf{C}_P(N_{PB}^*, \delta, T)] \\
& s.t. \\
& N_{PB}^* = \arg\max \mathbb{E}[\pi_{PB}^c(N)], \\
& \mathbb{E}[\pi_{PB}^c(N)] = \mathbb{E}[\mathbf{R}_P] + \kappa \mathbb{E}[\mathbf{B}(N_{PB}, \delta, T)] - Nw.
\end{aligned}
\tag{2.4}
$$

where $\mathbf{B}(N, \delta, T) = N(\mathbf{x}T - \delta)^+$, is the number of stockouts during the entire contract period.

The supplier's costs under PBC $\mathbf{C}_P(N, \delta, T) = Nc_p + N\delta c_s + c_h \mathbf{L}$. Since there is no reacquisition cost, we can verify that $\mathbb{E}[\mathbf{C}_{W+T}] \ge \mathbb{E}[\mathbf{C}_P]$.

Next, we discuss the solutions of the games under the different contracts and analyze how the decisions are impacted by the parameters.

2.5 Analysis with Exogenous δ

Solving the decision problems formulated by Equations (2.2)-(2.4), we derive the equilibrium solutions for the cases under both contracts, which is stated as Lemmas below.

Lemma 2.2. *(1) Under W+T, the customer's profit function is concave in N; the supplier's profit function incorporating the customer's decision is concave in p and in p_s.*

(2) Under W+T, the supplier's optimal solution is a price combination $\widehat{p}^*$, given as $\widehat{p}^* = p^* + p_s^* S(\delta, t, T) = \frac{DA_3(\delta,t,T) + C_{W+T}(\delta,t,T)}{2}$, where $S(\delta, t, T) = \mathbb{E}[\mathbf{S}(N, \delta, t, T)]/N$, and $C_{W+T}(\delta, t, T) = \mathbb{E}[\mathbf{C}_{W+T}(N, \delta, t, T)]/N$.

(3) The customer's optimal order quantity $N_{W+T}^* = \frac{DA_3(\delta,t,T) - C_{W+T}(\delta,t,T)}{4A_4(\delta,t,T)}$.

(4) $\partial \widehat{p}^*/\partial \delta > 0$, and $\partial \widehat{p}^*/\partial t > 0$.

The proof can be found in Appendix D.

At equilibrium, the prices are not unique and the supplier can choose from a menu of different product and service prices in order to satisfy an overall price for the product-service package. This is because the supplier makes a profit from selling both the product and the service. As a result, he can choose different combinations of the two prices depending on the customer's preference. For instance, the supplier may set a lower product price but a relatively higher service price to appeal to customers who prefer low purchasing cost and trust the reliability of the products. On the other hand, he can also set a higher product price and a lower service price for customers who focus more on the maintenance cost. From Lemma 2.2, we also know that prices are increasing in the number of spare parts stocks, and the length of warranty. This is because any increase of spare parts or the warranty duration implies extra cost for the supplier, which would induce the supplier to charge a higher price. Furthermore, we can verify $\partial N_{W+T}^*/\partial D > 0$, $\partial \widehat{p}^*/\partial D > 0$, implying that the growth of external demand leads to higher order quantity and higher prices.

On the other hand, we can verify that the total cost C_{W+T} is convex in δ. If $c_s + c_h F(\frac{\delta}{T}) > c_r[1 - F(\frac{\delta}{t})]$, C_{W+T} is increasing in δ, otherwise decreasing in δ. The intuition behind this inequality is that if the cost of stockout is higher than the cost of Lifetime-buy and leftover inventory holding cost, more inventory before obsolescence actually decreases the total costs. However, when the reacquisition cost is lower compared to the costs of Lifetime-buy plus leftover holding, the total costs increases with δ. Furthermore, we have $\partial \hat{p}^*/\partial t \geq 0$, which implies that a longer warranty period results in a higher transaction price. This confirms the intuition that the customer has to pay a higher price for a longer warranty.

Next, Lemma 2.3 gives the solutions for PBC.

Lemma 2.3. *(1) Under PBC, the customer's profit function is concave in N; the supplier's profit function is concave in w, regardless of penalty terms (lost revenue or the number of stockouts).*

(2) Under PBC-L, $\kappa^* = 0$ if it can be set endogenously by the supplier. Otherwise, $w^* = \frac{D\kappa^2 T(T-A_1)+DA_2[(2-\kappa)\kappa T+(1-\kappa)^2 A_1]+[\kappa T+A_2(1-\kappa)]C_P}{\kappa T+(2-\kappa)A_1}$. In addition, $\partial w^*/\partial \kappa > 0$.

(3) Under PBC-B, the supplier's optimal solution is given as $w^* - \kappa^* B(\delta, T) = \frac{DA_1(\delta,T)+C_P(\delta,T)}{2}$, where $B(\delta,T) = \mathbb{E}[\mathbf{B}(N,\delta,T)]/N$ and $C_P(\delta,T) = \mathbb{E}[\mathbf{C}_P(N,\delta,T)]/N$.

(4) Under PBC-L, endogenous κ and PBC-B, the customer's order quantity $N_{PB}^* = N_{PL}^* = \frac{DA_1(\delta,T)-C_P(\delta,T)}{4A_2(\delta,T)}$. Under PBC-L, exogenous κ, the customer's order quantity $N_{PL}^* = \frac{[DA_1(\delta,T)-C_P(\delta,T)]^2}{4\kappa T+4(2-\kappa)A_2(\delta,T)}$.

The proof can be found in Appendix E.

From Lemma 2.3, we can see that the contract terms are different if penalty is incurred on different performance measures. If the supplier pays a stockout penalty, similarly to the prices under W+T, he will create contract terms w^* and κ^* as a combination based on demand and costs. The difference is that under W+T, a higher price of product+warranty implies a lower service fee in TBC; while under PBC-B, a higher initial price results in a higher penalty rate. If the penalty is incurred on lost revenue, the optimal penalty rate is 0, if it can be set by the supplier. The optimal initial payment in this case equals the price combination under PBC-B. Even if the customer receives no penalty from the supplier, it is the optimal solution for the customer, the supplier and the supply chain. When the supplier is forced to compensate more

for lost revenue, i.e., the penalty rate is exogenously specified, we have $\partial w^*/\partial \kappa > 0$, which means the intimal payment is increasing in the penalty rate.

For the total costs under PBC, $\partial C_P(\delta, T)/\partial \delta > 0$. So the total costs are monotonously increasing in δ. This is because under PBC, the supplier does not incur any direct replenishment cost, but pays a penalty which is incorporated in the contract terms.

Although we have shown the impact of δ and t on the contract terms (prices/penalty rate), it is not straightforward to see how the Lifetime-buy ratio affects profits. We discuss this issue in the next section.

2.5.1   Comparison of Contracts

After we obtain the equilibrium solutions for the contract terms and the customer's order quantity, we can have the maximal profits for the supplier, the customer, and the supply chain. In particular, at equilibrium, the customer and the supplier shares a proportion of the supply chain's profit. In the sequel, We refer "profits" to the profit of the supplier, the customer, and the supply chain. By comparing the equilibrium profits across W+T and PBC, we have the following analytical results as stated in Proposition 2.4.

Proposition 2.4. *For a given spare parts Lifetime-buy ratio (i.e., δ),*

(1) *under PBC-L, $\pi_{PB}^{s(c)*} = \pi_{PL}^{s(c)*}$ if κ is endogenously set by the supplier (and $\kappa^* = 0$).*

(2) *under PBC-L, if κ is exogenously specified, $N_{PL}^* \leq N_{PB}^*$, and $\pi_{PL}^{s(c)*} \leq \pi_{PB}^{s(c)*}$. Furthermore, $\partial N_{PL}^*/\partial \kappa < 0$, $\partial \pi_{PL}^{s(c)*}/\partial \kappa < 0$, $\partial E_{PL}/\partial \kappa < 0$.*

(3) *if no stockouts occur during warranty, $\pi_{W+T}^{s(c)*} = \pi_{PB}^{s(c)*} \geq \pi_{PL}^{s(c)*}$.*

The proof can be found in Appendix F.

In Proposition 2.4, we first look at the impact of different penalty terms under PBC. Under PBC-L, the optimal penalty rate is 0, which makes the profits the same as PBC-B. If κ is forced to be more than zero, the profits under PBC-L will be less than PBC-B. This is because under PBC-B, the number of stockouts and the order quantity have the same manner of impact on the supplier's profit. Thus, penalizing stockouts is equivalent to reducing the order quantity.
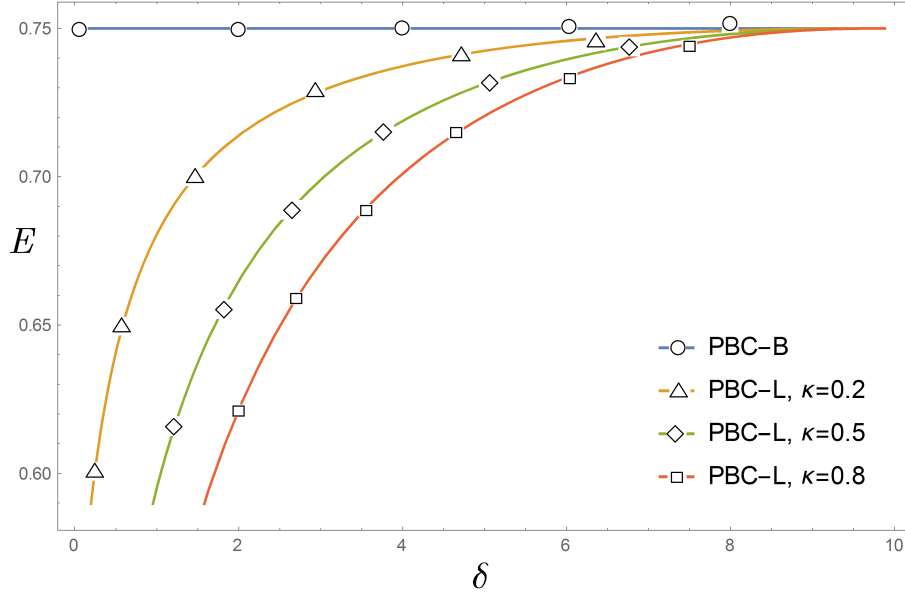
FIGURE 2.2: Contract efficiency under PBC-B and PBC-L, exogenous δ. **x** satisfies uniform distribution in [0,1]. $D = 50, T = 10, c_p = 30, c_s = 10, c_h = 5$.

For the customer, the price combination is the only factor that impacts the order quantity. So the supplier can choose different w^* and κ^* without affecting the customer's decision on N^*. However, if penalty is incurred on lost revenue, κ plays a different role on affecting the supplier's profit compared to the initial price w. The customer's lost revenue now depends on the order quantity as well as on the penalty rate. Since the revenue rate is not a constant, but a function of the order quantity, κ does not act as a product "price" to the customer, in the same manner as w. Consequently, w and κ cannot be treated as factors in a combination. Actually, w^* in such a case is increasing in κ^*, which consequently make the customer give a less order quantity and finally, profits become lower. As stated in part (2), the order quantity, profits, and contract efficiency are decreasing in κ. As shown in Figure 2.2, the efficiency curve of PBC-B is on top of those under PBC-L. Moreover, the efficiency decreases as κ increases. The efficiency of PBC-B remains constant at 75% due to the linear revenue function in our model. The curves converge at the point where δ is large enough to cover all the demand in the contract period, and thus no penalty occurs.

If we compare the profits across W+T and PBC, we can prove that if spare parts are sufficient to cover all the demand during warranty, W+T result in the same profit as PBC-B and PBC-L, endogenous κ. This is because if no spare parts reacquisition is required, the total costs as well as the product availability under W+T and PBC are the same, resulting the same price combinations and order quantity. Only under PBC-L, exogenous κ, the profits are lower.

However, if stockouts occur during warranty, the reacquisition cost c_r will play a role in the supply chain partners' decision. Unfortunately, we cannot have the analytical closed-form results for comparing the profits. We then conduct extensive numerical analysis to observe the impact of Lifetime-buy and reacquisition cost on the supply chain members' profits.

The profits under W+T are dependant on the warranty length t and reacquisition cost c_r. As shown in Figure 2.3, for a given δ, with low reacquisition cost ($c_r = 10$ in the figure), the profit is increasing in t when stockouts occur (where $t > \delta$ in the figure, under a uniform distributed $\mathbf{x}$), whereas with high reacquisition cost ($c_r = 80$ in the figure), it decreases in t when stockouts occur. However, if c_r is medium, the impact of t is implicit. Consider the margin of the product defined as $M^* = \pi^*/N^*$. In this model, we just use the supplier's margin to conduct our analysis since the individuals share part of the supply chain's profit. We have the margin of the supplier under W+T and PBC-B as $M_{W+T}^{s*} = \pi_{W+T}^{s*}/N_{W+T}^* = \frac{DA_3(\delta,t,T)-C_{W+T}(\delta,t,T)}{2}$ and $M_{PB}^{s*} = \pi_{PB}^{s*}/N_{PB}^* = \frac{DA_1(\delta,T)-C_P(\delta,T)}{2}$, respectively. Now we can clearly see that the profit is determined jointly by the margin and the order quantity. For W+T, both the margin and the order quantity are not monotonously affected by the warranty length t, but also depend on c_r . So given δ, different combinations of t and c_r may result in various changes in profit. As illustrated in Figure 2.4, when reacquisition is expensive ($c_r = 80$), both the order quantity and the margin are decreasing in t, and thus profit decreases with t. When reacquisition is cheap ($c_r = 10$), the order quantity almost remains the same but the margin is increasing, so profit increases with t. However, if the reacquisition cost is moderate ($c_r = 40$), the margin is increasing yet the order quantity is decreasing in t. Hence, the profit change caused by t can be in either direction.

Figure 2.5 shows the results of profit comparison under W+T and PBC, varying the Lifetime-buy ratio δ and warranty duration t. First, we can see that in all the subfigures, the profits under W+T are the same as PBC-B where $\delta > t$, i.e., spare parts are more than failure occurrence. This result validates our analytical conclusion stated in part (3), Proposition 2.4. Next, we discuss the cases when $\delta < t$. (i) When c_r is small, as shown in Figure 2.5(a), profit under W+T is higher than under PBC. Longer warranty implies higher product availability. In this case, the revenue increase caused by a higher product availability is more than the cost increase, given c_r is low. As a result, profit will be increasing in t. (ii) When c_r is large, as shown in Figure 2.5(d), profit under W+T is lower than under PBC-B, and in some cases when warranty is long, it is even lower than PBC-L. In this case, the cost of extending the warranty is expensive, because a much higher reacquisition cost is incurred, which decreases the profit. (iii)
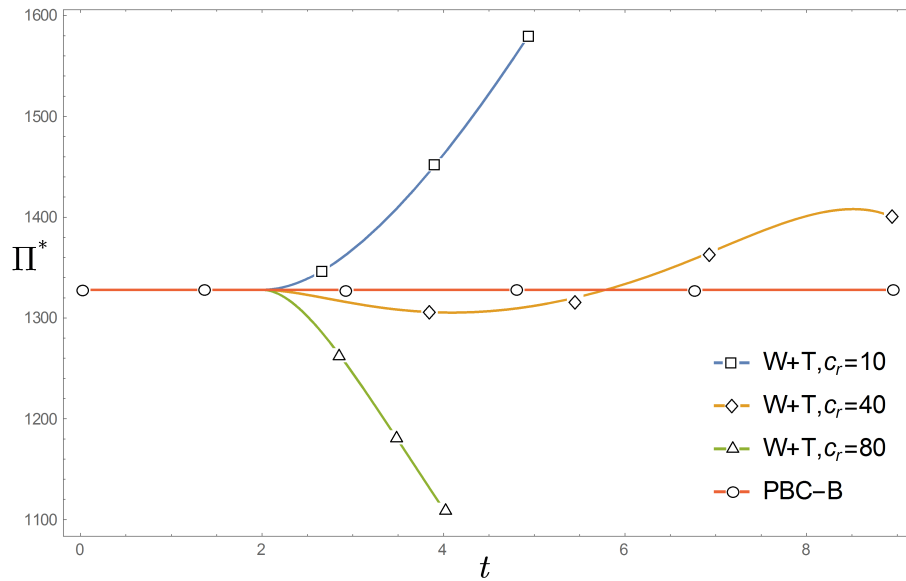
FIGURE 2.3: Profits under W+T and PBC-B, changing with warranty length t and various reacquisition cost c_r. **x** satisfies uniform distribution in [0,1]. $D = 50, T = 10, c_p = 30, c_s = 10, c_h = 5, \delta = 2$.
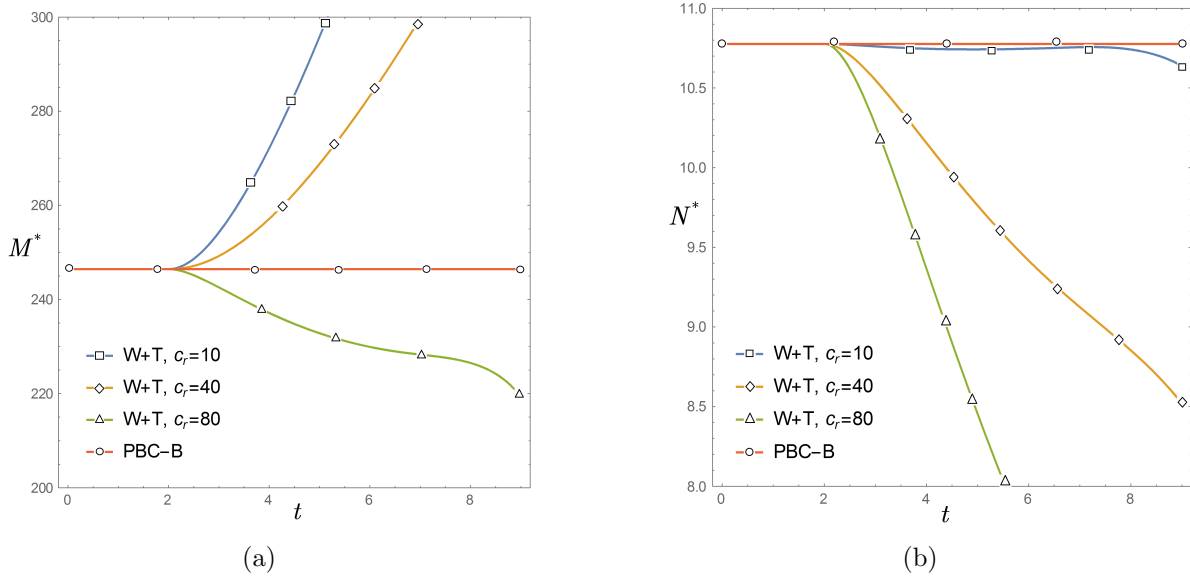
FIGURE 2.4: Margin (a) and order quantity (b) under W+T and PBC-B, changing with t and various reacquisition cost c_r. **x** satisfies uniform distribution in [0,1]. $D = 50, T = 10, c_p = 30, c_s = 10, c_h = 5, \delta = 2$.
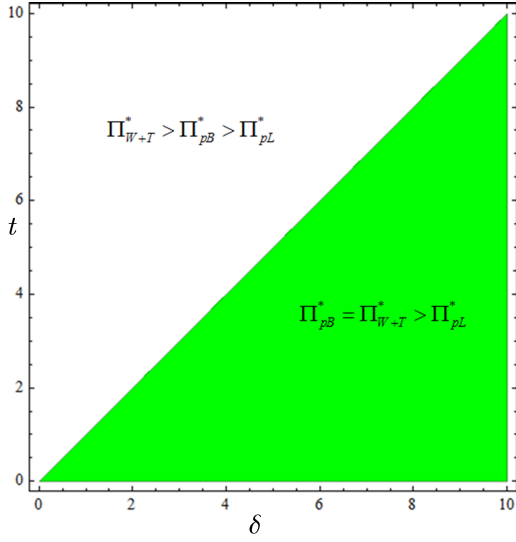
(a) $c_r = 10, \kappa = 0.2$
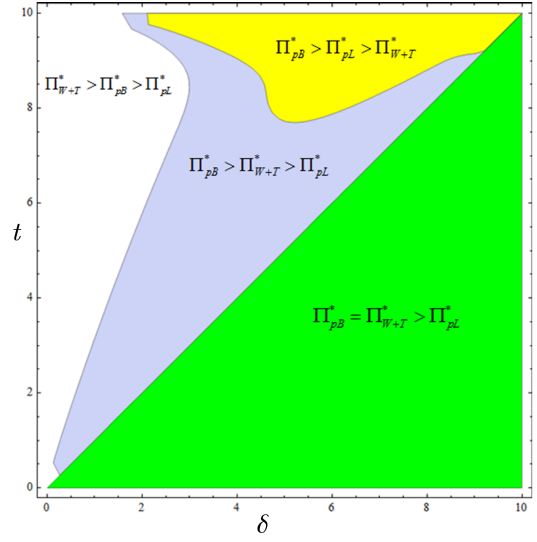
(b) $c_r = 40, \kappa = 0.2$
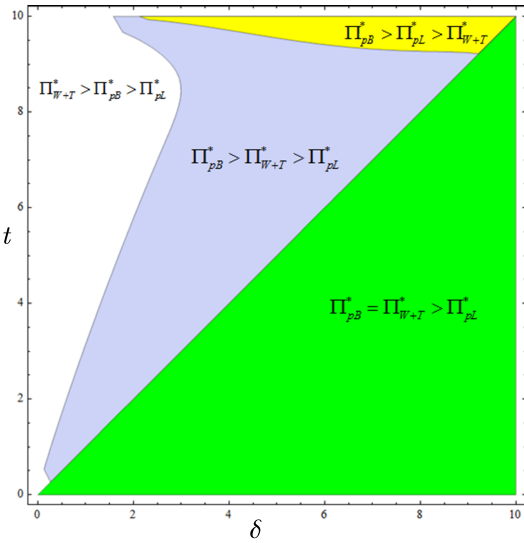
(c) $c_r = 40, \kappa = 0.8$
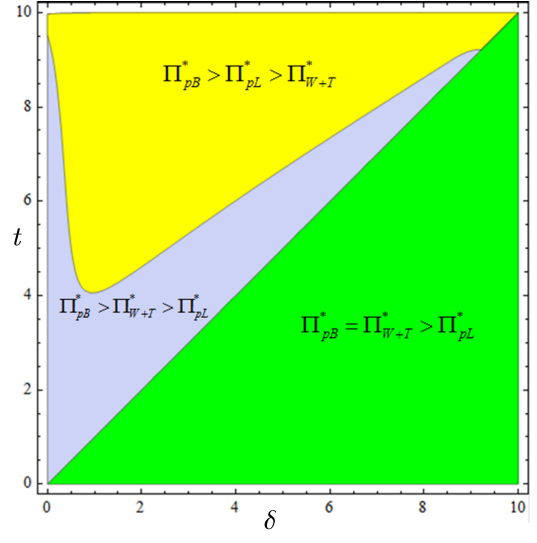
(d) $c_r = 80, \kappa = 0.2$

FIGURE 2.5: Regions of profit comparison under W+T and PBC (exogenous κ), changing with t and δ, with different reacquisition cost c_r. **x** satisfies uniform distribution in [0,1]. $D = 50, T = 10, c_p = 30, c_s = 10, c_h = 5$.

When c_r is medium, as shown in Figure 2.5(b)-(c), Counter-intuitively, W+T performs better when inventory is low but warranty is very long. This is mainly due to the rapid increase of the margin. In other words, with a relatively low reacquisition cost (e.g., the supplier has strong belief that the component will not be obsolete and replenishment is cheap), offering a longer warranty can generate more profit even if he does not have many pre-purchased spare parts.

Parameter sensitivity analysis with exogenous δ is summarized in Appendix B, Table 2.9-2.10. Specifically, the customer's willingness-to-pay (W_{W+T}, W_{PBC}) with respect to a marginal increase of the Lifetime-buy ratio can be viewed as $W_{W+T} = \partial(p^*, p_s^*)/\partial\delta$ under W+T and $W_{PBC} = \partial(w^*, \kappa^*)/\partial\delta$ under PBC. We have

$$W_{PBC} - W_{W+T} = \begin{cases} c_r[1 - F(\delta/T)] + D\displaystyle\int_{\frac{\delta}{t}}^{\infty} \frac{f(x)}{x}dx > 0, \ \text{if } t < \dfrac{T\delta}{1+\delta}, \\[4mm] c_r[1 - F(\delta/T)] + D\left(\displaystyle\int_{\frac{1+\delta}{T}}^{\infty} \frac{f(x)}{x}dx + \int_{\frac{\delta}{t}}^{\frac{1+\delta}{T}} (T - \frac{\delta}{x})f(x)dx\right) > 0, \ \text{if } t > \dfrac{T\delta}{1+\delta}. \end{cases}$$

In other words, under PBC, the customer's willingness-to-pay for a marginal increase of the spare parts is higher than W+T. This is because the prices in the contracts are determined by product availability and the supplier's cost. First, given a marginal increase of δ, the supplier's total cost under W+T has a lower increase than PBC, i.e., $\partial C_P(\delta, T)/\partial\delta > \partial C_{W+T}(\delta, t, T)/\partial\delta$. Second, product availability under PBC increases faster with δ than under W+T, i.e., $\partial A_P(\delta, T)/\partial\delta > \partial A_{W+T}(\delta, t, T)/\partial\delta$. Thus, under PBC, the customer is more willing to pay for a unit increase of the Lifetime-buy ratio.

To sum up, if the Lifetime-buy quantity for spare parts is exogenously specified, penalizing stockouts is better than the customer's lost revenue under PBC. On the other hand, the traditional W+T can be better, depending on the reacquisition cost. If it is relatively cheap to get the parts, offering a longer warranty can result in a higher profit, even with the risk of having insufficient inventory. However, when it is very expensive to reacquire the parts, it is better to go for PBC.

2.6   Analysis with Endogenous δ

In this section, we discuss the case when δ becomes an endogenous decision for the supplier. In such a case, the existence and uniqueness of the equilibrium depends on certain conditions,

because the concavity of profit function w.r.t δ is not always guaranteed. We come up with the solutions summarized in the following Lemma.

Lemma 2.5. *(1) Under W+T, the customer's profit function is concave in N; the supplier's profit function incorporating the customer's response is concave in p and p_s.*

(2) *Under W+T, the optimal Lifetime-buy ratio δ^*_{W+T} solves*

$$\frac{DA'_3(\delta^*_{W+T}, t, T) - C'_{W+T}(\delta^*_{W+T}, t, T)}{DA_3(\delta^*_{W+T}, t, T) - C_{W+T}(\delta^*_{W+T}, t, T)} = \frac{A'_4(\delta^*_{W+T}, t, T)}{2A_4(\delta^*_{W+T}, t, T)}.$$

(3) *Under PBC, the customer's profit function is concave in N; the supplier's profit function is concave in w and κ under PBC-B, and is concave in w under PBC-L.*

(4) *Under PBC-L, $\kappa^* = 0$ if it is endogenous.*

(5) *Under PBC-L, the optimal Lifetime-buy ratio δ^*_{PL} solves*

$$\frac{DA'_1(\delta^*_{PL}, T) - C'_P(\delta^*_{PL}, T)}{DA_1(\delta^*_{PL}, T) - C_P(\delta^*_{PL}, T)} = \frac{2(2 - \kappa)A'_2(\delta^*_{PL}, T)}{4\kappa T + 4(2 - \kappa)A_2(\delta^*_{PL}, T)}.$$

Under PBC-B, the optimal Lifetime-buy ratio δ^*_{PB} solves

$$\frac{DA'_1(\delta^*_{PB}, T) - C'_P(\delta^*_{PB}, T)}{DA_1(\delta^*_{PB}, T) - C_P(\delta^*_{PB}, T)} = \frac{A'_2(\delta^*_{PB}, T)}{2A_2(\delta^*_{PB}, T)}.$$

The optimal prices, order quantity, and profits are listed in Table 2.7.

The proof can be found in Appendix G.

From Lemma 2.5, we know that the optimal Lifetime-buy ratio is dependent on demand, costs, and the product availability functions. Specifically, under PBC-L, if the penalty rate is exogenously positive, κ also plays a role in setting δ^*_{PL}. Compared to the exogenous δ case, $\kappa^* = 0$ is still the optimal solution for PBC-L. Another similar result is that contract terms under W+T (p^*, p^*_s) and PBC-B (w^*, κ^*) are set in combinations. Hence, even if the supplier can set the Lifetime-buy quantity in his own interest, how the contract terms under W+T and PBC are given is in an analogous manner as the exogenous δ case. Next, we compare the optimal Lifetime-buy ratio and profits across contracts.

2.6.1 Comparison of Contracts

Although the fixed point equations for δ^* complicate the problem, we are still able to derive some conclusions based on analytical results.

Proposition 2.6. *If the Lifetime-buy ratio (i.e., δ) becomes endogenous for the supplier,*

(1) *under PBC-B, $\delta^*_{PB} = \delta^0$.*

(2) *under PBC-L, endogenous κ, same equilibrium profit can be achieved as PBC-B.*

(3) *under PBC-L, exogenous κ, $\delta^*_{PL} \geq \delta^*_{PB}$, and $\partial \delta^*_{PL}/\partial \kappa > 0$.*

The proof can be found in Appendix H.

Under PBC-B, the supplier's decision on Lifetime-buy quantity achieves the First Best solution. In the decentralized supply chain, efficiency is lost due to double marginalization of the supply chain partners. Specifically, when the customer and the supplier only maximize their own respective profits, order quantity will be less than the First Best case. However, for the Lifetime-buy ratio of the spare parts, the decentralized setting does not cause deviation from the First Best solution. The reason is twofold. First, the cost structure in the First Best and PBC-B is the same. In both cases, no reacquisition cost is incurred. Second, contract terms under PBC-B are set in combinations where the penalty rate κ does not have independent impact on setting the optimal δ. On the other hand, Under PBC-L, the optimal penalty is no penalty. If κ can be set by the supplier, $\kappa^* = 0$, resulting the same order quantity and profits as PBC-B. In addition, the contract efficiency remains constant at 75%, which is the same as the exogenous δ case. If κ is not allowed to be 0 so that the product performance must be evaluated, the supplier would set a higher Lifetime-buy ratio than the First Best and PBC-B, and the ratio is increasing in κ. In other words, a higher compensation rate of the lost revenue will induce the supplier to buy more spare parts. Consequently, we can conclude that penalizing the supplier based on the customer's lost revenue would induce the supplier to stock more spare parts, and thus result in a higher product availability.

Under W+T, the optimal Lifetime-buy ratio is affected by two more parameters: the reacquisition cost c_r and the duration of warrant t. With general product availability functions, we cannot obtain the closed-form solutions to compare contracts. We use numerical experiments to demonstrate the results of the optimal Lifetime-buy ratio and profits.

The results of PBC are verified by numerical experiments given in Table 2.3 and Table 2.4. On the other hand, for W+T, as in the exogenous δ case, the optimal inventory and profits at equilibrium are determined by warranty length and reacquisition cost. Numerical results can be found in Table 2.5. Parameter sensitivity analysis with endogenous δ is summarized in Appendix B, Table 2.11-2.12.

Firstly, Table 2.3 shows how the equilibrium and resulting profits under PBC-B are affected by parameters D, c_p, c_s and c_h. We can see that the optimal inventory, prices, and order quantity are all increasing in demand D. The total cost C_P increases as well because the supplier will have more spare parts inventory. However, if D is given, increasing costs (c_p, c_s and c_h) may cause different results. In general, any increase in cost will decrease order quantity and profits. In particular, the optimal inventory increases in product production cost c_p, but decreases in spare parts unit cost c_r and inventory holding cost c_h. Because c_p is the cost of the product and is irrelevant to spare parts, if it increases, the supplier needs more spare parts to achieve higher product availability and to guarantee the margin. Nevertheless, c_s and c_h are costs associated with spare parts. If the cost of the product remains the same, increasing the costs of spare parts intuitively reduces the level of inventory. In addition, since the optimal inventory under PBC-B is always the same as the First Best case, the efficiency remains constant at 75%.

Secondly, as shown in Table 2.4, under PBC-L, the optimal inventory is increasing in the penalty rate κ, if it is set exogenously. If the customer asks for more compensation on her lost revenue, the supplier will set a higher inventory level to reduce the penalty. However, this raises the initial price and decreases the order quantity and profits. Although product availability becomes higher in this case, the efficiency is below the level of the other case of PBC. It is generally argued that PBC can increase product availability, but our analysis here shows that it may depend on the penalty schemes. In addition, higher product availability can lead to lower efficiency.

Thirdly, under W+T, the warranty length t and reacquisition cost c_r jointly impact the decision on the optimal inventory. As shown in Table 2.5, if reacquisition is cheap ($c_r = 20$ in the Table), the optimal inventory can decrease in the warranty length t, and higher profits can be achieved. Given a longer warranty, the supplier does not always have to stock more spare parts. In such a case, the revenue increase caused by higher product availability due to a longer warranty period, is higher than the replenishment cost. So the total profit increases in t while δ^*_{W+T} can be low. However, if reacquisition becomes more expensive ($c_r = 40, 80$ in the Table), the supplier has more inventory, and the total profits decrease in c_r as well.

TABLE 2.3: Numerical experiment results of the equilibrium solution under PBC-B, endogenous δ. **x** satisfies uniform distribution in [0,1]. $T = 10$.

δ^*_{PB}	$w^* - \kappa^* B_P(\delta^*_{PB})$	N^*_{PB}	π^{s*}_{PB}	π^{c*}_{PB}	E	D	c_p	c_s	c_h	C_P
5.90	280.11	9.94	1812.85	906.43	75%	50	30	10	5	97.72
6.58	386.49	14.79	4138.76	2069.38	75%	70	30	10	5	106.64
7.03	491.10	19.68	7447.37	3723.69	75%	90	30	10	5	112.69
7.48	646.00	27.07	14267.60	7133.80	75%	120	30	10	5	118.84
5.95	286.02	9.65	1714.89	857.44	75%	50	40	10	5	108.39
6.00	291.89	9.37	1619.77	809.89	75%	50	50	10	5	119.03
6.05	297.72	9.09	1527.47	763.73	75%	50	60	10	5	129.67
6.10	303.52	8.81	1437.98	718.99	75%	50	70	10	5	140.28
5.10	277.54	9.41	1547.34	773.67	75%	50	30	15	5	113.04
4.42	271.65	8.96	1329.18	664.60	75%	50	30	20	5	123.26
3.83	263.23	8.56	1148.71	574.36	75%	50	30	25	5	129.37
3.31	253.00	8.26	998.66	499.33	75%	50	30	30	5	132.11
5.67	277.92	9.90	1779.55	889.78	75%	50	30	10	7	98.23
5.45	275.73	9.87	1748.62	874.31	75%	50	30	10	9	98.59
5.24	272.49	9.83	1706.04	853.02	75%	50	30	10	12	98.93

TABLE 2.4: Numerical experiment results of the equilibrium solution under PBC-L, endogenous δ, exogenous κ. **x** satisfies uniform distribution in [0,1]. $T = 10$.

δ^*_{PL}	κ	w^*	N^*_{PL}	π^{s*}_{PL}	π^{c*}_{PL}	E	D	c_p	c_s	c_h	C_P
6.46	0.1	289.65	9.76	1790.0	902.79	74.55%	50	30	10	5	104.98
6.68	0.2	294.79	9.68	1789.20	899.30	74.15%	50	30	10	5	107.98
6.81	0.3	298.07	9.63	1781.01	896.87	73.86%	50	30	10	5	109.66
6.88	0.4	300.43	9.59	1774.78	895.40	73.65%	50	30	10	5	110.58
6.92	0.5	302.44	9.56	1769.41	894.30	73.47%	50	30	10	5	111.22
6.96	0.6	304.27	9.53	1764.49	893.35	73.31%	50	30	10	5	111.75
6.99	0.7	305.93	9.50	1760.03	892.61	73.16%	50	30	10	5	112.14
7.03	0.8	307.60	9.48	1755.44	891.60	73.01%	50	30	10	5	112.67
7.05	0.9	309.08	9.46	1751.52	891.07	72.89%	50	30	10	5	112.92
7.07	1.0	310.39	9.44	1750.23	890.81	72.87%	50	30	10	5	113.32

TABLE 2.5: Numerical experiment results of the equilibrium under W+T, endogenous δ. **x** satisfies uniform distribution in [0,1]. $T = 10, D = 50, c_p = 30, c_s = 10, c_h = 5$.

δ^*_{W+T}	t	$p^* + p^*_s S(\delta^*_{W+T})$	N^*_{W+T}	π^{s*}_{W+T}	π^{c*}_{W+T}	E	c_r	C_{W+T}
5.90	1	280.11	9.94	1812.85	906.43	75.00%	20	97.72
5.90	3	280.11	9.94	1812.85	906.43	75.00%	20	97.72
5.90	5	280.11	9.94	1812.85	906.43	75.00%	20	97.72
5.18	7	275.29	10.05	1830.25	915.13	75.72%	20	93.26
3.78	9	295.60	10.01	1941.92	970.96	80.34%	20	101.65
5.90	1	280.11	9.94	1812.85	906.43	75.00%	40	97.72
5.90	3	280.11	9.94	1812.85	906.43	75.00%	40	97.72
5.90	5	280.11	9.94	1812.85	906.43	75.00%	40	97.72
6.11	7	286.30	9.84	1807.14	903.57	74.76%	40	102.71
6.21	9	306.04	9.54	1784.33	892.16	73.82%	40	119.04
5.90	1	280.11	9.94	1812.85	906.43	75.00%	80	97.72
5.90	3	280.11	9.94	1812.85	906.43	75.00%	80	97.72
5.90	5	280.11	9.94	1812.85	906.43	75.00%	80	97.72
6.57	7	291.94	9.73	1796.08	898.04	74.31%	80	107.51
7.56	9	312.40	9.27	1700.63	850.31	70.36%	80	129.10

In Figure 2.6, first we can see that δ^* under PBC-L is higher than PBC-B. As κ increases, the difference is even higher. This illustration validates our results as given in part (3), Proposition 2.6. Next, if we look at δ^* under W+T, we find that as the warranty duration increases, the supplier makes different decision on δ^* based on the reacquisition cost. If the reacquisition cost is relative low ($c_r = 20$ in the Figure), δ^* decreases in t. If the reacquisition cost is high ($c_r = 40, 80$ in the Figure), δ^* becomes increasing t. This result implies that longer warranty does not necessarily have to go with more spare parts. If replenishment is cheap, the supplier can give a longer warranty to the customer with fewer spare parts. The economic mechanism behind this is that the customer is willing to pay a higher price to have a longer warranty, and this increased price brings more income for the supplier, given reacquiring spare parts is relatively cheap.

Figure 2.7 shows the equilibrium profit under different contracts (we use the supply chain's profit in the Figure). First, PBC-L has a lower profit than PBC-B, and the profit under PBC-L is decreasing in κ. If we look back on the exogenous δ case, we find that the result is the same. Although product availability under PBC-L is higher, the profit will be drawn down by the penalty rate. It is generally argued that PBC can increase product availability, but our analysis here shows that it may depend on the penalty schemes. In addition, higher product

$$t \in [1,9], \ \kappa \in [0.1, 0.9]$$
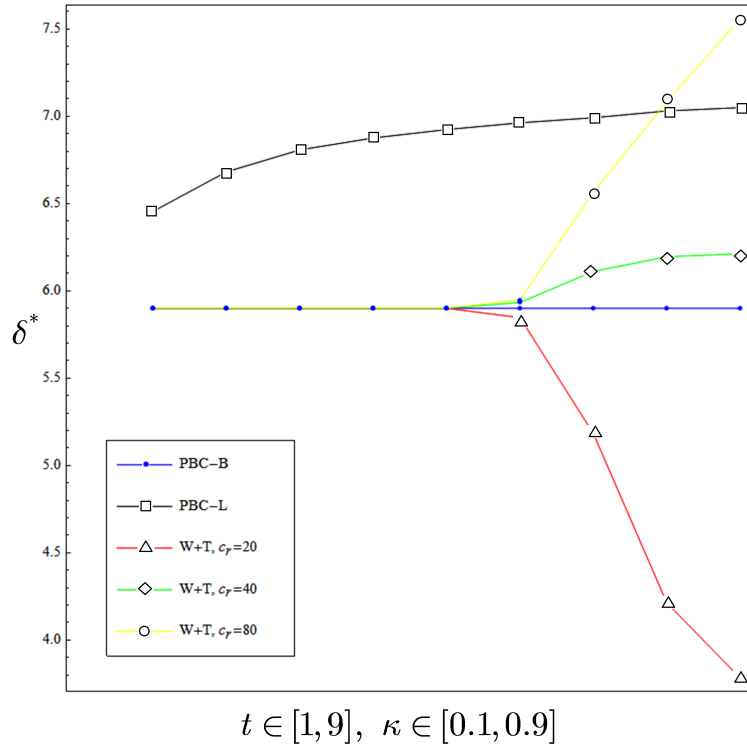
FIGURE 2.6: δ^* under W+T and PBC, changing in warranty length t and penalty rate κ, with various reacquisition cost c_r. **x** satisfies uniform distribution in [0,1]. $D = 50, T = 10, c_p = 30, c_s = 10, c_h = 5$.

availability might lead to lower profits and efficiency. Second, the profit under W+T is likewise impacted by c_r and t. Combining the results in Figure 2.6, we can see that as the warranty becomes longer, profit under W+T can be higher than PBC yet with a lower Lifetime-buy quantity, if the reacquisition cost is low. As c_r becomes larger, profit under W+T will become lower than PBC.

2.7 Some Extensions

2.7.1 A special case: the supplier reacquires all the stockouts under PBC

In previous sections, we have assumed that under PBC, the supplier is not mandated to reacquire the obsolete components but choose to pay a penalty. In this section, we discuss a special case when stockouts are not allowed to be untreated, and the supplier is requested to restore

$$t \in [1,9], \ \kappa \in [0.1, 0.9]$$
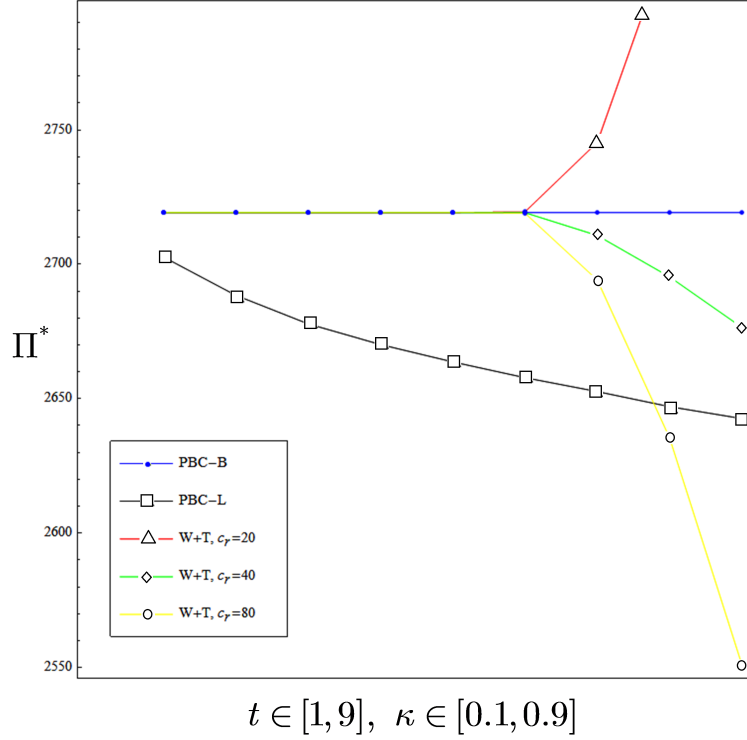
FIGURE 2.7: Profits at equilibrium under W+T and PBC, changing in warranty length t and penalty rate κ, with various reacquisition cost c_r. $\mathbf{x}$ satisfies uniform distribution in [0,1]. $D = 50, T = 10, c_p = 30, c_s = 10, c_h = 5$.

all failed products. In such a case, PBC become fixed-fee full warranty contracts. Hence, it is equivalent to the case of W+T, with $t = T$. Thus, following the previous main results, we have a corollary as follows (we use the subscript F to denote this full warranty contract).

Corollary 2.7. *(1) If δ is sufficiently large and there are no stockouts in $[0,T]$, profits under all contracts are equal, i.e., $\pi_F^{s(c)*} = \pi_{W+T}^{s(c)*} = \pi_{PB}^{s(c)*}$.*

(2) If stockouts do not occur in $[0,t]$ but occur in $(t,T]$, when c_r is small, $\pi_F^{s(c)*} > \pi_{W+T}^{s(c)*} = \pi_{PB}^{s(c)*}$; when c_r is large, $\pi_{W+T}^{s(c)*} = \pi_{PB}^{s(c)*} > \pi_F^{s(c)*}$.

(3) If stockouts occur in $[0,t]$, when c_r is small, $\pi_F^{s(c)*} > \pi_{W+T}^{s(c)*} > \pi_{PB}^{s(c)*}$; when c_r is large, $\pi_{PB}^{s(c)*} > \pi_{W+T}^{s(c)*} > \pi_F^{s(c)*}$.

Our former analysis tell us that longer warranty implies higher product availability by forcing the supplier to replenish spare parts, though incurring extra reacquisition cost. If there are no stockouts during the entire contract period $[0,T]$, profits under all contracts are equal. So even $t = T$, as long as spare parts are sufficient, the profits will be in no difference across contracts,

as stated in part (1), Corollary 2.7. However, if spare parts cannot satisfy all the demand during the entire contract period, the cost of reacquisition will affect the equilibrium profits. If c_r is small, profits are increasing in the warranty duration t. Otherwise, if c_r is large, the cost of part reacquisition exceeds the revenue increase due to a higher product availability, then profits are decreasing in t. Part (2) and (3) in Corollary 2.7 discuss such cases when stockouts may occur in $[0, T]$. With a small c_r, the benefit from high product availability is more than the cost of replenishing parts. Thus, the full warranty contract outperforms the partial warranty contract (i.e., W+T) and PBC. On the other side, if the reacquisition cost is substantially high, paying penalties is a better choice compared to giving long warranty.

2.7.2 Other Revenue Functions

We have thus far used a linear inverse revenue function for the customer. To test the robustness of the results, we use a Cobb-Douglas function and a Newsvendor-type frame to model the customer's revenue in the following analysis. We only look at the exogenous δ case due to tractability issues of the model.

2.7.2.1 Cobb-Douglas Revenue Function

In this section, we use the Cobb-Douglas model to form the customer's revenue as a function of external demand and number of working products. The Cobb-Douglas revenue function has also been used in other works, see for instance (Roels et al., 2010). Define

$$R = D^\alpha (\mathbb{E}[\int_0^T \mathbf{N}_v(\tau)d\tau])^{1-\alpha} = D^\alpha [NA_{P/W+T}(\delta, (t), T)]^{1-\alpha},$$

where α is the output elasticity, which measures the responsiveness of the output to a change in input values. So the revenue has two input factors – external demand and the working time capacity. In addition, we assume $0 < \alpha < 1$. This assumption guarantees the concavity of R w.r.t. N. By solving the new model with a Cobb-Douglas revenue function, we are able to obtain the unique equilibrium solutions under W+T and PBC (detailed results are listed in Table 2.8). Furthermore, the main insights are consistent with our previous findings. We give the numerical illustration in Figure 2.8.
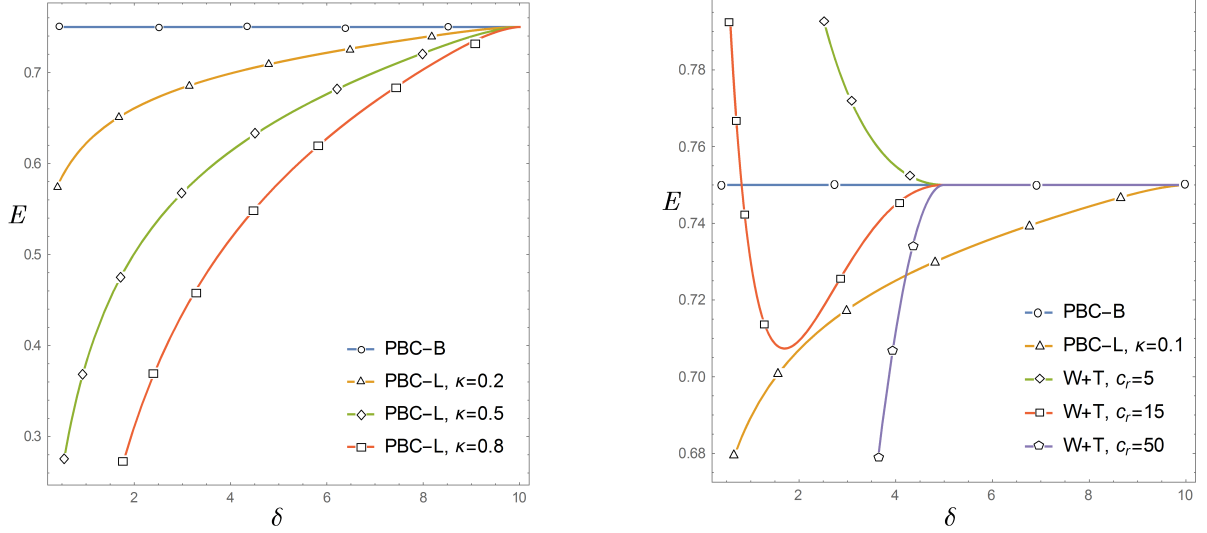
(a) Efficiency of PBC-B and PBC-L, exogenous κ.

(b) Efficiency of PBC and W+T.

FIGURE 2.8: Contracts efficiency under Cobb-Douglas revenue function. Given $\alpha = 0.5, T = 10$. (a): For PBC-L, $\kappa = 0.2, 0.5, 0.8$. (b): For PBC-L, $\kappa = 0.1$. For W+T, $t = 5$, and $c_r = 5, 15, 50$. Other parameters: $\mathbf{x}$ satisfies uniform distribution in $[0, 1]$, $c_p = 30, c_s = 10, c_h = 5$.

Under the Cobb-Douglas revenue function, α becomes an impacting parameter of the contracts efficiency. For example, under PBC-B, the efficiency $E_{PB} = \frac{(2-\alpha)(1-\alpha)^{\frac{1}{\alpha}}}{1-\alpha}$, which is not a constant, like in the case of linear revenue function. Nevertheless, the main results remain similar. As shown in 2.8(a), the efficiency of PBC-L is lower than PBC-B, and is decreasing in κ. Meanwhile, from Figure 2.8(b), we can see that c_r affects the contract efficiency in a similar manner. W+T can be more efficient than PBC given a low reacquisition cost ($c_r = 5$); when the reacquisition cost is high ($c_r = 50$), PBC-B works better than W+T. In sum, the Cobb-Douglas revenue function does not change the main insights of our model.

2.7.2.2 Newsvendor Customer

In this section, we model the customer's revenue using a Newsvendor type revenue function. Let r be the revenue rate of a working product, which is assumed to be constant. Demand $\boldsymbol{\xi}$ has a density function $f_\xi(y)$, which is continuously differentiable. Under a newsvendor framework, the revenue function can be written as

$$\mathbf{R} = r \int_0^T \min[\xi, \mathbf{N}_v(\tau)]d\tau.$$

It can be seen that the customer faces uncertainty on both the supply and demand sides. On one hand, part failures can result in supply uncertainty; on the other hand, external demand is another uncertainty. Then we have the expected revenue

$$\mathbb{E}(\mathbf{R}) = r \int_0^\infty f(x) \left(\int_0^\infty \left(\int_0^T \min[\xi, \mathbf{N}_v(\tau)]d\tau \right) f_\xi(y)dy \right) dx.$$

Proposition 2.8. *Under a newsvendor revenue function,*

(1) *The expected revenue functions under PBC and W+T are concave in N.*

(2) *A sufficient condition for the uniqueness of equilibrium is*

$$\int_0^N \xi^2 f_\xi(y)dy \leq \frac{N^3}{2} f_\xi(N).$$

The proof can be found in Appendix I.

If we model the revenue function under a newsvendor framework, closed-form solutions of the game are not feasible. However, we can come up with a sufficient condition for existence and uniqueness of the equilibrium, which is given in part (2), Proposition 2.8. However, we can verify that not all distributions can always satisfy this condition (e.g., for normal or exponential distributions, it depends on the parameters). Through numerical examples shown in Figure 2.9, we can see the equilibrium profits under W+T and PBC. Profits under PBC-L are lower than PBC-B. Moreover, profits under W+T can be higher with a low replenishment cost, and lower with high a replenishment cost. Hence, we can conclude that under a newsvendor type revenue function, the main insights are still in line with those from previous revenue functions.

2.7.3 Risk Aversion

We have so far assumed that the supply chain partners are risk neutral. However, in practice, most companies show some level of risk aversion. In this section, we discuss how the supply chain members' risk aversion affects the equilibrium outcomes. We assume the customer is risk averse to external demand and spare parts stockouts, and the supplier is risk averse to spare parts stockouts. Then under PBC, the customer's risk aversion $RA_P^c = rc_1 Var(\xi) + rc_2 Var(\mathbf{B}_P)$. The supplier's risk aversion $RA_P^s = rs_1 Var(\mathbf{B}_P)$. Under W+T, since warranty guarantees
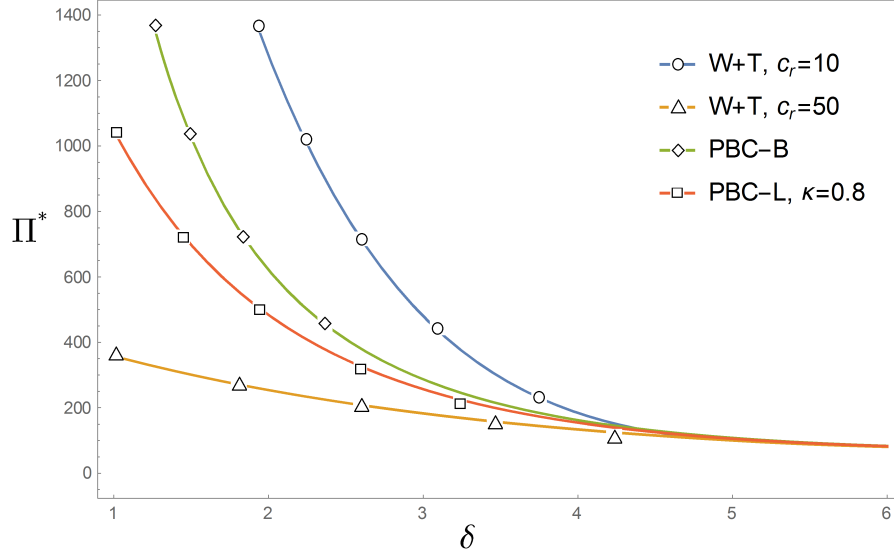
FIGURE 2.9: Equilibrium profits under W+T and PBC, under a newsvendor type of revenue function, exogenous δ.

full product availability, the customer is risk averse to the stockouts in the TBC period. So $RA^c_{W+T} = rc_3 Var(\mathbf{B_T})$. The supplier is risk averse to stockouts during warranty, i.e., $RA^s_{W+T} = rs_2 Var(\mathbf{B_W})$. Here rs and rc are the risk averse coefficient of the supplier and the customer. Then we have the utility functions under PBC and W+T.

Under PBC,

$$\mathbb{E}(u^c_P) = \mathbb{E}(\mathbf{R}) + \kappa\mathbb{E}(\mathbf{B}_P)(/\kappa\mathbb{E}(\mathbf{R}_L)) - Nw - rc_1 Var(\xi) - rc_2 Var(\mathbf{B}_P),$$
$$\mathbb{E}(u^s_P) = Nw - \kappa\mathbb{E}(\mathbf{B}_P)(/\kappa\mathbb{E}(\mathbf{R}_L)) - \mathbb{E}(\mathbf{C}_P) - rs_1 Var(\mathbf{B}_P).$$

Under W+T,

$$\mathbb{E}(u^c_{W+T}) = \mathbb{E}(\mathbf{R}) - Np - p_s\mathbb{E}(\mathbf{S}) - rc_3 Var(\mathbf{B}_T),$$
$$\mathbb{E}(u^s_{W+T}) = Np + p_s\mathbb{E}(\mathbf{S}) - rs_2 Var(\mathbf{B}_W).$$

The decision variables in the respective games remain the same. We can then derive the closed-form solutions for the equilibrium.

Proposition 2.9. *With risk aversion,*

(1) *under PBC-B, the optimal contract term is a combination of w^* and κ^*, based on demand, costs, and risk-averse factors. Under PBC-L, the optimal penalty rate $\kappa^* = 0$, and profits and efficiency are lower and decreasing in κ if it is exogenous.*

(2) *under W+T, the optimal contract term is a combination of p^* and p_s^*, based on demand, costs, and risk-averse factors.*

(3) *compared to the risk neutral case, efficiency is lower due to the increase in prices. Specifically, price increase under PBC is*

$$\Delta_{PB}^* = \frac{(DA_1 - C_P)rs_1\widetilde{B_P}}{4A_2 + 2(2rc_2 + rs_1)\widetilde{B_P}}$$

and

$$\Delta_{PL}^* = \frac{(DA_1 - C_P)[A_2((1-\kappa)rs_1 - \kappa rc_2) + \kappa T(rc_2 + rs_1)]\widetilde{B_P}}{[A_2(2-\kappa) + \kappa T][A_2(2-\kappa) + \kappa T + (2rc_2 + rs_1)\widetilde{B_P}]}.$$

Under W+T, price increase is

$$\Delta_{W+T}^* = \frac{(DA_3 - C_{W+T})rs_2\widetilde{B_W}}{4A_4 + 4rc_3\widetilde{B_T} + 2rs_2\widetilde{B_W}},$$

where $\widetilde{B}_{P(W)} = Var(\mathbf{B}_{P(W,T)})/N^2$.

The proof can be found in Appendix J.

From Proposition 2.9, we can see that when the supply chain partners become risk averse, the optimal decisions do not change. Risk aversion increases prices and thus reduces channel efficiency. This is consistent with the results in other OM papers (Chen et al., 2009; Agrawal and Seshadri, 2000). The price increase can be regarded as risk premiums that are set by the loss-averse decision makers.

2.8 Conclusion

In this chapter, we study joint contracting of product and the after-sales service under Warranty + Transaction-based contracts and Performance-based contracts. We also consider the problem of part obsolescence, and incorporate Lifetime-buy planning as part of the contracting problem. We look at the case of both exogenous and endogenous inventory requirements, as well as different penalty terms for product unavailability in PBC.

For PBC, we find that if penalty is based upon stockouts of spare parts, the initial price and penalty rate can be set in combination. However, if penalty is incurred on the customer's

lost revenue, zero penalty rate would be an ideal decision. However, in practice, product performance must be measured and tied to the customer's compensation. The penalty rate can be given by negotiation, or based on some industrial norms. In such a case, the penalty rate may reduce the supply chain partners' profits, although penalizing the supplier on the customer's revenue may result in a higher product availability, if the supplier can voluntarily chooses the number of spare part. We also compare PBC with the more traditional W+T, under which the warranty there compulsorily makes the supplier guarantee a certain period of full product availability. We find that if the supplier has more inventory than demand during the warranty period, W+T and PBC can have equivalent efficiency. However, if the supplier has to reacquire parts, the reacquisition cost will play a role in the decisions. For some parts which are easy and cheap to get, the supplier does not have to worry about stockouts, and offering longer warranty can make W+T result in higher profits than PBC. On the other hand, if the parts are very expensive to replenish once obsolete, it is better to go for PBC. We also show that the insights from our model are robust to other revenue functions.

Our analysis in this chapter provides insights to the implementation of service contracts. PBC are supposed to bring higher product availability, but our model show that it would be dependant on the penalty terms which may have different incentives. Moreover, sometimes higher product availability may not always go with higher profits. The insights of this chapter also shed light on contract selection between PBC and W+T. As PBC become increasingly popular, our analysis show that PBC are not always dominate W+T. Giving warranty to the customer can be beneficial to the supply chain in that warranty increases the overall product availability. The supplier even would like to offer a longer warranty with fewer spare parts, as long as replenishing components can be done at a low cost. However, when considering the part obsolescence issue, warranty may incur high acquisition cost of spare parts. If it is very expensive to reacquire the obsolete components, the benefit of warranty can be defeated by the cost of obsolescence, which makes PBC more preferable than W+T.

The model in this chapter also has limitations because some assumptions have been made to keep the model tractability. For example, we assume the supplier uses c_r in his planning when contracting with the customer. Sometimes, the expiring date of the component is given by the manufacturer in advance, and the buyer can make a last time order once he knows the exact date. In such a case, the total costs for the supplier may become lower. Including the End-of-life notice date in the model could be a direction for future research. Another assumption is that product availability is only dependent on the spare parts. In the model, as long as no

spare parts stockouts occur, or replenishment is made in time, product availability will not be affected. Actually, in some other cases, product downtime also includes the lead time of service completion time, which is determined by the service providers' repair capacity. Since both capacity and resource (i.e., spare parts) affect product performance, discussing the problem of concurrent management of these two items can be another interesting research topic.

2.9 Appendix

A. Equilibrium solutions

TABLE 2.6: Equilibrium solution under W+T and PBC, exogenous δ.

	W+T	PBC-P/PBC-L, endogenous κ	PBC-L, exogenous κ
Contract terms	$\begin{aligned} p^* &+ p_s^* S(\delta,t,T) \\ &= \frac{DA_3(\delta,t,T) + C_{W+T}(\delta,t,T)}{2} \end{aligned}$	$\begin{aligned} w^* &- \kappa^* B_P(\delta,T)/w^*(\kappa^* = 0) \\ &= \frac{DA_1(\delta,T) + C_P(\delta,T)}{2} \end{aligned}$	$\begin{aligned} w^* &= C_P(\delta,T) + D\kappa[T - A_1(\delta,T)] \\ &+ \frac{A_2(\delta,T)[DA_1(\delta,T) - C_P(\delta,T)]}{A_2(\delta,T)(\kappa - 2) + T\kappa} \end{aligned}$
Order quantity	$\dfrac{DA_3(\delta,t,T) - C_{W+T}(\delta,t,T)}{4A_4(\delta,t,T)}$	$\dfrac{DA_1(\delta,T) - C_P(\delta,T)}{4A_2(\delta,T)}$	$\dfrac{DA_1(\delta,T) - C_P(\delta,T)}{2\kappa T + 2(2-\kappa)A_2(\delta,T)}$
Supplier's profit	$\dfrac{[DA_3(\delta,t,T) - C_{W+T}(\delta,t,T)]^2}{8A_4(\delta,t,T)}$	$\dfrac{[DA_1(\delta,T) - C_P(\delta,T)]^2}{8A_2(\delta,T)}$	$\dfrac{[DA_1(\delta,T) - C_P(\delta,T)]^2}{4\kappa T + 4(2-\kappa)A_2(\delta,T)}$
Customer's profit	$\dfrac{[DA_3(\delta,t,T) - C_{W+T}(\delta,t,T)]^2}{16A_4(\delta,t,T)}$	$\dfrac{[DA_1(\delta,T) - C_P(\delta,T)]^2}{16A_2(\delta,T)}$	$\dfrac{[\kappa T + A_2(\delta,T)(1-\kappa)][DA_1(\delta,T) - C_p(\delta,T)]^2}{4[\kappa T + (2-\kappa)A_2(\delta,T)]^2}$
Contract efficiency	$\dfrac{3A_2(\delta,T)[DA_3(\delta,t,T) - C_{W+T}(\delta,t,T)]^2}{4A_4(\delta,t,T)[DA_1(\delta,T) - C_P(\delta,T)]^2}$	$3/4$	$\dfrac{A_2(\delta,T)[A_2(\delta,T)(3-2\kappa)+2\kappa T]}{[A_2(\delta,T)(2-\kappa)+\kappa T]^2}$

TABLE 2.7: Equilibrium solution under W+T and PBC, endogenous δ.

	W+T	PBC-B/PBC-L, endogenous κ	PBC-L, exogenous κ
δ^*	$\frac{DA_3'(\delta^*_{W+T},t,T)-C'_{W+T}(\delta^*_{W+T},t,T)}{DA_3(\delta^*_{W+T},t,T)-C_{W+T}(\delta^*_{W+T},t,T)} = \frac{A_4'(\delta^*_{W+T},t,T)}{2A_4(\delta^*_{W+T},t,T)}$	$\frac{DA_1'(\delta^*_{pB},T)-C'_p(\delta^*_{pB},T)}{DA_1(\delta^*_{pB},T)-C_p(\delta^*_{pB},T)} = \frac{A_2'(\delta^*_{pB},T)}{2A_2(\delta^*_{pB},T)}$	$\frac{DA_1'(\delta^*_{pL},T)-C'_p(\delta^*_{pL},T)}{DA_1(\delta^*_{pL},T)-C_p(\delta^*_{pL},T)} = \frac{2(2-\kappa)A_2'(\delta^*_{pL},T)}{4\kappa T+4(2-\kappa)A_2(\delta^*_{pL},T)}$
Contract terms	$p^* + p_s^* S(\delta^*_{W+T},t,T)$ $= \frac{DA_3(\delta^*_{W+T},t,T)+C_{W+T}(\delta^*_{W+T},t,T)}{2}$	$w^* - \kappa^* B_p(\delta^*_{pB},T)$ $= \frac{DA_1(\delta^*_{pB},T)+C_p(\delta^*_{pB},T)}{2}$	$w^* = C_P(\delta^*_{PL},T) + D\kappa[T - A_1(\delta^*_{PL},T)]$ $+ \frac{A_2(\delta^*_{PL},T)[DA_1(\delta^*_{PL},T) - C_P(\delta^*_{PL},T)]}{A_2(\delta^*_{PL},T)(\kappa - 2) + T\kappa}$
Order quantity	$\frac{DA_3(\delta^*_{W+T},t,T)-C_{W+T}(\delta^*_{W+T},t,T)}{4A_4(\delta^*_{W+T},t,T)}$	$\frac{DA_1(\delta^*_{pB},T)-C_p(\delta^*_{pB},T)}{4A_2(\delta^*_{pB},T)}$	$\frac{DA_1(\delta^*_{pL},T)-C_p(\delta^*_{pL},T)}{2\kappa T+2(2-k)A_2(\delta^*_{pL},T)}$
Supplier's profit	$\frac{[DA_3(\delta^*_{W+T},t,T)-C_{W+T}(\delta^*_{W+T},t,T)]^2}{8A_4(\delta^*_{W+T},t,T)}$	$\frac{[DA_1(\delta^*_{pB},T)-C_p(\delta^*_{pB},T)]^2}{8A_2(\delta^*_{pB},T)}$	$\frac{[DA_1(\delta^*_{pL},T)-C_p(\delta^*_{pL},T)]^2}{4\kappa T+4(2-\kappa)A_2(\delta^*_{pL},T)}$
Customer's profit	$\frac{[DA_3(\delta^*_{W+T},t,T)-C_{W+T}(\delta^*_{W+T},t,T)]^2}{16A_4(\delta^*_{W+T},t,T)}$	$\frac{[DA_1(\delta^*_{pB},T)-C_p(\delta^*_{pB},T)]^2}{16A_2(\delta^*_{pB},T)}$	$\frac{[\kappa T+A_2(\delta^*_{pL},T)(1-\kappa)](DA_1(\delta^*_{pL},T)-C_p(\delta^*_{pL},T))^2}{4[\kappa T+(2-\kappa)A_2(\delta^*_{pL},T)]^2}$
Contract efficiency	$\frac{3A_2(\delta^*_{pB},T)[DA_3(\delta^*_{W+T},t,T)-C_{W+T}(\delta^*_{W+T},t,T)]^2}{4A_4(\delta^*_{W+T},t,T)[DA_1(\delta^*_{pB},T)-C_p(\delta^*_{pB},T)]^2}$	$3/4$	$\frac{A_2(\delta^*_{pB},T)[A_2(\delta^*_{pL},T)(3-2\kappa)+2\kappa T][DA_1(\delta^*_{pL},T)-C_p(\delta^*_{pL},T)]^2}{[A_2(\delta^*_{pL},T)(2-\kappa)+\kappa T]^2[DA_1(\delta^*_{pB},T)-C_p(\delta^*_{pB},T)]^2}$

TABLE 2.8: Equilibrium solution under Cobb-Douglas revenue function, exogenous δ.

	W+T	PBC-B/PBC-L,endogenous κ	PBC-L, exogenous κ
Contract terms	$p^* + p_s^* S(\delta, t, T)$ $= \dfrac{C_{W+T}(\delta, t, T)}{1-\alpha}$	$(w^* - \kappa^* B_p(\delta, T)) / (w^*, \kappa^* = 0)$ $= \dfrac{C_p(\delta, T)}{1 - \alpha}$	$w^* = \dfrac{A_1{}^\alpha \kappa C_p(T - A_1) + A_1 C_p(T - A_1)^\alpha}{A_1(T - A_1)^\alpha(1 - \alpha) - A_1{}^\alpha \kappa(T - A_1)\alpha}$
Order quantity	$D\left(\dfrac{(1-\alpha)^2 A_{W+T}(\delta,t,T)^{1-\alpha}}{C_{W+T}(\delta,t,T)}\right)$	$D\left(\dfrac{(1-\alpha)^2 A_p(\delta,T)^{1-\alpha}}{C_p(\delta,T)}\right)$	$D\left(\dfrac{(1-\alpha)[A_1{}^{1-\alpha}(1-\alpha)-\kappa\alpha(T-A_1)^{1-\alpha}]}{C_p}\right)^{\frac{1}{\alpha}}$
Supplier's profit	$\dfrac{\alpha D C_{W+T}(\delta,t,T)}{1-\alpha}\left(\dfrac{A_{W+T}(\delta,t,T)^{1-\alpha}(1-\alpha)^2}{C_{W+T}(\delta,t,T)}\right)^{\frac{1}{\alpha}}$	$\dfrac{\alpha D C_p(\delta,T)}{1-\alpha}\left(\dfrac{A_p(\delta,T)^{1-\alpha}(1-\alpha)^2}{C_p(\delta,T)}\right)^{\frac{1}{\alpha}}$	$C_P \dfrac{\alpha}{1-\alpha}\left(\dfrac{D^\alpha(1-\alpha)(A_1{}^{1-\alpha}(1-\alpha)-\kappa\alpha(T-A_1)^{1-\alpha}}{C_P}\right)^{\frac{1}{\alpha}}$
Customer's profit	$\dfrac{D C_{W+T}(\delta,t,T)\alpha}{(1-\alpha)^2}\left(\dfrac{A_{W+T}(\delta,t,T)^{1-\alpha}(1-\alpha)^2}{C_{W+T}(\delta,t,T)}\right)^{\frac{1}{\alpha}}$	$\dfrac{D C_p(\delta,T)\alpha}{(1-\alpha)^2}\left(\dfrac{A_p(\delta,T)^{1-\alpha}(1-\alpha)^2}{C_p(\delta,T)}\right)^{\frac{1}{\alpha}}$	$\dfrac{\alpha C_P(A_1{}^\alpha \kappa(T-A_1)-A_1(T-A_1)^\alpha)\left(\dfrac{D^\alpha(1-\alpha)(A_1{}^{1-\alpha}(1-\alpha)-\kappa\alpha(T-A_1)^{1-\alpha}}{C_P}\right)^{\frac{1}{\alpha}}}{(1-\alpha)(A_1(T-A_1)^\alpha(1-\alpha)-A_1{}^\alpha\kappa(T-A_1)\alpha)}$
Contract efficiency	$\left(\dfrac{A_{W+T}(\delta,t,T)C_p(\delta,x,T)}{A_p(\delta,T)C_{W+T}(\delta,t,T)}\right)^{\frac{1-\alpha}{\alpha}}(2-\alpha)(1-\alpha)^{\frac{1-\alpha}{\alpha}}$	$\dfrac{(2-\alpha)(1-\alpha)^{\frac{1}{\alpha}}}{1-\alpha}$	$\dfrac{(A_1(T-A_1)^\alpha(2-\alpha)+A_1{}^\alpha\kappa(T-A_1)(1-\alpha))(1-(1+A_1{}^{-1+\alpha}\kappa(T-A_1)^{1-\alpha})\alpha)^{\frac{1}{\alpha}}}{A_1(T-A_1)^\alpha(1-\alpha)-A_1{}^\alpha\kappa(T-A_1)\alpha}$

B. Sensitivity of Parameters

TABLE 2.9: Parameter Sensitivity under W+T, exogenous δ

	D	c_p	c_s	c_h	c_r	δ
$p^* + p_s^* S(\delta)$	↑	↑	↑	↑	↑	↑
N_{W+T}^*	↑	↓	↓	↓	↓	⌢
π_{W+T}^{s*}	↑	↓	↓	↓	↓	⌢
π_{W+T}^{c*}	↑	↓	↓	↓	↓	⌢

TABLE 2.10: Parameter Sensitivity under PBC, exogenous δ

	D	c_p	c_s	c_h	δ
$w^* - \kappa^* B(\delta)/R_L$	↑	↑	↑	↑	↑
N_P^*	↑	↓	↓	↓	⌢
π_P^{s*}	↑	↓	↓	↓	⌢
π_P^{c*}	↑	↓	↓	↓	⌢

TABLE 2.11: Parameter Sensitivity under W+T, endogenous δ

	D	c_p	c_s	c_h	c_r
δ^*	↑	↑	↓	↓	≺
$p^* + p_s^* S(\delta^*)$	↑	↑	↑	↑	↑
N_{W+T}^*	↑	↓	⌢	⌢	≺
π_{W+T}^{s*}	↑	↓	⌢	⌢	≺
π_{W+T}^{c*}	↑	↓	⌢	⌢	≺

TABLE 2.12: Parameter Sensitivity under PBC, endogenous δ

	D	c_p	c_s	c_h
δ^*	↑	↑	↓	↓
$w^* - \kappa^* B(\delta^*)/R_L$	↑	↑	↑	↑
N_P^*	↑	↓	⌢	⌢
π_P^{s*}	↑	↓	⌢	⌢
π_P^{c*}	↑	↓	⌢	⌢

C. Proof of Proposition 2.1.

Given the average number of faillures $\mathbf{x}$, the product failure process under W+T and PBC is shown in Figure 2.10. For a given spare parts inventory s, T_s is the time when the sth failure

occurs. In other words, before T_s, all the failed parts can be replaced with working units. Thus, until T_s the number of working products $N_v = N$. After T_s, no spare parts are available and N_v decreases as more component failure occurs. Under PBC, as shown in Figure 2.10(a), depending on $\mathbf{x}$, case (a) and (b) occur when N or fewer than N products have failed before T, respectively. If $T_s > T$, it implies that spare parts inventory is sufficient to cover all the demand in $[0, T]$, resulting in case (c). Figure 2.10(b) shows the failure process under W+T. The difference between W+T and PBC is that the warranty length t plays a role in W+T. If $t < T_s$, i.e., there are no stockouts in the warranty period, the process is the same as PBC. However, if stockout occurs in warranty ($t' > T_s$ in Figure 2(b)), the supplier has to reacquire the parts at a higher cost c_r to replace the failed ones, until the warranty expires (we ignore the lead time of parts replenishment + repair). Once the warranty has expired, no additional parts are reacquired, and each subsequent failure results in a decrease in the number of working products N_v.

Firstly, in Figure 2.10(a), since T_s is the sth failure, we have $T_s = \delta/\mathbf{x}$. Case (c) occurs when $T_s > T$, i.e., $\mathbf{x} < \delta/T$. Case (b) occurs when $\delta/T < \mathbf{x} < (1+\delta)/T$, and case (a) occurs when $\mathbf{x} > (1+\delta)/T$. Then we have the expected product availability under PBC

$$A_P(\delta, T) = \int_0^{\frac{\delta}{T}} T f(x) dx + \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \left(\frac{\delta}{x} + \int_{\frac{\delta}{x}}^{T} [1 - (\tau x - \delta)] d\tau \right) f(x) dx$$
$$+ \int_{\frac{1+\delta}{T}}^{\infty} \left(\frac{\delta}{x} + \int_{\frac{\delta}{x}}^{\frac{1+\delta}{x}} [1 - (\tau x - \delta)] d\tau \right) f(x) dx.$$

Secondly, in Figure 2.10(b), we can see the warranty length t under W+T affects $N_v(\tau)$. If $t < T_s$, it is equivalent to the case in Figure 2.10(a). If $t > T_s$ (t' in Figure 2.10(b)), $N_v(\tau)$ decreases from t' (rather than from T_s in Figure 2.10(a). We have

- when $t \leq T \frac{\delta}{1+\delta}$,

$$A_{W+T}(\delta, t, T) = \int_0^{\frac{\delta}{T}} T f(x) dx + \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \left(\frac{\delta}{x} + \int_{\frac{\delta}{x}}^{T} [1 - (\tau x - \delta)] d\tau \right) f(x) dx$$
$$+ \int_{\frac{1+\delta}{T}}^{\frac{\delta}{t}} \left(\frac{\delta}{x} + \int_{\frac{\delta}{x}}^{\frac{1+\delta}{x}} [1 - (\tau x - \delta)] d\tau \right) f(x) dx + \int_{\frac{\delta}{t}}^{\infty} \left(t + \int_{t}^{t+\frac{1}{x}} [1 - x(\tau - t)] d\tau \right) f(x) dx.$$
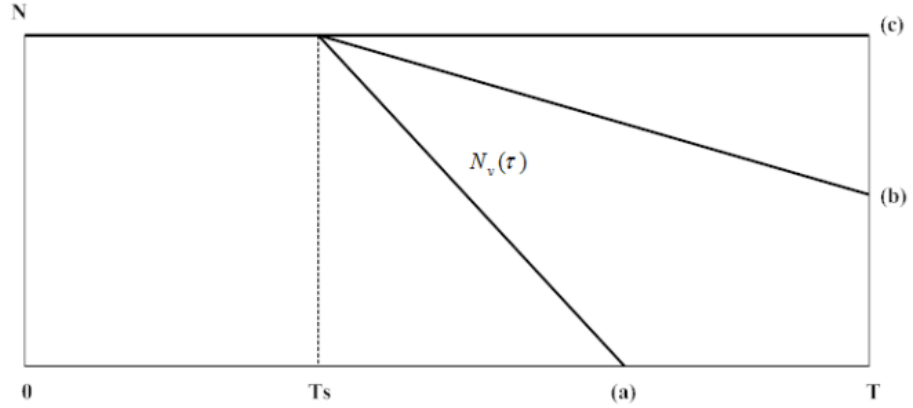
(a) Failure process under PBC.
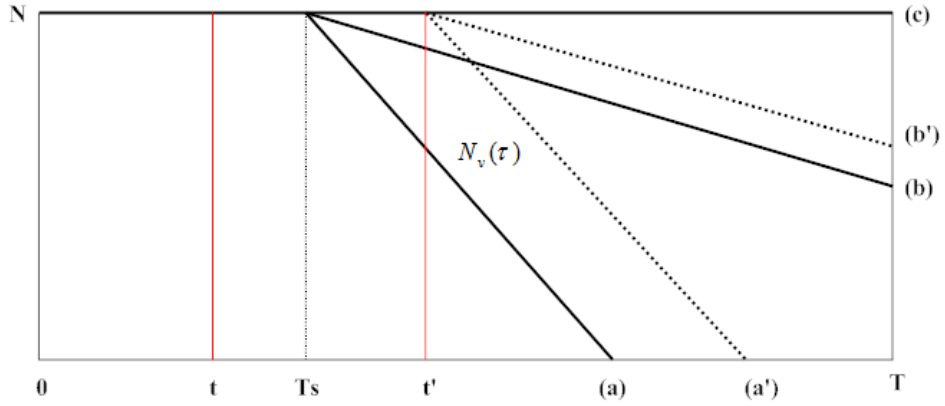
(b) Failure process under W+T.

FIGURE 2.10: Failure process under PBC and W+T. X axis is the time interval. Y axis is the number of working products. Contract duration is from 0 to T. Case (a), (b) and (c) occurs when $N_v(T) = 0, 0 < N_v(T) < N$ and $N_v(T) = N$, respectively.

- when $t > T\frac{\delta}{1+\delta}$,

$$A_{W+T}(\delta, t, T) = \int_0^{\frac{\delta}{T}} Tf(x)dx + \int_{\frac{\delta}{T}}^{\frac{\delta}{t}} \left(\frac{\delta}{x} + \int_{\frac{\delta}{x}}^{T} [1 - (\tau x - \delta)]d\tau \right) f(x)dx$$

$$+ \int_{\frac{\delta}{t}}^{\frac{1}{T-t}} \left(t + \int_t^T [1 - x(\tau - t)]d\tau \right) f(x)dx + \int_{\frac{1}{T-t}}^{\infty} \left(t + \int_t^{t+\frac{1}{x}} [1 - x(\tau - t)]d\tau \right) f(x)dx.$$

Differentiating $A_P(\delta, T)$ w.r.t. δ, we have (i) the first term

$$\frac{\partial \int_0^{\frac{\delta}{T}} Tf(x)dx}{\partial \delta} = f(\frac{\delta}{T});$$

the second term

$$\int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \left(\frac{\delta}{x} + \int_{\frac{\delta}{x}}^{T} [1 - (\tau x - \delta)]d\tau \right) f(x)dx = \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \left(T - \frac{T^2 x}{2} + T\delta - \frac{\delta^2}{2x} \right) f(x)dx,$$

so

$$\frac{\partial \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \left(T - \frac{T^2 x}{2} + T\delta - \frac{\delta^2}{2x} \right) f(x)dx}{\partial \delta} = -f(\frac{\delta}{T}) + \frac{\left(T + T\delta - \frac{T\delta^2}{2(1+\delta)} - \frac{1}{2}T(1+\delta) \right) f(\frac{1+\delta}{T})}{T}$$

$$+ \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \left(T - \frac{\delta}{x} \right) f(x)dx = -f(\frac{\delta}{T}) + \frac{(1+2\delta) f(\frac{1+\delta}{T})}{2(1+\delta)} + \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \left(T - \frac{\delta}{x} \right) f(x)dx;$$

(iii) the third term

$$\int_{\frac{1+\delta}{T}}^{\infty} \left(\frac{\delta}{x} + \int_{\frac{\delta}{x}}^{\frac{1+\delta}{x}} [1 - (\tau x - \delta)]d\tau \right) f(x)dx = \int_{\frac{1+\delta}{T}}^{\infty} \left(\frac{\delta}{x} + \frac{1}{2x} \right) f(x)dx,$$

so

$$\frac{\partial \int_{\frac{1+\delta}{T}}^{\infty} \left(\frac{\delta}{x} + \frac{1}{2x} \right) f(x)dx}{\partial \delta} = -\frac{\left(\frac{T}{2(1+\delta)} + \frac{T\delta}{1+\delta} \right) f(\frac{1+\delta}{T})}{T} + \int_{\frac{1+\delta}{T}}^{\infty} \frac{f(x)}{x}dx$$

$$= -\frac{(1+2\delta) f(\frac{1+\delta}{T})}{2(1+\delta)} + \int_{\frac{1+\delta}{T}}^{\infty} \frac{f(x)}{x}dx.$$

Adding the above three terms together, we have

$$\frac{\partial A_P(\delta, T)}{\partial \delta} = f(\frac{\delta}{T}) - f(\frac{\delta}{T}) + \frac{(1+2\delta) f(\frac{1+\delta}{T})}{2(1+\delta)} + \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \left(T - \frac{\delta}{x} \right) f(x)dx$$

$$- \frac{(1+2\delta) f(\frac{1+\delta}{T})}{2(1+\delta)} + \int_{\frac{1+\delta}{T}}^{\infty} \frac{f(x)}{x}dx$$

$$= \int_{\frac{1+\delta}{T}}^{\infty} \frac{f(x)}{x}dx + \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} (T - \frac{\delta}{x})f(x)dx > 0.$$

The second derivative

$$\frac{\partial^2 A_P(\delta, T)}{\partial \delta^2} = -\frac{f(\frac{1+\delta}{T})}{1+\delta} - f(\frac{\delta}{T}) + f(\frac{1+\delta}{T}) + f(\frac{\delta}{T}) - \frac{\delta f(\frac{1+\delta}{T})}{1+\delta} - \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \frac{f(x)}{x}dx$$

$$= -\int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \frac{f(x)}{x}dx < 0.$$

So $A_P(\delta, T)$ is increasing in concave in δ.

Similarly, differentiating $A_{W+T}(\delta, t, T)$ w.r.t. δ and t, we have

- when $t \leq T\frac{\delta}{1+\delta}$,

$$\frac{\partial A_{W+T}(\delta, t, T)}{\partial \delta} = \int_{\frac{1+\delta}{T}}^{\frac{\delta}{t}} \frac{f(x)}{x}dx + \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} (T - \frac{\delta}{x})f(x)dx > 0,$$

$$\frac{\partial A_{W+T}(\delta, t, T)}{\partial t} = 1 - F\left(\frac{\delta}{t}\right) > 0.$$

- when $t > T\frac{\delta}{1+\delta}$,

$$\frac{\partial A_{W+T}(\delta, t, T)}{\partial \delta} = \int_{\frac{\delta}{T}}^{\frac{\delta}{t}} (T - \frac{\delta}{x})f(x)dx > 0,$$

$$\frac{\partial A_{W+T}(\delta, t, T)}{\partial t} = 1 - F\left(\frac{1}{T-t}\right) + \int_{\frac{\delta}{t}}^{\frac{1}{T-t}} (T - t)xf(x) > 0.$$

Let $t = T\frac{\delta}{1+\delta}$, it can be verified that

$$\left.\frac{\partial A_{W+T}(\delta, t, T)}{\partial \delta}\right|_{t \leq T\frac{\delta}{1+\delta}} = \left.\frac{\partial A_{W+T}(\delta, t, T)}{\partial \delta}\right|_{t > T\frac{\delta}{1+\delta}},$$

$$\left.\frac{\partial A_{W+T}(\delta, t, T)}{\partial t}\right|_{t \leq T\frac{\delta}{1+\delta}} = \left.\frac{\partial A_{W+T}(\delta, t, T)}{\partial t}\right|_{t > T\frac{\delta}{1+\delta}}.$$

Then we can conclude that $A_{W+T}(\delta, t, T)$ is continuous and differentiable w.r.t. δ and t, as well as increasing in δ and t.

Since $A_{W+T}(\delta, t, T)$ is increasing in t, we have when $t = 0$, $A_{W+T}(\delta, t, T) = A_P(\delta, T)$. So given δ and t, $A_{W+T}(\delta, t, T) \geq A_P(\delta, T)$. $\square$

D. Proof of Lemma 2.2. Differentiating $\mathbb{E}[\pi^c_{W+T}(N)]$ in Equation (2.2) w.r.t. N, we have

$$\frac{\partial^2 \mathbb{E}(\pi^c_{W+T})}{\partial N^2} = -2A_4(T, t, \delta) < 0.$$

So the customer's profit function is concave in N. By solving the First Order Condition (FOC), we have the customer's best response $N^*_{W+T} = \frac{DA_3(\delta, t, T) - p - p_s S(\delta, t, T)}{2A_4(\delta, t, T)}$. Substituting N^*_{W+T} into

the supplier's profit function, and differentiating it w.r.t. p and p_s, we have

$$\frac{\partial^2 \mathbb{E}(\pi_{W+T}^s)\big|_{N=N_{W+T}^*}}{\partial p^2} = -\frac{1}{A_4(\delta, t, T)} < 0,$$

$$\frac{\partial^2 \mathbb{E}(\pi_{W+T}^s)\big|_{N=N_{W+T}^*}}{\partial p_s^2} = -\frac{S(\delta, t, T)^2}{A_4(\delta, t, T)} < 0.$$

Hence, the supplier's profit function is concave in p and p_s. The FOC w.r.t. p and p_s is

$$\begin{cases} \dfrac{DA_3(\delta, t, T) + C_{W+T}(\delta, t, T) - 2[p + p_s S(\delta, t, T)]}{2A_4(\delta, t, T)} = 0 \\[3mm] \dfrac{S(\delta, t, T)(DA_3(\delta, t, T) + C_{W+T}(\delta, t, T) - 2[p + p_s S(\delta, t, T)])}{2A_4(\delta, t, T)} = 0 \end{cases}$$

Solving the system equations, we have

$$\begin{cases} p^* = \dfrac{DA_3(\delta, t, T) + C_{W+T}(\delta, t, T)}{2} - S(\delta, t, T)p_s, \\[3mm] p_s^* = p_s \end{cases}$$

Then the optimal solution of p and p_s satisfies

$$p^* + p_s^* S(\delta, t, T) = \frac{DA_3(\delta, t, T) + C_{W+T}(\delta, t, T)}{2}.$$

Substituting into the customer's best response, we have $N_{W+T}^* = \frac{DA_3(\delta, t, T) - C_{W+T}(\delta, t, T)}{4A_4(\delta, t, T)}$. Because of the concavity, p^*, p_s^* and N_{W+T}^* are the unique optimal solutions that maximize the agents' decision functions.

In addition, we have $\partial N_{W+T}^*/\partial D = \frac{A_3(\delta, t, T)}{2A_4(\delta, t, T)} > 0$. For the price combination, we have $\partial \widehat{p}^*/\partial D = \frac{A_3(\delta, t, T)}{2} > 0$, $\partial \widehat{p}^*/\partial \delta = p_s[F(\delta/t) - F(\delta/T)] > 0$, and $\partial \widehat{p}^*/\partial t = \frac{D}{2}(\partial A_3/\partial t + \partial C_{W+T}/\partial t) > 0$.
□

E. Proof of Lemma 2.3. First, we prove the results of PBC-B. Differentiating $\mathbb{E}[\pi_{PB}^c(N)]$ in Equation (2.4) w.r.t. N, we have

$$\frac{\partial^2 \mathbb{E}(\pi_P^c)}{\partial N^2} = -2A_2(\delta, T) < 0.$$

So the customer's profit function is concave in N. By solving the FOC, we have the customer's best response $N_{PB}^* = \frac{DA_1(\delta, T) - w + \kappa B(\delta, T)}{2A_2(\delta, T)}$. Substituting N_{PB}^* into the supplier's profit function

and differentiate it w.r.t. w, κ, we have

$$\frac{\partial^2 \mathbb{E}(\pi_{PB}^s)|_{N=N_{PB}^*}}{\partial w^2} = -\frac{1}{A_2(\delta, T)} < 0,$$

$$\frac{\partial^2 \mathbb{E}(\pi_{PB}^s)|_{N=N_{PB}^*}}{\partial \kappa^2} = -\frac{B(\delta, x, T)^2}{A_2(\delta, T)} < 0.$$

So the supplier's profit function is concave in w and κ. Then solving the FOC, we can have the unique equilibrium solution as stated in Table 2.6.

Second, under PBC-L, we have

$$\frac{\partial^2 \mathbb{E}(\pi_p^c)}{\partial N^2} = -2[\kappa T + (1-\kappa)A_2(\delta, T)] < 0.$$

So the customer's profit function is concave in N. Substituting the customer's best response $N_{PL}^* = \frac{D\kappa T - w + D(1-\kappa)A_1(\delta, T)}{2[\kappa T + (1-\kappa)A_2(\delta, T)]}$ into the supplier's profit function in Equation (2.3), if κ is a decision variable, we have the Lagrangian of the supplier's problem

$$L(w, \kappa, \mu_1, \mu_2) = N_{PL}^* w - \kappa \mathbb{E}[R_L(N_{PL}^*)] - \mathbb{E}[C_P(N_{PL}^*)] + \mu_1(1-\kappa) + \mu_2 \kappa.$$

Using the Karush-Kuhn-Tucker condition, we have the optimality conditions

$$\begin{cases} \dfrac{\partial L(w, \kappa, \mu_1, \mu_2)}{\partial w} = 0 \\[2mm] \dfrac{\partial L(w, \kappa, \mu_1, \mu_2)}{\partial \kappa} = 0 \\[2mm] \mu_1(1-\kappa) = 0 \\[1mm] \mu_2 \kappa = 0 \\[1mm] 0 \leq \kappa \leq 1 \\[1mm] \mu_1, \mu_2 \geq 0 \end{cases}$$

(1) $\mu_1 = 0, \mu_2 > 0, \kappa^* = 0$. We have the solution

$$w^* = \frac{DA_1(\delta, T) + C_P(\delta, T)}{2}, \mu_2 = \frac{(T - A_2(\delta, T))[DA_1(\delta, T) - C_P(\delta, T)]^2}{16A_2(\delta, T)^2} > 0,$$

which is feasible.

(2) $\mu_2 = 0, \mu_1 > 0, \kappa^* = 1$. We have the solution

$$w^* = \frac{T[DT - DA_1(\delta, T) + DA_2(\delta, x, T) + C_P(\delta, T)]}{T + A_2(\delta, T)},$$

$$\mu_1 = -\frac{[T - A_2(\delta, T)][DA_1(\delta, T) - C_P(\delta, T)]^2}{4(T + A_2(\delta, T))^2} < 0,$$

which is not feasible.

(3) $\mu_1 > 0, \mu_2 > 0$. No solutions.

(4) $\mu_1 = 0, \mu_2 = 0$. No solutions.

Hence, the unique optimal solution

$$w^* = \frac{DA_1(\delta, T) + C_P(\delta, T)}{2}, \kappa^* = 0.$$

If κ is given, we have

$$\frac{\partial^2 \mathbb{E}(\pi_{PL}^{s*})|_{N=N_{PL}^*}}{\partial w^2} = \frac{-\kappa T + (-2 + \kappa)A_2(\delta, T)}{2[\kappa T + A_2 - \kappa A_2(\delta, T)]^2} < 0.$$

So the supplier's profit function incorporating the customer's best response is concave in w. Then solving the FOC, we can have the unique optimal solution w^*, and then the equilibrium of the game. $\square$

F. Proof of Proposition 2.4. (1) For PBC-L, if κ is set by the supplier, $\kappa^* = 0$. Comparing the solutions in Table 2.6, we have the results $N_{PL}^* = N_{PB}^*$, $\pi_{PL}^* = \pi_{PB}^*$. (2) For PBC-L, if κ is exogenous, we have

$$\frac{\partial N_{PL}^*}{\partial \kappa} = -\frac{[T - A_2(\delta, T)][DA_1(\delta, T) - C_P(\delta, T)]}{2[\kappa T + (2 - \kappa)A_2(\delta, T)]^2} < 0,$$

$$\frac{\partial \pi_{PL}^*}{\partial \kappa} = -\frac{[T - A_2(\delta, T)][DA_1(\delta, T) - C_P(\delta, T)]^2}{4[\kappa T + (2 - \kappa)A_2(\delta, T)]^2} < 0.$$

Since $N_{PL}^*(\kappa = 0) = N_{PB}^*$, $\pi_{PL}^*(\kappa = 0) = \pi_{PB}^*$, , we have $N_{PB}^* \geq N_{PL}^*$, $\pi_{PB}^{s(c)*} \geq \pi_{PL}^{s(c)*}$. (3) For W+T, if no stockout occurs in warranty, i.e., $\mathbb{E}[\mathbf{B}_W] = 0$, then we have $A_1(\delta, T) = A_3(\delta, t, T)$ and $C_{W+T}(\delta, t, T) = C_P(\delta, T)$. So $N_{W+T}^* = N_{PB}^*$, $\pi_{W+T}^{s(c)*} = \pi_{PB}^{s(c)*}$. Since we have just proved that $\pi_{PB}^{s(c)*} \geq \pi_{PL}^{s(c)*}$, we have $\pi_{W+T}^{s(c)*} = \pi_{PB}^{s(c)*} \geq \pi_{PL}^{s(c)*}$. $\square$

G. Proof of Lemma 2.5. The concavity w.r.t. N, p and p_s remains the same as the exogenous case. The supplier's objective function incorporating the customer's best response is

$$\max_{p,p_s,\delta} \quad -\frac{[p - DA_3(\delta) + p_s S(\delta)][p - C_{W+T}(\delta) + p_s S(\delta)]}{2A_4(\delta)}.$$

We have the determinant of the Hessian of the objective function $Det(H) = 0$. Then we first solve the FOC of p and p_s, and then substitute p^* and p_s^* into the FOC of δ. We have

$$\frac{[DA_3(\delta) - C_{W+T}(\delta)][2A_4(DA_3'(\delta) - C_{W+T}'(\delta)) - (DA_3(\delta) - C_{W+T}(\delta))A_4'(\delta)]}{8A_4(\delta)^2} = 0.$$

We can easily verify that δ^* that solves $DA_3(\delta^*) - C_{W+T}(\delta^*) = 0$ is not the maximum value point. Then δ^* that solves

$$2A_4[DA_3'(\delta^*) - C_{W+T}'(\delta^*)] = [DA_3(\delta^*) - C_{W+T}(\delta^*)]A_4'(\delta^*) \tag{2.5}$$

is the optimal solution that maximizes the objective function.

For PBC, the concavity w.r.t. N, w and κ remains the same as the exogenous case. (i) Under PBC-B, the supplier's objective function incorporating the customer's best response is

$$\max_{w,\kappa,\delta} \frac{[DA_1(\delta) + \kappa B(\delta) - w][w - \kappa B(\delta) - C_P(\delta)]}{2A_2(\delta)}.$$

We have the determinant of the Hessian of the above function $Det(H) = 0$. So by looking at the Hessian can not directly show the existence of the optima. We then first solve the FOC w.r.t. w, κ, and have the FOC w.r.t. δ given w^* and κ^* as

$$\frac{[DA_1(\delta) - C_P(\delta)][(-DA_1(\delta) + C_P(\delta))A_2'(\delta) + 2A_2(\delta)(DA_1'(\delta) - C_P'(\delta))]}{8A_2(\delta)^2} = 0.$$

It is straightforward to verify that profit is 0 if $DA_1(\delta^*) - C_P(\delta^*) = 0$. So δ^* which satisfies

$$2A_2(\delta^*)[DA_1'(\delta^*) - C_P'(\delta^*)] = [DA_1(\delta^*) - C_P(\delta^*)]A_2'(\delta^*) \tag{2.6}$$

is the maximum point. (ii) Under PBC-L, if κ is exogenous, we have the supplier's objective function incorporating the customer's best response is

$$\max_{w,\delta}$$
$$-\frac{(D\kappa T - w + (D - D\kappa)A_1(\delta))(D\kappa T - w)(\kappa T + (2 - \kappa)A_2(\delta)) - D\kappa A_1(\delta)(T + \kappa T + A_2(\delta) - \kappa A_2(\delta)] + 2(\kappa T + A_2(\delta) - \kappa A_2(\delta))C_P(\delta))}{4[\kappa T + A_2(\delta) - \kappa A_2(\delta)]^2}$$

We first solve the FOC of w, and have the FOC of δ incorporating w^* as

$$\frac{[DA_1(\delta) - C_P(\delta)](2D(\kappa T + (2 - \kappa)A_2)A_1'(\delta) - (2 - \kappa)(DA_1(\delta) - C_P(\delta))A_2'(\delta) - 2(\kappa T + (2 - \kappa)A_2(\delta))C_P'(\delta))}{4[\kappa T + (2 - \kappa)A_2(\delta)]^2}.$$

Similarly, δ^* that solves

$$2[\kappa T + (2 - \kappa)A_2(\delta^*)]DA_1'(\delta^*) - C_P'(\delta^*)] = (2 - \kappa)A_2'(\delta^*)[DA_1(\delta^*) - C_P(\delta^*)] \qquad (2.7)$$

is the optimal solution that maximizes the objective function. If κ is a decision variable, we have the Lagrangian of the supplier's problem

$$L(w, \kappa, \delta, \mu_1, \mu_2) = N_{PL}^* w - \kappa \mathbb{E}[R_L(N_{PL}^*)] - \mathbb{E}[C_P(N_{PL}^*)] + \mu_1(1 - \kappa) + \mu_2\kappa.$$

Using the Karush-Kuhn-Tucker condition, we have the optimality conditions

$$\begin{cases} \dfrac{\partial L(w, \kappa, \delta, \mu_1, \mu_2)}{\partial w} = 0 \\[2mm] \dfrac{\partial L(w, \kappa, \delta, \mu_1, \mu_2)}{\partial \kappa} = 0 \\[2mm] \dfrac{\partial L(w, \kappa, \delta, \mu_1, \mu_2)}{\partial \delta} = 0 \\[2mm] \mu_1(1 - \kappa) = 0 \\[1mm] \mu_2\kappa = 0 \\[1mm] 0 \leq \kappa \leq 1 \\[1mm] \mu_1, \mu_2 \geq 0 \end{cases}$$

(1) $\mu_1 = 0, \mu_2 > 0, \kappa^* = 0$. We have the solution

$$2[\kappa T + (2 - \kappa)A_2(\delta^*)]DA_1'(\delta^*) - C_P'(\delta^*)] = (2 - \kappa)A_2'(\delta^*)[DA_1(\delta^*) - C_P(\delta^*)],$$

$$w^* = \frac{DA_1(\delta^*) + C_P(\delta^*)}{2}, \mu_2 = \frac{(T - A_2(\delta^*))[DA_1(\delta^*) - C_P(\delta^*)]^2}{16A_2(\delta^*)^2} > 0,$$

which is feasible.

(2) $\mu_2 = 0, \mu_1 > 0, \kappa^* = 1$. We have the solution

$$2[\kappa T + (2 - \kappa)A_2(\delta^*)]DA_1^{'}(\delta^*) - C_P^{'}(\delta^*)] = (2 - \kappa)A_2^{'}(\delta^*)[DA_1(\delta^*) - C_P(\delta^*)],$$

$$w^* = \frac{T[DT - DA_1(\delta^*) + DA_2(\delta^*) + C_P(\delta^*)]}{T + A_2(\delta^*)}, \mu_1 = -\frac{[T - A_2(\delta^*)][DA_1(\delta^*) - C_p(\delta^*)]^2}{4(T + A_2(\delta^*))^2} < 0,$$

which is not feasible.

(3) $\mu_1 > 0, \mu_2 > 0$. No solutions.

(4) $\mu_1 = 0, \mu_2 = 0$. No solutions.

Hence, the optimal solution is

$$2[\kappa T + (2 - \kappa)A_2(\delta^*)]DA_1^{'}(\delta^*) - C_P^{'}(\delta^*)] = (2 - \kappa)A_2^{'}(\delta^*)[DA_1(\delta^*) - C_P(\delta^*)],$$

$$w^* = \frac{DA_1(\delta^*) + C_P(\delta^*)}{2}, \kappa^* = 0.$$

$\square$

H. Proof of Proposition 2.6. (1) Comparing Equation (2.6) with the solution of the First Best case in Table 2.2, we can see that $\delta_{PB}^* = \delta^0$. (2) For PBC-L, if κ is endogenous, then $\kappa^* = 0$. It is straightforward to see that $\delta_{PB}^* = \delta_{PL}^*$, $N_{PB}^* = N_{PL}^*$ and $\pi_{PB}^{s(c)*} = \pi s(c)_{PL}^*$. (3) The left-hand-side of the equations solving optimal δ under exogenous and endogenous κ is the same. Let

$$LHS(\delta) = \frac{DA_1^{'}(\delta) - C_p^{'}(\delta)}{DA_1(\delta) - C_p(\delta)}.$$

We have

$$\frac{\partial LHS(\delta)}{\partial \delta} = \frac{-[DA_1^{'}(\delta) - C_p^{'}(\delta)]^2 + [DA_1(\delta) - C_p(\delta)][DA_1^{''}(\delta) - C_p^{''}(\delta)]}{[DA_1(\delta) - C_p(\delta)]^2}.$$

From Proposition 2.1, we know $A_1(\delta)$ is concave in δ, i.e., $DA_1^{''}(\delta) < 0$. Since $DA_1(\delta) - C_P(\delta) > 0$ always holds, we have $\partial LHS(\delta)/\partial \delta < 0$. So $LHS(\delta)$ is decreasing. Then we look at the right-hand-side of the equations. For endogenous κ, we can see

$$RHS_{en-\kappa}(\delta) = \frac{2(2 - \kappa)A_2^{'}(\delta)}{4\kappa T + 4(2 - \kappa)A_2(\delta)},$$

and $RHS_{en-\kappa=0}(\delta) = RHS_{ex-\kappa}(\delta)$. Further, we have

$$\frac{\partial RHS_{en-\kappa}(\delta)}{\partial \kappa} = -\frac{TA_2{'}(\delta)}{[A_2(\delta)(2-\kappa)+\kappa T]^2} < 0.$$

Then $RHS_{en-\kappa}(\delta) \leq RHS_{ex-\kappa}(\delta)$. Because $LHS(\delta)$ is decreasing, the intersection with $RHS_{ex-\kappa}(\delta)$ is further than that with $RHS_{en-\kappa}(\delta)$ (Figure 2.11). Hence, δ^* under exogenous κ is larger. □
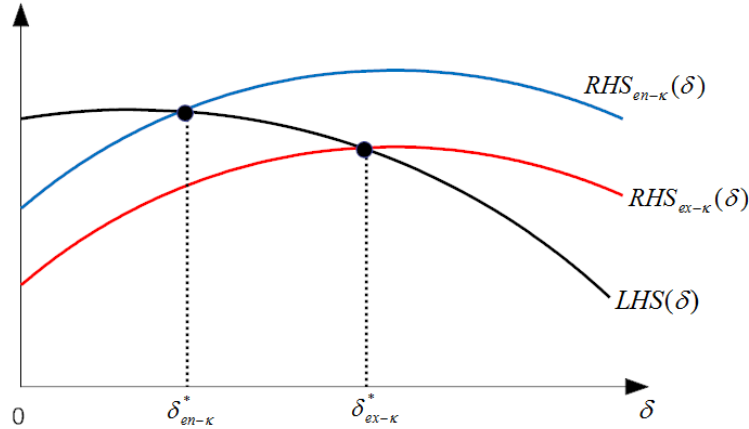
FIGURE 2.11: Optimal δ under exogenous and endogenous κ.

I. Proof of Proposition 2.8. (1) Under PBC, the number of working products in $[0, T]$ can be cases (a)(b) and (c) in Figure 2.10(a), depending on **x**. If $\mathbf{x} < \delta/T$,

$$\mathbb{E}_c(\mathbf{R}) = rT \int_0^{\frac{\delta}{T}} f(x)dx \left(\int_0^N \xi f_\xi(y)dy + N \int_N^\infty f_\xi(y)dy \right).$$

If $\delta/T < \mathbf{x} < (1+\delta)/T$,

$$\mathbb{E}_b(\mathbf{R}) = r \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} f(x) \left(\int_0^\infty \left(\int_0^{\frac{\delta}{x}} \min[\xi, N]d\tau + \int_{\frac{\delta}{x}}^T \min[\xi, N(1-\tau x + \delta)]d\tau \right) f_\xi(y)dy \right) dx$$

$$= r \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} f(x)\frac{\delta}{x} \left(\int_0^N \xi f_\xi(y)dy + N \int_N^\infty f_\xi(y)dy \right) dx + r \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \frac{N(2-Tx+\delta)(Tx-\delta)}{2x} f(x)dx \int_N^\infty f_\xi(y)dy$$

$$+ r \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} f(x) \left(\int_0^N \frac{\xi^2 - 2\xi N + N^2(1-Tx+\delta)^2}{2Nx} f_\xi(y)dy \right) dx.$$

If $\mathbf{x} > (1 + \delta)/T$,

$$\mathbb{E}_a(\mathbf{R}) = r \int_{\frac{1+\delta}{T}}^{\infty} f(x) \left(\int_0^{\infty} \left(\int_0^{\frac{\delta}{x}} \min[\xi, N] d\tau \right) f_\xi(y) dy + \int_0^{\infty} \left(\int_{\frac{\delta}{x}}^{\frac{1+\delta}{x}} \min[\xi, N(1 - \tau x + \delta)] d\tau \right) f_\xi(y) dy \right) dx$$

$$= r \int_{\frac{1+\delta}{T}}^{\infty} f(x) \frac{\delta}{x} \left(\int_0^N \xi f_\xi(y) dy + N \int_N^{\infty} f_\xi(y) dy \right) dx + r \int_{\frac{1+\delta}{T}}^{\infty} f(x) \left(\int_0^N \frac{N^2 + 2\xi(N - \xi)}{2Nx} f_\xi(y) dy \right) dx$$

$$+ r \int_{\frac{1+\delta}{T}}^{\infty} \frac{N f(x)}{2x} dx \int_N^{\infty} f_\xi(y) dy$$

Then the expected revenue

$$\mathbb{E}(\mathbf{R}_P) = \mathbb{E}_a(\mathbf{R}) + \mathbb{E}_b(\mathbf{R}) + \mathbb{E}_c(\mathbf{R}).$$

Differentiating $\mathbb{E}(\mathbf{R}_P)$ twice w.r.t N, we have

$$\frac{\partial^2 \mathbb{E}(\mathbf{R}_P)}{r \partial N^2} = -f_\xi(N) \left(T \int_0^{\frac{\delta}{T}} f(x) dx + \int_{\frac{\delta}{T}}^{\infty} \frac{\delta f(x)}{x} dx \right) - \int_0^N \frac{\xi^2 f_\xi(y) dy}{N^3} \int_{\frac{\delta}{T}}^{\infty} \frac{f(x)}{x} dx < 0.$$

Hence, $\mathbb{E}(\mathbf{R}_P)$ is concave in N. Similar, we can prove the concavity of the expected revenue function under W+T.

(2) Under PBC, the customer's problem is

$$\max_N \ \mathbb{E}(\mathbf{R}_{PB}) - Nw + \kappa NB(\delta).$$

It can be verified that the profit function is concave in N. Solving the FOC, we can have the customer's best response and equivalently the supplier's decision on price as

$$w - B(\delta)\kappa = r \left(\begin{array}{l} \left(T \int_0^{\frac{\delta}{T}} f(x) + \delta \int_{\frac{\delta}{T}}^{\infty} \frac{f(x)}{x} dx \right) \int_N^{\infty} f_\xi(y) dy + \int_{\frac{1+\delta}{T}}^{\infty} f(x) \left(\int_N^{\infty} \frac{f_\xi(y)}{2x} dy + \int_0^N \frac{\xi^2 f_\xi(y)}{2N^2 x} dy \right) dx \\ - \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} f(x) \left(\int_0^N \frac{N^2(1 - Tx + \delta) - \xi^2}{2xN^2} f_\xi(y) dy + \int_N^{\infty} \frac{(2 - Tx + \delta)(Tx - \delta)}{2x} f_\xi(y) dy \right) dx. \end{array} \right)$$

The supplier's problem becomes

$$\mathbb{E}(\pi_{PB}^s) = [w - B(\delta)\kappa](N)N - NC_P(\delta),$$

where $[w - B(\delta)\kappa](N)$ is determined by the above equation. Differentiating w.r.t. N, we have

$$\frac{\partial \mathbb{E}(\pi_{PB}^s)}{r\partial N} = -\frac{C_P}{r} + \left(T\int_0^{\frac{\delta}{T}} f(x)dx + \int_{\frac{\delta}{T}}^{\infty} \frac{\delta f(x)}{x}dx \right) \int_N^{\infty} f_\xi(y)dy + \int_{\frac{1+\delta}{T}}^{\infty} \frac{f(x)}{2x}\left(\int_N^{\infty} f_\xi(y)dy + \int_0^N \frac{\xi^2 f_\xi(y)}{N^2}dy \right) dx$$

$$+ \int_{\frac{\delta}{T}}^{\frac{1+\delta}{T}} \frac{f(x)}{2x}\left(\int_N^{\infty} (2 - Tx + \delta)(Tx - \delta)f_\xi(y)dy + \int_0^N \frac{[N^2(1 - Tx + \delta)^2 - \xi^2]f_\xi(y)}{N^2}dy \right) dx$$

$$- Nf_\xi(N)\left(T\int_0^{\frac{\delta}{T}} f(x)dx + \int_{\frac{\delta}{T}}^{\infty} \frac{\delta f(x)}{x}dx \right) - \int_{\frac{\delta}{T}}^{\infty} \frac{f(x)}{x}dx \int_0^N \frac{\xi^2 f_\xi(y)}{N^2}dy$$

This equation is concave (unimodal) if the last term

$$\int_{\frac{\delta}{T}}^{\infty} \frac{f(x)}{x}dx \int_0^N \frac{\xi^2 f_\xi(y)}{N^2}dy$$

is nondecreasing (because all other terms are decreasing). Because

$$\partial \int_0^N \frac{\xi^2 f_\xi(y)}{N^2}dy / \partial N = f_\xi(N) - \int_0^N \frac{2\xi^2 f_\xi(y)}{N^3}dy,$$

so if $\int_0^N \xi^2 f_\xi(y)dy \leq \frac{N^3}{2}f_\xi(N)$, the game has the unique equilibrium. Analogously, we can prove the results under W+T. $\square$

J. Proof of Proposition 2.9. With risk averse agents (exogenous δ), we have

$$\frac{\partial^2 \mathbb{E}(u_{PB}^c)}{\partial N^2} = -2A_2(\delta) - 2rc_2\widetilde{B}_P(\delta) < 0,$$

$$\frac{\partial^2 \mathbb{E}(u_{PB}^s(N_{PB}^*))}{\partial w^2} = -\frac{2A_2(\delta) + (2rc_2 + rs_1)\widetilde{B}_P(\delta)}{2[A_2(\delta) + rc_2\widetilde{B}_P(\delta)]^2} < 0,$$

$$\frac{\partial^2 \mathbb{E}(u_{PB}^s(N_{PB}^*))}{\partial \kappa^2} = -\frac{B_P(\delta)^2[2A_2(\delta) + (2rc_2 + rs_1)\widetilde{B}_P(\delta)]}{2[A_2(\delta) + rc_2\widetilde{B}_P(\delta)]^2} < 0.$$

So the utility functions are concave in N, w and κ under PBC. Similarly, we can prove the utility functions under W+T are concave in N, p and p_s. Then for PBC, penalty incurred on backorder, we have the equilibrium prices

$$w_{RA}^* = \frac{A_2(C_P + DA_1 + 2B_P\kappa^*) + (C_Prc_2 + DA_1(rc_2 + rs_1) + B_P\kappa^*(2rc_2 + rs_1))\widetilde{B}_P}{2A_2 + \widetilde{B}_P(2r_{c2} + r_{s1})}.$$

Taking the difference w_{RA}^* and w^* under risk neutral, we can have the result Δ_{PB}^* in Proposition 2.9. Similarly, we can also obtain Δ_{PL}^* and Δ_{W+T}^*. $\square$

Chapter 3

Coordinating Product Support Supply Chains under Outcome-based Compensations

3.1 Introduction

In recent years, outcome-based contracts, also known as performance-based contracts (PBC), performance-based logistics (PBL), or Power-by-the-Hour contacts, are increasingly used in industry (Cohen et al., 2006), and change companies' business model for providing products and services. For example, in the defence and military industry, PBL has become a mandatory acquisition strategies for weapon systems (DoD, 2003). Boeing applies PBL contracts with the US air force for maintenance of aircraft[1]. Rolls-Royce provides a Power-by-the-Hour strategy with the customers for aircraft engines (Smith, 2013). The core concept of an out-come based contract is that users only pay for what they use, or for product availability, rather than the ownership of the assets (Booz Allen Hamilton, 2005). Since customers only pay for outcomes, practitioners usually hope to realize higher product availability and lower operating cost (Aerospace Systems Division, 2007). Under such contracts, the Original Equipment Manufacturer (OEM) is usually the integrated service provider who is accountable for servicing the products for the end customers, which implies that the OEM will be penalized whenever

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[1]http://www.boeing.com/boeing/defense-space/support/business_overview/pbr.page

product downtime occurs. Hence, managing product availability is a highly important and challenging task for the OEM (Kim and Tomlin, 2013).

Dealing with both customers and the suppliers in order to guarantee product availability adds complexity to the OEM. Usually, the OEM assembles the final product. He is the contractor who is responsible to the customer. However, the quality of the product also depends on the subsystems or components that are manufactured by the suppliers. For example, BMW manufactures engines and the car. But the sensors of the motor management systems are supplied by Bosch. The car is not running if the sensors are faulty. The customer only attributes the liability to BMW, even if a product failure is caused by the parts from the suppliers. Outcome-based contracts target at the availability improvement for the final product, which depends on the quality of the components and the response speed once failure occurs. Typically, the OEM can invest in quality improvement on his own product to reduce failure occurrence, and in service capacity to reduce the repair completion lead time (Kim and Tomlin, 2013). However, the supplier's activities on quality management are usually beyond the OEM's direct control. In order to improve the quality of the final product, the OEM has to give the supplier appropriate incentives to make the supplier's parts more reliable.

Without outcome-based contracts, the OEM usually tries to impact the supplier's effort by quality inspection and imposing cost penalties (Baiman et al., 2001; Hwang et al., 2006). However, better ways may exist to incentivize the supplier and result in a higher profit for the OEM. In this chapter, we discuss three types of contracts which are offered by an OEM to the supplier: price-only contracts (PO), repair cost sharing contracts (RCS), and penalty sharing contracts (PS). Under the price-only contracts, the OEM buys parts from the supplier at a fixed price. Once the product has been delivered to the customer, the OEM is in charge of product support, and the supplier is not involved in the service process. Under the repair cost sharing contracts, the supplier has to bear the service cost which is incurred due to the supplier's part. Under the penalty sharing contract, the supplier does not have to pay for the repair cost, but compensates the OEM by sharing the penalty which is caused by downtime of the supplier's part. We study the impact of different contracts on the OEM's and the supplier's individual activities on quality improvement and service capacity management, and analyze the supply chain members' profit and channel efficiency. Our main findings are as follows.

1) If only the OEM has the repair service capacity, price-only contracts and repair cost sharing contracts cause under-investment in failure reduction for the supplier, which results in a lower product availability. Penalty sharing contracts achieve the First Best solution for the OEM

and for the supply chain. If the OEM also invests in service capacity, price-only contracts and repair cost sharing contracts lead to over-investment in service capacity for the OEM and under-investment in effort of failure reduction for the supplier. Penalty sharing contracts optimize the supplier's decision on failure reduction and the OEM's decision on capacity setting, which achieves the highest efficiency.

2) If both the supplier and the OEM have service capacity to repair their own part, paying a fixed service fee to the supplier leads to under-investment in the supplier's effort to reduce failure rate. On the other hand, penalty sharing can realize the First Best solution by setting an optimal penalty rate. If the product is nonseparable and repair cost/penalty is shared by a predefined proportion between the OEM and the supplier, repair cost sharing contracts still result in under-investment in the supplier's effort of failure reduction, whereas penalty sharing contracts achieve the First Best results and coordinate the supply chain.

3) If information about the quality of the part becomes asymmetric, i.e., only the supplier has the private information about the failure rate of his part, penalty sharing contracts cannot coordinate the supply chain, however, the OEM's expected profit under penalty sharing contracts is the highest, compared to the other two type of contracts.

The remaining of this chapter is organized as follows. In §3.2, we present a brief literate review and summarize the contribution of this paper. Then, we introduce the model setting in §3.3 and establish the basic model in §3.4. Next, we give the equilibrium solutions and analyze the comparison across contracts in §3.5. In §3.6, we examine the robustness of the results by extending the model with different settings and introducing information asymmetry. Finally, we give a summary of findings and potential future research directions in §3.7.

3.2 Literature Review

This chapter focuses on incentives and contracting issues in the product and service management fields. Our models are developed based on streams of literature regarding product quality management and (after-sales) service operations. First, product quality management is a significant research topic in Operations Management (OM). The game-theoretic quality management model in this chapter is similar to the following papers in terms of methodology. Reyniers and Tapiero (1995) study contracting on a supplier's quality level and a buyers's inspection policy.

Tagaras and Lee (1996) discuss the relation of quality and cost in vendor selection. Chen et al. (1998) study the optimal inspection procedure for products under warranty. More recently, Baiman et al. (2000) discuss the impact of information availability on the efficiency of a supply chain where the supplier is responsible for quality improvement and the buyer for quality appraisal. Later, Baiman et al. (2001) examine the relationship between product architecture, supply-chain performance metrics, and supply-chain efficiency in a setting where the supplier makes an effort in process improvement and the customer appraises the supplier's components. Hwang et al. (2006) also study the impact of the supplier's performance measure on product quality management. They develop a moral hazard model and compare two regimes - certification and appraisal. Zhu et al. (2007) study the problem in which both the buyer and the supplier incur quality-related costs and therefore have incentives to invest in quality improvement. They examine how quality-improvement decisions affects the buyer's order quantity and the supplier's production lot size. Moreover, Balachandran and Radhakrishnan (2005) also study the quality control problem when the supplier's penalty is based on the buyer's inspection or external failure. They develop single and double moral hazard models and analyze system efficiency under different cases. Dai et al. (2012) study the impact of warranty period setting on the efficiency of the supply chain, where the supplier controls the product quality and the customer decides on order quantity. They show that when the warranty period is determined by the firm sharing the larger proportion of total warranty costs, the supply chain can achieve greater system-wide profit.

Those above-mentioned papers mainly focus on ensuring suppliers' quality management by different evaluation or penalty policies. However, product architecture has been ignored, i.e., the supplier is the single producer of the final product. As a contrast, we discuss a setting when both the supplier and the manufacturer (OEM) produce their own parts which form the final product. In such a sense, the setting of this paper resembles (Chao et al., 2009). In their model, the manufacturer and the supplier both produce components that compose the final product, but they focus on the recall problem, while the impacting factors are cost sharing contracts. Unlike Chao et al. (2009), we consider a three stage supply chain where the supplier's and the manufacturer's efforts both affect the product availability on which the customer's profit relies. Furthermore, we also incorporate service capacity decision next to the supply chain members' efforts on quality improvement.

The after-sales service setting in this chapter is that the compensation between the manufacturer and the customer is under an outcome-based contract. In the streams of literature studying

performance-based contract, Kim et al. (2007) study how performance-based contracts affect the supplier's spare parts inventory decision. Roels et al. (2010) discuss collaborating with the suppliers under different service contracts. Kim et al. (2010) investigate service capacity decisions under performance-based contracts. Although the outcome -based payment setting is similar, our work mainly focuses on how to transfer the accountability of the performance of the final product to the supplier whose quality control activities are noncontractible. In (Kim and Tomlin, 2013), they study failure reduction and capacity setting of parallel parties who make simultaneous decisions. They emphasize the joint failure setting while our work assumes independent failures and studies sequential games between the supply chain members under certain types of contracts.

Finally, the model in this paper is based on game theory and contract theory. For the application of game theory in OM, Cachon and Netessine (2004) introduce general game-theoretic models in a SCM setting. Furthermore, Cachon (2004) gives extensive overview on supply chain coordination with contracts. Our work contributes to this field by combining quality and service decisions within the context of outcome-based compensations. Unlike revenue sharing (Cachon and Lariviere, 2005) in the retailing supply chain, we come up with penalty sharing mechanism for coordinating the after-sales service supply chain.

3.3 Model Setting

We consider a supply chain that consists of a customer, an OEM and a supplier, as shown in Figure 3.1. The customer buys a durable product (such as industrial equipment) from the OEM to generate revenue. The customer pays the OEM under an *Outcome-based Compensation* contract. Under this type of contract, the OEM receives payment from the customer based on product performance. Normally, product availability is the metric which is adopted by practitioners to measure product performance. In our model, we assume that the payment framework between the customer and the OEM is "initial payment + penalty". At the beginning, the customer pays the OEM a lump sum w. During the contract period, once the product fails, the OEM is responsible for repairing the failed product. Since the product cannot generate revenue for the customer during the failure-repairing period, the OEM has to pay a penalty to the customer with rate κ, which is the penalty per unit downtime.
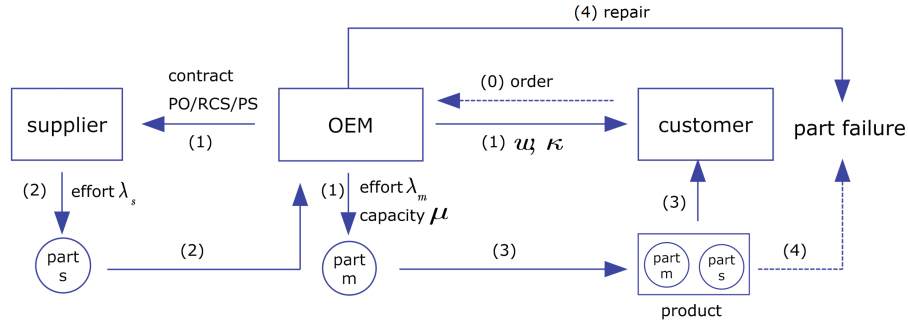
FIGURE 3.1: Sequence of events.

In order to avoid product downtime, the OEM aims at optimizing two managements: improving product quality to have fewer failure occurrences and investing service capacity to have a quicker failure repair. In practice, industrial machines are often assembled from many subsystems and components which are produced by the suppliers. Furthermore, the quality of the final product is dependent on the various components. To capture this feature, in this chapter, we assume the final product consists of two parts - part s and part m which are manufactured by the supplier and the OEM respectively. The OEM acquires part s from the supplier and assembles it with part m to form the final product. In addition, We assume that the product has a series structure and the malfunction of any of the parts leads to a product failure. The OEM can exert efforts on improving the quality of part m, however, the OEM is unable to dictate the supplier to improve the quality of part s. Therefore, the OEM has to design appropriate contracts that can incentivize the supplier to product part s with a higher quality[2].

The sequence of events is as shown in Figure 3.1. (1) The OEM receives an order from the customer[3]. The product and its after-sales service are delivered based on an outcome-based contract. Without loss of generality, we normalize the contract period to 1. The OEM sets the contract terms w and κ. Meanwhile, the OEM exert quality improvement effort and manufactures part m. At the same time, the OEM offers a take-it-or-leave-it contract to the supplier to acquire part s. The contract between the OEM and the supplier can be PO, RCS, or PS. Specifically,

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[2]In practice, asking the supplier to improve the part quality can be very difficult to implement because the supplier's effort is hardly possible to be monitored in an easy and cost-effective way, especially when there are multiple suppliers or under global sourcing. That is the reason why such cases usually fall into the Moral-Hazard setting, which is captured by Principal-Agent models in this chapter.

[3]To simplify the model and emphasize the main focus, we first assume there is a single product in the supply chain, which is similar to the setting in (Kim et al., 2010).

- under PO, the OEM pays the supplier a price p_1 to buy part s. After that, the supplier is not involved in the after-sales support for the product.

- under RCS, the OEM first pays the supplier a price p_2 to buy part s. During the contract period, the supplier has to bear the repair cost incurred by failures of part s.

- under PS, the OEM first pays the supplier a price p_3 to buy part s. Instead of sharing repair cost, the supplier has to share the OEM's penalty which is caused by the downtime of part s.

In addition, the OEM also has a service capacity μ to conduct repairing product failures during the product support process. In the model, μ can be a given parameter (exogenous) or can be set by the OEM (endogenous). (2) If the supplier accepts the contract, he manufactures part s. Depending on the OEM's contracts, the supplier may also exert quality improvement effort on part s. Then the supplier delivers part s to the OEM. (3) The OEM assembles part m and part m into the final product and delivers it to the customer. (4) The customer starts running the product and the OEM commences after-sales product support. In case failure occurs, the OEM repairs the defective parts[4] and pays a penalty based on the duration of downtime.

As a general custom in literature, we assume that part s and part m fail independently according to two Poisson processes. Given a quality level, the initial failure rate is λ_{s0} and λ_{m0}. After the quality improvement effort is exerted, the failure rate becomes λ_s and λ_m. Let N denote the total number of part failures during the contract period. The expected value $\mathbb{E}[N] = \lambda_s + \lambda_m$. The repair lead time for each failure is $S_i, i = 1, .., N$, which are independent and identically distributed with rate $1/\mu$, where μ is the service capacity. It is straightforward to see that a large capacity implies short downtime. Since each part failure leads to a system offline, the total system downtime can be formulated as $\sum_{i=1}^{N} S_i$, and the uptime is $1 - \sum_{i=1}^{N} S_i$. It can be proved that $\sum_{i=1}^{N} S_i$ satisfies a Compound Poisson distribution with parameters $(\lambda_i + \lambda_s, 1/\mu)$.

Finally, the cost breakdown is as follows. The cost of the quality improvement effort is $C_i(\Delta\lambda_i)$, where $\Delta\lambda_i = \lambda_{i0} - \lambda_i$, $i = s, m$ denoting the supplier and the OEM respectively. We assume that $C_i(\Delta\lambda)$ is increasing and convex in $\Delta\lambda$. In order to make the model tractable for further analysis, following literature such as (Heese and Swaminathan, 2006; Ju and Wan, 2012), we use quadratic functions to represent the effort cost. Let $C_s(\Delta\lambda_s) = c_s(\lambda_{s0} - \lambda_s)^2$ and $C_m(\Delta\lambda_m) =$

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[4]In §3.6.1, we also look at the setting when the OEM and the supplier have individual service capacity and repair their own part.

$c_m(\lambda_{m0} - \lambda_m)^2$, where c_s and c_m are effort cost coefficients. Besides, the production cost of part s for the supplier is K_s, and the production cost of the final product (including the cost of manufacturing part m) for the OEM is K_m. Furthermore, c_r is the cost for each repair and c is the unit service capacity cost.

3.4 Contracting

3.4.1 First Best

As the benchmark, we first analyze the First Best (FB) case when the supplier, the OEM, and the customer are integrated as a single decision maker. The risk-neutral firm maximizes his expected profit by setting the level of effort of failure rate reduction for part s and part m (service capacity μ can be exogenous or endogenous).

$$\max_{\lambda_s,\lambda_m,(\mu)} r(1 - \mathbb{E}[\sum_{i=1}^{N} S_i \,\Big|\, \lambda_s, \lambda_m, \mu]) - (K_s + K_m) - c\mu - c_r\mathbb{E}[N\,|\,\lambda_s, \lambda_m] - C_s(\Delta\lambda) - C_m(\Delta\lambda_m). \quad (3.1)$$

The first terms in Equation (3.1) is the total revenue generated by the product. We notice that the product only generates revenue during uptime $1 - \mathbb{E}(\sum_{i=1}^{N} S_i \,\Big|\, \lambda_s, \lambda_m, \mu)$, and r is the product revenue rate in a unit working time. The remainders are the total costs, which consist of production cost $K_m + K_s$, service capacity cost $c\mu$, cost of repair $c_r\mathbb{E}(N\,|\,\lambda_s, \lambda_m)$, and cost of the failure rate reduction efforts $C_s(\Delta\lambda)$, $C_m(\Delta\lambda_m)$.

Lemma 3.1. *(1) The firm's profit function is concave*[5] *in λ_s, λ_m, and μ.*

(2) The optimal solution of FB is as summarized in Table 3.1.

The proof can be found in Appendix A.

From Lemma 3.1, we know the optimal effort level and the service capacity for the supply chain. Furthermore, we can understand how parameters affect the effort level. Let $\Delta\lambda_m = $

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[5]not jointly concave

TABLE 3.1: Optimal solution of FB.

	Exogenous μ	Endogenous μ
μ^*	-	$c_m(r^2 + r(c_r - 2c_s(\lambda_{m0} + \lambda_{s0}))\mu^{FB*})$ $+ 2cc_s\mu^{FB*3}) = c_sr(r + c_r\mu^{FB*}),$ if $\mu^{FB*} < \dfrac{3(c_m + c_s)r}{2(c_m(2c_s(\lambda_{m0} + \lambda_{s0}) - c_r) - c_rc_s)}.$
λ_m^*, λ_s^*	$\lambda_m^{FB*} = \lambda_{m0} - \dfrac{r + c_r\mu}{2c_m\mu},$ $\lambda_s^{FB*} = \lambda_{s0} - \dfrac{r + c_r\mu}{2c_s\mu}.$	$\lambda_m^{FB*} = \lambda_{m0} - \dfrac{r + c_r\mu^{FB*}}{2c_m\mu^{FB*}},$ $\lambda_s^{FB*} = \lambda_{s0} - \dfrac{r + c_r\mu^{FB*}}{2c_s\mu^{FB*}}.$

$\lambda_{m0} - \lambda_{m(FB)}^*$. If μ is exogenous, we have $\partial\Delta\lambda_m/\partial r > 0$, $\partial\Delta\lambda_m/\partial c_r > 0$, $\partial\Delta\lambda_m/\partial c_m < 0$, and $\partial\Delta\lambda_m/\partial\mu < 0$. If the customer's revenue rate r increases, the central firm will exert more effort on reducing the failure rate. If the repair cost increases, similarly, the effort level will increase as well. This is because a lower failure rate implies fewer failure occurrences. If the repair cost becomes higher, it is optimal to reduce the number of failures. However, if the effort cost c_m, or capacity increases, the effort level will decrease. It is intuitive that if the effort cost c_m increases, the effort level will becomes less. We also observe that increasing service capacity can decrease the effort level. This is because higher capacity means quicker repair for each failure; the total downtime will therefore be shortened. This implies the firm can invest less in reducing the failure rate.

The optimal solutions for λ_s^* and λ_m^* are symmetric. We only need to change the impacting parameter c_m into c_s in the denominator. Furthermore, if μ becomes endogenous, i.e., the service capacity can be set by the decision maker, we can obtain the optimal capacity by solving the polynomial equation with respect to μ^{FB*} in Table 3.1. However, this characteristic reduces the tractability of the model and complicates our analysis, especially for contract comparison. Next, we discuss the decentralized setting where the OEM offers contracts to the supplier. We first analyze the case with exogenous μ, and then we discuss the endogenous μ case by numerical analysis.

3.4.2 Decentralized Supply Chain

Once the setting becomes decentralized, the supplier's effort becomes incontractible. Let T be the internal payment between the OEM and the supplier. We have the OEM's problem as

$$
\max_{\lambda_m, w, \kappa, (\mu)} \; w - \kappa \mathbb{E}\left(\sum_{i=1}^{N} S_i \,\middle|\, \lambda_m, \lambda_s^*, \mu\right) - K_m - c\mu - c_r \mathbb{E}[N | \lambda_s, \lambda_m^*] - C_m(\Delta \lambda_m) - T
$$

s.t.

$$
r[1 - \mathbb{E}\left(\sum_{i=1}^{N} S_i \,\middle|\, \lambda_m, \lambda_s^*, \mu\right)] - w + \kappa \mathbb{E}\left(\sum_{i=1}^{N} S_i \,\middle|\, \lambda_m, \lambda_s^*, \mu\right) \geq 0, \; \text{(IR-C)} \tag{3.2}
$$

$$
\pi_s(\lambda_s^*) = T - K_s - C_s(\Delta \lambda_s^*) \geq 0, \; \text{(IR-S)}
$$

$$
\lambda_s^* = \arg\max \pi_s(\lambda_s). \; \text{(IC-S)}
$$

The objective function of the OEM is to maximize the expected profit by setting the optimal effort level λ_m, the PBC contract terms (w and κ) while taking the supplier's decision λ_s^* into account. We can interpret the OEM's profit as the revenue received from the customer $w - \kappa \mathbb{E}\left(\sum_{i=1}^{N} S_i \,\middle|\, \lambda_m, \lambda_s^*, \mu\right)$ minus the total costs which consist of the production cost K_m, service capacity cost $c\mu$, repair cost $c_r \mathbb{E}[N | \lambda_s, \lambda_m^*]$, effort cost $C_m(\Delta \lambda_m)$, and the expected payment transferred to the supplier T. The form of the internal payment T depends on the type of contracts.

(i) Under PO, the OEM only pays the supplier a selling price of part s. Then $T = p_1$.

(ii) Under RCS, the supplier has to bear the repair cost of part s. The repair cost of part s is $c_r N_s$, where N_s is the number of failures of part s. So we have $T = p_2 - c_r \lambda_s^*$.

(iii) Under PS, the supplier shares the penalty with the OEM. The penalty is allocated by the downtime caused by failure of the parts. The downtime due to failure of part s is $\sum_{i=1}^{N_s} S_i^s \,\big|\, \lambda_s^*, \mu$, where S_i^s is the ith failure of part s, and N_s is the total failures of part s. Then, we have $T = p_3 - \kappa \mathbb{E}(\sum_{i=1}^{N_s} S_i^s \,\big|\, \lambda_s^*, \mu)$.

Compared to the FB, there are more constraints in the OEM's problem. The first IR-C is the Individual Rationality constraint for the customer, which states that the customer's profit should be no less than a reserved level (normalized to 0). Similarly, the second IR-S constraint

states that the supplier's profit should also be no less than 0. The IR constraints ensure the participation of the customer and the supplier. In addition, there is an IC-S constraint. In the decentralized supply chain setting, the supplier just chooses an effort level which is best for his own profit, rather than the desired level for the OEM. Here the IC-S is the Incentive Compatibility constraint to capture the supplier's individual interest.

3.5 Analysis of Contracts

3.5.1 Exogenous μ

First, we start with the exogenous μ case. The OEM's service capacity is exogenously specified. The OEM designs the contract terms and the effort level of quality improvement for part m. Given the contracts, the supplier sets the effort level on part s.

Lemma 3.2. *(1) Unique equilibrium solutions can be obtained under all contracts. The results are as summarized in Table 3.2.*

(2) IR-C and IR-S constraints are binding at equilibrium.

The proof can be found in Appendix B.

TABLE 3.2: Equilibrium solution under contracts, exogenous μ.

	PO	RCS	PS
λ_m^*	$\lambda_{m0} - \frac{r+c_r\mu}{2c_m\mu}$	$\lambda_{m0} - \frac{r+c_r\mu}{2c_m\mu}$	$\lambda_{m0} - \frac{r+c_r\mu}{2c_m\mu}$
λ_s^*	λ_{s0}	$\lambda_{s0} - \frac{c_r}{2c_s}$	$\lambda_{s0} - \frac{r+c_r\mu}{2c_s\mu}$
$p_{1,2,3}^*$	K_s	$K_s + c_r\lambda_{s0} - \frac{c_r^2}{4c_s}$	$K_s + c_r\lambda_{s0} - \frac{c_r^2}{4c_s} - \frac{r(r+2(c_r-2c_s\lambda_{s0})\mu)}{4c_s\mu^2}$
w^*, κ^*	$w^* = r - \frac{(r-\kappa^*)(\lambda_m^*+\lambda_{s0})}{\mu}$	$w^* = r - \frac{(r-\kappa^*)(\lambda_m^*+\lambda_s^*)}{\mu}$	$\kappa^* = r + c_r\mu$

By analyzing the results in Lemma 3.2, we have the following Proposition.

Proposition 3.3. *(1) The OEM's effort is independent of the type of contracts which are offered to the supplier.*

(2) *Under PO, the supplier exerts no effort on failure rate reduction. RCS and PS can induce the supplier to exert effort on failure rate reduction.*

(3) *Under RCS, the supplier's effort level is increasing in c_r and decreasing in c_s.*

(4) *Under PS, the supplier's effort level is increasing in r and c_r, and is decreasing in c_s and μ.*

The proof can be found in Appendix C.

From Table 3.2, we can see that the OEM's effort remains the same across contracts. In other words, the supplier's activity does not impact the OEM's decision on the optimal effort level. This is because the failure processes of part s and part m are independent. As the leader in contracting, the OEM's activity is not affected by the supplier's following-up action. However, the supplier's effort level is affected by the type of contracts. PO cannot induce the supplier to exert effort. On the other hand, the supplier is willing to invest in failure reduction under RCS and PS. Under RCS, the supplier has to pay for the repair cost of part s. Thus, if λ_s is reduced, the supplier's service cost payment will become less. We have $\partial \Delta \lambda_s^{RCS}/\partial c_r > 0$, and $\partial \Delta \lambda_s^{RCS}/\partial c_s < 0$. As c_r increases, the supplier will pay more for the repair cost. So the supplier should reduce the failure rate to a lower level. Similarly, under PS, the supplier's payment on penalty will decrease if downtime is reduced. Since lowering failure rate can reduce downtime, the supplier is also incentivized to invest in failure reductions. In Proposition 3.3, we also notice that the supplier's effort is not affected by μ under RCS, but is decreasing in it under PS. The intuition is that under RCS, the supplier only pays for the repair cost which is only related to the number of failures; under PS, the supplier pays for the penalty which is determined both by the number of failures and the repair lead time. An increase in capacity can reduce the product downtime, but does not affect the failure rate. Hence, the supplier's effort is decreasing in μ under PS and is independent of μ under RCS.

Proposition 3.4. *(1) Under PO and RCS, the supplier **under-invests** in the effort of failure rate reduction on part s.*

(2) *For the OEM, $\pi_m^{PS*} > \pi_m^{RCS*} > \pi_m^{PO*}$.*

(3) *The OEM's profit is concave in μ if $\lambda_{m0} < 2\lambda_{s0}$.*

(4) *PS achieve the First Best profits and coordinate the supply chain.*

The proof can be found in Appendix D.

Although both RCS and PS can motivate the supplier to exert an effort, the supply chain members' profits are different. Proposition 3.4 gives the results of contract comparison. First of all, compared to FB, PO and RCS lead to under-investment in the effort of failure rate reduction. Since there is no effort under PO, it is obvious that the optimal failure rate under PO is higher than FB. For RCS, the supplier's effort is less than FB. Concretely, we have $\Delta\lambda_s^{PS} - \Delta\lambda_s^{RCS} = \frac{r}{2c_s\mu}$. Since $\partial\frac{r}{2c_s\mu}/\partial\mu < 0$, we know that the under-investment under RCS is decreasing in μ. Furthermore, the supplier's effort under PS is equivalent to the FB. In other words, PS can have the same efficiency as FB in inducing the supplier's quality improvement effort. As shown in Figure 3.2, the reduced failure rate under PS is the lower than RCS. As μ increases, the gap between the two is decreasing.

FIGURE 3.2: Optimal failure rate of part s at equilibrium under contracts, exogenous μ.
$r = 100$, $c_s = 20$, $c_m = 20$, $\lambda_{m0} = 1$, $\lambda_{s0} = 1$, $c = 1$, $c_r = 10$.

Because the supplier's effort under PS achieves FB, we can conclude that the efficiency of PS is the highest. RCS can incentivize the supplier and thus it's efficiency is higher than PO. Furthermore, since IR-S constraints are all binding at equilibrium, the supplier has no surplus whereas the OEM obtains all supply chain's profit. Consequently, for the OEM's profit, we have PS > RCS > PO. Specifically, we can prove that

$$\pi_m^{PS*} - \pi_m^{PO*} = \frac{(r + c_r\mu)^2}{4c_s\mu^2}$$

and

$$\pi_m^{PS*} - \pi_m^{RCS*} = \frac{r^2}{4c_s\mu^2}.$$

Because $\partial\frac{(r+c_r\mu)^2}{4c_s\mu^2}/\partial\mu < 0$, and $\partial\frac{r^2}{4c_s\mu^2}/\partial\mu < 0$, the profit difference between PO, RCS and PS are decreasing in μ. Figure 3.3 illustrates how the OEM's profit changes in μ under contracts. In general, $\pi_m^{PS*} > \pi_m^{RCS*} > \pi_m^{PO*}$. As μ increases, the difference of profits becomes smaller. In addition, we give the sensitivity analysis of the OEM's profit with respect to parameters in Table 3.3. The OEM's profit is increasing in r, decreasing in costs (c_m, c_s, c_r, c), and concave in μ. If r increases, the OEM can offer a higher price to the customer, and then the OEM's revenue increases. Any increase in cost will make the profit lower. In addition, as μ increases, the repair lead time becomes less. However, the failure reduction effort also becomes lower. As a result, when μ is small, the profit benefits more from the reduction of repair lead time. When μ becomes sufficiently large, the profit can be decreasing in μ in that the effort level decreases largely which makes the failure rate become higher.

Another conclusion that we can verify is $\partial p_1^*/\mu = \partial p_2^*/\mu = 0$, $\partial p_3^*/\mu < 0$. That is to say that under PO and RCS, the price of the supplier's part is not affected by the OEM's service capacity. However, under PS, the OEM can pay less for a part if he has a higher capacity. The intuition of this result is that under PO or RCS, how quickly a product failure can be restored is beyond the supplier's concern. But, if the supplier has to share product downtime penalty, he is willing to accept a lower price if the OEM's service rate is increased, because a shorter repair lead time implies a less penalty for the supplier.

TABLE 3.3: Sensitivity of the OEM's profit at equilibrium under contracts, exogenous μ.

	r	c_s	c_m	c_r	c	μ
π_m^{PO*}	↑	-	↓	↓	↓	⌢
π_m^{RCS*}	↑	↓	↓	↓	↓	⌢
π_m^{PS*}	↑	↓	↓	↓	↓	⌢

3.5.2 Endogenous μ

In previous sections, we have assumed that μ is exogenously specified. In this section, we incorporate μ as an decision variable for the OEM , i.e., the OEM can set an optimal level of
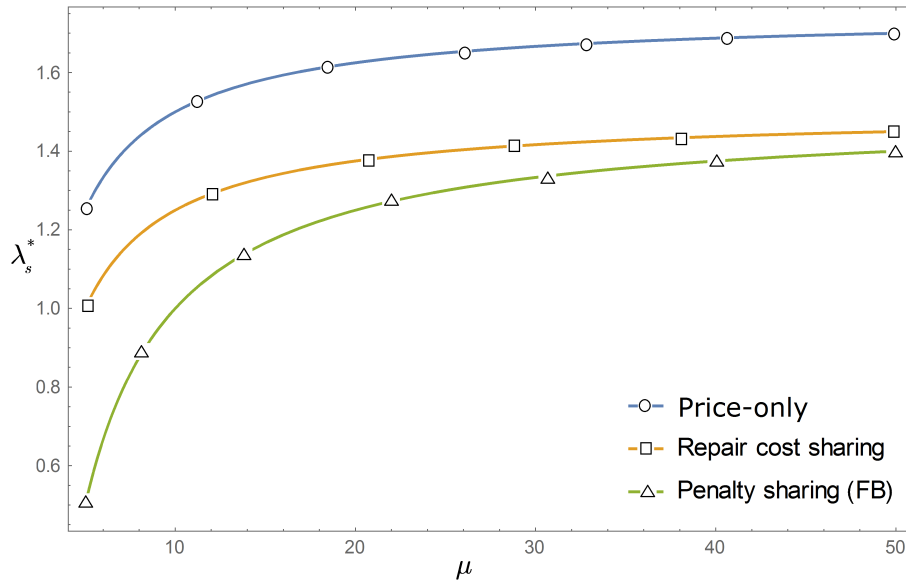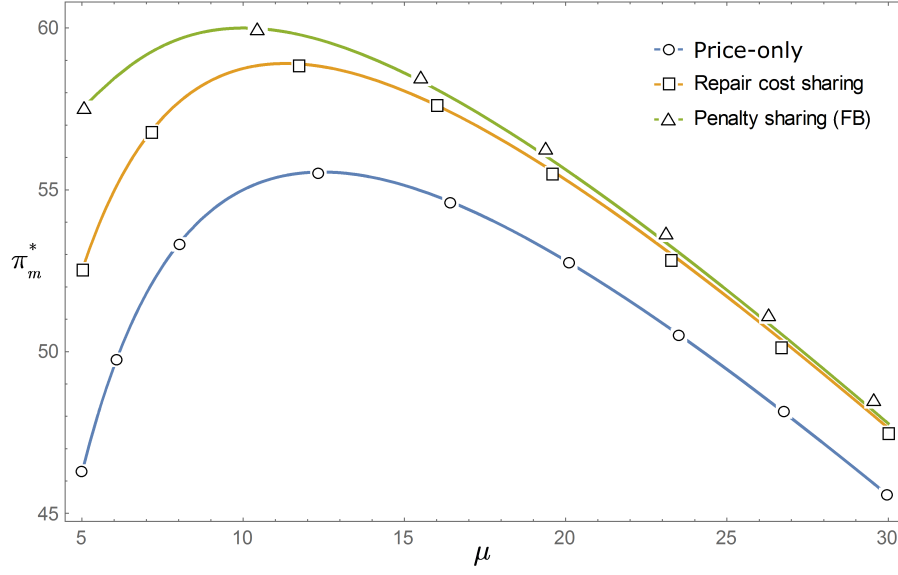
FIGURE 3.3: The OEM's profit at equilibrium under contracts, exogenous μ. $r = 100$, $c_s = 20$, $c_m = 20$, $\lambda_{m0} = 1$, $\lambda_{s0} = 1$, $c = 1$, $c_r = 10$.

the service capacity. Although endogenizing μ reduces the tractability of the model, we can show the existence and uniqueness of the equilibrium, as summarized in Lemma 3.5.

Lemma 3.5. *(1) Under PO, the supplier's effort is 0. The optimal capacity μ^{PO*} satisfies*
$$r(2c_m(\lambda_{m0} + \lambda_{s0}) - c_r)\mu^{PO*} - r^2 - 2cc_m\mu^{PO*^3} = 0.$$

(2) Under RCS, $\lambda_s^{RCS*} = \lambda_{s0} - \frac{c_r}{2c_s}$. The optimal capacity μ^{RCS*} satisfies $c_s r^2 + (c_m c_r r + c_s r(c_r - 2c_m(\lambda_{m0} + \lambda_{s0})))\mu^{RCS*} + 2cc_m c_s \mu^{RCS*^3} = 0$.

(3) Under PS, the optimal capacity μ^{PS*} satisfies $r^2(c_m + c_s) + (c_r c_s r + c_m r(c_r - 2c_s(\lambda_{m0} + \lambda_{s0})))\mu^{PS*} + 2cc_m c_s \mu^{PS*^3} = 0$, and $\lambda_s^{PS*} = \lambda_{s0} - \frac{r}{2c_s\mu^{PS*}}$.

The proof can be found in Appendix E.

Once μ can be set by the OEM, the optimal capacity under contracts can be obtained by solving the equations as given in Lemma 3.5. We notice that under PO and RCS, endogenizing μ does not impact the supplier's decision on the effort. Similar to the exogenous μ case, the supplier does not invest in failure rate reduction under PO. Under RCS, the supplier exerts an effort, nevertheless, the level of effort is independent of μ^*. The correlation of λ_s^* and μ^* is only seen under PS. Moreover, we have $\partial\lambda_s^{PS*}/\partial\mu^{PS*} > 0$, i.e., the game under PS is supermodular with resect to μ and λ_s. After obtaining the equilibrium solution, we can compare the results across contacts.

Proposition 3.6. *(1) Under PO and RCS, the OEM **over-invest** in service capacity, i.e.,*
$\mu^{PO*} > \mu^{FB*}$, $\mu^{RCS*} > \mu^{FB*}$.

*(2) Under PO and RCS, the supplier **under-invest** in failure rate reduction, i.e.,* $\lambda_s^{PO*} > \lambda_s^{FB*}$, $\lambda_s^{RCS*} > \lambda_s^{FB*}$.

(3) PS achieves the First Best decision in both capacity and effort, and coordinate the supply chain.

The proof can be found in Appendix F.

Once the decision structure becomes decentralized, PO and RCS lead to over-investment in service capacity, whereas PS realize the FB solution. For the level of effort, under PO and RCS, the supplier's investment in failure reduction effort is less than the FB, while PS can result in the equivalent solution. Under PO, the supplier does not exert any effort. Under RCS, the supplier's effort level is lower than the FB. The product failure rate under these two contracts are higher than the FB and PS, as shown in Figure 3.5. Consequently, the OEM have to invest more in capacity to deal with higher failure rate. In such a case, over-investment in capacity appears. In Figure 3.4, we can see that $\mu^{PO*} > \mu^{RCS*} > \mu^{PS*}$. More importantly, PS has the property of coordinating the supply chains as the exogenous μ case. Even if the OEM can set up the capacity for his own good, the supplier can still exert the failure reduction effort to the optimal level, which realizes the maximum profit for the OEM. Because μ^* is determined by solving the cubic functions, we can not show the closed-form solutions and compare the OEM's profit analytically. However, since PS achieve the FB, and PO have the highest efficiency loss, it can be concluded that for the OEM's profit, $\pi_m^{PS*} > \pi_m^{RCS*} > \pi_m^{PO*}$. This argument is also verified by numerical experiment. In Figure 3.6, we give the OEM's profit at equilibrium under contracts. From the Figure, we can see that profits are decreasing in λ_{m0}. The OEM's profit is always the highest under PS. Profit becomes lower under RCS, and PO are even worse than RCS.
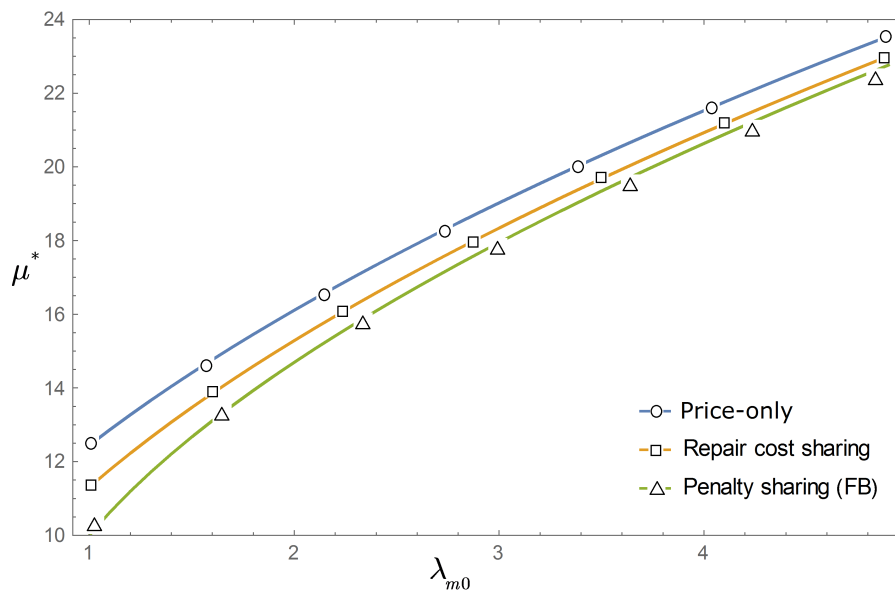
FIGURE 3.4: Optimal service capacity at equilibrium under contracts, endogenous μ.
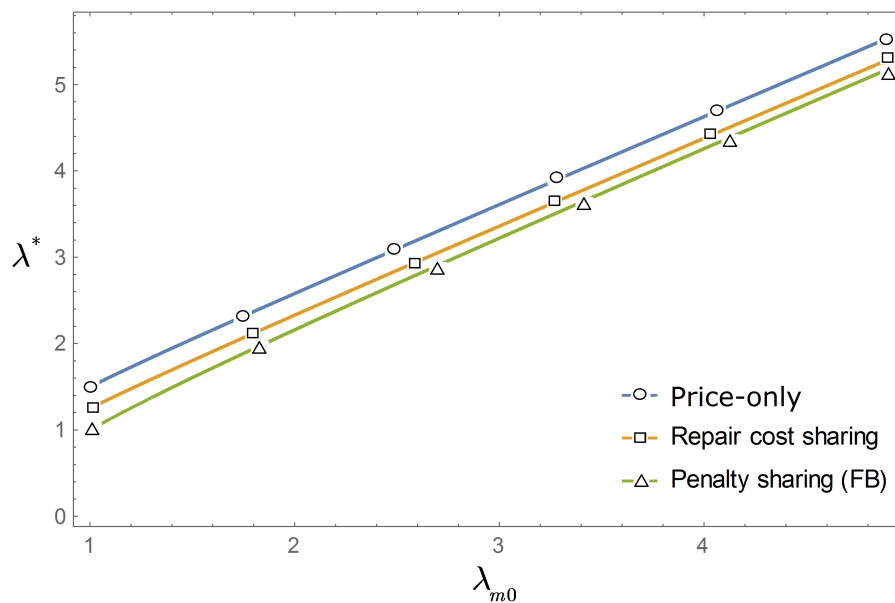
FIGURE 3.5: Optimal failure rate at equilibrium under contracts, endogenous μ. $\lambda^* = \lambda_s^* + \lambda_m^*$.
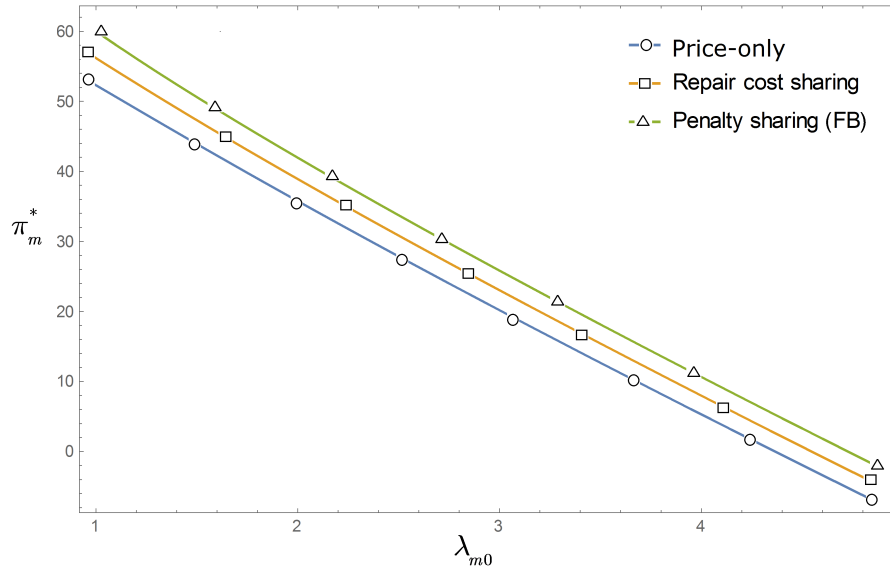
FIGURE 3.6: The OEM's profit at equilibrium under contracts,endogenous μ.

3.6 Some Extensions

3.6.1 Individual Capacity

In previous sections, we have assumed that the OEM has the capacity to repair both part m and part s. In practice, the manufacturer can be the unique service provider for the part. For the setting in this chapter, the OEM may not have the resource to repair part s once failure occurs. On the contrary, the supplier and the OEM have individual capacity μ_m and μ_s to repair their own part. The OEM is sill the accountable service provider for the customer, yet he has to ask the supplier to maintenance part s. Under this circumstance, we discuss two types of contracts - Service Outsourcing (SO) and Penalty Sharing (PS). Under the former, if part s fails, the OEM calls on the supplier to repair it, and pays p_r for each service; under the latter, the supplier still has to repair part s and shares the penalty with the OEM. Then, the OEM's problem becomes

$$\max_{\lambda_m,w,\kappa,p_r,(\mu_m)} w - \kappa \mathbb{E}(\sum_{i=1}^{N} S_i \Big| \lambda_m, \lambda_s^*, \mu_m, \mu_s) - K_m - c\mu_m - c_r\lambda_m - C_m(\Delta\lambda_m) - p_r\lambda_s^*$$

$$s.t.$$

$$r[1 - \mathbb{E}(\sum_{i=1}^{N} S_i \Big| \lambda_m, \lambda_s^*, \mu_m, \mu_s)] - w + \kappa\mathbb{E}(\sum_{i=1}^{N} S_i \Big| \lambda_m, \lambda_s^*, \mu_m, \mu_s) \geq 0, \ (\text{IR-C})$$

$$\pi_s(\lambda_s^*) = p_r\lambda_s^* - K_s - c_r\lambda_s^* - c\mu_s - C_s(\Delta\lambda_s^*) \geq 0, \ (\text{IR-S})$$

$$\lambda_s^* = \arg\max \pi_s(\lambda_s). \ (\text{IC-S})$$

(3.3)

and

$$\max_{\lambda_m,w,\kappa,p,(\mu_m)} w - \kappa \mathbb{E}(\sum_{i=1}^{N_m} S_i^m \Big| \lambda_m, \mu_m) - K_m - c\mu_m - c_r\lambda_m - C_m(\Delta\lambda_m) - p_4$$

$$s.t.$$

$$r[1 - \mathbb{E}(\sum_{i=1}^{N} S_i \Big| \lambda_m, \lambda_s^*, \mu_m, \mu_s)] - w + \kappa\mathbb{E}(\sum_{i=1}^{N} S_i \Big| \lambda_m, \lambda_s^*, \mu_m, \mu_s) \geq 0, \ (\text{IR-C})$$

$$\pi_s(\lambda_s^*) = p_4 - \kappa\mathbb{E}(\sum_{i=1}^{N_s} S_i^s \Big| \lambda_s^*, \mu_s) - K_s - c_r\lambda_s^* - c\mu_s - C_s(\Delta\lambda_s^*) \geq 0, \ (\text{IR-S})$$

$$\lambda_s^* = \arg\max \pi_s(\lambda_s). \ (\text{IC-S})$$

(3.4)

From Equation (3.3) and (3.4), we can see that the OEM and the supplier build their own capacity μ_m and μ_s, and incurs the repair cost $c_r\lambda_m$ and $c_r\lambda_s$. Under SO, the OEM pays the service fee $p\lambda_s^*$ to the supplier. Under PS, the OEM and the supplier repairs their own part and share the penalty based on the downtime of part s ($\mathbb{E}(\sum_{i=1}^{N_s} S_i^s \Big| \lambda_s, \mu_s)$) and part m ($\mathbb{E}(\sum_{i=1}^{N_m} S_i^m \Big| \lambda_m, \mu_m)$). In this setting, endogenous μ becomes highly untractable. We have shown that endogenizing μ does not change the insights significantly, we only look at the exogenous μ case for this individual capacity setting. We have the unique equilibrium solution summarized in Table 3.4.

Proposition 3.7. *(1) Under SO, the supplier does not invest in failure reduction.*

(2) The OEM's profit under PS is higher than SO.

TABLE 3.4: Equilibrium solution of the individual capacity case, exogenous μ.

	FB	SO	PS
λ_m^*	$\lambda_{m0} - \frac{r+c_r\mu_m}{2c_m\mu_m}$	$\lambda_{m0} - \frac{r+c_r\mu_m}{2c_m\mu_m}$	$\lambda_{m0} - \frac{r+c_r\mu_m}{2c_m\mu_m}$
λ_s^*	$\lambda_{s0} - \frac{r+c_r\mu_s}{2c_s\mu_s}$	λ_{s0}	$\lambda_{s0} - \frac{r+c_r\mu_s}{2c_s\mu_s}$
$p_{r,4}^*$	-	$c_r + \frac{K_s+c\mu_s}{\lambda_{s0}}$	$K_s - \frac{c_r{}^2}{4c_s} + c_r\lambda_{s0} + c\mu_s$ $- \frac{r(r + 2(c_r - 2c_s\lambda_{s0})\mu_s)}{4c_s\mu_s{}^2}$
w^*, κ^*	-	/	$w^* = r, \kappa^* = r$

(3) PS achieve the FB and coordinate the supply chain.

The proof can be found in Appendix G.

Once the supplier has his own service capacity to repair part s, the supplier does not invest in the effort of reducing failure rate. Recall that in the setting where only the OEM has service capacity, the supplier has investment in effort under RCS. However, in this individual capacity setting, even if the supplier still bears the repair cost of part s, the supplier cannot be induced to exert any effort on quality improvement. This is because the the service fee received from the OEM covers the repair cost. The supplier's revenue depends on the failure occurrence of part s. Intuitively, the supplier's revenue increases with failure rate of part s. Thus, the supplier has no incentives to make part s more reliable. As such, SO have efficiency loss due to the under-investment in the supplier's effort. On the other hand, PS achieve FB and coordinate the supply chain. Compared to the single capacity setting, the optimal w and κ are not given in combinations. The OEM sets $w^* = r, \kappa^* = r$, i.e., both the initial payment and the penalty rate are equivalent to the customer's revenue rate. In this way, the customer's revenue loss due to downtime of part s is shifted to the supplier's profit. As such, the supplier exerts the effort to a level as the FB. From Figure 3.7, we can see that $\pi_m^{PS*} > \pi_m^{SO*}$. This result shows the robustness of the PS's coordinating property. Even if the OEM totally outsources the service of part s to the supplier, he is still able to induce the supplier to reduce the failure rate of part s to the desired level, by letting the supplier share the penalty of product downtime.
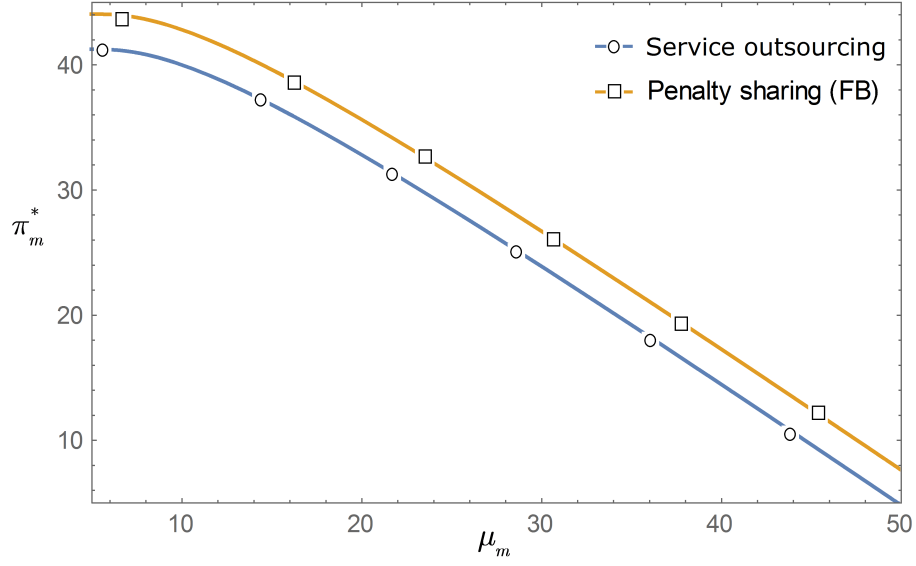
FIGURE 3.7: The OEM's profit at equilibrium, individual capacity, endogenous μ.

3.6.2 Nonseparable Product

In this section, we discuss the case that once part s and part m have been assembled, the product becomes nonseparable. In other words, when the product is down, it cannot be figured out which part causes the product failure. Nevertheless, the RCS and PS contract could also work. Under RCS, the OEM repairs all the failures, however, he shares the total repair cost with the supplier in the proportion of θ, $(0 < \theta < 1)$ and $1 - \theta$, respectively. Similarly, for the PS, they share the penalty with the proportion θ and $1 - \theta$.

Proposition 3.8. *(1) Under PO, $\lambda_s^* = \lambda_{s0}^*$; under RCS, $\lambda_s^* = \lambda_{s0} - \frac{c_r(1-\theta)}{2c_s}$; under PS, $\lambda_s^* = \lambda_{s0} - \frac{r+c_r\mu}{2c_s\mu}$.*

*(2) Under PO and PCS, the supplier **under-invests** in failure rate reduction. If $\theta = 1$, $\lambda_s^{RCS*} = \lambda_s^{PO*}$.*

(3) For the OEM, $\pi_m^{PS*} > \pi_m^{RCS*} > \pi_m^{PO*}$.

(4) PS achieve the First-Best and coordinate the supply chain.

The proof can be found in Appendix H.

If the product becomes nonseparable, the root cause of product failure cannot be figured out. Thus, it becomes unclear about the contribution of part s and part m to the total system

downtime. Under RCS, the OEM and the supplier share the total repair cost. Similarly, they share the total penalty under PS. In this setting, we can show that PO, the same as previous, cannot induce the supplier to reduce failure rate. Under RCS, the supplier's effort level is less than the FB. In addition, we have $\partial \lambda_s^{RCS*} / \partial \theta > 0$. In other words, if the OEM bears more service cost, the supplier will exert less effort. If $\theta = 1$, RCS=W, i.e., if the OEM pays all the repair cost, the supplier's effort is 0. Furthermore, under PS, the supplier's effort level is independent of θ and archives the FB. For the OEM's profit, PS> RCS > PO.

The OEM does not have to worry that if θ becomes higher, he will pay more penalty to the customer under PS. On the contrary, the OEM can change the penalty-sharing proportion by changing κ. Specifically, $\kappa^* = \frac{r + c_r \mu}{1 - \theta}$. If θ increases, the penalty rate increases as well. In this way, the penalty is optimally shared by the OEM and the supplier, and the supplier's effort is independent of θ. As such, the supplier's failure rate reduction can achieve the FB. The conclusion of this setting enhances the robustness of PS. Even if the liability of product downtime cannot be identified, PS can still achieve the best outcome for the OEM.

3.6.3 Asymmetric Information

In this section, we introduce Information Asymmetry (AI) into the model. In practice, the OEM may not have the knowledge about the quality type of the parts manufactured by the supplier. Within the context of this setting, we assume that there are two types of quality for part s, λ_{s0H} and λ_{s0L}, $\lambda_{s0H} > \lambda_{s0L}$. However, the OEM does not know the true type of part s beforehand. He estimates the initial failure rate to be λ_{s0H} with probability ρ, and to be λ_{s0L} with probability $1 - \rho$. Then the OEM offers a menu of contracts for λ_{s0H} and λ_{s0L} respectively, and the supplier selects the contracts based on the quality type of part s. The OEM's problem

under AI can be written as

$$
\max_{\substack{\lambda_{mH}, w_H, \kappa_H, T_H, \\ \lambda_{mL}, w_L, \kappa_L, T_L}} \rho \left(w_H - \kappa_H \mathbb{E}(\sum_{i=1}^{N} S_i \bigg| \lambda_{mH}, \lambda_{sH}^*, \mu) - K_m - c\mu - c_r(\lambda_{sH}^* + \lambda_{mH}) - C_m(\Delta\lambda_{mH}) - T_H \right) +
$$

$$
(1-\rho) \left(w_L - \kappa_L \mathbb{E}(\sum_{i=1}^{N} S_i \bigg| \lambda_{mL}, \lambda_{sL}^*, \mu) - K_m - c\mu - c_r(\lambda_{sL}^* + \lambda_{mL}) - C_m(\Delta\lambda_{mL}) - T_L \right)
$$

$s.t.$

$$
r[1 - \mathbb{E}(\sum_{i=1}^{N} S_i \bigg| \lambda_{sH(L)}^*, \mu)] - w_{H(L)} + \kappa_{H(L)} \mathbb{E}(\sum_{i=1}^{N} S_i \bigg| \lambda_{sH(L)}^*, \mu) \geq 0, \ \text{(IR-C)}
$$

$$
\pi_{sH(L)}(\lambda_{sH(L)}^*) = T_{H(L)} - K_s - C_s(\Delta\lambda_{sH(L)}^*) \geq 0, \ \text{(IR-S)}
$$

$$
\lambda_{sH(L)}^* = \arg\max \pi_{sH(L)}(\lambda_{sH(L)}), \ \text{(IC1)}
$$

$$
\pi_{sH}(\lambda_{sH}^*) > \pi_{sH}(\lambda_{sL}^*), \ \pi_{sL}(\lambda_{sL}^*) > \pi_{sL}(\lambda_{sH}^*). \ \text{(IC2)}
$$

$$(3.5)$$

The objective function in Equation (3.5) is the OEM's expected profit under AI. The OEM has to design two sets of contract terms T_H and T_L for the supplier, and (w_H, κ_H) and (w_L, κ_L) for the customer, in response to the supplier's specification of the quality type of part s. In other words, if the supplier states that the quality type of part s is λ_{s0H}, the OEM will offer a contract with term T_H, and vice versa. Contract term $T_{H(L)}$, depends on the contract type. Specificity, under PO, $T_{H(L)} = p_{1H(L)}$; under RCS, $T_{H(L)} = p_{2H(L)} - c_r(\lambda_{sH(L)}^*)$; under PS, $T_{H(L)} = p_{3H(L)} - \kappa_{H(L)} \mathbb{E}(\sum_{i=1}^{N_s} S_i \bigg| \lambda_{sH(L)}^*, \mu)$. For the constraints, IR-C and IR-S are the Individual Rationality constraints for the customer and the supplier. Both sets of contract terms have to guarantee the participation of the customer and the supplier. IC1 is the one of the Incentive Compatibility constraints for the supplier, which captures that the supplier's effort maximizes his own profit. IC2 is the other Incentive Compatibility constraint in this AI setting. Based on the Revelation Principle, IC2 makes the supplier "tell the truth" regarding the quality type of part s. If the supplier lies, his profit will be lower than that if he tells the truth. For example, assuming the true quality type of part s is λ_{s0H}, the optimal reduced effort is λ_{sH}^*. If the supplier tells the truth, his payoff will be $T_H(\lambda_{sH}^*) - K_s - C_s(\Delta\lambda_{sH}^*)$. Otherwise, if the supplier gives the false information, his profit will be $T_H(\lambda_{sL}^*) - K_s - C_s(\Delta\lambda_{sL}^*)$. The IC2 constraint forces $\pi_{sH}(\lambda_{sH}^*) > \pi_{sH}(\lambda_{sL}^*)$, so that the supplier can only be better off if he reveals the information truthfully.

Again, due to tractability issues, we only discuss the exogenous μ case.

Lemma 3.9.   *(1) Under PO, $\lambda^*_{sH(L)} = \lambda_{s0H(L)}$, and $\pi^*_{sH} = \pi^*_{sL} = 0$.*

(2) *Under RCS, $p^*_{2H} = p^*_{2L} = c_r\lambda_{s0H} - \frac{c_r^2}{4c_s}$, $\lambda^*_{sH(L)} = \lambda_{s0H(L)} - \frac{c_r}{2c_s}$. In addition, $\pi^*_{sH} = 0$, $\pi^*_{sL} = c_r(\lambda_{s0H} - \lambda_{s0L}) > 0$.*

(3) *Under PS, $\lambda^*_{sH} = \frac{\lambda_{s0H} - \lambda_{s0L}(1-\rho)}{\rho} - \frac{r+c_r\mu}{2c_s\mu}$, $\lambda^*_{sL} = \lambda_{s0L} - \frac{r+c_r\mu}{2c_s\mu}$. The penalty rate $\kappa^*_H = (r + c_r\mu) - 2c_s(\lambda_{s0H} - \lambda_{s0L})\mu\frac{(1-\rho)}{\rho}$, $\kappa^*_L = r + c_r\mu$. Furthermore, $\pi^*_{sH} = 0$, $\pi^*_{sL} = (\lambda_{s0H} - \lambda_{s0L})\left(\frac{r+c_r\mu}{\mu} - \frac{2c_s(\lambda_{s0H}-\lambda_{s0L})(1-\rho)}{\rho}\right) > 0$.*

The proof can be found in Appendix I.

Like the Symmetric Information (SI) case, PO cannot make the supplier exert any effort. Under RCS, the supplier's effort level depends on the initial failure rate, repair cost c_r, and effort cost c_s. From Lemma 3.9, we also notice that the repair prices for the two failure rate types are the same. Only the high failure rate λ_{s0H} affects repair pricing. Besides, IR-S constraint for low failure rate is not binding. For both PO and RCS, the OEM's estimation probability ρ does not impact the supplier's effort level. However, under PS, given a high failure rate, $\lambda^*_{sH} = \frac{\lambda_{s0H} - \lambda_{s0L}(1-\rho)}{\rho} - \frac{r+c_r\mu}{2c_s\mu}$. But, if $\lambda_{s0} = \lambda_{s0L}$, i.e., given a low failure rate, the customer's estimating probability does not affect the supplier's decision, and the supplier's profit is greater than 0.

Let the centralized setting under AI be the Second Best (SB) case. Under SB, there is no internal payment, but information asymmetry still exists. The central planner maximizes the expected profit, estimating $\lambda_{s0} = \lambda_{s0H}$ with probability θ, and $\lambda_{s0} = \lambda_{s0L}$ with probability $1-\theta$. SB can be the comparing benchmark for the decentralized setting under AI.

Proposition 3.10.   *(1) Under PO and RCS, $\lambda^*_{sH(L)} > \lambda^{FB*}_{sH(L)}$, i.e., the supplier **under-invests** in the effort of failure rate reduction.*

(2) *Under PS, $\lambda^*_{sH} > \lambda^{FB*}_{sH}$, $\lambda^*_{sL} = \lambda^{FB*}_{sL}$, i.e., the supplier **under-invests** in the effort of failure rate reduction under a high failure rate but achieves SB solution under a low failure rate.*

(3) *PS cannot coordinate the supply chain. For the OEM's profit, $\Pi^{FB*} > \pi^{PS*}_m > \pi^{RCS*}_m > \pi^{PO*}_m$.*

(4) *Under a low failure rate, the supplier can receive extra profit under RCS and PS.*

The proof can be found in Appendix J.

As in the SI case, the supplier still invests less effort under PO and RCS. Furthermore, the level of the under-investment does not become worse duo to hidden information. In other words, ρ does not affect the supplier's effort under PO and RCS. However, information asymmetry decreases the efficiency of PS. Under SI, PS can achieve the FB solution, whereas under AI, PS can only achieve SB with regards to the supplier's effort when the failure rate of part s is low. If part s has a high failure rate, the supplier under-invests in the effort level. Moreover, we have $\partial \Delta \lambda_{sH}^{*} / \partial \rho = \frac{\lambda_{s0H} - \lambda_{s0L}}{\rho^2} > 0$, i.e., the supplier's effort level under a high failure rate is increasing in ρ. If the OEM believes that it is more likely that the failure rate of part s is high, the supplier will exert more effort to reduce failure rate. As shown in Figure 3.8, given a high failure rate, the supplier's effort under PO and RCS is lower than FB, and are independent of ρ. For PS, the supplier's effort level increases in ρ. Particularly, PS converges with FB if $\rho = 1$. The increasing effort implies that the supplier's decision comes closer to the SB if he has more "correct" estimation. Specificity, if the true failure rate is high, and the OEM believes it is high, i.e., $\rho = 1$, the supplier can make the optimal decision.
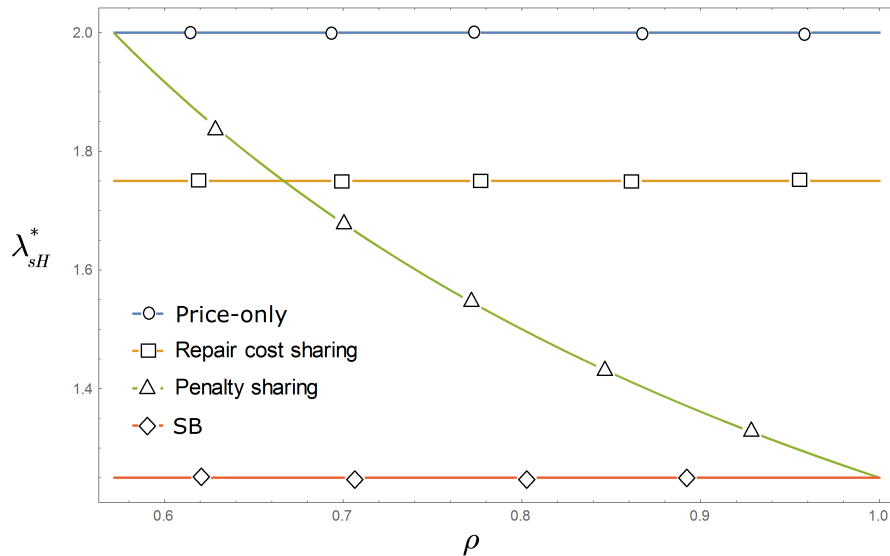
FIGURE 3.8: Optimal failure rate of part s under AI, $\lambda_{s0} = \lambda_{s0H}$.

Although the supplier's effort can achieve SB when failure rate is low, PS cannot coordinate the supply chain under AI. If we compare the OEM's profit under three contracts, we have PS > RCS > PO. Thus, efficiency loss under PS is the lowest. This result resembles the SI case. As shown in Figure 3.9, the OEM's profit under contracts are lower than the SB case. On the other hand, PS has the highest profit compared to RCS and PO. Another result can
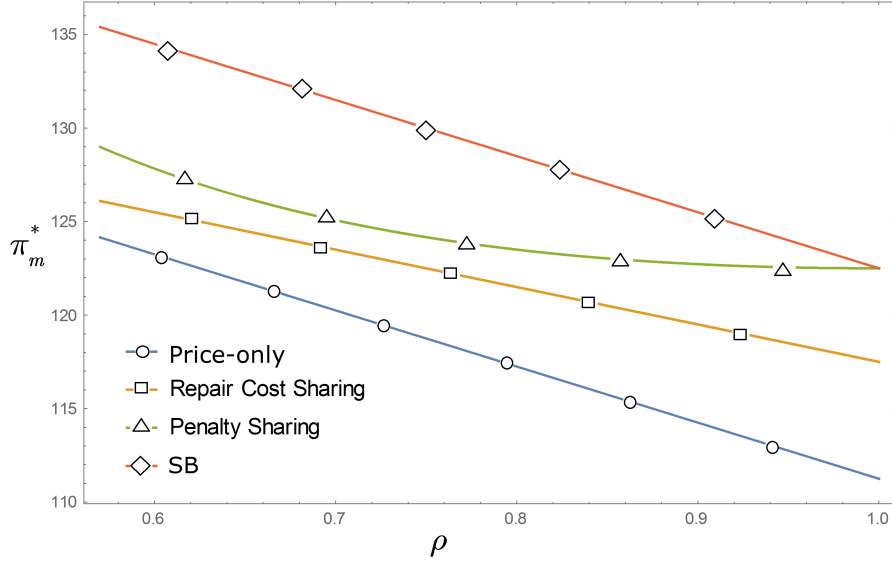
FIGURE 3.9: The OEM's expected profit under AI.

be seen is that the OEM's profits are decreasing in ρ. Analytically, we have $\partial \Pi^{SB*}/\partial \rho = -\frac{(\lambda_{s0H}-\lambda_{s0L})(r+c_r\mu)}{\mu} < 0$, $\partial \pi_m^{PS*}/\partial \rho = -\frac{c_s(\lambda_{s0H}-\lambda_{s0L})^2(1-\rho^2)}{\rho^2} < 0$, $\partial \pi_m^{RCS*}/\partial \rho = -\frac{r(\lambda_{s0H}-\lambda_{s0L})}{\mu} < 0$, and $\partial \pi_m^{PO*}/\partial \rho = -\frac{(\lambda_{s0H}-\lambda_{s0L})(r+c_r\mu)}{\mu} < 0$. In addition, PS converges with SB at $\rho = 1$. Finally, the supplier can have a surplus under RCS and PS if the failure rate of part s is low. This is the "*information rent*" caused by information asymmetry. Without this extra payment, the supplier will over-state the quality of part s, i.e., the supplier will state part s has a low failure rate even if the actual failure rate is high. In addition, we have $\partial \pi_{sL}^{RCS*}/\partial \rho = 0$, $\partial \pi_{sL}^{PS*}/\partial \rho = \frac{2c_s(\lambda_{s0H}-\lambda_{s0L})^2}{\rho^2} > 0$. In other words, the supplier's extra profit is independent of ρ under RCS, and is increasing in ρ under PS.

3.6.4 Multiple Products

In previous sections, the model is established based on the setting where there is a single product in the supply chain. In this section, we relax this assumption and allow for a multiple products case. Assuming the customer buys M (identical) products from the OEM. Each product consists one part s and one part m, with failure rate λ_s and λ_m respectively. Therefore, there are M part s and M part m in the supply chain. The failure process of part s and part m are independent. Whenever a failure occurs (it can be part s and part m), the product enters the OEM's repair facility. The service capacity is μ. We assume the OEM has ample servers and the failure-repair process can be seen as a $M/M/\infty$ queue (similar assumption as in Kim et al., 2007). Then,

each failure cause a $1/\mu$ downtime. For M part s and part m, the total expected number of failures is $M(\lambda_s + \lambda_m)$, and the expected total system downtime is $\frac{M(\lambda_s+\lambda_m)}{\mu}$. The decision structure and business sequence are the same as the previous case. Solving the problem with multiple products, we have the results summarized in Proposition 3.11.

Proposition 3.11. *(1) Under PO, $\lambda_s^{PO*} = \lambda_{s0}$; under RCS, $\lambda_s^{RCS*} = \lambda_{s0} - \frac{Mc_r}{2cs}$; under PS,* $\lambda_s^{PS*} = \lambda_{s0} - \frac{M(r+c_r\mu)}{2c_s\mu}$.

*(2) $\lambda_s^{FB*} = \lambda_s^{PS*} < \lambda_s^{RCS*} < \lambda_s^{PO*}$, i.e., the supplier **under-invests** in the effort of failure rate reduction under PO and RCS, whereas PS achieve the FB.*

The proof can be found in Appendix K.

From Proposition 3.11, we know that if even considering multiple products, which captures the fact that quality improvement effort is usually exerted on the whole production system or product design so that all the products can be affected, PS still achieve the First Best result in aligning the supplier's incentives.

3.7   Conclusion

Manufacturing and servicing durable products involve collaboration of supply chain partners. The OEM usually faces challenges of incentivizing the suppliers to improve part reliability with the objective to improve product availability. In this chapter, we focus on a three stage supply chain setting, where an outcome-based payment is adopted between the OEM and the customer. The performance of the final product depends on both the OEM and the supplier's effort, yet the OEM cannot directly dictate the supplier's quality improvement efforts. We introduce three types of contracts - price-only contracts, repair cost sharing contracts, and penalty sharing contracts between the OEM and the supplier, and investigate how the OEM designs the optimal contract terms to induce the supplier to exert effort on part failure reduction and to maximize the profit.

First, we develop a moral hazard model to capture the setting when the OEM has the capacity to repair both of the parts, and the repair capacity level is exogenous. We show that price-only contracts cannot induce the supplier to exert any quality improvement effort. Repair cost sharing contracts cause under-investment in the supplier's effort. Penalty sharing contracts

overcome the under-investment problem and achieve the First Best solution for the supply chain. Only allocating the service cost to the supplier based on the failures of his part does not give sufficient incentive to the supplier to invest in the effort to an optimal level. However, asking the supplier to share the OEM's penalty due to the downtime of the final product can make the supplier improve the part quality to the desired level. Next, we endogenize the service capacity as one of the OEM's decision variables. We show that price-only contracts and repair cost sharing contracts lead to over-investment in service capacity for the OEM, and under-investment in the effort of failure reduction for the supplier. Penalty sharing can achieve the First Best solution and generate the highest profit for the OEM and the supply chain. The main managerial insights derived from our model give suggestions for the performance-based contractors on how to collaborate and share the responsibility with their business partners. Linking the performance measure of the final product to part suppliers can result in a most efficient supply chain.

Subsequently, we examine the robustness of the model by introducing more settings. First, we introduce the individual capacity setting, where the supplier and the OEM take care of the service of their own parts. Under such circumstances, paying the supplier a service fee for each repair cannot induce the supplier to invest in the part quality improvement. On the other hand, penalty sharing can realize the First Best solution by setting an appropriate penalty rate. Then, we discuss the case when the final product is nonseparable once it has been assembled. In this setting, the liability of product failure cannot be allocated to the certain part. For repair cost sharing and penalty sharing contracts, the supplier and the OEM share the total repair cost/penalty by a predefined proportion. Our model shows that repair cost sharing contracts still result in under-investment for the supplier's effort level, and penalty sharing contracts coordinate the supply chain. The extensions of the above two cases expand the application area of the penalty sharing contracts. Even if the OEM has no capacity to repair the supplier's part, or the failure root cause analysis is not applicable to the final product, sharing penalties can realize the best outcome for the OEM and the supply chain.

Finally, we put our model into an asymmetric information setting. The supply has private information on his part, and the customer offers a menu of contracts to screen the part quality. We develop a screening game model with moral hazard and analyze how information asymmetry affects the supply chain partners' decision. We find that penalty sharing contracts outperform other contracts, however, all contracts have efficiency loss and due to information asymmetry.

In this chapter, we compare price-only contracts, repair cost sharing contracts and penalty sharing contracts in a decentralized supply chain setting. We show that only penalty sharing contracts can lead to the First Best results, while the other two have efficiency loss. However, we do observe that in practice penalty sharing is not the single dominate contract; on the contrary, price-only contracts and cost sharing contracts are widely used by companies. This may due to the easiness of the implementation. First, the implementation of the price-only contract is the easiest. Both cost sharing and penalty sharing require deep involvement of the supplier. The supplier has to verify the cause of the failure of the final product, which adds lots of administrative burden. Second, compared to penalty sharing, cost sharing contracts have less hassle. The supplier only has to confirm the failure of his part. However, under penalty sharing, the penalty is based on product downtime. Even if the product breakdown is duo to the supplier's part, how quickly the failure can be repaired may depend on the OEM. As such, the supplier has to verify the OEM's repair activity. We can see that the cost of contract implementation may affect the choice of contract, which is assumed away in our model. However, as the fast development of information technology, the supply chain partners' activity will become more and more transparent and easy to monitor. We believe the benefit of the penalty sharing contracts will manifest in the long run.

In the model, we have made some assumptions in order to keep its tractability and facilitate contracts comparison. We can extend the model by relaxing some of the assumptions in future research. For example, we have assumed that all supply chain members are risk neutral agents. But in practice, companies with different size, market power, etc. may have various risk appetite. Risk aversion can be introduced into the model to study how the risk appetite affects the supply chain members' decision. Another limitation of the model is that we only consider a single contracting period. However, the supply chain members may have multiple periods contracts and contract terms can be renegotiated at the beginning of each period. We can add dynamics to our model to study the repeated interactions among the supply chain members. Last but not least, we consider a supply chain formed by a single customer, a single manufacturer (OEM), and a single supplier. We can extend the model by adding more parties in each stage to investigate the supply chain members behavior under competition.

3.8 Appendix

A. Proof of Lemma 3.1. The supply chain's expected profit is

$$\Pi = r(1 - \frac{\lambda_s + \lambda_m}{\mu}) - (K_s + K_m) - c\mu - c_r(\lambda_s + \lambda_m) - c_s(\lambda_{s0} - \lambda_s)^2 - c_m(\lambda_{m0} - \lambda_m)^2.$$

Taking the second derivative with respect to (w.r.t.) λ_m, λ_s, and μ, we have

$$\frac{\partial\Pi^2}{\partial\lambda_m{}^2} = -2c_m < 0,$$

$$\frac{\partial\Pi^2}{\partial\lambda_s{}^2} = -2c_s < 0,$$

$$\frac{\partial\Pi^2}{\partial\mu^2} = -\frac{2r(\lambda_m + \lambda_s)}{\mu^3} < 0.$$

So the supply chain's profit is concave (not jointly concave) in λ_m, λ_s, and μ. Note that joint concavity for all λ_m, λ_s, and μ is not necessarily needed to obtain the optimal solution.

1) If μ is exogenous, the Hessian of Π is

$$H_{FB} = \begin{pmatrix} -2c_m & 0 \\ 0 & -2c_s \end{pmatrix}$$

which is negative definite. So Π is jointly concave in λ_m and λ_s. Thus, solving the First Order Condition (FOC) with w.r.t. λ_m and λ_s, we can obtain the optimal solution that maximizes Π.

2) If μ is endogenous, the Hessian of Π w.r.t. λ_m, λ_s, and μ is

$$H_{FB} = \begin{pmatrix} -2c_m & 0 & \frac{r}{\mu} \\ 0 & -2c_s & \frac{r}{\mu} \\ \frac{r}{\mu} & \frac{r}{\mu} & -\frac{2r(\lambda_m + \lambda_s)}{\mu^3} \end{pmatrix}$$

Solving the FOC w.r.t. λ_m, λ_s, and μ, we have λ_m^{FB*}, λ_s^{FB*}, and μ^{FB*}. As long as the Hessian given λ_m^{FB*}, λ_s^{FB*}, and μ^{FB*} is negative definite, λ_m^{FB*}, λ_s^{FB*}, and μ^{FB*} are the optimal solution

maximizing Π. Placing μ with λ_m^{FB*} in the above H_{FB}, we have

$$H_{FB} = \begin{pmatrix} -2c_m & 0 & \frac{r}{\mu^{FB*2}} \\ 0 & -2c_s & \frac{r}{\mu^{FB*2}} \\ \frac{r}{\mu^{FB*2}} & \frac{r}{\mu^{FB*2}} & \frac{r(c_s(r+c_r\mu^{FB*})+c_m(r+(c_r-2c_s(\lambda_{m0}+\lambda_{s0}))\mu^{FB*}))}{c_m c_s \mu^{FB*4}} \end{pmatrix}$$

H_{FB} is negative definite iff the determinant of H_{FB} is negative. We have

$$\det[H_{FB}] = \frac{6(c_m + c_s)r^2 + 4r(c_r c_s + c_m(c_r - 2c_s(\lambda_{m0} + \lambda_{s0})))\mu^{FB*}}{\mu^{FB*4}}.$$

Hence, if $6(c_m + c_s)r^2 + 4r(c_r c_s + c_m(c_r - 2c_s(\lambda_{m0} + \lambda_{s0})))\mu^{FB*} < 0$, H_{FB} is negative definite, and μ_{FB}^* is the maximum point. $\square$

B. Proof of Lemma 3.2. We solve the OEM's problem under PS. For RCS and W, we can use similar methods to find out the equilibrium solution. Based on Equation (3.2), we have the OEM's problem under PS as

$$\max_{\lambda_m, w, \kappa} \ w - \kappa\frac{\lambda_m}{\mu} - K_m - c\mu - c_r(\lambda_m + \lambda_s) - c_m(\lambda_{m0} - \lambda_m)^2 - p_3$$

$$s.t.$$

$$r(1 - \frac{\lambda_s + \lambda_m}{\mu}) - w + \kappa\frac{\lambda_s + \lambda_m}{\mu} \geq 0, \ \text{(IR-C)}$$

$$p_3 - K_s - c_s(\lambda_{s0} - \lambda_s)^2 - \kappa\frac{\lambda_s}{\mu} \geq 0, \ \text{(IR-S)}$$

$$\lambda_s = \arg\max\{p_3 - K_s - c_s(\lambda_{s0} - \lambda_s)^2 - \kappa\frac{\lambda_s}{\mu}\}. \ \text{(IC-S)}$$

We solve the game by backward induction. Since the OEM is the leader, we first solve the supplier's problem, and then put the supplier's best response into the OEM's profit function and constraints. The supplier's problem is

$$\max_{\lambda_s} \ p_3 - K_s - c_s(\lambda_{s0} - \lambda_s)^2 - \kappa\frac{\lambda_s}{\mu}.$$

The supplier's profit function is concave in λ_s and we have the best response of λ_s^*. Substituting λ_s^* into the OEM's problem, we have

$$\max_{\lambda_m, w, \kappa} \frac{c_r(\kappa - 2c_s(\lambda_m + \lambda_{s0})\mu) - 2c_s(\kappa\lambda_m + \mu(K_m + p_3 - w + c_m(\lambda_m - \lambda_{m0})^2 + c\mu))}{2c_s\mu}$$

s.t.

$$r - w + \frac{\kappa(-\kappa + r)}{2c_s\mu^2} + \frac{(\kappa - r)(\lambda_m + \lambda_{s0})}{\mu} \geq 0, \text{ (IR-C)}$$

$$p_3 + \frac{\kappa(\kappa - 4c_s\lambda_{s0}\mu)}{4c_s\mu^2} - K_s \geq 0. \text{ (IR-S)}$$

Then we solve the problem by using Karush-Kuhn-Tucker (K-K-T) condition. The Lagrangian of the problem is

$$L(\lambda_m, w, \kappa, l_1, l_2) = \frac{c_r(\kappa - 2c_s(\lambda_m + \lambda_{s0})\mu) - 2c_s(\kappa\lambda_m + \mu(K_m + p_3 - w + c_m(\lambda_m - \lambda_{m0})^2 + c\mu))}{2c_s\mu}$$

$$+ l_1(r - w + \frac{\kappa(-\kappa + r)}{2c_s\mu^2} + \frac{(\kappa - r)(\lambda_m + \lambda_{s0})}{\mu}) + l_2(p_3 + \frac{\kappa(\kappa - 4c_s\lambda_{s0}\mu)}{4c_s\mu^2} - K_s)$$

where l_1, l_2 are the Lagrange multipliers. We have the K-K-T condition as

$$\begin{cases} \dfrac{\partial L(w, \kappa, \lambda_m, l_1, l_2)}{\partial w} = 1 - l_1 = 0, \\[2mm] \dfrac{\partial L(w, \kappa, \lambda_m, l_1, l_2)}{\partial \kappa} = \dfrac{\kappa(-2l_1 + l_2) + (c_r - 2c_s(\lambda_m + l_2\lambda_{s0}))\mu + l_1(r + 2c_s(\lambda_m + \lambda_{s0})\mu)}{2c_s\mu^2} = 0, \\[2mm] \dfrac{\partial L(w, \kappa, \lambda_m, l_1, l_2)}{\partial p_3} = -1 + l_2 = 0, \\[2mm] \dfrac{\partial L(w, \kappa, \lambda_m, l_1, l_2)}{\partial \lambda_m} = -\dfrac{\kappa - \kappa l_1 + l_1 r + (c_r + 2c_m\lambda_m - 2c_m\lambda_{m0})\mu}{\mu} = 0, \\[2mm] l_1(r - w + \dfrac{\kappa(-\kappa + r)}{2c_s\mu^2} + \dfrac{(\kappa - r)(\lambda_m + \lambda_{s0})}{\mu}) = 0, \\[2mm] l_2(p_3 + \dfrac{\kappa(\kappa - 4c_s\lambda_{s0}\mu)}{4c_s\mu^2} - K_s) = 0, \\[2mm] l_1 \geq 0, \ l_2 \geq 0. \end{cases}$$

Solving the above systems of equations, we have $w^*, \kappa^*, p^*, \lambda_s^*, \lambda_m^*$ as shown in Table 3.2, and $l_1 = 1 > 0, l_2 = 1 > 0$, which satisfies the constraint specified in the last inequality of the K-K-T condition. Substituting the solution into the equations of the IR-C and IR-S constraints, we have the left-hand-side equations are 0, i.e., IR-C and IR-S are binding. $\quad\square$

C. Proof of Proposition 3.3. (1) From Lemma 3.2, we have $\lambda_m^* = \lambda_{m0} - \frac{r+c_r\mu}{2c_m\mu}$ under all contracts. It is straightforward to see that λ_s^* is independent of c_s. (2) From Table 3.2, we have under PO, $\lambda_s^* = \lambda_{s0}$; under RCS, $\lambda_s^* = \lambda_{s0} - \frac{c_r}{2c_s}$; under PS, $\lambda_s^* = \lambda_{s0} - \frac{r+c_r\mu}{2c_s\mu}$. Therefore, the supplier's effort is 0 under PO and greater than 0 under RCS and PS. (3) Under RCS, we have $\partial\lambda_s^*/\partial c_s = \frac{c_r}{2c_s^2} > 0$, $\partial\lambda_s^*/\partial c_r = -\frac{1}{2c_s} < 0$. So the effort is is decreasing in c_r and increasing in c_s. (4) Under PS, we have $\partial\lambda_s^*/\partial r = -\frac{1}{2c_s\mu} < 0$, $\partial\lambda_s^*/\partial c_r = -\frac{1}{2c_s} < 0$, $\partial\lambda_s^*/\partial c_s = \frac{r+c_r\mu}{2c_s^2\mu} > 0$, and $\partial\lambda_s^*/\partial\mu = \frac{r}{2c_s\mu^2} > 0$. $\square$

D. Proof of Proposition 3.4. (1) The optimal effort level for part s of the FB is $\lambda_s^{FB*} = \lambda_{s0} - \frac{r+c_r\mu}{2c_s\mu}$. Comparing to PO, RCS, and PS, we have

$$\lambda_s^{FB*} - \lambda_s^{PO*} = -\frac{r+c_r\mu}{2c_s\mu} < 0,$$

$$\lambda_s^{FB*} - \lambda_s^{RCS*} = -\frac{r}{2c_s\mu} < 0,$$

$$\lambda_s^{FB*} - \lambda_s^{PS*} = 0.$$

So we can conclude that PO and RCS lead to under-investment in effort and PS achieve FB. (2) For the OEM's profit, we have

$$\pi_m^{PS*} - \lambda_s^{PO*} = \frac{(r+c_r\mu)^2}{4c_s\mu^2} > 0,$$

$$\pi_m^{PS*} - \lambda_s^{RCS*} = \frac{r^2}{4c_s\mu^2} > 0,$$

and $\frac{(r+c_r\mu)^2}{4c_s\mu^2} > \frac{r^2}{4c_s\mu^2}$, So we have $\pi_m^{PS*} > \lambda_s^{RCS*} > \lambda_s^{PO*}$. (3) Differentiating π_m^* w.r.t. μ, we have

$$\frac{\partial^2\pi_m^{PO*}}{\mu^2} = \frac{r(3r + 2(c_r - 2c_m(\lambda_{m0} + \lambda_{s0}))\mu)}{2c_m\mu^4},$$

$$\frac{\partial^2\pi_m^{RCS*}}{\mu^2} = \frac{r(2c_mc_r\mu + c_s(3r + 2(c_r - 2c_m(\lambda_{m0} + \lambda_{s0}))\mu))}{2c_mc_s\mu^4},$$

$$\frac{\partial^2\pi_m^{PS*}}{\mu^2} = \frac{r(3(c_m + c_s)r + 2(c_rc_s + c_m(c_r - 2c_s(\lambda_{m0} + \lambda_{s0})))\mu)}{2c_mc_s\mu^4}.$$

If $\lambda_{m0} < 2\lambda_{s0}$, we have $\frac{\partial^2\pi_m^{PO*}}{\mu^2} < 0$, $\frac{\partial^2\pi_m^{RCS*}}{\mu^2} < 0$, $\frac{\partial^2\pi_m^{PS*}}{\mu^2} < 0$, and thus π_m^* is concave in μ. $\square$

E. Proof of Lemma 3.5. Under PO, the supplier's problem is $\max_{\lambda_s} p_1 - K_s - c_s(\lambda_{s0} - \lambda_s)^2$. So μ does not affect the supplier's decision. Similarly, the supplier's problem under RCS is $\max_{\lambda_s} p_2 - K_s - c_s(\lambda_{s0} - \lambda_s)^2 - c_r\lambda_s$. The supplier's problem remains the same as the exogenous

μ case. Under PS, the supplier's problem becomes $\max_{\lambda_s} p_3 - K_s - c_s(\lambda_{s0} - \lambda_s)^2 - \kappa\frac{\lambda_s}{\mu}$. Here we can see that the OEM's decision on μ plays a role in the supplier's decision. Solving the optimization problem, we have the supplier's best response as $\lambda_s^* = \lambda_{s0} - \frac{\kappa}{2c_s\mu}$. Substituting the supplier's best response to the OEM's objective functions and constraints and solve the FOCs, we can obtain the equilibrium solutions given in Lemma 3.5.

F. Proof of Proposition 3.6. Solving the FOC of FB, we have λ_s^* and μ^* satisfies that $\lambda_s^* = \lambda_{s0} - \frac{r+c_r\mu^*}{2c_s\mu^*}$ and $r^2(c_m + c_s) + (c_rc_sr + c_mr(c_r - 2c_s(\lambda_{m0} + \lambda_{s0})))\mu^* + 2cc_mc_s\mu^{*3} = 0$. In addition, for all the cases under three contracts, the best response of the OEM's decision on μ^* w.r.t. the supplier's effort λ_s^* satisfies

$$\mu^* = \frac{\sqrt{r}\sqrt{\lambda_m^* + \lambda_s^*}}{\sqrt{c}}.$$

1) Solving the problem under PS, we have $\kappa^* = r + c_r\mu^*$, and λ_s^*, μ^* the same as the solution of FB. 2) Under PO, $\lambda_s^* = \lambda_{s0}$, so $\lambda_s^{PO*} > \lambda_s^{FB*}$, and $\mu^{PO*} > \mu^{FB*}$. 3) Similarly, under RCS, we have $\lambda_s^{FB*} - \lambda_s^{RCS*} = -\frac{r}{2c_s\mu^*} < 0$. Thus, $\mu^{RCS*} > \mu^{FB*}$. Hence, under PO and RCS, the supplier under-invests in the effort, and the OEM over-invests in capacity. □

G. Proof of Proposition 3.7. A hidden constraint is that $0 < \lambda_s^* \leq \lambda_{s0}$, i.e., after being exerted the effort, the failure rate can only be less or equal to the initial value. Solving the K-K-T condition of Equation (3.3), we have this constraint is binding, i.e., $\lambda_s^* = \lambda_{s0}$. Furthermore, we can obtain $w^* = r$, $\kappa^* = r$, and $\lambda_s^* = \lambda_{s0} - \frac{r+c_r\mu_s}{2c_s\mu_s}$. Then we have

$$\pi_s^{PS*} - \pi_s^{SO*} = \frac{(r + c_r\mu_s)^2}{4c_s\mu_s^2} > 0,$$

i.e., $\pi_s^{PS*} > \pi_s^{SO*}$. □

H. Proof of Proposition 3.8. If the product is nonseparable, the problem under PO is the same as before. Under RCS, the supplier's problem becomes

$$\max_{\lambda_s} p - K_s - c_s(\lambda_{s0} - \lambda_s)^2 - (1 - \theta)c_r(\lambda_s + \lambda_m).$$

Then the best response of the supplier is $\lambda_s^* = \lambda_{s0} - \frac{c_r(1-\theta)}{2c_s}$. So if $\theta = 1$, $\lambda_s^* = \lambda_{s0}$. Moreover, we have

$$\lambda_s^{FB*} - \lambda_s^{RCS*} = -\frac{r + c_r\theta\mu}{2c_s\mu} < 0,$$

i.e., the supplier's effort is less than the FB. Similarly, we can solve the problem under PS. We have the equilibrium solution $\kappa = \frac{r+c_r\mu}{1-\theta}$, $\lambda_s^* = \lambda_{s0} - \frac{r+c_r\mu}{2c_s\mu}$. Furthermore, we have

$$\pi_m^{PS*} - \pi_m^{RCS*} = \frac{(r + c_r\theta\mu)^2}{4c_s\mu^2},$$

$$\pi_m^{PS*} - \pi_m^{PO*} = \frac{(r + c_r\mu)^2}{4c_s\mu^2}.$$

Because $0 < \theta < 1$, $\frac{(r+c_r\theta\mu)^2}{4c_s\mu^2} < \frac{(r+c_r\mu)^2}{4c_s\mu^2}$. Thus, $\pi_m^{PS*} > \pi_m^{RCS*} > \pi_m^{PO*}$. $\square$

I. Proof of Lemma 3.9. We only show the proof of the results under PS. For PO and RCS, the proof is simpler and can be done in similarly methods. First, we solve the supplier's problem, IC1 in Equation (3.5).

$$\max_{\lambda_{sH}}\ p_H - \kappa_H\frac{\lambda_{sH}}{\mu_s} - c_r\lambda_{sH} - c_s(\lambda_{s0H} - \lambda_{sH})^2,$$

$$\max_{\kappa_L}\ p_L - \kappa_L\frac{\lambda_{sL}}{\mu_s} - c_r\lambda_{sL} - c_s(\lambda_{s0L} - \lambda_{sL})^2.$$

We have

$$\lambda_{sH}^* = \lambda_{s0H} - \frac{\kappa_H}{2c_s\mu}, \quad \lambda_{sL}^* = \lambda_{s0L} - \frac{\kappa_L}{2c_s\mu}.$$

Then, we put λ_{sH}^*, λ_{sL}^* into the objective function and constraints IR-S, IR-C. Next, we expand constraint IC2. For $\lambda_{s0} = \lambda_{s0H}$, we have the supplier's profit of telling the truth (left-hand-side term) is

$$p_H - cs(\lambda_{s0H} - \lambda_{sH})^2 - \frac{\kappa_H}{\mu}(\lambda_{sH})\Bigg|_{\lambda_{sH}=\arg\max p_H - cs(\lambda_{s0H}-\lambda_{sH})^2 - \frac{\kappa_H}{\mu}(\lambda_{sH})} = p_H + \frac{\kappa_H(\kappa_H - 4c_s\lambda_{s0H}\mu)}{4c_s\mu^2}.$$

The supplier's profit of lying (right-hand-side term) is

$$p_L - c_s(\lambda_{s0H} - \lambda_{sH})^2 - \frac{\kappa_L}{\mu}(\lambda_{sH})\Bigg|_{\lambda_{sH}=\arg\max p_L - c_s(\lambda_{s0H}-\lambda_{sH})^2 - \frac{\kappa_L}{\mu}(\lambda_{sH})} = p_L + \frac{\kappa_L(\kappa_L - 4c_s\lambda_{s0H}\mu)}{4c_s\mu^2}.$$

Then, for $\lambda_{s0} = \lambda_{s0H}$, IC2 can be reduced to

$$p_H - p_L + \frac{(\kappa_H - \kappa_L)(\kappa_H + \kappa_L)}{4c_s\mu^2} - \frac{(\kappa_H - \kappa_L)\lambda_{s0H}}{\mu} > 0.$$

Similarly, we can reduce IC2 for $\lambda_{s0} = \lambda_{s0L}$ as

$$p_L - p_H - \frac{(\kappa_H - \kappa_L)(\kappa_H + \kappa_L - 4c_s\lambda_{s0L}\mu)}{4c_s\mu^2} > 0.$$

Then we solve the K-K-T condition and obtain the equilibrium

$$\kappa_H^* = \frac{2c_s(\lambda_{s0H} - \lambda_{s0L})\mu(-1 + \rho) + (r + c_r\mu)\rho}{\rho}, \quad \kappa_L^* = r + c_r\mu,$$

$$\lambda_{sH}^* = \frac{2c_s\mu(\lambda_{s0H} + \lambda_{s0L}(-1 + \rho)) - (r + c_r\mu)\rho}{2c_s\mu\rho}, \quad \lambda_{sL}^* = \lambda_{s0L} - \frac{r + c_r\mu}{2c_s\mu}.$$

Moreover, $\pi_{sH}^* = 0$, $\pi_{sL}^* = (\lambda_{s0H} - \lambda_{s0L})\frac{\kappa_H^*}{\mu} > 0$. $\square$

J. Proof of Proposition 3.10. (1) Because

$$\lambda_{sH(L)}^{PO*} - \lambda_{sH(L)}^{SB*} = \frac{r + c_r\mu}{2c_s\mu} > 0,$$

$$\lambda_{sH(L)}^{RCS*} - \lambda_{sH(L)}^{SB*} = \frac{r}{2c_s\mu} > 0,$$

$$\lambda_{sH}^{PS*} - \lambda_{sH}^{SB*} = \frac{(\lambda_{s0H} - \lambda_{s0L})(1 - \rho)}{\rho} > 0,$$

$$\lambda_{sH}^{PS*} - \lambda_{sH}^{SB*} = 0,$$

we can conclude that under PO and RCS, the supplier under-invests in the effort. (2) Under PS, if $\lambda_{s0} = \lambda_{s0H}$, the supplier under-invests in effort; if $\lambda_{s0} = \lambda_{s0L}$, the supplier's effort achieves SB. (3) Comparing the OEM's effort at equilibrium, we have

$$\Pi^{SB} - \pi_m^{PS*} = \frac{(\lambda_{s0H} - \lambda_{s0L})(1 - \rho)(cs(\lambda_{s0H} - \lambda_{s0L})\mu(1 - \rho) + (r + c_r\mu)\rho)}{\mu\rho} > 0,$$

$$\pi_m^{PS*} - \pi_m^{RCS*} = \frac{r^2\rho + 4c_s(\lambda_{s0H} - \lambda_{s0L})\mu(1 - \rho)(cs(\lambda_{s0H} - \lambda_{s0L})\mu(1 - \rho) + r\rho)}{4c_s\mu^2\rho} > 0,$$

$$\pi_m^{RCS*} - \pi_m^{PO*} = \frac{c_r(2r + \mu(c_r + 4c_s(\lambda_{s0H} - \lambda_{s0L})(1 - \rho)))}{4c_s\mu} > 0.$$

So $\Pi^{SB} > \pi_m^{PS*} > \pi_m^{RCS*} > \pi_m^{PO*}$. (4) Under RCS, $\pi_{sL}^* = c_r(\lambda_{s0H} - \lambda_{s0L}) > 0$. Under PS, $\pi_{sL}^* = (\lambda_{s0H} - \lambda_{s0L})\frac{\kappa_H^*}{\mu} > 0$. $\square$

K. Proof of Proposition 3.11. If there are M products in the supply chain, the problem for the intergraded firm (FB) becomes

$$\max_{\lambda_s,\lambda_m} \Pi = Mr(1 - \frac{\lambda_s + \lambda_m}{\mu}) - M(K_s + K_m) - c\mu - Mc_r(\lambda_s + \lambda_m) - c_s(\lambda_{s0} - \lambda_s)^2 - c_m(\lambda_{m0} - \lambda_m)^2.$$

The profit function is concave in λ_s, and solving the FOC, we have $\lambda_s^{FB*} = \lambda_{s0} - \frac{M(r + c_r\mu)}{2c_s\mu}$. The problem of the decentralized supply chain becomes

$$\max_{w,k,\lambda_s,\lambda_m} Mw - \kappa[1 - \frac{M(\lambda_s + \lambda_m)}{\mu}] - MK_m - c\mu - Mc_r(\lambda_s + \lambda_m) - c_m(\lambda_{m0} - \lambda_m)^2 - T$$

$s.t.$

$$Mr(1 - \frac{\lambda_s + \lambda_m}{\mu}) - Mw + \kappa[1 - \frac{M(\lambda_s + \lambda_m)}{\mu}] \geq 0, \text{ IRC}$$

$$\pi_s(\lambda_s^*) = T - MK_s - c_s(\lambda_{s0} - \lambda_s^*)^2 \geq 0, \text{ IRS}$$

$$\lambda_s^* = \arg\max \pi_s(\lambda_s). \text{ ICS}$$

where

$$T = \begin{cases} Mp_1, & \text{PO} \\ Mp_2 - Mc_r\lambda_s, & \text{RCS} \\ Mp_3 - \kappa\frac{M\lambda_s}{\mu}. & \text{PS} \end{cases}$$

Solving the above optimization problem, we can obtain the solutions as given in Proposition 3.11. $\square$

Chapter 4

Contract Choice for Product Support

4.1 Introduction

In recent decades, service outsourcing has been world-widely witnessed in manufacturing industry (Kedia and Lahiri, 2007). By outsourcing service to suppliers, companies can focus on their core competence and develop new value to customers. As a consequent result, there has been ongoing rapid growth in service sectors (Fixler and Siegel, 1999). Many traditional manufacturing companies have shifted their business to service-oriented domains. For example, IBM reduces its manufacturing business by selling department of personal computers to Lenovo[1], and expand the global service unit to the world's largest business and technology services provider. Apart from the IT service, repair and maintenance outsourcing are also commonly seen in manufacturing industries (Jain et al., 2013). For example, Dell, Apple, Fujitsu, and Honeywell, etc. outsource their product repair center to DBK[2]. In such setting, DBK has to establish the capacity (repair technicians, error testing equipment, spare parts, etc.) for failure restoration.

Although outsourcing service enables the product users to develop their own advantageous business, the customer may lose direct control of the service process, which creates misaligned objectives for the customer and the supplier. The customer usually wants the product availability to be maintained at a high level, whereas the supplier may not be willing to do so since it increases the service cost and even reduces revenue, especially when repairing failures is the supplier's revenue stream. The interactions between the customer and the supplier depends

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[1]http://www.techhive.com/article/120670/article.html

[2]http://www.dbk.com/repair-center_transparent_oem_repairs.htm

on the compensation schemes, i.e., the service contracts. Under different contracts, the service supplier's repair activity is paid by the customer in different manners. Typically, service outsourcing is based on the Time & Material contracts (T&M) or the Performance-based contracts (PBC) (Roels et al., 2010). Under T&M, the customer pays a service fee to the supplier for each repair. Under PBC, the customer only pays for product availability. Within PBC, the way of how the customer's payment is transferred can also be various. The customer can only pay for product uptime during the contract period. For example, Wagner Equipment offers Power by the Hour contracts, under which the customers have 24 hours access to the machines but only pays for the hours they use. If the product is inoperable, the customer does not have to pay for it[3]. We call this type of contracts *PBC-U*. Under the other type of PBC, the supplier pays downtime penalties to the customer, i.e., if the product performance is under the predefined level, the supplier is penalized for the under-performance[4]. We refer to this type of contracts *PBC-P*. Although PBC are becoming more attractive, the other type of service contracts like T&M are also used in many occasions[5].

In the context of oursourcing, the most challenging task for the customer is to deal with the supplier's moral hazard problem, i.e., the customer is usually unable to control and monitor the supplier's repair and maintenance activities. Although PBC are stated to be able to align the incentives for the customer and the supplier (Cohen et al., 2006), the suppliers may worry about the financial impact of PBC because compared to traditional T&M, PBC mostly shift risks to the suppliers, which makes them "reluctant to sign up"[6]. As a matter of fact, designing optimal contracts becomes a critical task for the contract offerer. In this chapter, we build stylized models to address the problems of service contract choice and answer the following research questions: (1) What are the optimal contract terms in T&M and PBC? (2) Which type of contracts is better for the supplier/customer under what circumstance? (3) How service contracts affect the supplier's and the customer's individual activities in capacity setting and product failure prevention?

We find that if both supplier and the customer are risk neutral, T&M and PBC are equivalent for the supplier. Under T&M, the supplier invests in service capacity by asking for a higher service fee. Under PBC, incentives are generated by the penalty terms or uptime payments,

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[3]http://wagnerequipment.com/contact/power
[4]http://reason.org/news/show/performance-based-contracting
[5]http://www.niaid.nih.gov/researchfunding/sop/pages/contracttype.aspx
[6]http://knowledge.wharton.upenn.edu/article/pay-for-what-you-get-putting-performance-based-contracting-to-the-test/

which makes the supplier to set up the appropriate capacity to guarantee the optimal product uptime. This finding shows that T&M may not be less effective than PBC in incentivizing the supplier to invest in capacity for failure restoration. If the supplier becomes risk averse to the variance of his revenue, the optimal contract is PBC-P, where the penalty rate is 0, i.e., a fixed fee contract is optimal for the supplier. A zero penalty eliminates the impact of uncertainty caused by product failures, and the risk-free lump sum can satisfy the risk neutral customer's participation requirement. As such, PBC-P realize the highest profit for the supplier as the risk neutral case. On the other hand, if the failure rate is relatively low, the supplier has a higher profit under PBC-U than T&M, whereas the customer has more surplus under T&M than PBC-U. If the product failure rate is high, the result is reversed.

Next, we discuss the case when the customer exerts failure prevention effort by performing preventive maintenance. If the supplier's capacity is exogenous, PBC-P achieve the First Best (FB) solution. For PBC-U vs. T&M, the customer over-invests in the effort under T&M and under-invests in it under PBC-U. The supplier has a higher profit under T&M than PBC-U if the failure rate is high, and has a lower profit under T&M than PBC-U if the failure rate is low. If the supplier's capacity becomes endogenous, PBC-P also achieve FB. Under T&M, the customer over-invests in the effort of failure reduction, and the supplier under-invests in capacity. Under PBC-U, the situation is reversed. In addition, the supplier has the lowest profit under T&M.

Finally, we extend the model with more settings. If the penalty rate is forced be to above 0 under PBC-P, we find that the penalty rate leads to over-investment in capacity and decreases the supplier's profit. If the penalty rate is sufficiently large, the supplier's profit can be lower than T&M or PBC-U. Another finding is that when the customer becomes risk averse, PBC dominates T&M If the supplier is risk neutral.

The rest of this chapter is organized as follows. After a brief survey of related literature in §4.2, we develop the basic model of risk neutral in §4.3. Next, in §4.4, we discuss the setting when the supplier becomes risk averse. Subsequently, we investigate the impact of contracts on the customer's failure prevention effort in §4.5. Then, we extend the model in more settings in §4.6. Finally, we summarize the major findings and discuss future directions for research in §4.7.

4.2 Literature Review

The model in this chapter is formulated based on principal-agent framework and focuses on service outsourcing context. Traditional OM literature mainly study service process management based on queueing models. Some examples are Gilbert and Weng. (1998), Plambeck and Zenios (2003), Ren and Zhou (2008), and Lu et al. (2009). Although principle-agent relationship is considered in those papers, the main focus is congestion management by changing number of servers or service rate. In this chapter, we emphasize the impact of different service contracts on the supply chain members' decisions on pricing and service rate setting.

For failure restoration, spare parts inventory management is a traditional area in OM literature. Muckstadt (2005) gives an extensive overview of this research. Compared to the model in this chapter, incentive and contracting are often not incorporated in the spare parts inventory literature. An exception is Kim et al. (2007), where they study Performance-based contracting by constructing multiple-agents model with repairable spare parts inventory decisions. Our work differs from theirs in several aspects. The model in this chapter focuses on failure restoration by setting service capacity, rather than spare parts inventory. In addition, we discuss Time & Material contracts by considering a service fee for each repair. Similar cases occur when comparing this chapter with (Kim et al., 2010). The failure restoration model in this chapter is in line with Kim et al. (2010), but the setting of the problems are different. In (Kim et al., 2010), the customer sets prices of the PBC, and two performance measures are investigated. In this chapter, we deal with the case where the supplier sets prices, which is more often seen in commercial sectors, and study two payment schemes- paying penalty for downtime and paying only for uptime under PBC. Moreover, we compare PBC with T&M where in (Kim et al., 2010), only PBC is discussed.

In this chapter, efforts which affect product reliability are discussed in a principal-agent model. Incentives of effort on product quality improvement have been studied in literature such as (Balachandran and Radhakrishnan, 2005), (Zhu et al., 2007), and (Kim and Tomlin, 2013). The settings in those papers are either the customer incentivizes the supplier or both the customer and the supplier exerts quality improvement effort. As a contrast, in this chapter, we focus on the customer's preventive failure reduction before service outsourcing, and explore how T&M and PBC affect the customer's effort.

Finally, this chapter also relates to classical moral-hazard problem and supply chain contracting literature. Extensive reviews of principal-agents models and supply chain contracting can be found in (Bolton and Dewatripont, 2005) and (Cachon, 2004). In this chapter, we apply the principal-agent and contracting analysis to contracting for product support, which also enriches the research in this area.

4.3 The Basic Model

We first consider a setting in which a risk-neutral customer (she) has a fleet of identical products which need support and maintenance by a single service supplier (he). We first assume the product quantity is 1. The product is subject to random failures during the contract period. Following the common assumptions in literature, we model the failure occurrence as a Poisson process with rate λ which is determined by some exogenous activities such as product design, manufacturing, and preventive maintenance[7]. If we normalize the contract period to 1, we have the expected value of the total number of failures (denoted by N) $\mathbb{E}[N] = \lambda$. Once system downtime occurs, the supplier has to repair the failed product. The repair lead time of the ith failure is S_i ($i = 1, .., N$). We assume S_i is independent and identically distributed (i.i.d.) with a mean $1/\mu$, where μ is the supplier's capacity for conducting service activities. We assume μ is set by the supplier at the beginning and remains unchanged during the contract period. In addition, let $\underline{\mu}$ be the default level of the service capacity. The supplier may choose to set up a higher capacity or maintain the default level. The unit cost of the service capacity is c. To avoid trivial cases, for the relation between μ and λ, we assume $1/\lambda > 1/\mu$, which can be interpreted that the service capacity is ample and the Mean Time Between Failure (MTBF) is longer than the repair lead time. Otherwise, the product will remain inoperable all the time. Then we can formulate the downtime of the product as $\sum_{i=1}^{N} S_i$ and uptime as $1 - \sum_{i=1}^{N} S_i$. Once product downtime occurs, the customer will suffer a revenue loss. Let r be the revenue rate per unit uptime. The customer's total revenue derived from running the product is $r(1 - \sum_{i=1}^{N} S_i)$.

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[7]We endogenize the customer's effort on failure reduction in §4.5.

4.3.1 Contracting

As we observed in practice, the supplier offers two types of service contracts to the customer - T&M and PBC. Under T&M, the customer pays a service fee to the supplier for each repair. Let p be the service fee. Then the supplier's revenue during the contract period is pN. Under PBC, the supplier's revenue is tied to some performance measure of the product. In most of the cases, product uptime is the performance indicator for the supplier. In particular, the transfer of payment under PBC can be based on a *lump sum + penalty* or an *uptime revenue* framework. In this chapter, we use PBC-P and PBC-U to represent the former and the latter case respectively. Under PBC-P, the customer first pays a initial fee to the supplier. After the uptime of the product has been measured at the end of the contract period, the supplier pays a penalty to the customer based on product downtime. Let w denote the lump sum and κ be the penalty rate. Then the supplier's revenue under PBC-P is $w - \kappa \sum_{i=1}^{N} S_i$. Under PBC-U, the customer only pays for product uptime. Let v be the unit uptime price. The supplier's revenue under PBC-U is $v(1 - \sum_{i=1}^{N} S_i)$.

The business sequence is as follows. First, the supplier offers a contract with specified contract terms (p under T&M, w, κ under PBC-P, or v under PBC-U) to the customer. The customer can accept or reject the contract. If the contract is accepted, the supplier sets up the service capacity at the beginning and starts to provide product support. Whenever a product failure occurs, the supplier has to restore the failed product by using his service capacity. The customer then transfers payment to the supplier based on the type of contracts.

4.3.2 The supplier's Objective

Given such a sequence of events, we can model the supplier's problem as

- Under T&M,

$$
\begin{aligned}
&\max_{p,\ \mu}\ \mathbb{E}(pN) - c(\mu - \underline{\mu}) \\
&s.t. \\
&\mathbb{E}[r(1 - \sum_{i=1}^{N} S_i|\lambda,\mu) - pN] \geq 0,\ \text{(IR)} \\
&\mu \geq \underline{\mu}.
\end{aligned}
\tag{4.1}
$$

- Under PBC-P,

$$\max_{w,\,\kappa,\,\mu}\ \mathbb{E}[w - \kappa \sum\nolimits_{i=1}^{N} S_i | \lambda, \mu] - c(\mu - \underline{\mu})$$

$$s.t.$$

$$\mathbb{E}[r(1 - \sum\nolimits_{i=1}^{N} S_i | \lambda, \mu) - (w - \kappa \sum\nolimits_{i=1}^{N} S_i | \lambda, \mu)] \geq 0,\ (\text{IR})$$

$$\mu \geq \underline{\mu}.$$

(4.2)

- Under PBC-U,

$$\max_{v,\,\mu}\ \mathbb{E}[v(1 - \sum\nolimits_{i=1}^{N} S_i | \lambda, \mu)] - c(\mu - \underline{\mu})$$

$$s.t.$$

$$\mathbb{E}[(r - v)(1 - \sum\nolimits_{i=1}^{N} S_i | \lambda, \mu)] \geq 0,\ (\text{IR})$$

$$\mu \geq \underline{\mu}.$$

(4.3)

The supplier's problem is to maximize the expected profit, which equals the payment received from the customer minus the service capacity cost, by setting the contract terms and capacity. In Equations (4.1)-(4.3), the Individual Rationality (IR) constraints state that the customer's expected profit (revenue generated by running the product minus the payment transferred to the supplier) has to be greater than a reserved level (here normalized to 0), which guarantees the participation of the customer. It can be proved that $\sum_{i=1}^{N} S_i$ satisfies a Compound Poisson distribution, and $\mathbb{E}(\sum_{i=1}^{N} S_i | \lambda, \mu) = \lambda/\mu$.

Lemma 4.1. *(1) The profit functions under PBC are concave in μ.*

(2) The IR constraints are binding at optimality.

(3) The optimal solutions are as shown in Table 4.1-4.2.

The proof can be found in Appendix A.

TABLE 4.1: The optimal solution of the basic model under T&M.

	$\lambda \leq \frac{c\underline{\mu}^2}{r}$	$\lambda > \frac{c\underline{\mu}^2}{r}$
contract terms	$p^* = r(\frac{1}{\lambda} - \frac{1}{\mu})$, $\mu^* = \underline{\mu}$	$p^* = \frac{r - \sqrt{c}\sqrt{r}\sqrt{\lambda}}{\lambda}$, $\mu^* = \sqrt{\frac{r\lambda}{c}}$
π_s^*	$r(1 - \frac{\lambda}{\underline{\mu}})$	$r - 2\sqrt{cr\lambda} + c\underline{\mu}$
π_c^*	0	0

TABLE 4.2: The optimal solution of the basic model under PBC.

	PBC-P		PBC-U	
	$\lambda \leq \frac{c\underline{\mu}^2}{r}$	$\lambda > \frac{c\underline{\mu}^2}{r}$	$\lambda \leq \frac{c\underline{\mu}^2}{r}$	$\lambda > \frac{c\underline{\mu}^2}{r}$
contract terms	$w^* = r - \frac{(r - \kappa^*)\lambda}{\mu}$, $\mu^* = \underline{\mu}$	$w^* = r - (r - \kappa^*)\sqrt{\frac{c\lambda}{r}}$, $\mu^* = \sqrt{\frac{r\lambda}{c}}$	$v^* = r$, $\mu^* = \underline{\mu}$	$v^* = r$, $\mu^* = \sqrt{\frac{r\lambda}{c}}$
π_s^*	$r(1 - \frac{\lambda}{\underline{\mu}})$	$r - 2\sqrt{cr\lambda} + c\underline{\mu}$	$r(1 - \frac{\lambda}{\underline{\mu}})$	$r - 2\sqrt{cr\lambda} + c\underline{\mu}$
π_c^*	0	0	0	0

When both the supplier and the customer are risk-neutral, we notice that the IR constraint under each case is binding at optimality. The supplier can set the prices which give him the maximum profit while enable the customer to accept the contract. Next, we discuss how the optimal solution changes with parameters and compare contracts.

Proposition 4.2. *(1) If $\lambda > \frac{c\underline{\mu}^2}{r}$, the supplier will invest more service capacity, and $\partial\mu^*/\partial r > 0$, $\partial\mu^*/\partial\lambda > 0$, $\partial\mu^*/\partial c < 0$. Otherwise, if $\lambda < \frac{c\underline{\mu}^2}{r}$, the supplier will keep the capacity at the default level $\underline{\mu}$.*

(2) Under T&M, $\partial p^*/\partial\mu^* > 0$; Under PBC-P, w^* and κ^* are given as combinations; Under PBC-U, v^* is independent of μ^*.

(3) The supplier's profit is decreasing in λ, i.e., $\partial\pi_s^*/\partial\lambda > 0$.

(4) The product uptime and the supplier's profit under each contract are equivalent.

The proof can be found in Appendix B.

The supplier's optimal service capacity only depends on the relation between λ and $c\mu^2/r$. If the failure rate is large ($\lambda > \frac{c\underline{\mu}^2}{r}$), the supplier will set a capacity which is higher than the initial level. This is intuitive for PBC because a high failure rate implies more system downtime, which needs more capacity to restore the product to avoid penalty. Our model shows that this is also true for T&M. Since the supplier's revenue only depends on the number of failures rather than system downtime, it seems intuitive that the supplier has no incentive to invest in service capacity. However, the results of our model shows that whether the supplier invests in capacity does not depend on the types of contract, but on the relative value of the failure rate. Under T&M, the optimal service fee, p^*, is the product of the customer's revenue rate r and the difference between MTBF and repair lead time. Furthermore, we have $\partial p^*/\partial\mu^* > 0$. In other words, a quicker repair lead time can result in a higher service fee. This is the reason why the supplier will invest in service capacity under T&M. Under PBC, prices are set differently under the corresponding payment methods. Under PBC-P, w^* and κ^* are given in combinations as $w^* = r - \frac{(r-\kappa^*)\lambda}{\mu^*}$. And we have $\partial w^*/\partial\kappa^* > 0$. Under PBC-U, the optimal price v^* is independent of the service capacity and remains the same value as the revenue rate r.

Even though prices are different across contracts, the product uptime and the supplier's profit is the same. From Lemma 4.1, we can see that the supplier's optimal capacity under each contract is identical. Since the expected product uptime $\mathbb{E}(1 - \sum_{i=1}^{N} S_i | \lambda, \mu^*) = 1 - \lambda/\mu^*$, we know that given the same λ and μ^*, the product uptime will be indentical. Moreover, the results show that all IR constraints are binding, which means that the customer has 0 surplus in the game and the supplier takes all the profit of the supply chain. The supply chain's profit is given by $r(1 - \lambda/\mu^*) - c(\mu^* - \underline{\mu})$. Therefore, for a given value of μ^*, profit under different contracts will be equivalent.

The basic model captures the setting when decision-makers are risk-neutral. Although T&M and PBC create different incentives for the supplier, our model shows that the supplier's investment in service capacity is independent of the types of contract. Under T&M, the supplier's revenue does not link to product downtime apparently. However, he can have a higher service fee if he invests in capacity, which results in a higher profit. Meanwhile the increased service fee does not hurt the customer's profit ($\pi_c^* \geq 0$ can be satisfied.)

Generally, although one may argue that PBC can incentivize the supplier to improve product availability, our analysis states that PBC may not have superior performance over T&M, if both supply chain partners are risk-neutral and prices are set by the supplier.

4.4 Risk-averse Agents

In the basic model, we have assumed that both the supplier and the customer are risk-neutral. In practice, the bilateral relationship between the supplier and the customer can be various. The financial status of some small service suppliers can be tightly connected with a single large customer. In such a case, the supplier would be risk averse to the variance of the payment that is received from the customer. In this section, we discuss how the supplier's' risk aversion affects the choice of contracts.

Let γ_s, $\gamma_s > 0$, be the supplier's coefficient of risk aversion, where the subscript s represents the supplier. Following literature, we assume that a risk-averse agent has a mean-variance utility function in the decision problem. Under risk-aversion, we have the supplier's problem as

- Under T&M,

$$
\begin{aligned}
&\max_{p,\,\mu} \ \mathbb{E}(pN) - \gamma_s \operatorname{var}(pN) - c(\mu - \underline{\mu}) \\
&s.t. \\
&\mathbb{E}[r(1 - \sum_{i=1}^{N} S_i | \lambda, \mu) - pN] \geq 0, \ (\text{IR}) \\
&\mu \geq \underline{\mu}.
\end{aligned}
\tag{4.4}
$$

- Under PBC-P,

$$
\begin{aligned}
&\max_{w,\,\kappa,\,\mu} \ \mathbb{E}[w - \kappa \sum_{i=1}^{N} S_i | \lambda, \mu] - \gamma_s \operatorname{var}[\kappa \sum_{i=1}^{N} S_i | \lambda, \mu] - c(\mu - \underline{\mu}) \\
&s.t. \\
&\mathbb{E}[r(1 - \sum_{i=1}^{N} S_i | \lambda, \mu) - (w - \kappa \sum_{i=1}^{N} S_i | \lambda, \mu)] \geq 0, \ (\text{IR}) \\
&\mu \geq \underline{\mu}.
\end{aligned}
\tag{4.5}
$$

- Under PBC-U,

$$\max_{v,\,\mu} \ \mathbb{E}[v(1 - \sum\nolimits_{i=1}^{N} S_i | \lambda, \mu)] - \gamma_s \operatorname{var}[v(1 - \sum\nolimits_{i=1}^{N} S_i | \lambda, \mu)] - c(\mu - \underline{\mu})$$

$$s.t.$$

$$\mathbb{E}[(r - v)(1 - \sum\nolimits_{i=1}^{N} S_i | \lambda, \mu)] \geq 0, \ \text{(IR)}$$

$$\mu \geq \underline{\mu}. \tag{4.6}$$

In order to obtain the closed-form solutions for comparing contracts, we assume that the repair lead time, S_i, satisfies exponential distribution with parameter μ. Then we have the variance term in the supplier's utility functions as summarized in the following Lemma.

Lemma 4.3. *(1) Under T&M,* $\operatorname{var}(pN) = p^2 \lambda$;

(2) *Under PBC-P,* $\operatorname{var}[\kappa \sum_{i=1}^{N} S_i | \lambda, \mu] = \frac{2\lambda \kappa^2}{\mu^2}$;

(3) *Under PBC-U,* $\operatorname{var}[v(1 - \sum_{i=1}^{N} S_i | \lambda, \mu)] = \frac{2\lambda v^2}{\mu^2}$.

The proof can be found in Appendix C.

Solving Equations (4.4-4.6), we can obtain the optimal solutions. Firstly, the optimal capacity is determined as follows.

Lemma 4.4. *(1) Under T&M, if* $\lambda > \frac{2r^2 \gamma_s \underline{\mu} + c\underline{\mu}^3}{2r^2 \gamma_s + r\underline{\mu}}$, μ^* *satisfies* $2r^2 \gamma_s(\lambda - \mu^*) + r\lambda\mu^* - c\mu^{*3} = 0$.
   If $\lambda < \frac{2r^2 \gamma_s \underline{\mu} + c\underline{\mu}^3}{2r^2 \gamma_s + r\underline{\mu}}$, $\mu^* = \underline{\mu}$.

(2) *Under PBC-P, if* $\lambda > \frac{c\underline{\mu}^2}{r}$, $\mu^* = \sqrt{\frac{r\lambda}{c}}$. *If* $\lambda < \frac{c\underline{\mu}^2}{r}$, $\mu^* = \underline{\mu}$.

(3) *Under PBC-U, if* $\frac{c\underline{\mu}^3}{4r^2 \gamma_s + r\underline{\mu}} < \lambda < \frac{\underline{\mu}^2}{\underline{\mu} + 4r\gamma_s}$, μ^* *satisfies* $r\lambda(4r\gamma_s + \mu^*) - c\mu^{*3} = 0$. *If* $\lambda > \frac{\underline{\mu}^2}{\underline{\mu} + 4r\gamma_s}$
   or $\lambda < \frac{c\underline{\mu}^3}{4r^2 \gamma_s + r\underline{\mu}}$, $\mu^* = \underline{\mu}$.

The proof can be found in Appendix D.

Compared to the risk-neutral setting, the risk-averse coefficient plays a role in setting the optimal service capacity, except PBC-P. Generally, the supplier will invest in capacity if the failure rate is relatively high. We also notice that there is a special case under PBC-U. If $\lambda > \frac{\underline{\mu}^2}{\underline{\mu} + 4r\gamma_s}$, the supplier will not invest in capacity. We interpret this counter-intuitive result in the following discussions.

Secondly, prices are set as given in Lemma 4.5.

Lemma 4.5. *(1) Under T&M, If $\lambda < \frac{2r\gamma_s\underline{\mu}}{2r\gamma_s+\underline{\mu}}$, $p^* = \frac{1}{2\gamma_s}$.*

If $\frac{2r\gamma_s\underline{\mu}}{2r\gamma_s+\underline{\mu}} < \lambda < \frac{2r^2\gamma_s\underline{\mu}+c\underline{\mu}^3}{2r^2\gamma_s+r\underline{\mu}}$, $p^* = \frac{r(\underline{\mu}-\lambda)}{\lambda\underline{\mu}}$.

If $\lambda > \frac{2r^2\gamma_s\underline{\mu}+c\underline{\mu}^3}{2r^2\gamma_s+r\underline{\mu}}$, $p^* = \frac{r(\mu^*-\lambda)}{\lambda\mu^*}$.

(2) Under PBC-P, If $\lambda > \frac{c\underline{\mu}^2}{r}$, $w^* = r - \sqrt{cr\lambda}$, $\kappa^* = 0$.

If $\lambda < \frac{c\underline{\mu}^2}{r}$, $w^* = r(1 - \frac{\lambda}{\underline{\mu}})$, $\kappa^* = 0$.

(3) Under PBC-U, If $\lambda > \frac{\underline{\mu}^2}{\underline{\mu}+4r\gamma_s}$, $v^* = \frac{\underline{\mu}(\underline{\mu}-\lambda)}{4\gamma_s\lambda}$.

If $\lambda < \frac{\underline{\mu}^2}{\underline{\mu}+4r\gamma_s}$, $v^* = r$.

The proof can be found in Appendix D.

The most remarkable change with risk aversion lies in the prices in PBC-P. Under risk neutrality, w^* and κ^* are set as combinations. However, under risk aversion, $\kappa^* = 0$ if it can be set by the supplier. A zero penalty rate implies the supplier's compensation does not link to the product's performance. PBC-P becomes a fixed-fee contract. For PBC-U, we can see that if the failure rate is high ($\lambda > \frac{\underline{\mu}^2}{\underline{\mu}+4r\gamma_s}$), a price $\frac{\underline{\mu}(\underline{\mu}-\lambda)}{4\gamma_s\lambda}$, which is lower than r, is optimal for the supplier.

From Lemma 4.4 and Lemma 4.5, we can see that risk aversion has changed the supplier's decision on setting the optimal service capacity and contract price. Next, we compare the service capacity level to the case of risk neutrality.

Proposition 4.6. *Let the subscript RA and RN denotes the case of risk aversion and risk neutrality. If the supplier is risk-averse,*

*(1) under T&M, if $\lambda > \frac{c\underline{\mu}^2}{r}$, the supplier **under-invests** in capacity, i.e., $\mu^*_{RA} < \mu^*_{RN}$. If $\lambda < \frac{c\underline{\mu}^2}{r}$, $\mu^*_{RA} = \mu^*_{RN}$.*

(2) under PBC-P, $\mu^*_{RA} = \mu^*_{RN}$.

*(3) under PBC-U, if $\frac{c\underline{\mu}^3}{4r^2\gamma_s+r\underline{\mu}} < \lambda < \frac{\underline{\mu}^2}{\underline{\mu}+4r\gamma_s}$, the supplier **over-invests** in capacity, i.e., $\mu^*_{RA} > \mu^*_{RN}$; otherwise, $\mu^*_{RA} = \mu^*_{RN}$.*

The proof can be found in Appendix E.

According to Proposition 4.6, under PBC-P, the supplier sets the penalty rate $\kappa^* = 0$. As a result, the variance term in the utility function is eliminated. At the same time, the supplier also cuts the link between his payment and product downtime. In this way, he can achieve the same capacity level as the risk neutral case. Compared to the risk neutrality case, T&M can lead to under-investment while PBC-U can lead to over-investment in capacity due to risk aversion if failure rate is high. Consequently, we can conclude that PBC-U results in the highest product uptime, and T&M results in the lowest product uptime. However, both of them deviate from the solution under risk neutrality, which can be achieved under PBC-P. As shown in Figure 4.1, when $\lambda > \frac{c\underline{\mu}^3}{4r^2\gamma_s + r\underline{\mu}}$, the supplier invests in capacity under PBC-U. The capacity level is higher than PBC-P. On the contrary, when $\lambda > \frac{2r^2\gamma_s\underline{\mu} + c\underline{\mu}^3}{2r^2\gamma_s + r\underline{\mu}}$, the supplier invests in capacity under T&M. The capacity level is lower than PBC-P.
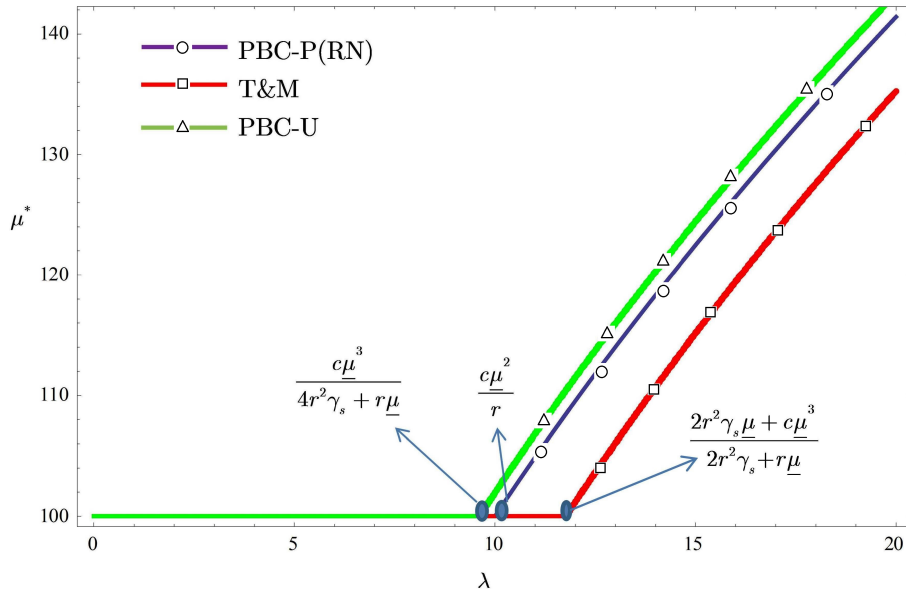
FIGURE 4.1: Optimal service capacity under risk aversion. $r = 1000, c = 1, \underline{\mu} = 100, \gamma_s = 0.001$.

Although both PBC-P and PBC-U have performance linkage on the supplier's revenue, the payment framework creates differences under risk aversion. Under PBC-P, the supplier pays a penalty based on the product downtime. However, he can charge a lump sum beforehand. By setting $\kappa^* = 0$, the supplier exempts himself from paying penalty; at the same time, he also eliminate the variance caused by the uncertainty. w^* is a risk-free revenue for the supplier. As long as the customer's participation condition is satisfied, the supplier can set w^* to an optimal value which maximizes his utility (with 0 variance). Hence, PBC-P can achieve the same profit

for the supplier as the risk neutrality case[8]. However, this is not the case for PBC-U. Since the customer only pays for uptime, the variance of the supplier's revenue can never be eliminated. As a result, risk aversion affects the decisions on pricing and capacity setting.

Proposition 4.7. *(1) Under PBC-P, IR constraint is always binding. Under T&M, $\pi_c^* > 0$ if $\lambda < \frac{2r\gamma_s\underline{\mu}}{2r\gamma_s + \underline{\mu}}$. Under PBC-U, $\pi_c^* > 0$ if $\lambda > \frac{\underline{\mu}^2}{\underline{\mu} + 4r\gamma_s}$.*

(2) For the supplier's profit, PBC-P is higher than PBC-U and T&M. Furthermore, if $\lambda > (\sqrt{2} - 1)\underline{\mu}$, $\pi_s^*(T\&M) > \pi_s^*(PBC - U)$. If $\lambda < (\sqrt{2} - 1)\underline{\mu}$, $\pi_s^*(T\&M) < \pi_s^*(PBC - U)$.

(3) For the customer's profit, PBC-P is lower than or equal to PBC-U and T&M. Furthermore, if $\lambda > (\sqrt{2} - 1)\underline{\mu}$, $\pi_c^*(T\&M) < \pi_c^*(PBC - U)$. If $\lambda < (\sqrt{2} - 1)\underline{\mu}$, $\pi_c^*(T\&M) > \pi_c^*(PBC - U)$.

The proof can be found in Appendix F.

Proposition 4.7, part (1) compares the customer's profit under risk aversion with the risk neutral case. When the supply chain members are risk-neutral, the customer can never obtain any surplus. But if the supplier becomes risk averse, the customer can derive some benefits out of it. Specifically, under T&M, if $\lambda < \frac{2r\gamma_s\underline{\mu}}{2r\gamma_s + \underline{\mu}}$, the supplier can set an optimal price $p^* = \frac{1}{2\gamma_s}$ which maximizes his utility while keeps the customer has a positive profit $r(1 - \frac{\lambda}{\underline{\mu}}) - \frac{\lambda}{2\gamma_s}$. In addition, we have $\partial\pi_c^*/\partial\lambda < 0$. Because a high failure rate leads to a lower product uptime, the customer's profit is decreasing in λ. For PBC-U, when $\lambda > \frac{\underline{\mu}^2}{\underline{\mu} + 4r\gamma_s}$, the customer can get a positive profit.

From Proposition 4.7, part (2), we know that the supplier has the highest profit under PBC-P. As stated in a previous argument, the variance of the supplier's revenue is eliminated and thus PBC-P achieve the highest utility. If we compare PBC-U and T&M, we see that if $\lambda > (\sqrt{2}-1)\underline{\mu}$, the supplier's profit under T&M is higher than PBC-U, as shown in Figure 4.2(a). Unlike the risk neutral case where the supplier's profit is monotonically decreasing in λ, the supplier's utility is increasing in λ under T&M. This is in line with the fact that the customer pays for each repair, and a higher failure rate leads to more failures and thus a higher revenue for the supplier. Here risk aversion changes the way how the repair price is set. Our models shows that if the failure rate is sufficiently large ($\lambda > (\sqrt{2} - 1)\underline{\mu}$), the supplier's profit under T&M

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[8]Recall that under risk neutrality, w^* and κ^* are set in combinations. In that case, if we let $\kappa^* = 0$, w^* is the same as the risk averse case.

(a)

(b)

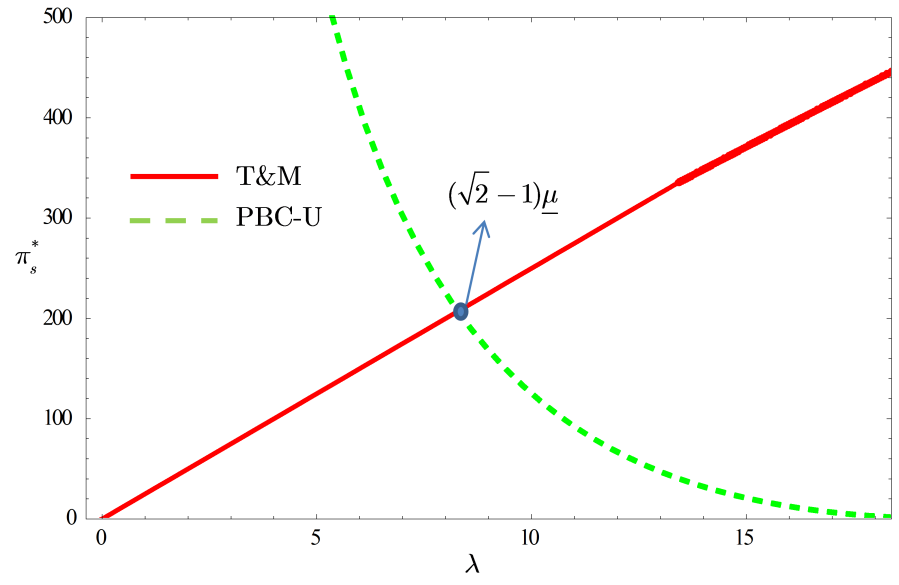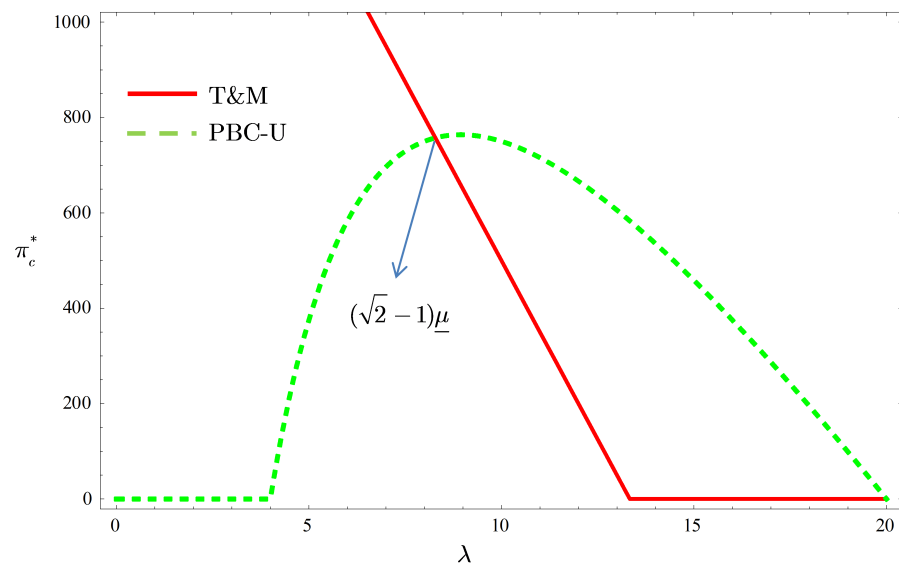FIGURE 4.2: The supplier's (a) and the customer's (b) optimal profit under T&M and PBC-U. $r = 2000, c = 1, \underline{\mu} = 20, \gamma_s = 0.01$.

can be higher than PBC-U. For the customer's profit, as stated in Proposition 4.7, part (3), the result is reversed. Under T&M, the customer can have a positive profit if the failure rate is low; under PBC-U, the customer can have a positive profit if the failure is high. Thus, as shown in Figure 4.2(b), If the failure rate is high, i.e., $\lambda > (\sqrt{2} - 1)\underline{\mu}$, the customer's profit under T&M is lower than PBC-U.

4.5 Preventive Maintenance by the Customer

In previous sections, we have assumed that the failure rate of the product is exogenous. In this section, we discuss the case when the product reliability can be improved by the customer. Before outsourcing product support to the supplier, the customer can exert effort on reducing the product failure by performing preventive maintenance. Let λ_0 be the initial product failure and λ_c be the failure rate after the customer's effort. Then the change of failure rate is $\Delta\lambda = \lambda_0 - \lambda_c$. The cost of the failure rate reduction is $C(\Delta\lambda)$. We assume that $C(\cdot)$ is continuously differentiable and convex in $\Delta\lambda$. In order to obtain the closed form solutions to compare contracts analytically, we use a quadratic function $K(\lambda_0 - \lambda_c)^2$, where K is the coefficient of effort cost, to model the customer's preventive maintenance cost. We assume that information is symmetric in the supply chain, i.e., the initial failure rate of the product is a public knowledge of the supply chain members . As the contract offerer, the supplier sets the prices anticipating the customer's effort level on reducing failure rate. Then the supplier's problem becomes

- Under T&M,

$$
\begin{aligned}
&\max_{p,\,(\mu)}\ \mathbb{E}(p\,N\,|\,\lambda_c^*) - c(\mu - \underline{\mu}) \\
&s.t. \\
&\mathbb{E}[r(1 - \sum\nolimits_{i=1}^{N} S_i | \lambda_c^*, \mu) - p\,N\,|\,\lambda_c^*] \geq 0,\ \text{(IR)} \\
&\lambda_c^* = \arg\max \mathbb{E}[r(1 - \sum\nolimits_{i=1}^{N} S_i | \lambda, \mu) - p\,N\,|\,\lambda],\ \text{(IC)} \\
&\mu \geq \underline{\mu}.
\end{aligned}
\tag{4.7}
$$

- Under PBC-P,

$$\max_{w,\,\kappa,\,(\mu)} \; \mathbb{E}[w - \kappa \sum_{i=1}^{N} S_i | \lambda_c^*, \mu] - c(\mu - \underline{\mu})$$

$$s.t.$$

$$\mathbb{E}[r(1 - \sum_{i=1}^{N} S_i | \lambda_c^*, \mu) - (w - \kappa \sum_{i=1}^{N} S_i | \lambda_c^*, \mu)] \geq 0, \;\; (IR)$$

$$\lambda_c^* = \arg\max \mathbb{E}[r(1 - \sum_{i=1}^{N} S_i | \lambda, \mu) - (w - \kappa \sum_{i=1}^{N} S_i | \lambda, \mu)], \;\; (IC)$$

$$\mu \geq \underline{\mu}.$$

(4.8)

- Under PBC-U,

$$\max_{v,\,(\mu)} \; \mathbb{E}[v(1 - \sum_{i=1}^{N} S_i | \lambda_c^*, \mu)] - c(\mu - \underline{\mu})$$

$$s.t.$$

$$\mathbb{E}[(r - v)(1 - \sum_{i=1}^{N} S_i | \lambda_c^*, \mu)] \geq 0, \;\; (IR)$$

$$\lambda_c^* = \arg\max \mathbb{E}[(r - v)(1 - \sum_{i=1}^{N} S_i | \lambda, \mu)] \geq 0, \;\; (IC)$$

$$\mu \geq \underline{\mu}.$$

(4.9)

With the customer's failure reduction effort, the decision becomes a sequential game. When setting prices and capacity, the supplier expects that the customer will make some effort on reducing the failure rate. We have the Incentive Compatibility (IC) constraints in Equations (4.7)-(4.9). These IC constraints capture the customer's individual objective. When she exerts an effort, she will reduce the failure rate to an optimal level λ_c^* which maximizes her own profit.

4.5.1 Exogenous μ

First, for the sake of tractability, we discuss the case when the supplier's service capacity level is exogenous, i.e., μ is given as a parameter in the supplier's objective functions. If the supplier and the customer is integrated as a centralized firm, it is the First Best (FB) case in this setting. The central firm solves for an optimal failure rate λ_{sc}^* which maximizes the supply chains' profit, where $\Pi_{sc} = r[1 - \mathbb{E}(\sum_{i=1}^{N} S_i \big| \lambda_{sc}, \mu)] - c\mu - K(\lambda_0 - \lambda_{sc})^2$. It is not difficult to see that the supply

chains' profit function is concave in λ_{sc} and we have the FB solution as:

$$\lambda_{sc}^* = \lambda_0 - \frac{r}{2K\mu}, \Pi_{sc}^* = r + \frac{r^2}{4K\mu^2} - \frac{r\lambda_0}{\mu} - c\mu.$$

Next, we solve the supplier's problem under each type of contract and the equilibrium solutions are given in Lemma 4.8.

Lemma 4.8. *(1) The customer's profit function is concave in λ_c. The supplier's profit function given the customer's best response of λ_c^* is concave in p, κ, and v under T&M, PBC-P and PBC-U, respectively.*

(2) *Under T&M, if $\lambda_0 \leq \frac{2\sqrt{r(r+12K\mu^2)}-r}{6K\mu}$, $\lambda_c^* = \frac{\lambda_0}{2} - \frac{r}{4K\mu}$. If $\lambda_0 > \frac{2\sqrt{r(r+12K\mu^2)}-r}{6K\mu}$, $\lambda_c^* = \sqrt{\lambda_0{}^2 - \frac{r}{K}}$.*

(3) *Under PBC-P, $\lambda_c^* = \lambda_0 - \frac{r}{2K\mu}$. Particularly, $\kappa^* = 0$.*

(4) *Under PBC-U, if $\lambda_0 \leq \mu - \frac{r}{2K\mu}$, $\lambda_c^* = \lambda_0$. If $\lambda_0 > \mu - \frac{r}{2K\mu}$, $\lambda_c^* = \frac{\lambda_0+\mu}{2} - \frac{r}{4K\mu}$.*

The proof can be found in Appendix G.

We can see that, except for the case PBC-U when $\lambda_0 < \mu - \frac{r}{2K\mu}$, the customer tries to lower the failure rate under all contracts, yet the levels of effort are different. In addition, we have $\partial\lambda_c^*/\partial K > 0$. In other words, if the cost of failure reduction increases, the level of effort decreases. Subsequently, we can compare the results of contracts by analyzing the equilibrium solutions. Proposition 4.9 gives the results of contract comparison with regard to the customer's effort level and the supplier's and the customer's profit.

Proposition 4.9. *(1) PBC-P achieve the FB solution for the customer's optimal effort level, i.e., $\lambda_c^* = \lambda_{sc}^*$. Under T&M, the customer **over-invests** in the effort of failure reduction, i.e., $\lambda_c^* < \lambda_{sc}^*$. Under PBC-U, the customer **under-invests** in effort of failure reduction, i.e., $\lambda_c^* > \lambda_{sc}^*$.*

(2) *The customer has a surplus under T&M, if $\lambda_0 < \frac{2\sqrt{r(r+12K\mu^2)}-r}{6K\mu}$, and under PBC-U, if $\lambda_0 > \mu - \frac{r}{2K\mu}$. Otherwise, $\pi_c^* = 0$.*

(3) *For the supplier's profit, $\pi_s^*(PBC\text{-}P)$ is the highest, and $\pi_s^*(PBC\text{-}P) = \Pi_{sc}^*$. Furthermore, if $\lambda_0 > \frac{\mu}{2} + \frac{r}{2K\mu}$, $\pi_s^*(T\&M) > \pi_s^*(PBC\text{-}U)$. If $\lambda_0 \leq \frac{\mu}{2} + \frac{r}{2K\mu}$, $\pi_s^*(T\&M) \leq \pi_s^*(PBC\text{-}U)$.*

The proof can be found in Appendix H.

As shown in Figure 4.3, under T&M, the optimal failure rate is lower than PBC-P (also the FB). Under PBC-U, the optimal failure rate is higher than PBC-P (also the FB). Under T&M, the customer has to pay the service fee for each repair. Hence, she would like to have lower failure rate so as to cut down the service cost. In a decentralized setting, the customer tends to over-invest in reducing product failure rate. The situation is reversed under PBC-U. Under PBC-U, the supplier does not receive payment from the customer if the product is down. Intuitively, at the customer's end, she has no direct inventive to reduce the product failure rate because it seems the supplier's duty to shorten product downtime. Our model confirms this argument. Lemma 4.8 states that if $\lambda_0 < \mu - \frac{r}{2K\mu}$, $\lambda_c^* = \lambda_0$. However, we also notice that the customer can make a failure reduction investment if $\lambda_0 > \mu - \frac{r}{2K\mu}$. This is because if this condition holds, the customer can get a surplus by exerting effort. Meanwhile, we have proved that even if the customer can lower the failure rate, $\lambda_c^* > \lambda_{sc}^*$, i.e., the customer's effort under PBC-U is lower than the FB.

Similar to the risk aversion setting, the supplier can achieve the FB solution under PBC-P by setting $\kappa^* = 0$. Generally, efficiency loss is caused by "Double Marginalization". The internal payment that affects the players' decision makes the system less efficient. Under PBC-P, only κ is related to the customer's effort because penalty is based on downtime. The customer can receive more penalty from the supplier in the presence of a higher κ. However, if the supplier sets $\kappa = 0$, the customer's "moral hazard"-type action, i.e., not investing in failure rate reduction in this setting, will be eliminated. Consequently, PBC-P can achieve the FB solution and the supplier can get the highest profit.

If we look at the supplier's profit, as shown in Figure 4.4, PBC-P have the highest profit which equals the FB. The supplier's profit is decreasing in λ_0 under PBC. The supplier's profit under T&M is not monotonic. Because the supplier's revenue is generated from product failures, the supplier's profit is increasing in λ_c. However, the customer keeps reducing λ_0 as it increases. Moreover, the customer's effort is nonlinear in λ_0 (shown in Figure 4.3). If we compare T&M with PBC-U, an important result is that if failure rate is sufficiently large, i.e., $\lambda_0 > \frac{\mu}{2} + \frac{r}{2K\mu}$, T&M can be better than PBC-U for the supplier. This is because when λ_0 is low, the customer's over-investment is significantly high. A lower failure rate brings the supplier less revenue. As λ_0 increases, the acceleration of the customer's effort becomes lower. The supplier receives more revenue than the loss under PBC-U. Thus, the supplier's profit becomes higher than PBC-U.
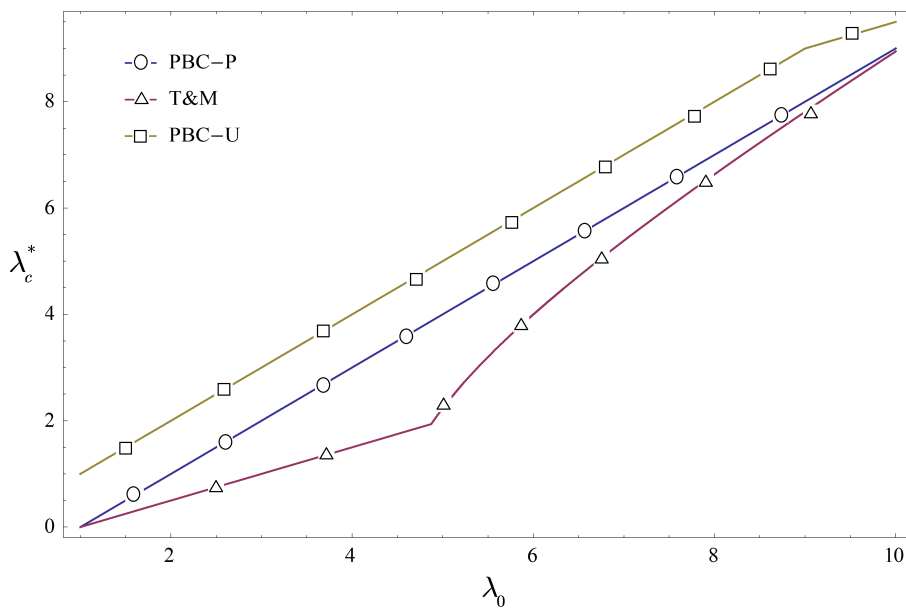
FIGURE 4.3: The optimal failure rate under contracts, exogenous μ. $r = 1000, K = 200, \mu = 20$.
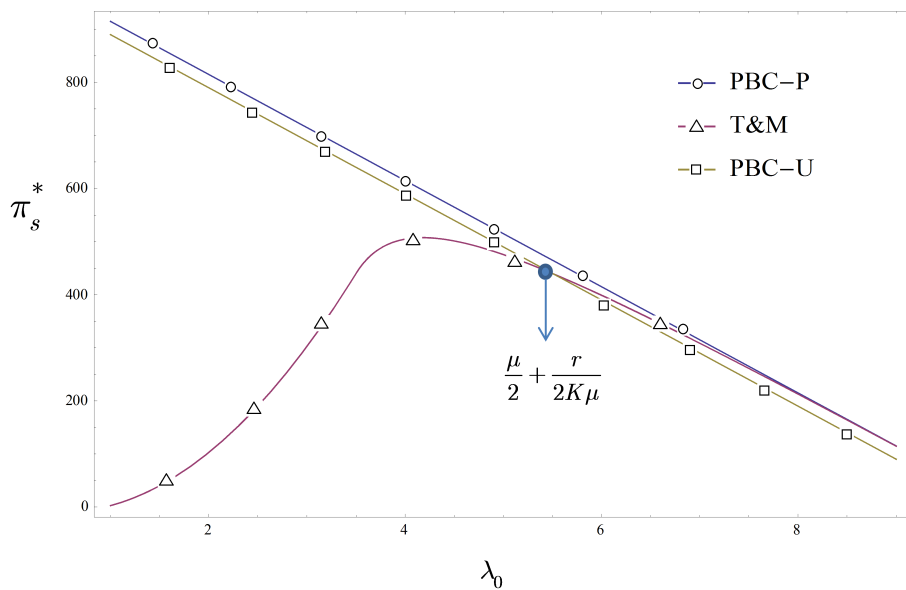

FIGURE 4.4: The supplier's profit at equilibrium under contracts, exogenous μ. $r = 1000, K = 100, \mu = 10$.

4.5.2 Endogenous μ

In this section, we incorporate capacity μ as an endogenous decision variable for the supplier. First of all, we have the solution for the FB case as summarized in Table 4.3. The effort of failure reduction depends on the level of capacity. We have $\partial \Delta \lambda / \partial \mu^* < 0$, i.e., level of effort is decreasing in capacity. Whether to invest in capacity depends on the initial failure rate. If λ_0 is large ($\lambda_0 > \frac{c\underline{\mu}^2}{r} + \frac{r}{2K\underline{\mu}}$) , it is optimal to increase capacity to a level which is defined by the equation $2cK\mu_{sc}^{*3} - 2rK\lambda_0\mu_{sc}^* + r^2 = 0$.

TABLE 4.3: Optimal solution of FB, endogenous μ.

	$\lambda_0 < \frac{c\underline{\mu}^2}{r} + \frac{r}{2K\underline{\mu}}$	$\lambda_0 > \frac{c\underline{\mu}^2}{r} + \frac{r}{2K\underline{\mu}}$
$\lambda_{sc}^*,\ \mu_{sc}^*$	$\lambda_{sc}^* = \lambda_0 - \frac{r}{2K\underline{\mu}},\ \mu_{sc}^* = \underline{\mu}$	$\lambda_{sc}^* = \lambda_0 - \dfrac{r}{2K\mu_{sc}^*},$ $2cK\mu_{sc}^{*3} - 2rK\lambda_0\mu_{sc}^* + r^2 = 0.$
Π_{sc}^*	$\frac{r(r+4K\underline{\mu}(\underline{\mu}-\lambda_0))}{4K\underline{\mu}^2}$	$r + \frac{r^2}{4K\mu_{sc}^{*2}} - \frac{r\lambda_0}{\mu_{sc}^*} - c(\mu_{sc}^* - \underline{\mu})$

Next, we explore how the suppliers' decision on capacity and the customer's decision on failure rate reduction jointly affect the contract choice in the decentralized setting. Unlike the exogenous μ case, the model's tractability drops a lot and we cannot obtain closed-form solutions. In order to simply the model, we assume $\underline{\mu} = 0$. This assumption implies that the supplier cannot keep an initial capacity level unchanged but has to invest in it. The simplification does not change the aim of the model which is to understand the joint impact of capacity and effort on supply chain contracting.

Lemma 4.10. *(1) Under T&M, if* $4 + \frac{\sqrt{cKr}}{(K(K\lambda_0-r))^{3/4}} - \frac{2K\lambda_0}{\sqrt{K(K\lambda_0-r)}} > 0$, $\mu^* = \sqrt{\frac{r}{cK}}(K(K\lambda_0^2 - r))^{1/4}$,
$\lambda_c^* = \sqrt{\lambda_0^2 - \frac{r}{K}}$.
If $4 + \frac{\sqrt{cKr}}{(K(K\lambda_0-r))^{3/4}} - \frac{2K\lambda_0}{\sqrt{K(K\lambda_0-r)}} < 0$, $\lambda_c^* = \frac{\lambda_0}{2} - \frac{r}{4K\mu^*}$, *and* μ^* *solves* $r^2 - 2Kr\lambda_0\mu^* + 4cK\mu^{*3} = 0$.

(2) Under PBC-P, $\lambda_c^* = \lambda_0 - \frac{r}{2K\mu^*}$, *and* μ^* *solves* $2cK\mu^{*3} - 2rK\lambda_0\mu^* + r^2 = 0$.

(3) Under PBC-U, if $K > \frac{c(r+\sqrt{cr\lambda_0})}{2\lambda_0(r-c\lambda_0)}$, $\lambda_c^* = \lambda_0$, $\mu^* = \sqrt{\frac{r\lambda_0}{c}}$. *If* $K < \frac{c(r+\sqrt{cr\lambda_0})}{2\lambda_0(r-c\lambda_0)}$, $\lambda_c^* = \frac{2K\mu^*(\lambda+\mu^*)-r}{4K\mu^*}$, *and* μ^* *solves* $4cK\mu^{*3} = (r - 2K\mu^{*2})(2K\mu^*(\mu^* - \lambda_0) - r)$.

The proof can be found in Appendix I.

Lemma 4.10 gives the equilibrium solutions of the decentralized setting. Under PBC-P, the optimal failure rate always depends on the service capacity. In addition, $\partial \lambda_c^* / \partial \mu^* = \frac{r}{2K\mu^{*2}} > 0$. Under T&M, solving the customer's problem, we have $\lambda_c^* = -\frac{r + p\mu^* - 2K\lambda_0\mu^*}{2K\mu^*}$. Then, $\partial \lambda_c^* / \partial \mu^* = \frac{r}{2K\mu^{*2}} > 0$. Under PBC-U, there are two cases. 1) $\lambda_c^* > \lambda_0$, and $\partial \lambda_c^* / \partial \mu^* = 0$. 2) $\lambda_c^* = \frac{2K\mu^*(\lambda_0 + \mu^*) - r}{4K\mu^*}$, and $\partial \lambda_c^* / \partial \mu^* = \frac{1}{4}(2 + \frac{r}{K\mu^{*2}})$. Although there are various cases under different contracts, we can see that the basic trend is that $\partial \lambda_c^* / \partial \mu^* \geq 0$. In other words, if the supplier sets a higher capacity, the customer will invest less in failure reduction. If we compare the decentralized setting to the FB, we have the main insights as given in Proposition 4.11.

Proposition 4.11. *(1) PBC-P achieve the FB results. The optimal penalty rate $\kappa^* = 0$.*

> *(2) Under T&M, the customer **over-invests** in failure rate reduction; the supplier **under-invests** in service capacity, i.e., $\lambda_c^*(T\&M) < \lambda_c^*(FB)$, $\mu^*(T\&M) < \mu^*(FB)$.*

> *(3) Under PBC-U, the customer **under-invests** in failure rate reduction; the supplier **over-invests** in service capacity, i.e., $\lambda_c^*(PBC - U) > \lambda_c^*(FB)$, $\mu^*(PBC - U) > \mu^*(FB)$.*

Like the exogenous μ case, PBC-P achieve the FB solution, and $\kappa^* = 0$. The reason is similar as explained in the previous section. $\kappa^* = 0$ eliminates the impact of the customer's unilateral action. By setting an independent lump sum w^*, the supplier can realize the highest profit while ensure the customer's participation. However, both T&M and PBC-U have efficiency loss. Similar to the exogenous μ case, T&M leads to over-investment while PBC-U leads to under-investment in failure reduction. The results for capacity is the other way round. Compared to the FB, the supplier's investment is less under T&M and more under PBC-U. Numerical exhibit can be seen from Figure 4.5 and Figure 4.6. The reverse situation of T&M and PBC-U can be explained by the extended finding from Lemma 4.10, i.e., $\partial \lambda_c^* / \partial \mu^* > 0$. A higher service capacity results in a lower effort level. Under PBC-P, since the customer under-invests in effort, i.e., λ_c^* is higher, the supplier will set up a higher service capacity, which causes over-investment. The same reasoning applies to the T&M.

Although we have derived the analytical results of contract comparison with respect to effort and capacity, we are not able to compare the supplier's profit under T&M and PBC-U analytically. Since PBC-P achieve the FB, we can conclude that the supplier's profit under PBC-P is the highest. We then compare T&M and PBC-U by numerical analysis. As shown in Figure 4.7, the supplier's profit is decreasing in λ under PBC-U whereas unmonotonic under T&M. Compared to the exogenous μ case (Figure 4.4), we have not seen T&M > PBC-U. On the contrary, the
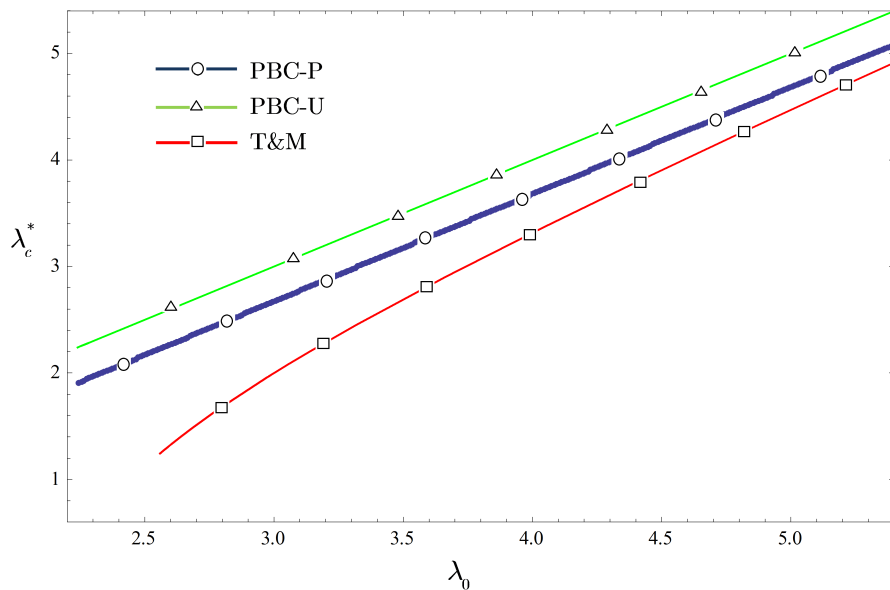
FIGURE 4.5: The optimal failure rate under contracts, endogenous μ. $r = 1000, K = 100, c = 1$.
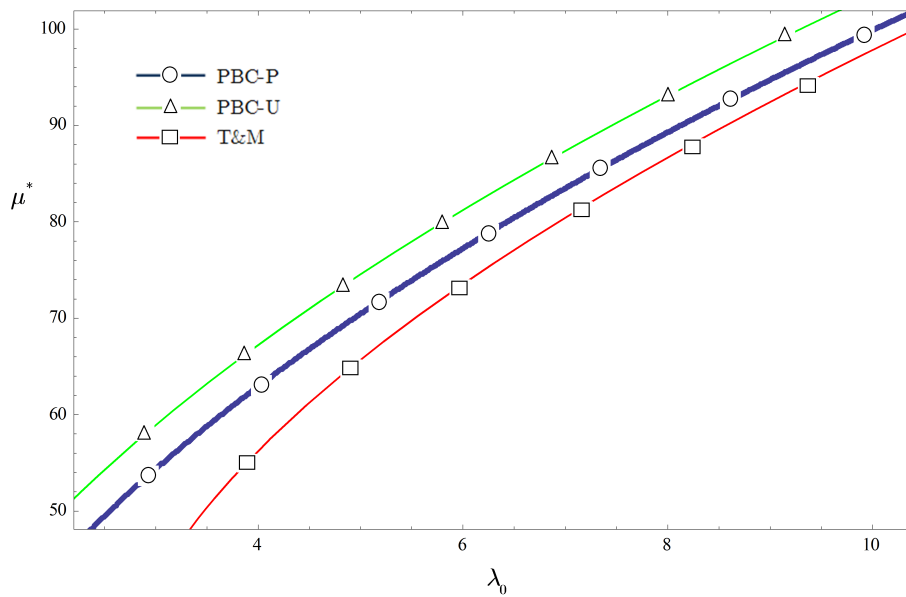
FIGURE 4.6: The optimal service capacity under contracts, endogenous μ. $r = 1000, K = 100, c = 1$.

supplier's profit under PBC-U is higher than T&M in the entire range of λ_0. To examine the convergence, we look at the extreme case when $\lambda_0 \to \infty$. We can prove that

$$\lim_{\lambda_0 \to \infty} \pi_s^*(\text{PBC-U}) - \pi_s^*(\text{T \& M}) =$$

$$\lim_{\lambda_0 \to \infty} (r - 2\sqrt{cr\lambda_0}) - 2(r - \frac{\sqrt{cr}(K(K\lambda_0^2 - r))^{1/4}}{\sqrt{K}} + \lambda_0(\sqrt{K(K\lambda_0^2 - r)} - K\lambda_0))$$

$$= 0.$$

Thus, T&M and PBC-U converges at infinity. Consequently, we can argue that in most of the cases, the supplier's profit under PBC-U is higher than T&M.
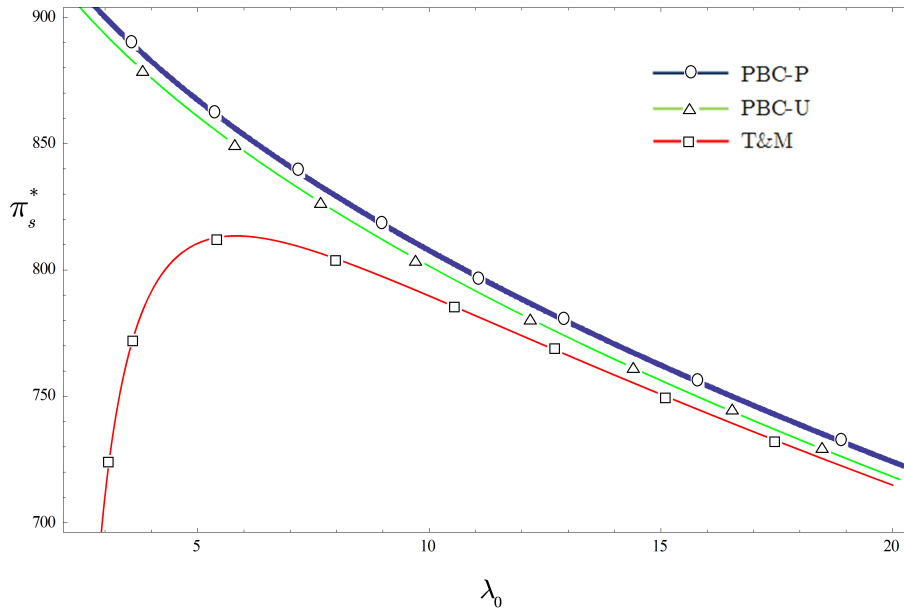
FIGURE 4.7: The supplier's profit at equilibrium under contracts, endogenous μ. $r = 1000, K = 100, c = 1$.

4.6 Extensions

4.6.1 Bargaining

In previous sections, we have assumed that the supplier is the contract offerer in the supply chain. In other words, the supplier acts as the leader in the business sequence. In this section, we discuss another setting under which the supplier and the customer bargain with each on the

contract terms. As such, the supplier and the customer make simultaneous actions and thus, we model the decision process based on a Nash Bargaining framework. First, we discuss the case when both supply chain members are risk neutral. The problem under contracts are as follows.

Under T&M,

$$\max_{p,\,\mu} \left[p\mathbb{E}(N) - c\mu\right]^{\alpha} \left[\mathbb{E}(r(1 - \sum\nolimits_{i=1}^{N} S_i|\lambda,\mu) - pN)\right]^{1-\alpha} \tag{4.10}$$

Under PBC-P,

$$\max_{w,\kappa,\mu} \left[w - \kappa\mathbb{E}(\sum\nolimits_{i=1}^{N} S_i|\lambda,\mu) - c\mu\right]^{\alpha} \left[\mathbb{E}(r(1 - \sum\nolimits_{i=1}^{N} S_i|\lambda,\mu) - w + \kappa\sum\nolimits_{i=1}^{N} S_i|\lambda,\mu)\right]^{1-\alpha} \tag{4.11}$$

Under PBC-U,

$$\max_{v,\mu} \left[v\mathbb{E}(\sum\nolimits_{i=1}^{N} S_i|\lambda,\mu) - c\mu\right]^{\alpha} \left[\mathbb{E}((r-v)(1 - \sum\nolimits_{i=1}^{N} S_i|\lambda,\mu))\right]^{1-\alpha} \tag{4.12}$$

In Equations (4.10)-(4.12), α and $1 - \alpha$ represents the bargaining power of the supplier and the customer respectively ($0 < \alpha < 1$). The supplier and the customer bargain over prices (p, w, κ, v) and service capacity μ under contracts. Nash products are formed by the supply chain members' respective profits. Nash equilibrium can be obtained by solving the solutions maximizing the Nash products.

Proposition 4.12. *(1) Under T&M, $p^* = \frac{r\alpha + \sqrt{cr\lambda}(1-2\alpha)}{\lambda}$, $\mu^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$.*

(2) Under PBC-P, $w^* = \kappa^* + r\alpha + \frac{\sqrt{c\lambda}}{\sqrt{r}}(r - 2r\alpha - \kappa^*)$, $\mu^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$. .

(3) Under PBC-U, $v^* = \frac{r(\sqrt{r}\alpha + \sqrt{c\lambda}(1-2\alpha))}{\sqrt{r}-\sqrt{c\lambda}}$, $\mu^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$.

(4) The supplier's and the customer's profits are equivalent under contacts. $\pi_s^* = \alpha(r - 2\sqrt{rc\lambda})$, $\pi_c^* = (1-\alpha)(r - 2\sqrt{rc\lambda})$.

The proof can be found in Appendix J.

If both the supplier and the customer are risk neutral, the optimal service capacity at equilibrium $\mu^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$ under all contracts. Unlike in the basic leader-follower model, where the

supplier takes all the profit, the customer and the supplier share the supply chain's profit ($r - 2\sqrt{cr\lambda}$) with proportion $1 - \alpha$ and α. Although the bargaining power determines the supply chain members' profits, different contracts do not play a role in the profit structure. In other words, even if prices are different, the supplier and the customer share the supply chain's profit in a same manner under different contracts. This result is in line with the basic model where the supplier moves first as the contract designer. We can conclude that risk neutrality equates contracts.

Next, we discuss the setting when the supplier becomes risk averse. Unfortunately, variance terms in the supply chain members' profit function make the model untractable for contract comparison. We conduct numerical experiments to observe how contracts change the supplier's and the customer's utility. From Figure 4.8, we can see that first, the supplier's utility is decreasing under PBC and is concave under T&M. Secondly, the supplier's utility under PBC-P is always higher than PBC-U and T&M. On the other hand, when λ is large, the supplier's utility under T&M is higher than PBC-U. Compared to Figure 4.2 and Figure 4.4, we have similar trend for contract comparison. By setting $\kappa^* = 0$, the supplier avoids revenue risk and generates the highest utility. When the variance terms can not be removed under T&M and PBC-U, the supplier prefers PBC-U when failure rate is low and prefers T&M when failure rate is high. If we look at the customer's profit, the result is reversed. Figure 4.9 tells us that the customer's utility is the lowest under PBC-P. Furthermore, under T&M, the utility is higher than PBC-U when the failure rate is low. Recalling the argument in Proposition 4.7 and Proposition 4.9, we know that the customer has no surplus under PBC-P, but can have a higher profit under T&M with a low failure rate and under PBC-U with a high failure rate. Under this bargaining setting, we have seen similar results. If the supplier is risk averse, PBC-P are preferable for the supplier, but result in the lowest profit for the customer. On the other hand, when the product failure rate is low, T&M are better for the customer and PBC-U generate a higher profit for the supplier. When the product failure rate is relatively high, the result is the other way round.

4.6.2 Exogenous κ in PBC-P

Thus far, we have shown that PBC-P turn out to be fixed-fee contracts when the supplier is risk averse, or the customer performs preemptive maintenance to reduce product failure rate. The basic logic is that $\kappa = 0$ can eliminate the supplier's profit variance and the moral hazard

FIGURE 4.8: The supplier's utility at equilibrium under Nash bargaining, risk averse supplier. $r = 100, \mu = 10, c = 1, \gamma_s = 0.01, \alpha = 0.5$.
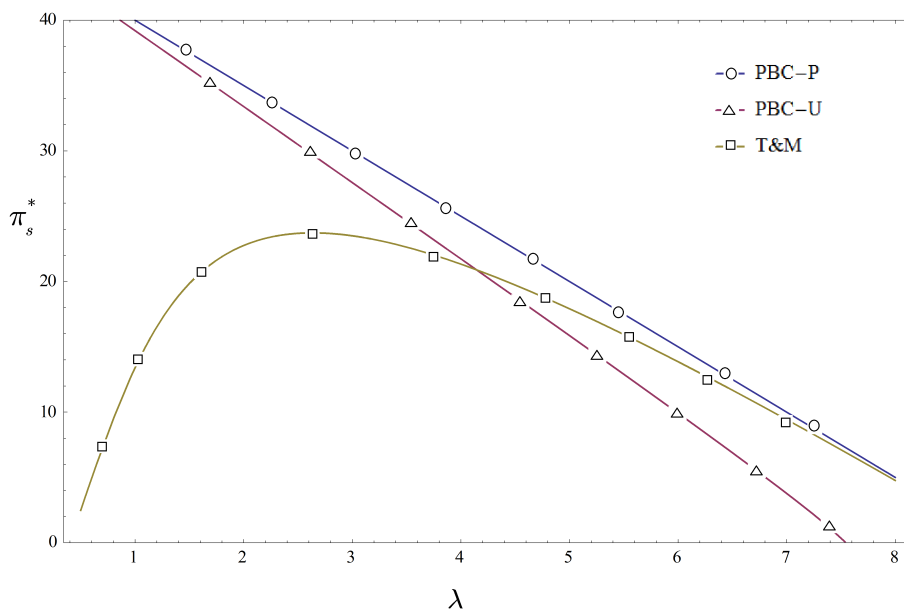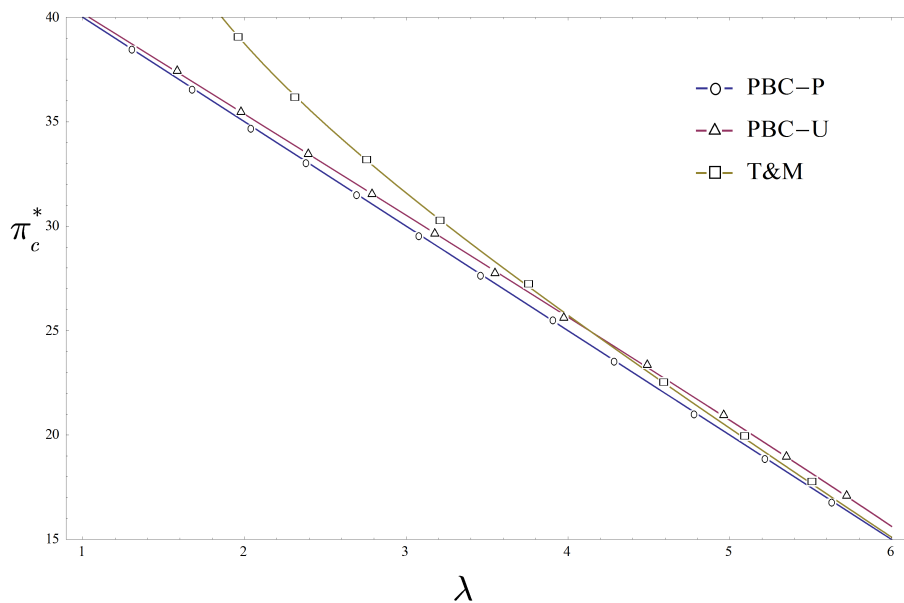

FIGURE 4.9: The customer's utility at equilibrium under Nash bargaining, risk averse supplier. $r = 100, \mu = 10, c = 1, \gamma_s = 0.01, \alpha = 0.5$.

environment for the customer. However, in practice, the customer may not accept 0 penalty even though the customer's reserved profit can be realized theoretically. In this circumstance, penalty rate κ can be specified by negotiation. In this section, we discuss the case when κ is exogenous under PBC-P.

Proposition 4.13. *(1) If both supply chain members are risk neutral, the supplier's decision on service capacity is not affected by κ. Equivalent profits can be realized across all contracts.*

*(2) If the supplier is risk averse, the optimal service capacity μ^* under PBC-P satisfies $\lambda(4\kappa^2\gamma_s + r\mu^*) - c\mu^{*3} = 0$ if $\lambda > \frac{c\underline{\mu}^3}{4\kappa^2\gamma_s + r\underline{\mu}}$, and the supplier **over-invests** in μ.*

The proof can be found in Appendix K.

If both the supplier and the customer are risk neutral, the penalty rate has no impact on the supplier's investment in capacity. The supplier can raise the lump sum w to overcome the increased penalty. Consequently, like the endogenous κ case, the supplier can derive equivalent profits under all contracts. However, if the supplier becomes risk averse, κ deteriorates the supplier's utility by forcing the supplier to over-invest in service capacity, as stated in Proposition 4.13. We have proved that $\kappa = 0$ is the best solution for the risk averse supplier. Hence, any increase in κ should be detrimental. If $\lambda < \frac{c\underline{\mu}^3}{4\kappa^2\gamma_s + r\underline{\mu}}$, we have $\pi_s^* = r - \frac{2\kappa^2\gamma_s\lambda}{\underline{\mu}^2} - \frac{r\lambda}{\underline{\mu}}$, and $\partial\pi_s^*/\partial\kappa = -\frac{4\kappa\gamma_s\lambda}{\underline{\mu}^2} < 0$, i.e., the supplier's profit is decreasing in κ. If $\lambda > \frac{c\underline{\mu}^3}{4\kappa^2\gamma_s + r\underline{\mu}}$, we can not obtain the closed-form solution. However, since κ increases the over-investment, we can infer that if κ is sufficiently large, the supplier's profit under PBC-P can be lower than T&M or PBC-U. For example, we can compare PBC-P with T&M over the case where there are closed-form solutions. Under T&M, if $\lambda < \frac{2r\gamma_s\underline{\mu}}{2r\gamma_s + \underline{\mu}}$, $\pi_s^*(T\&M) = \frac{\lambda}{4\gamma_s}$. Under PBC-P, if $\lambda < \frac{c\underline{\mu}^3}{4\kappa^2\gamma_s + r\underline{\mu}}$, $\pi_s^*(PBC-P) = r - \frac{2\kappa^2\gamma_s\lambda}{\underline{\mu}^2} - \frac{r\lambda}{\underline{\mu}}$. Then, if $\kappa > \sqrt{\frac{\underline{\mu}(4r\gamma_s(\underline{\mu}-\lambda)-\lambda\underline{\mu})}{8\gamma_s^2\lambda}}$, $\pi_s^*(PBC-P) < \pi_s^*(T\&M)$. This result explains the phenomenon that many non-performance-based contracts are still adopted by service providers. If penalty is forced to be incurred, the supplier may have lower profit and offers other types of contracts.

4.6.3 Risk averse Customer

In previous sections, we have assumed that the customer is risk neural in all cases. In this section, we relax this assumption and discuss the case when the customer becomes risk averse.

Let γ_c be the risk averse coefficient of the customer. The customer's profit variance is given in the following Lemma.

Lemma 4.14. *(1) Under T&M,* $\operatorname{var}(\pi_c) = 2r^2\lambda/\mu^2 + p^2\lambda + 2rp\lambda/\mu.$

(2) Under PBC-P, $\operatorname{var}(\pi_c) = 2(r - \kappa)^2\lambda/\mu^2.$

(3) Under PBC-U, $\operatorname{var}(\pi_c) = 2(r - v)^2\lambda/\mu^2.$

The proof can be found in Appendix L.

Proposition 4.15. *(1) If the supplier is risk neutral, under PBC-P, $w^* = r, \kappa^* = r$; under PBC-U, $v^* = r$. The supplier has equivalent profit under PBC-P and PBC-U while has lower profit under T&M, where the supplier over-invests in μ.*

(2) If the supplier is risk averse, under PBC-P, $\kappa^* = \frac{r\gamma_c}{\gamma_c + \gamma_s}$. Furthermore, $\pi_s(PBC - P) > \pi_s(T\&M)$.

The proof can be found in Appendix M.

If the supplier is risk neutral, risk aversion of the customer can be eliminated under PBC. In particular, the supplier sets $\kappa = r$ under PBC-P and $v = r$ under T&M. Consequently, the customer's profit variance is 0, which does not affect the supplier's capacity level. Then, the supplier can have the same profit level as the risk neutrality case. However, the service fee p under T&M cannot be adjusted in a manner which can eliminate the customer's profit variance. As such, the supplier has redundant capacity resulting in a lower profit. On the other hand, if both the supplier and the customer are risk averse, we have under PBC-P, penalty rate $\kappa^* = \frac{r\gamma_c}{\gamma_c + \gamma_s}$, i.e., the customer's risk aversion affects the penalty rate setting. Analytically, we can prove that the supplier has a higher profit under PBC-P than T&M. However, analysis under PBC-U becomes untractable. To sum up, if the customer becomes risk averse, PBC mostly have a higher efficiency than T&M.

4.6.4 Discounting Effect

In previous sections, we normalize the contract period to 1 and use a constant revenue rate r in the model. However, in practice, contract period can be as long as decades. In such a case, the

supplier and the customer should consider discounted cash flow when planning the contract. In this section, we investigate whether incorporating discounting would change the results.

Assuming the supply chain members consider two periods for discounting cash flows. Let d be the discounting factor. Then the supplier's problem[9] under T&M and PBC can be written as

$$\max_{p,\mu} \ p\lambda - c(\mu - \underline{\mu})$$

$$s.t.$$

$$r(1 - \frac{\lambda}{\mu})\left(\frac{1}{2} + \frac{1}{2d^2}\right) - p\lambda \geq 0,$$

$$\mu > \underline{\mu}.$$

and

$$\max_{w,\kappa,\mu} \ w - \kappa\frac{\lambda}{\mu} - c(\mu - \underline{\mu})$$

$$s.t.$$

$$r(1 - \frac{\lambda}{\mu})\left(\frac{1}{2} + \frac{1}{2d^2}\right) - (w - \kappa\frac{\lambda}{\mu}) \geq 0,$$

$$\mu > \underline{\mu}.$$

The solution of the optimal capacity under the two contracts is $\mu^* = \frac{\sqrt{1+d^2}}{\sqrt{2}d}\frac{\sqrt{r\lambda}}{\sqrt{c}}$ or $\mu^* = \underline{\mu}$. We can see that although the discounting factor d will affect the level of the service capacity, but its impact is in a same manner across contracts. Thus, the insights of contract comparison that we have derived from the previous sections will not change.

4.7 Conclusion

In this chapter, we study two commonly used product support contracts - Time & Material contracts and Performance-based contracts and explore the different incentives created by contracts under several decision-making settings. In our model, the supplier sets prices and invests in repair service capacity. Under T&M, the customer pays a service fee to the supplier for each repair. Under PBC, there are two payment frameworks - lump sum + penalty (PBC-P) and paying for uptime (PBC-U). Under PBC-P, the customer first pays a lump sum to the supplier

---

[9]Here we just look at the basic model with discounting.

to initiate the contract. Then, the supplier pays the customer a penalty based on the total downtime of the product during the contract period. Under PBC-U, the customer's payment is only based on product uptime. We find that under different circumstances, the supplier may prefer different contracts.

First, we develop a basic model to discuss the setting when both the supplier and the customer are risk neutral, i.e., decisions are made upon maximizing the expected profits. We find that under risk neutrality, equivalent maximal profits can be realized across contracts for the supplier. Under T&M, although the supplier only receives payment for each repair, which does not give obvious incentive to the supplier to invest in service capacity, the supplier still would like to set up a higher capacity to have a quicker repair, in that the customer will agree to pay a higher service fee, which gives the supplier a higher profit. This finding states that PBC may not be more effective than T&M in incentivizing the supplier to reduce product downtime.

Next, we discuss the setting when the supplier becomes risk averse to the variance of his revenue caused by the uncertainty in product failures. The results of our model show that if the penalty rate under PBC-P can be set by the supplier, the optimal value of the penalty rate is 0, and PBC-P then become fixed fee contracts. Because the variance of the supplier's profit is caused by the uncertainty in product downtime, setting a zero penalty eliminates the impact of product failures. On the other hand, the risk-free lump sum can satisfy the risk neutral customer's participation requirement. As such, PBC-P realize the highest profit for the supplier as the risk neutral case. However, under T&M and PBC-U, the impact of product failures can not be eliminated. We find that if the failure rate is relatively low, the supplier has a higher profit under PBC-U than T&M, whereas the customer has more surplus under T&M than PBC-U. If the product failure rate is high, the result is reversed.

Subsequently, we develop a moral hazard model to endogenize the product failure rate by incorporating the customer's preventive maintenance. Before the customer hands over product support to the supplier, the customer performs preventive maintenance to reduce the product failure rate. If the supplier's capacity is exogenous, PBC-P achieve the FB solution with penalty rate equals 0. On the other hand, the customer over-invests in the effort of failure reduction under T&M and under-invests in it under PBC-U. With regard to the supplier's profit, the supplier has a higher profit under T&M than PBC-U if the failure rate is high, and has a lower profit under T&M than PBC-U if the failure rate is low. If the supplier's capacity becomes endogenous, PBC-P also achieve FB. Under T&M, the customer over-invests in the effort of failure reduction, and the supplier under-invests in capacity. Under PBC-U,

the customer under-invests in failure reduction while the supplier over-invests in repair service capacity. Moreover, the supplier has the lowest profit under T&M.

Finally, we extend the model with more settings. If the supplier and the customer do not have a leader-follower relationship, but can bargain on the contracting terms, we obtain similar results. Under risk neutrality, the supplier and the customer have identical profits. Under risk aversion, PBC-P generate the highest profits, and the supplier prefers PBC-U while the customer prefers T&M if the product failure rate is low, and the choice is the opposite if the product failure is high. Then, we discuss the case when penalty rate is forced be to above 0 under PBC-P. We find that the penalty rate leads to over-investment in service capacity and decreases the supplier's profit. If the penalty rate is sufficiently large, the supplier's profit can be lower than T&M or PBC-U. At last, we discuss the case when the customer becomes risk averse. If the supplier is risk neutral, the supplier's profit under PBC is higher than T&M. If the supplier is also risk averse, some analysis becomes untractable. However, we can show that in most of the case, the supplier's profit under PBC-P is higher than T&M.

The assumptions in our model can be regard as shortcomings. For example, we assume there is a single product which needs maintenance by the supplier. We believe that out model can be easily extended to capture the multiple identical products setting. However, if the products are heterogenous with different failure rates and requires different types of repair capacities, the results are not straightforward to see, which can be one of the streams for future research. Another potential direction is to incorporate various contract durations. For short and long contract terms, the supplier may make different choices on the service contracts. Last but not least, the impact of information asymmetry can be explored by extending the model.

4.8 Appendix

A. Proof of Lemma 4.1. The Lagrangian of Equation (4.1) is

$$L(p, \mu, l_1, l_2) = p\lambda - c(\mu - \underline{\mu}) + l_1(r(1 - \frac{\lambda}{\mu}) - p\lambda) + l_2(\mu - \underline{\mu}).$$

Next, the K-K-T condition of the problem is

$$
\begin{cases}
\dfrac{\partial L(p, \mu, l_1, l_2)}{\partial p} = \lambda(1 - l_1) = 0, \\[2mm]
\dfrac{\partial L(p, \mu, l_1, l_2)}{\partial \mu} = -c + l_2 + \dfrac{l_1 r \lambda}{\mu^2} = 0, \\[2mm]
l_1\left(r\left(1 - \dfrac{\lambda}{\mu}\right) - p\lambda\right) = 0, \\[2mm]
l_2(\mu - \underline{\mu}) = 0, \\[2mm]
l_1 \geq 0, \ l_2 \geq 0.
\end{cases}
$$

Solving the above system of equations, we have the solutions: 1) $p = \frac{r - \sqrt{c}\sqrt{r}\sqrt{\lambda}}{\lambda}$, $\mu = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$, $l_1 = 1$, $l_2 = 0$. 2) $p = r\left(\frac{1}{\lambda} - \frac{1}{\underline{\mu}}\right)$, $\mu = \underline{\mu}$, $l_1 = 1$, $l_2 = c - \frac{r\lambda}{\underline{\mu}^2}$. Because the constraint $l_1 \geq 0$, $l_2 \geq 0$ have to be satisfied, in solution 2), we have to let $l_2 = c - \frac{r\lambda}{\underline{\mu}^2} > 0$, i.e., $\lambda < \frac{c\underline{\mu}^2}{r}$. For both solutions 1) and 2), we have the IR constraint

$$
r\left(1 - \frac{\lambda}{\mu^*}\right) - p^*\lambda = 0.
$$

So IR is binding.

For Equation (4.2), we have

$$
\frac{\partial^2 \pi_s(\text{PBC-P})}{\partial \mu^2} = -\frac{2k\lambda}{\mu^3} < 0.
$$

So the supplier's profit function is concave in μ. The K-K-T condition of Equation (4.2) is

$$
\begin{cases}
1 - l_2 = 0, \\[2mm]
\dfrac{(-1 + l_2)\lambda}{\mu} = 0, \\[2mm]
\dfrac{l_2 r \lambda + k(\lambda - l_2\lambda) + (-c + l_1)\mu^2}{\mu^2} = 0, \\[2mm]
l_1(\mu - \underline{\mu}) = 0, \\[2mm]
l_2(r(1 - \lambda/\mu) - w + k\lambda/\mu) = 0, \\[2mm]
l_1 \geq 0, \ l_2 \geq 0.
\end{cases}
$$

Solving this system of equations, we can have the results of PBC-P as in Table 4.2.

For Equation (4.3), we have

$$\frac{\partial^2 \pi_s(\text{PBC-U})}{\partial \mu^2} = -\frac{2v\lambda}{\mu^3} < 0.$$

So the supplier's profit function is concave in μ. The K-K-T condition of Equation (4.3) is

$$
\begin{cases}
\dfrac{(-1 + l_2)(\lambda - \mu)}{\mu} = 0, \\[2mm]
- c + l_1 + \dfrac{l_2(r - v)\lambda}{\mu^2} + \dfrac{v\lambda}{\mu^2} = 0, \\[2mm]
l_1(\mu - \underline{\mu}) = 0, \\[2mm]
l_2((r - v)(1 - \lambda/\mu) = 0, \\[2mm]
l_1 \geq 0, \ l_2 \geq 0.
\end{cases}
$$

Solving this system of equations, we can have the results of PBC-U as in Table 4.2. □

B. Proof of Proposition 4.2. (1) From Table 4.1, we can see that if $\lambda > \frac{c\underline{\mu}^2}{r}$, $\mu^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}} > \underline{\mu}$. In addition, we have

$$\frac{\partial \mu^*}{\partial r} = \frac{\sqrt{\lambda}}{2\sqrt{c}\sqrt{r}} > 0,$$

$$\frac{\partial \mu^*}{\partial \lambda} = \frac{\sqrt{r}}{2\sqrt{c}\sqrt{\lambda}} > 0,$$

$$\frac{\partial \mu^*}{\partial c} = -\frac{\sqrt{r}\sqrt{\lambda}}{2c^{3/2}} < 0.$$

if $\lambda > \frac{c\underline{\mu}^2}{r}$, $\mu^* = \underline{\mu}$.

(2) Because IR is always binding, under T&M, we have

$$r(1 - \frac{\lambda}{\mu^*}) - p^*\lambda = 0.$$

Equivalently,

$$p^* = r(\frac{1}{\lambda} - \frac{1}{\mu^*}).$$

Then we have

$$\frac{\partial p^*}{\partial \mu^*} = \frac{r}{\mu^2} > 0.$$

Under PBC-P, w^* and κ^* are given as combinations, as shown in Table 4.2. Under PBC-U, $v^* = r$, so v^* is independent of μ^*.

(3) For all the cases, if $\lambda > \frac{c\mu^2}{r}$, $\pi_s^* = r - 2\sqrt{c}\sqrt{r}\sqrt{\lambda} + c\underline{\mu}$. Then, $\partial \pi_s^*/\partial \lambda = -\frac{\sqrt{c}\sqrt{r}}{\sqrt{\lambda}} < 0$. If $\lambda < \frac{c\mu^2}{r}$, $\pi_s^* = r(1 - \frac{\lambda}{\underline{\mu}})$. Then we have $\partial \pi_s^*/\partial \lambda = -\frac{r}{\underline{\mu}} < 0$.

(4) The expected product uptime is $1 - \frac{\lambda}{\mu^*}$. Since μ^* is the same under contracts, we can conclude that the uptime is equivalent under contracts, and so as the supplier's profit. $\square$

C. Proof of Lemma 4.3. (1) $\operatorname{var}(pN) = p^2 \operatorname{var}(N)$. Because N satisfies a Poisson distribution with parameter λ, we have $\operatorname{var}(pN) = p^2 \operatorname{var}(N) = p^2 \lambda$. (2) $\operatorname{var}(\kappa \sum_{i=1}^N S_i | \lambda, \mu) = \kappa^2 \operatorname{var}(\sum_{i=1}^N S_i | \lambda, \mu)$. Because $\sum_{i=1}^N S_i | \lambda, \mu$ satisfies a Compound Poisson distribution, we have $\operatorname{var}(\sum_{i=1}^N S_i | \lambda, \mu) = \mathbb{E}(N)\mathbb{E}(S_i^2)$. Because S_i satisfies a exponential distribution with parameter $1/\mu$, we have $\mathbb{E}(S_i^2) = 2/\mu^2$. So $\operatorname{var}(\sum_{i=1}^N S_i | \lambda, \mu) = \frac{2\lambda}{\mu^2}$, and $\operatorname{var}(\kappa \sum_{i=1}^N S_i | \lambda, \mu) = \frac{2\lambda\kappa^2}{\mu^2}$. (3) $\operatorname{var}[v(1 - \sum_{i=1}^N S_i | \lambda, \mu)] = v^2 \operatorname{var}(\sum_{i=1}^N S_i | \lambda, \mu) = \frac{2v^2\lambda}{\mu^2}$. $\square$

D. Proof of Lemma 4.4 and Lemma 4.5. The Lagrangian of Equation (4.4) can be written as

$$L(p, \mu, l_1, l_2) = p\lambda - c(\mu - \underline{\mu}) - \gamma_s p^2 \lambda + l_1(r(1 - \lambda/\mu) - pN) + l_2(\mu - \underline{\mu}).$$

1) $l_1 = 0, l_2 > 0$. We have

$$\frac{\partial L(p, \mu, l_1, l_2)}{\partial p} = \lambda - 2p\gamma_s\lambda,$$

$$\frac{\partial L(p, \mu, l_1, l_2)}{\partial \mu} = -c + l_1.$$

We have $p^* = \frac{1}{2\gamma_s}$, $\mu^* = \underline{\mu}$. Since $l_1 = 0$, IR is not binding, so

$$r(1 - \lambda/\mu) - p\lambda|_{p^* = \frac{1}{2\gamma_s}, \mu^* = \underline{\mu}} = r - \frac{\lambda}{2\gamma_s} - \frac{r\lambda}{\underline{\mu}} > 0$$

has to hold. Rearrange the above inequality, we have $\lambda < \frac{2r\gamma_s\underline{\mu}}{2r\gamma_s+\underline{\mu}}$.

2) $l_1 > 0, l_2 > 0$. Both constraints are binding, we have $p^* = r(\frac{1}{\lambda} - \frac{1}{\underline{\mu}})$, $\mu^* = \underline{\mu}$. Furthermore,

$$l_1 = 1 - \frac{2r\gamma_s}{\lambda} + \frac{2r\gamma_s}{\underline{\mu}}, \quad l_2 = c - \frac{r(2r\gamma_s(\lambda - \underline{\mu}) + \lambda\underline{\mu})}{\underline{\mu}^3}.$$

Then we have

$$\frac{2r\gamma_s\underline{\mu}}{2r\gamma_s + \underline{\mu}} < \lambda < \frac{2r^2\gamma_s\underline{\mu} + c\underline{\mu}^3}{2r^2\gamma_s + r\underline{\mu}}.$$

3) $l_1 > 0, l_2 = 0$. IR is binding, i.e, $r(1 - \lambda/\mu) - p\lambda = 0$. We have $p^* = r(\frac{1}{\lambda} - \frac{1}{\mu^*})$. We have

$$\left.\frac{\partial \pi_s}{\partial \mu}\right|_{p=r(\frac{1}{\lambda} - \frac{1}{\mu})} = \frac{2r^2\gamma_s(\lambda - \mu) + r\lambda\mu - c\mu^3}{\mu^3}.$$

So μ^* solves $2r^2\gamma_s(\lambda - \mu^*) + r\lambda\mu^* - c\mu^{*3} = 0$.

4) $l_1 = 0, l_2 = 0$. We have $\frac{\partial L(p,\mu,l_1,l_2)}{\partial p} = \lambda - 2p\gamma_s\lambda$, $\frac{\partial L(p,\mu,l_1,l_2)}{\partial \mu} = -c$. There are no feasible solutions. Summarizing the cases of 1)-4), we have the conditions and solutions for T&M as shown in Lemma 4.4 and Lemma 4.5. Similarly, we can have the solutions for PBC-P and PBC-U by solving Equations (4.5) and (4.6).   □

E. Proof of Proposition 4.6. (1) From Lemma 4.1, we know that under T&M, $\mu_{RN}^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$ if $\lambda > \frac{c\underline{\mu}^2}{r}$. Otherwise, $\mu_{RN}^* = \underline{\mu}$. From Lemma 4.4, we have if $\lambda > \frac{2r^2\gamma_s\underline{\mu}+c\underline{\mu}^3}{2r^2\gamma_s+r\underline{\mu}}$, μ_{RA}^* satisfies $2r^2\gamma_s(\lambda - \mu_{RA}^*) + r\lambda\mu_{RA}^* - c\mu_{RA}^{*3} = 0$. Otherwise, $\mu_{RA}^* = \underline{\mu}$. First,

$$\frac{2r^2\gamma_s\underline{\mu} + c\underline{\mu}^3}{2r^2\gamma_s + r\underline{\mu}} - \frac{c\underline{\mu}^2}{r} = \frac{2\gamma_s\underline{\mu}(r - c\underline{\mu})}{2r\gamma_s + \underline{\mu}} > 0.$$

Then if $\frac{c\underline{\mu}^2}{r} < \lambda < \frac{2r^2\gamma_s\underline{\mu}+c\underline{\mu}^3}{2r^2\gamma_s+r\underline{\mu}}$, $\mu_{RA}^* < \mu_{RN}^*$. Next, if $\lambda > \frac{2r^2\gamma_s\underline{\mu}+c\underline{\mu}^3}{2r^2\gamma_s+r\underline{\mu}}$, μ_{RA}^* is the fixed point of the equation $2r^2\gamma_s(\lambda - \mu_{RA}^*) + r\lambda\mu_{RA}^* - c\mu_{RA}^{*3} = 0$. Rearranging the equation, we have $2r^2\gamma_s(\lambda - \mu_{RA}^*) + r\lambda\mu_{RA}^* = c\mu_{RA}^{*3}$. So μ_{RA}^* is the intersecting point of function $c\mu^3$ and $2r^2\gamma_s(\lambda - \mu) + r\lambda\mu$. If $\gamma_s = 0$, we have $\mu_{RA}^* = \sqrt{r\lambda/c} = \mu_{RN}^*$. Thus μ_{RN}^* is the intersecting point of $c\mu^3$ and $r\lambda\mu$. If we compare the slope the two lines, we have

$$\partial(r\lambda\mu)/\partial\mu = r\lambda,$$
$$\partial(2r^2\gamma s(\lambda - \mu) + r\lambda\mu)/\partial\mu = r\lambda - 2r^2\gamma_s.$$

If we let $r\lambda\mu = 2r^2\gamma_s(\lambda - \mu) + r\lambda\mu$, we have $\mu = \lambda$. So $\forall \mu > \lambda$, $r\lambda\mu > 2r^2\gamma_s(\lambda - \mu) + r\lambda\mu$. Since $r\lambda\mu$ and $c\mu^3$ insects at $\mu_{RN}^* = \sqrt{r\lambda/c}$, the intersection point of $2r^2\gamma_s(\lambda - \mu) + r\lambda\mu$ and $c\mu^3$, i.e., $\mu_{RA}^* < \mu_{RN}^*$, as shown in Figure 4.10.

(2) Under PBC-P, from Lemma 4.4, we know $\mu_{RA}^* = \sqrt{r\lambda/c} = \mu_{RN}^*$. (3) Under PBC-U, similarly, we can prove $\mu_{RA}^* > \mu_{RN}^*$.   □

F. Proof of Proposition 4.7. (1) Under PBC-P, $\kappa^* = 0, w^* = r(1 - \lambda/\mu^*)$. Substituting to the customer's profit function, we have $\pi_c^* = 0$. Under T&M, from Appendix D, we have
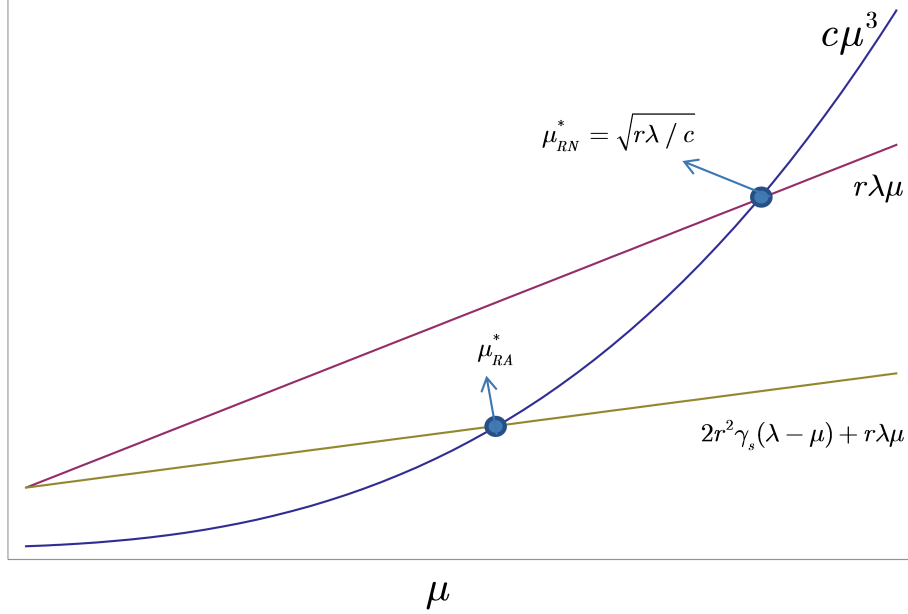
FIGURE 4.10: μ^*_{RA} and μ^*_{RN} under T&M.

if $\lambda < \frac{2r\gamma_s\mu}{2r\gamma_s+\mu}$, IR is not binding. $\pi^*_c = r - \frac{\lambda}{2\gamma_s} - \frac{r\lambda}{\mu} > 0$. Under PBC-P, if $\lambda > \frac{\mu^2}{4r\gamma_s+\mu}$, $v^* = \frac{\mu^2-\lambda\mu}{4\gamma_s\lambda}$, $\mu^* = \underline{\mu}$. In such a case, $\pi^*_s = (1 - \frac{\lambda}{\underline{\mu}})(r - \frac{(\underline{\mu}-\lambda)\underline{\mu}}{4\gamma_s\lambda}) > 0$.

(2) Because PBC-P achieves FB, we can conclude that the supplier's profit is the highest under PBC-P. Then, we compare PBC-U and T&M. Under T&M, cases separated by conditions are 1) $\lambda < \frac{2r\gamma_s\mu}{2r\gamma_s+\underline{\mu}}$, 2) $\frac{2r\gamma_s\mu}{2r\gamma_s+\underline{\mu}} < \lambda < \frac{2r^2\gamma_s\mu+c\mu^3}{2r^2\gamma_s+r\underline{\mu}}$, 3) $\lambda > \frac{2r^2\gamma_s\mu+c\mu^3}{2r^2\gamma_s+r\underline{\mu}}$. Under PBC-P, cases separated by conditions are 1) $\lambda > \frac{\mu^2}{\underline{\mu}+4r\gamma_s}$, 2) $\frac{c\mu^3}{4r^2\gamma_s+r\underline{\mu}} < \lambda < \frac{\mu^2}{\underline{\mu}+4r\gamma_s}$, 3) $\lambda < \frac{c\mu^3}{4r^2\gamma_s+r\underline{\mu}}$. Then, for T&M 1) vs. PBC-U 1), we have $\pi^*_s(T\&M) = \frac{\lambda}{4\gamma_s}$, $\pi^*_s(PBC\text{-}P) = \frac{(\underline{\mu}-\lambda)^2}{8\gamma_s\lambda}$. We have if $(\sqrt{2}-1)\underline{\mu} < \lambda < \frac{2r\gamma_s\mu}{2r\gamma_s+\underline{\mu}}$, $\pi^*_s(T\&M) > \pi^*_s(PBC\text{-}U)$. If $\frac{\mu^2}{4r\gamma_s+\underline{\mu}} < \lambda < (\sqrt{2}-1)\underline{\mu}$, $\pi^*_s(T\&M) < \pi^*_s(PBC\text{-}U)$. For T&M 1) vs. PBC-U 3), we have if $\lambda < \frac{c\mu^3}{4r^2\gamma_s+r\underline{\mu}}$, $\pi^*_s(T\&M) < \pi^*_s(PBC\text{-}U)$. Because PBC-U 2) > PBC-U-3), we have for T&M 1) vs. PBC-U 2), $\pi^*_s(T\&M) < \pi^*_s(PBC\text{-}U)$. For T&M 2) vs. PBC-U 1), we have if $\frac{2r\gamma_s\mu}{2r\gamma_s+\underline{\mu}} < \lambda < \frac{2r^2\gamma_s\mu+c\mu^3}{2r^2\gamma_s+r\underline{\mu}}$, $\pi^*_s(T\&M) > \pi^*_s(PBC\text{-}U)$. Because T&M 3) > T&M 2), we have for T&M 3) vs. PBC-U 1), if $\lambda > \frac{2r^2\gamma_s\mu+c\underline{\mu}^3}{2r^2\gamma_s+r\underline{\mu}}$, $\pi^*_s(T\&M) > \pi^*_s(PBC\text{-}U)$. For T&M 2) vs. PBC-U 2) and T&M 2) vs. PBC-U 3), conditions are not overlapping. To sum up, we have if $\lambda > (\sqrt{2}-1)\underline{\mu}$, $\pi^*_s(T\&M) > \pi^*_s(PBC\text{-}U)$. If $\lambda < (\sqrt{2}-1)\underline{\mu}$, $\pi^*_s(T\&M) < \pi^*_s(PBC\text{-}U)$.

(3) For the customer's profit, since IR constraint is always binding under PBC-P, the customer's profit under PBC-P is the lowest. For T&M and PBC-U, similarly, we can compare the customer's profit case by case and have the results stated in Proposition 4.7. $\square$

G. Proof of Lemma 4.8. (1) Under all contracts, we have $\partial^2 \pi_c / \partial \lambda_c^2 = -2K < 0$. Thus, the customer's profit is concave in λ_c. Under T&M, solving the customers problem, we have the customer's best response is $\lambda_c^* = -\frac{r + p^* \mu - 2K\lambda_0 \mu}{2K\mu}$. Substituting into the supplier's objective function, we have $\pi_s = p\lambda_0 - \frac{p(r + p\mu)}{2K\mu} - c(\mu - \underline{\mu})$. Then, $\partial^2 \pi_s / \partial p^2 = -1/K < 0$, i.e., π_s is concave in p. Similarly, we have under PBP-P, $\partial^2 \pi_s / \partial \kappa^2 = -\frac{1}{K\mu^2} < 0$. Under PBC-U, $\partial^2 \pi_s / \partial v^2 = -\frac{1}{K\mu^2} < 0$.

(2) Under T&M, the Lagrangian of the supplier's problem incorporating the customer's best response is

$$L(p, l_1) = p\lambda_0 - c\mu - \frac{p(r + p\mu)}{2K\mu} + l_1 \left(\frac{r^2 + p(p - 4K\lambda_0)\mu^2 + 2r\mu(p + 2K(\mu - \lambda_0))}{4K\mu^2} \right).$$

1) If $l_1 = 0$, we solve $\partial L(p, l_1) / \partial p = \lambda_0 - \frac{r + 2p\mu}{2K\mu} = 0$. We have $p^* = K\lambda_0 - \frac{r}{2\mu}$, $\lambda_c^* = \frac{\lambda_0}{2} - \frac{r}{4K\mu}$, $\pi_s^* = \frac{1}{8}(4K\lambda_0^2 + \frac{r^2}{K\mu^2} - \frac{4r\lambda_0}{\mu} - 8c\mu)$, and $\pi_c^* = r - \frac{3K\lambda_0^2}{4} + \frac{r^2}{16K\mu^2} - \frac{r\lambda_0}{4\mu}$. Since $\pi_c^* > 0$, we have $\lambda_0 < \frac{2\sqrt{r(r + 12K\mu^2)} - r}{6K\mu}$. 2) If $l_1 > 0$, i.e., IR is binding. Then we solve $\frac{r^2 + p(p - 4K\lambda_0)\mu^2 + 2r\mu(p + 2K(\mu - \lambda_0))}{4K\mu^2} = 0$. We have $p^* = 2K\lambda_0 - 2\sqrt{K(K\lambda_0^2 - r)} - \frac{r}{\mu}$. Subsequently, we have $\lambda_c^* = \sqrt{\lambda_0^2 - \frac{r}{K}}$, $\pi_s^* = 2\lambda_0(\sqrt{K(K\lambda_0^2 - r)} - K\lambda_0) + r(2 - \frac{\sqrt{K(K\lambda_0^2 - r)}}{K\mu}) - c\mu$, and $\pi_c^* = 0$. Similarly, we can have the equilibrium solutions of PBC-P and PBC-U. $\square$

H. Proof of Proposition 4.9. (1) Comparing the results in Lemma 4.8 with the solution of FB where $\lambda_{sc}^* = \lambda_0 - \frac{r}{2K\mu}$, we have under PBC-P, $\lambda_c^* = \lambda_{sc}^*$. Under T&M, if $\lambda_0 < \frac{2\sqrt{r(r + 12K\mu^2)} - r}{6K\mu}$, $\lambda_c^* = \frac{\lambda_0}{2} - \frac{r}{4K\mu} = \lambda_{sc}^*/2$. If $\lambda_0 > \frac{2\sqrt{r(r + 12K\mu^2)} - r}{6K\mu}$, $\lambda_c^* = \sqrt{\lambda_0^2 - \frac{r}{K}} < \lambda_{sc}^*$. Under PBC-U, if $\lambda_0 < \mu - \frac{r}{2K\mu}$, $\lambda_c^* = \lambda_0 > \lambda_{sc}^*$. If $\lambda_0 > \mu - \frac{r}{2K\mu}$, we have

$$\lambda_c^* - \lambda_{sc}^* = \frac{\lambda_0 + \mu}{2} - \frac{r}{4K\mu} - (\lambda_0 - \frac{r}{2K\mu}) = \frac{r + 2K\mu(\mu - \lambda_0)}{4K\mu} > 0.$$

(2) Under T&M, solving the supplier's problem, we have if $\lambda_0 < \frac{2\sqrt{r(r + 12K\mu^2)} - r}{6K\mu}$, IR is not binding. Similarly, under PBC-U, if $\lambda_0 > \mu - \frac{r}{2K\mu}$, IR is not binding, and $\pi_c^* = \frac{(r - 2K(\mu - \lambda_0)\mu)(r + 6K\mu(\mu - \lambda_0))}{16K\mu^2}$.

(3) Since PBC-P achieves the FB, the supplier's profit is higher than T&M and PBC-U. For T&M and PBC-U, we have under T&M, 1) If $\lambda_0 < \frac{2\sqrt{r(r + 12K\mu^2)} - r}{6K\mu}$, $\pi_s^*(\text{T \& M}) = \frac{1}{8}(4K\lambda_0^2 + \frac{r^2}{K\mu^2} - \frac{4r\lambda_0}{\mu} - 8c\mu)$. 2) If $\lambda_0 > \frac{2\sqrt{r(r + 12K\mu^2)} - r}{6K\mu}$, $\pi_s^* = 2\lambda_0(\sqrt{K(K\lambda_0^2 - r)} - K\lambda_0) + r(2 - \frac{\sqrt{K(-r + K\lambda_0^2)}}{K\mu}) - c\mu$. Under PBC-U, 1) if $\lambda_0 > \mu - \frac{r}{2K\mu}$,

$\pi_s^* = \frac{r^2 + 4Kr\mu(\mu - \lambda_0) + 4K\mu^2(K(\mu - \lambda_0)^2 - 2c\mu)}{8K\mu^2}$. 2) If $\lambda_0 < \mu - \frac{r}{2K\mu}$, $\pi_s^* = r(1 - \frac{\lambda}{\mu}) - c\mu$. For T&M 1) vs. PBC-U 1), if $\lambda_0 < \frac{\mu}{2} + \frac{r}{2K\mu}$, π_s^*(T&M) $< \pi_s^*$(PBC-U). If $\lambda_0 > \frac{\mu}{2} + \frac{r}{2K\mu}$, π_s^*(T&M) $> \pi_s^*$(PBC-U). For T&M 1) vs. PBC-U 2), if conditions of T&M 1) and PBC-U 2) are satisfied, π_s^*(T&M) $<$ π_s^*(PBC-U). For T&M 2) vs. PBC-U 1), if conditions T&M 2) and PBC-U 1) are satisfied, π_s^*(T&M) $> \pi_s^*$(PBC-U). For T&M 2) vs. PBC-U 2), if $\lambda_0 < \frac{\mu}{2} + \frac{r}{2K\mu}$, π_s^*(T&M) $< \pi_s^*$(PBC-U). If $\lambda_0 > \frac{\mu}{2} + \frac{r}{2K\mu}$, π_s^*(T&M) $> \pi_s^*$(PBC-U). $\quad\square$

I. Proof of Lemma 4.10. Under T&M, the Lagrangian of the supplier's problem is

$$L(p, \mu, l_1) = p\lambda_0 - c\mu - \frac{p(r + p\mu)}{2K\mu} + l_1\left(\frac{r^2 + p(p - 4K\lambda_0)\mu^2 + 2r\mu(p + 2K(\mu - \lambda_0))}{4K\mu^2}.\right)$$

1) $l_1 > 0$. IR is binding. We solve $\frac{r^2 + p(p - 4K\lambda_0)\mu^2 + 2r\mu(p + 2K(-\lambda_0 + \mu))}{4K\mu^2} = 0$. We have $p = 2K\lambda_0 - 2\sqrt{K(K\lambda_0^2 - r)} - \frac{r}{\mu}$. Then we solve for μ and l_1 by

$$\left.\frac{\partial L(p, \mu, l_1)}{\partial \mu}\right|_{p = 2K\lambda_0 - 2\sqrt{K(K\lambda_0^2 - r)} - \frac{r}{\mu}} = 0,$$

$$\left.\frac{\partial L(p, \mu, l_1)}{\partial p}\right|_{p = 2K\lambda_0 - 2\sqrt{K(K\lambda_0^2 - r)} - \frac{r}{\mu}} = 0.$$

We have $\mu^* = \frac{\sqrt{r}(K(K\lambda_0^2 - r))^{1/4}}{\sqrt{cK}}$, $l_1 = \frac{1}{2}(4 + \frac{\sqrt{cKr}}{(K(K\lambda_0^2 - r))^{3/4}} - \frac{2K\lambda_0}{\sqrt{K(K\lambda_0^2 - r)}})$. Let $l_1 > 0$, we have $4 + \frac{\sqrt{cKr}}{(K(K\lambda_0^2 - r))^{3/4}} - \frac{2K\lambda_0}{\sqrt{K(K\lambda_0^2 - r)}} > 0$. 2) $l_1 = 0$. We solve

$$\frac{\partial L(p, \mu, l_1)}{\partial \mu} = K\lambda_0 - \frac{r}{2\mu} = 0,$$

$$\frac{\partial L(p, \mu, l_1)}{\partial p} = -c + \frac{pr}{2K\mu^2} = 0.$$

We have $p^* = K\lambda_0 - \frac{r}{2\mu^*}$, μ^* solves $r^2 - 2Kr\lambda_0\mu^* + 4cK\mu^{*3} = 0$. Similarly, we can have the solution of PBC-U and PBC-P. $\quad\square$

J. Proof of Proposition 4.12. Under T&M, the FOC of the objective function w.r.t. p and μ is

$$
\begin{cases}
\dfrac{\lambda(r - p\lambda - \frac{r\lambda}{\mu})^{-\alpha}(p\lambda - c\mu)^{-1+\alpha}(r\alpha(\lambda - \mu) + \mu(p\lambda + c(-1+\alpha)\mu))}{\mu} = 0 \\[3mm]
\dfrac{(r - p\lambda - \frac{r\lambda}{\mu})^{-\alpha}(p\lambda - c\mu)^{-1+\alpha}(cr\mu(\lambda - 2\alpha\lambda + \alpha\mu) + p\lambda(r(-1+\alpha)\lambda - c\alpha\mu^2))}{\mu^2} = 0
\end{cases}
$$

Solving the system equations, we have $p^* = \frac{\sqrt{r}(\sqrt{r}\alpha + \sqrt{c}(1-2\alpha)\sqrt{\lambda})}{\lambda}$, $\mu^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$, and $\pi_s^* = \alpha(r - 2\sqrt{cr\lambda})$, $\pi_c^* = (1-\alpha)(r - 2\sqrt{cr\lambda})$. Furthermore, the determinant of the Hessian of the objective function is

$$
-\frac{2r(1-\alpha)^2\alpha\lambda^3(r - p\lambda - \frac{r\lambda}{\mu})^{-2\alpha}(p\lambda - c\mu)^{-2+2\alpha}(r(\lambda - \mu) + c\mu^2)^2}{\mu^4(r(\mu - \lambda) + p\lambda\mu)} < 0.
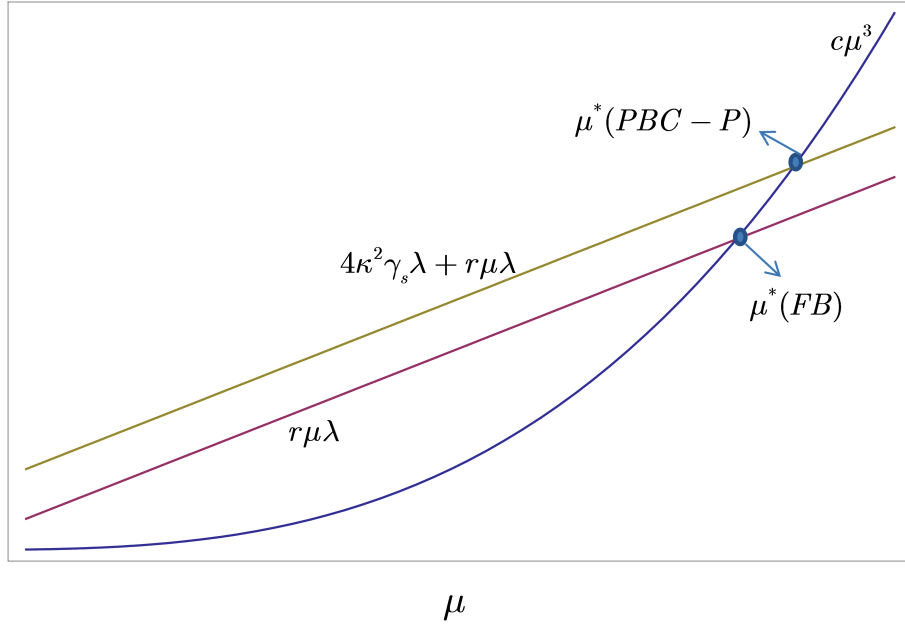$$

So (p^*, μ^*) maximizes the objective function. Similarly, we solve the problem under PBC-P and PBC-U and have the solutions stated in Proposition 4.12. $\square$

K. Proof of Proposition 4.13. (1) If the supplier's is risk-neutral, solving the supplier's problem under PBC-P, we have $w^* = r - \frac{(r-\kappa)\lambda}{\mu^*}$, and $\mu^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$ or $\mu^* = \underline{\mu}$, which are the same as the endogenous κ case.

(2) If the supplier is risk-averse, we have if $\lambda < \frac{c\underline{\mu}^3}{4\kappa^2\gamma_s + r\underline{\mu}}$, $\mu^* = \underline{\mu}$. If $\lambda > \frac{c\underline{\mu}^3}{4\kappa^2\gamma_s + r\underline{\mu}}$, μ^* solves $4\kappa^2\gamma_s\lambda + r\lambda\mu^* = c\mu^{*3}$. If $\kappa = 0$, solving $4\kappa^2\gamma_s\lambda + r\lambda\mu = c\mu^3$, we have $\mu^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$ which is the same as the FB and the endogenous κ case. If $\kappa > 0$, $4\kappa^2\gamma_s\lambda + r\lambda\mu > r\lambda\mu$. Then, the intersecting point of $4\kappa^2\gamma_s\lambda + r\lambda\mu$ and $c\mu^3$ is larger than $\mu = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$, as shown in Figure 4.11. So we can conclude that the supplier's capacity is higher than FB if $\kappa > 0$. $\square$

L. Proof of Lemma 4.14. (1) Under T&M,

$$
\begin{aligned}
\mathrm{var}(\pi_c) &= \mathrm{var}(r(1 - \sum\nolimits_{i=1}^{N} S_i) - pN) = \mathrm{var}(r\sum\nolimits_{i=1}^{N} S_i + pN) \\
&= r^2\,\mathrm{var}(\sum\nolimits_{i=1}^{N} S_i) + p^2\,\mathrm{var}(N) + 2rp\,\mathrm{cov}(\sum\nolimits_{i=1}^{N} S_i, N) \\
&= r^2 2\lambda/\mu^2 + p^2\lambda + 2rp\left(E(N\sum\nolimits_{i=1}^{N} S_i) - \lambda^2/\mu\right) \\
&= r^2 2\lambda/\mu^2 + p^2\lambda + 2rp\left(E(N^2)E(S_i) - \lambda^2/\mu\right) \\
&= r^2 2\lambda/\mu^2 + p^2\lambda + 2rp\left((\lambda + \lambda^2)/\mu - \lambda^2/\mu\right) \\
&= 2r^2\lambda/\mu^2 + p^2\lambda + 2rp\lambda/\mu
\end{aligned}
$$

FIGURE 4.11: μ_{FB}^* and μ_{PBC-P}^*, exogenous κ.

(2) Under PBC-P,

$$
\text{var}(\pi_c) = \text{var}(r(1 - \sum_{i=1}^{N} S_i) - w + \kappa \sum_{i=1}^{N} S_i) = \text{var}((\kappa - r) \sum_{i=1}^{N} S_i)
$$
$$
= 2(\kappa - r)^2 \lambda / \mu^2
$$

(3) Under PBC-U, $\text{var}(\pi_c) = \text{var}((r - v)(1 - \sum_{i=1}^{N} S_i)) = 2(\kappa - v)^2 \lambda / \mu^2$ $\square$

M. Proof of Proposition 4.15. (1) Under T&M, solving the supplier's problem, we have if $\lambda \leq \frac{c\underline{\mu}^2}{r}$, $p^* = \frac{-2r\gamma_c \lambda \underline{\mu} - \lambda \underline{\mu}^2 + \sqrt{\lambda \underline{\mu}^2 (-4r^2\gamma_c^2 \lambda + 4r\gamma_c \underline{\mu}^2 + \lambda \underline{\mu}^2)}}{2\gamma_c \lambda \underline{\mu}^2}$, $\mu^* = \underline{\mu}$. If $\lambda > \frac{c\underline{\mu}^2}{r}$, we have $p^* = \frac{-2r\gamma_c \lambda \mu^* - \lambda \mu^{*2} + \sqrt{\lambda \mu^{*2} (-4r^2\gamma_c^2 \lambda + 4r\gamma_c \mu^{*2} + \lambda \mu^{*2})}}{2\gamma_c \lambda \mu^{*2}}$, μ^* solves $\frac{2r^2\gamma_c \lambda^2 \mu^*}{\sqrt{(4r\gamma_c + \lambda)\mu^{*2}\lambda - 4r^2\gamma_c^2\lambda^2}} + r\lambda \mu^* = c\mu^{*3}$. If $\gamma_c = 0$, solving the equation, we have $\mu^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$, which is the FB solution. If $\gamma_c > 0$, the intersecting point of $\frac{2r^2\gamma_c \lambda^2 \mu}{\sqrt{(4r\gamma_c + \lambda)\mu^2\lambda - 4r^2\gamma_c^2\lambda^2}} + r\lambda \mu$ and $c\mu^3$ is larger than that with $r\lambda \mu$, i.e., $\mu^*(T\&M) > \mu^*(FB)$.

Under PBC-P and PBC-U, solving the supplier's problem, we have if $\lambda > \frac{c\underline{\mu}^2}{r}$, $\mu^* = \frac{\sqrt{r}\sqrt{\lambda}}{\sqrt{c}}$, $w^* = r$, $\kappa^* = r$, $v^* = r$. If $\lambda \leq \frac{c\underline{\mu}^2}{r}$, $\mu^* = \underline{\mu}$, $w^* = r$, $\kappa^* = r$, $v^* = r$. Then, we can see that the supplier's capacity achieves the FB under PBC-P and PBC-U. Compared to T&M, where the supplier over-invests in capacity, the supplier's profit under PBC is higher than T&M.

(2) If the supplier becomes risk-averse, under T&M, we have if $\lambda < \frac{4r\gamma_s{}^2\underline{\mu}^2}{8r^2\gamma_c\gamma_s{}^2+4r\gamma_s(\gamma_c+\gamma_s)\underline{\mu}+(\gamma_c+2\gamma_s)\underline{\mu}^2}$, $p^* = \frac{1}{2\gamma_s}$, $\mu^* = \underline{\mu}$, and $\pi_s^* = \frac{\lambda}{4\gamma_s}$. For other cases, we do not have closed-form solutions. If we look at the supplier's profit function, we have $\partial^2\pi_s/\partial p^2 = -2\gamma_s\lambda, \partial\pi_s/\partial\mu = -c$. So π_s is decreasing in μ and concave in p. Thus, solving the FOC w.r.t. p, we have $p^* = \frac{1}{2\gamma_s}$, and $\mu^* = \underline{\mu}$ maximize π_s among all cases. In other words, without considering the conditions, $\pi_s^*|_{p^*=\frac{1}{2\gamma_s},\ \mu^*=\underline{\mu}} = \frac{\lambda}{4\gamma_s}$ is the highest profit for the supplier.

Under PBC-P, we have the closed-form solution if $\lambda < \frac{(\gamma_c+\gamma_s)c\underline{\mu}^3}{r(4r\gamma_c\gamma_s+(\gamma_c+\gamma_s)\underline{\mu})}$, $\kappa^* = \frac{r\gamma_c}{\gamma_c+\gamma_s}, \mu^* = \underline{\mu}$, and $\pi_s^* = r(1-\frac{\lambda}{\underline{\mu}} - \frac{2r\gamma_c\gamma_s\lambda}{(\gamma_c+\gamma_s)\underline{\mu}^2})$. Because the supplier does not invests in capacity, π_s^* is lower than the other case when $\mu^* > \underline{\mu}$. If we compare the highest $\pi_s^*(T\&M)$ and the lowest $\pi_s^*(PBC-P)$, we have if the conditions for the two cases hold, $\pi_s^*(PBC-P) > \pi_s^*(T\&M)$. Then, we can conclude for other cases, the supplier's profit under PBC-P is higher than T&M as well. $\quad\square$

Chapter 5

Conclusion and Future Research

Various types of contracts for after-sales service and product support are employed by companies. Traditionally, Warranty and Time & Material contracts (T&M) are commonly used for after-sales service. More recently, Performance-based service contracts have become increasingly popular in industry. Different contracts not only offers multiple choices for the customer, but also require different business models for the manufacturer and service providers. Therefore, companies need to learn more about the impact of different contracts on their operations management framework and make wise decisions accordingly. In this thesis, we develop theoretical frameworks of how to model and analyze service contracts and come up with optimal solutions for several important issues in after-sales and product support management.

5.1 Conclusion

In Chapter 2, we study joint contracting for products and the after-sales service between a customer and a supplier while the supplier simultaneously has to plan the spare part inventory, as the part may expire. The customer makes decisions on how many products to buy to satisfy the external demand. At the same time, the customer must take the after-sales service into account since only working products can generate revenue. On the supplier's side, when selling the products, spare parts need to be well planned to service the product after the sale, because those parts may expire and become difficult to reacquire once stockouts occur. We establish game-theoretic models for the cases of Warranty + Transaction-based contracts (W+T) and

Performance-based contracts (PBC) and make comparisons. We discuss different penalty terms under PBC - penalty based on number of spare parts stockouts and penalty based on the customer's lost revenue. We also discuss the cases when the quantity of Lifetime-buy spare parts are exogenous and endogenous. The main findings of Chapter 2 are as follows.

- Under PBC, if a penalty is incurred on spare parts stockouts, the penalty rate can be set in combination with the product price; if a penalty is incurred on the customer's lost revenue, the optimal PBC become fixed fee contracts, i.e., penalty rate should be 0. If penalty is exogenously given, the penalty rate on the customer's lost revenue increases spare parts inventory and product availability, but reduces supply chain members' profits.

- Under W+T, increasing duration of warranty can lead to higher product availability, but may result in different impact on profits. If the cost of reacquisition of spare parts is low, a longer warranty will generate more profits even if spare parts stockouts occur during warranty. As the reacquisition cost increases, profits (of the supplier, the customer, and the supply chain) can be reduced.

- Comparing W+T with PBC, we find that if spare parts are sufficient and no stockouts occur during warranty, PBC and W+T can result in equal profits. However, if spare parts stockouts occur during warranty, the cost of reacquisition plays a role. The supplier should choose W+T if the reacquisition cost is low and choose PBC if the reacquisition cost is high.

In Chapter 3, we study the supply chain coordination problems under the outcome-based contracts, when the OEM is responsible for the availability of the final product, while the supplier's effort also impacts the final product's performance. The OEM can not directly control the supplier's quality improvement activity, but only use contracts to induce the supplier to exert effort on reducing the failure rate of his part. We discuss three types of contracts - price-only contracts, repair cost sharing contracts, and penalty sharing contracts between the OEM and the supplier, and propose a penalty sharing contract as a channel coordination mechanism. First, we discuss the setting where the OEM has the capacity to repair both of the parts, and the repair capacity level can be exogenous or endogenous. Then, we study the individual capacity setting, where the supplier and the OEM take care of the service of their own parts. Next, we explore the setting when the final product is nonseparable, i.e., once the product is assembled, it is not possible to identify the root cause of product failure. Finally, we discuss the impact

of information asymmetry on the choice of contracts. The main findings are summarized as follows.

- If the repair capacity is exogenous, price-only contracts cannot induce the supplier to exert any quality improvement effort; repair cost sharing contracts cause under-investment in the supplier's effort. If repair capacity is endogenous, price-only contracts and repair cost sharing contracts lead to over-investment in service capacity. For both settings, penalty sharing contracts achieve the First Best solution for the supply chain.

- If the repair capacity becomes decentralized, paying a service fee for each repair can not incentive the supplier to improve part quality. However, sharing product downtime penalty can induce the supplier to exert the failure rate reduction effort to the optimal level.

- If the final product is nonseparable, the supplier and the OEM share the repair cost or penalty with a predefined proportion. In such a case, repair cost sharing contracts still result in under-investment in the supplier's effort level, and penalty sharing contracts can coordinate the supply chain.

Furthermore, we also discuss the case when the quality of the supplier's part is unknown information for the OEM. We show that although the efficiency loss due to information asymmetry cannot be solved completely, the OEM has the highest profit under penalty sharing contracts compared the other two.

In Chapter 4, we systematically compare Time & Material contracts with two types of Performance-based contracts - PBC with downtime penalty (PBC-P) and PBC with uptime payment (PBC-U), and investigate how decision-makers' risk appetite and the customer' effort of failure prevention affect the choice of contracts. First, we study the contracting problem when both the supplier and the customer are risk neutral. Then, we introduce risk aversion to the supplier. Next, we discuss the case when the customer performs preventive maintenance on the product to reduce it's failure rate. Finally, we extend the model by adding cases when the customer and the supplier bargain on contract prices, and when the customer becomes risk averse. The main findings are as follows.

- If both the customer and the supplier are risk neutral, equivalent product availability and profits can be achieved under PBC and T&M, which infers that product availability under T&M may not be lower than PBC.

- If the supplier becomes risk averse, the optimal contract is a fixed fee contract, i.e., PBC-P with 0 penalty rate. For PBC-U and T&M, the preference of the supplier and the customer are opposite. If product failure rate is high, the supplier has a higher profit under T&M, whereas the customer has a surplus under PBC. If product failure rate is high, the result is reversed.

- If the customer does preventive maintenance before the supplier taking over the product support, PBC-P still dominate other contacts (with 0 penalty rate). T&M cause over-investment and PBC-U cause under-investment for the customer's failure reduction effort. Moreover, the supplier has the lowest profit under T&M.

- If the penalty rate under PBC-P is exogenous, the supplier's profit decreases in the penalty rate. On the other hand, if the customer becomes risk averse, PBC is more preferable to the supplier.

5.2 Future Research

Sevitization and business model innovation in manufacturing and service operations create many research opportunities in OM. In this thesis, we study several service contracting issues concerning spare parts and capacity management, service pricing, and supply chain coordination. However, there is ample room for future research. Next, we give a brief discussion on some potential directions.

Performance-based contracting in networks. The models in this thesis mainly consider single customer / single supplier supply chains. However, as we have observed in practice, a manufacturer usually faces multiple customers with heterogenous products and demand. For example, under a short-term contract period, the customer would go for Transaction-based contracts, while under a long-term contract period, the customers may choose Performance-based contracts. Because different contracts may generate conflicting incentives, it becomes much more challenging for the manufacturer to make the best of common service capacity and resources to serve various customers under different contacts. In Economics literature,

papers like (Pavan and Calzolari, 2009; Attar et al., 2010), discuss multiple principals/agents moral hazard problems, which can be used to study Performance-based contracting in this circumstance.

Repeated interaction in service contracting. In this thesis, we only consider single-period contracting, where the customer and the supplier make one-shot decisions. However, in practice, the relationship between supply chain members can be long term and cover multi-periods. Repeated interaction and renegotiation are not rare to be seen. For example, in (Swinney and Netessine, 2009) , a two-period game model is built to study long-term contracts. As a matter of fact, the dynamics in the supply chain members' situation may vary in different periods, which may induce them to make different decisions. Applying theory and models of repeated games can be another direction for future research.

Performance-based contracting and Sustainability. In modern business, Sustainability is an important part of operations management. For both commercial and public sectors, people are expecting organizations to implement sustainable policies in the long run. As such, performance-based contracting applying Sustainability metrics is becoming increasingly popular. For example, the Department of Transportation of the USA is conducting PBC to achieve energy saving[1]. Moreover, as stated in (Drake and Spinler, 2013), Sustainability will be a continuous topic in both practice and theoretic research. Hence, performance-based contracting and Sustainability will be a promising field for future research.

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[1]http://www.dot.gov/mission/sustainability/performance-based-contracts

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Summary

Nowadays, service business contributes more than 70% of GDP in most developed countries. Even in traditional manufacturing sectors, the value of service has been explored dramatically by companies when they sell their products. Over the past decades, business model innovation in product and its service has been growing rapidly, especially for durable goods. Companies shift their strategies from selling physical products to delivering solutions and performance for customers. Within this context, the outcome-based service contracts, such as Performance-based Logistics and Power-by-the-Hour, have been developed in both public and commercial industry. At the same time, traditional service contracts such as Warranty and Time & Material contracts are still being used in many occasions. Under various business models, managing the after-sales service and product support becomes increasingly challenging.

In this thesis, we study several nontrivial service contracting problems concerning optimal design of contract terms, spare parts inventory and service capacity management, and service outsourcing control. We provide managerial insights for selecting the best service contract, choosing the right performance measurement, setting the optimal service resources (spare parts and repair capacity), and incentivizing the supplier to improve the profits of the supply chain.

In Chapter 2, we study joint contracting for products and the after-sales service under two types of contracts: Warranty + Transaction-based contracts (W+T) and Performance-based contracts (PBC) with different penalty terms. We also consider part obsolescence problem in the setting, where the spare parts inventory determines product availability. We formulate Stackelberg leader-follower models to capture the setting where the customer makes decision on product order quantity while taking the after-sales service into account; and the supplier sets the contract terms when the lifetime buy inventory affects product availability. The equilibrium solutions show that under PBC, penalty based on the customer's lost revenue leads to a lower

profit, and a longer warranty contract is better than PBC when the cost of part reacquisition is low.

In Chapter 3, we study service outsourcing management under an outcome-based contract for product support. In such a setting, the final product is formulated by parts which are manufactured by the Original Equipment Manufacturer (OEM) and the supplier. As a result, the quality of the final product is affected by the effort of the supplier and the OEM. Meanwhile, the OEM is responsible for the product support but cannot directly control the supplier's quality management activity. We formulate Principle-Agent models to fit the moral hazard setting and capture the supply chain members's decision under price-only contracts, repair cost sharing contracts, and penalty sharing contracts. We show that penalty sharing contracts lead to the first-best solutions which coordinate the supply chain, while price-only contracts and repair cost sharing contracts result in efficiency loss due to over-investment in service capacity and under-investment in part failure rate reduction. The results hold even when repair capacity is decentralized or the final product is nonseparable.

In Chapter 4, we study comparisons between Performance-based contracts (PBC) and Time & Material contracts (T&M) under various settings where the supplier makes decisions on designing contract terms and setting service capacity level. Under PBC, we discuss two types of payment framework - downtime penalty (PBC-P) and uptime payment (PBC-U). Under the former, the customer pays the supplier a lump sum and the supplier pays penalty to the customer based on product downtime; under the latter, the customer pays the supplier based on product uptime. We show that if the supply chain members are risk neutral, equivalent profits can be realized across contracts for the supplier. If the supplier becomes risk averse to the profit variance caused by uncertainty of product failures, PBC-P is the best contract for the supplier while T&M is better than PBC-U if the product failure rate is high. PBC-P also dominate when the customer exerts effort on reducing the product failure rate, yet T&M lead to the lowest profit in this occasion.

Finally, we give conclusions of the thesis and discuss potential directions for future research in Chapter 5. In sum, the research in this thesis contributes to the theoretical study in Contracting and Operations Management, and the findings provide useful insights for management of the after-sales service and product support in real business environment.

Samenvatting (Summary in Dutch)

Tegenwoordig omvat de dienstensector in de meeste ontwikkelde landen meer dan 70% van het BBP. De waarde van diensten bij de verkoop van producten is uitvoerig bekeken en onderzocht in veel industriesectoren. Gedurende de afgelopen decennia is het aantal innovaties in business modellen zowel voor producten als diensten sterk toegenomen, in het bijzonder voor duurzame goederen. Bedrijven verschuiven hun strategie van het verkopen van tastbare producten naar het leveren van oplossingen en prestaties aan de klant. Binnen deze context zijn uitkomst-gebaseerde dienstencontracten, zoals prestatie-gebaseerde logistiek en "Power-by-the-Hour", ontwikkeld in zowel de publieke als de commerciële sector. Tegelijkertijd worden traditionele dienstencontracten zoals garantie- en tijd- & materiaalcontracten nog steeds gebruikt in veel situaties. Bij een aantal business modellen wordt het beheren van de after-sales dienst- en productondersteuning een steeds grotere uitdaging.

In dit proefschrift bestuderen we ontwerp en vergelijking van service-contracten, met betrekking tot de optimale opzet van de contractvoorwaarden, het management van de reserve-onderdelenvoorraad en service capaciteit, en het beheersen van de service bij uitbesteding. We bieden management inzichten voor het selecteren van het beste service contract, het kiezen van de juiste prestatiemaatstaf, het bepalen van de optimale service resources (reserve onderdelen en reparatieca-paciteit), en het structureren van contractvoorwaarden voor de leverancier.

In hoofdstuk 2 bestuderen we de gezamelijke contractering voor producten en de after-sales diensten voor twee soorten contracten: Garantie + Transactie-gebaseerde contracten (War-ranty + Transaction, W+T) en Prestatie-gebaseerde contracten (PBC) met verschillende boete-voorwaarden. We beschouwen ook het onderdeelverouderingsprobleem in een situatie, waar de reserve-onderdelenvoorraad de productbeschikbaarheid bepaalt. We formuleren Stackelberg leider-volger modellen om de situaties te beschrijven, waar de klant beslist over product order hoeveelheid, waarbij hij de after sales diensten in zijn beslissing meeneemt en de leverancier de

contractvoorwaarden stelt in het geval dat de lifetime buy voorraad de productbeschikbaarheid beïnvloedt. De evenwichtsoplossingen tonen aan dat bij PBC, een penalty gebaseerd op de verloren omzet van de klant leidt tot een lagere winst, en een contract met langere garantie beter is dan PBC als de kosten van het opnieuw verwerven van onderdelen laag is.

In hoofdstuk 3 bestuderen we service outsourcing management voor een uitkomst-gebaseerd contract voor productondersteuning. We veronderstellen dat uiteindelijke product samengesteld wordt uit onderdelen die geproduceerd zijn door de "Original Equipment Manufacturer" (OEM) en de leverancier. Het gevolg is dat de kwaliteit van het uiteindelijke product beïnvloedt wordt door de inzet van de leverancier en de OEM. De OEM is verantwoordelijk voor de productondersteuning, maar heeft geen directe controle over de kwaliteitsmanagement-activiteiten van de leverancier. We formuleren Principal-Agent modellen die passen bij deze moral hazard situatie en beschrijven daarmee de beslissingen van de supply chain leden voor price-only contracten, repair cost sharing contracten en penalty sharing contracten. We laten zien dat penalty sharing contracten leiden tot de beste oplossingen waarmee de supply chain volledig gecoordineerd kan worden, terwijl price-only contracten en repair cost sharing contracten resulteren in efficiëntieverlies vanwege een teveel aan investeringen in service capaciteit en vanwege te lage investeringen in verbetering van het faalgedrag van onderdelen. De resultaten gelden ook als de reparatiecapaciteit gedecentraliseerd is.

In hoofdstuk 4 vergelijken we Prestatie-gebaseerde (PBC) en Tijd & Materiaalcontracten (T&M) voor situaties waarin de leverancier beslist over de opzet van contractvoorwaarden en het zetten van de hoogte van de service capaciteit. Voor PBC bediscussieren we twee soorten betaalframeworks - downtime penalty (PBC-P) en uptime payment (PBC-U). Bij PBC-P betaalt de klant de leverancier een hoofdsom en de leverancier betaalt een penalty aan de klant gebaseerd op de product downtime; voor PBC-U betaalt de klant de leverancier gebaseerd op de product uptime. We tonen aan dat als de supply chain leden risico-neutraal zijn, de leverancier dezelfde winsten kan verkrijgen voor alle soorten contracten. Als de leverancier risico-avers is met betrekking tot de winstvariantie veroorzaakt door de onzekerheid van productfalen, dan is PBC-P het beste contract voor de leverancier, terwijl T&M beter is dan PBC-U als de mate van productfalen hoog is. PBC-P domineert ook als de klant tracht de faalkans van de producten te verlagen, maar T&M leidt tot de laagste winst in dit geval.

Tenslotte trekken we conclusies voor dit proefschift en bediscussieren we mogelijke richtingen voor toekomstig onderzoek in hoofdstuk 5. Kort samengevat draagt dit proefschrift bij aan de theoretische kennis in Contracting and Operations Management, en de bevindingen bieden

nuttige inzichten voor management van after-sales diensten- en productondersteuning in echte bedrijfsomgevingen.

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