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Vertical Dynamic Stiffness of Offshore Foundations

Chiara Latini¹, Michele Cisternino¹, Varvara Zania¹

¹Civil Engineering Department, Technical University of Denmark
Lyngby, Denmark

ABSTRACT

Nowadays, pile and suction caisson foundations are widely used to support offshore structures which are subjected to vertical dynamic loads. The dynamic soil-structure interaction of floating foundations (foundations embedded in a soil layer whose height is greater than the foundation length) is investigated by numerical analyses of representative finite element models. The 3D numerical model is compared and validated with existing analytical solutions. A parametric study is carried out analyzing the effect of the slenderness ratio H_p/d and the height and the stiffness of the soil layer on the dynamic stiffness and damping.

KEY WORDS: soil-structure interaction; dynamic stiffness; damping; floating foundations; numerical modelling; elastodynamic analytical solution.

INTRODUCTION

Nowadays, the overall concept design for offshore wind farms is developing towards different foundation solutions. So far the support structures for offshore wind turbines have been classified into two main types: fixed (or grounded to the seabed) and floating. The majority of installed or operating turbines are supported on fixed foundation system (Bhattacharya, 2014), while deep installations require jackets structures with floating piles or with suction caissons. These types of foundations are subjected to dynamic load such as wind.

Houlsby et al. (2005) investigated the possibility of deploying suction caissons as offshore wind turbine foundations. In his work it was showed that suitable soil conditions are required and the functionality of suction caissons is limited up to water depth of approximately 40m. Suction caissons are skirted shallow foundations characterized by a slenderness ratio (foundation embedded length to foundation width) lower than 4 and they are put in place by creating negative pressure inside the caisson skirt by pumping out the water (Byrne and Houlsby, 2006). Consequently, suction caissons are preferred to driven piles, since their installation does not require heavy duty equipment. Due to its features this type of foundation is receiving more attention in the current research on deep water installations.

The way a foundation interacts during vibrations with the surrounding

soil influences considerably the dynamic characteristics of the foundation (Kramer, 1996). Hence it is fundamental to accurately evaluate the dynamic stiffness and damping of the soil-foundation system. Considering the estimation of the vertical dynamic soil-pile interaction many studies have been carried out by previous researchers by applying analytical solutions and numerical methods. Most of the past studies are based on the assumption that the soil around the foundation is a linear elastic single-phase medium. They can be categorized as follows:

- Rigorous analytical continuum solutions for end bearing piles (Nogami and Novak, 1976), where the soil was modelled as viscoelastic layer. In this formulation the displacement and the resistance factor of the soil layer were obtained neglecting the radial displacement of the soil medium.
- Winkler type analytical solution (Novak, 1974; Novak et al., 1978; Mylonakis, 2001; Hu et al., 2004; Wu et al., 2013; Zheng et al., 2014). For dynamic problems the use of Winkler foundation coefficients based on Baranov's equation for in plane and out plane vibration of a disk has been recommended by Novak (1974). An improved model incorporating in the analysis the normal and shear stresses acting on the upper and lower faces of a horizontal soil element by integrating the governing equations over the thickness of the soil layer has been developed by Mylonakis (2001). Wu et al. (2013) and Zheng et al. (2014) provided an extended model to study the vertical dynamic response of an end bearing pile by considering both the radial and the vertical displacement of the soil layer.
- Numerical continuum finite element solutions (Roesset & Angelides, 1980), where the soil is treated as an elastic continuum and the pile is assumed to have a rigid cross section and it is modelled as series of regular beam segments.

The abovementioned studies are founded on the assumption that the pile is embedded in a single-phase medium. However, the offshore environment is characterized by fully saturated soil and by water pressure acting on the foundation.

In literature there are a few works in which the dynamic response of pile foundations installed in a saturated elastic layer over a rigid bedrock was investigated, see Li et al. (2004) and Liu et al. (2014).

On the other hand the response of floating piles has been investigated either numerically (Kuhlemeyer, 1979) or analytically (Novak, 1977;

Nozoe et al., 1988; Deng et al., 2014; Zheng et al., 2015). However, the dynamic response of suction caissons has received less attention (Liingaard, 2006). In the work of Liingaard (2006) the dynamic stiffness coefficients were determined, considering linear viscoelastic soil and modelling the suction caisson using a coupled BE/FE model in homogeneous halfspace comparing the obtained results with analytical solutions for surface foundations.

The purpose of the current study is to examine the vertical dynamic response of floating piles and suction caissons in different soil conditions for the estimation of the dynamic stiffness and damping coefficients with respect to the frequency. Consequently, 3D FE models were established and the dynamic stiffness to vertical loading was determined. The results of the numerical models have been compared and validated respectively with the rigorous analytical solutions of soil-end bearing pile vibration by Nogami & Novak (1976), Hu et al. (2004), Wu et al. (2013), Zheng et al. (2014). Thereafter a parametric analysis investigated the effects of the stiffness and height of the soil layer on the soil-foundation system response. Moreover, the dynamic stiffness and damping are analyzed varying the slenderness ratio H_p/d . The frequency dependent stiffness and damping of suction caissons illustrates the effect of the cap on the vertical vibration.

METHODOLOGY

3D finite element models in the commercial software ABAQUS (Simulia, 2013) have been developed to investigate the dynamic impedances of the suction caisson.

The following assumptions are considered in the numerical models: 1) linear elastic isotropic behavior of the pile; 2) linear viscoelastic isotropic behavior of soil with hysteretic type damping and 3) perfect contact between the foundation and the soil during the analysis.

The symmetry of the problem has allowed taking into account only half of the foundation and the surrounding soil. The pile consists of steel with diameter $d=1\text{m}$, length $H_p=10\text{m}$, Young's modulus $E_p = 210\text{ GPa}$ and Poisson's ratio $\nu=0.35$. The pile foundation has thickness of $t=d/100$. Two different piles modelling approaches are used: 1) shell pile, where the foundation is modelled by its shell and 2) equivalent solid pile, for which equivalent material properties are applied to match the axial stiffness. The foundation is embedded in a soil layer with hysteretic type damping of $\zeta=5.0\%$ and constant profile of shear wave velocity $V_s=250\text{-}500\text{m/s}$.

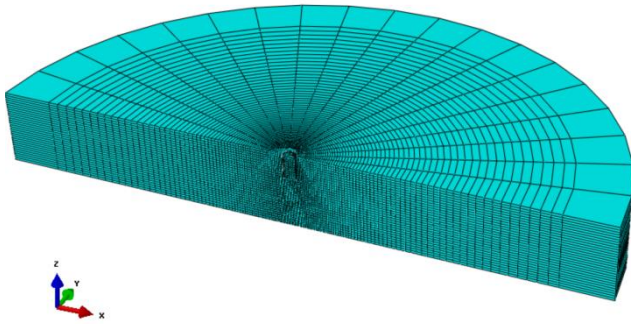


Fig. 1 Finite element model of the pile and the surrounding soil.

Hexahedral elements are deployed to discretize the soil domain of diameter $100d$ and height $H_s=30d=30\text{m}$. The boundaries are modelled by placing infinite elements in order to simulate the far field soil and avoid spurious reflection. Full contact among the pile lateral surface and the surrounding soil is ensured to prevent relative motion between them. Steady state linearized response of the model subject to harmonic excitation in the frequency domain is performed. The dynamic impedance K_v at the level of the pile head is then directly calculated as

axial force N , when the head of the pile is subjected to unit vertical displacement, v . The mesh size needs to be small enough to capture the stress wave accurately. A mesh size of at least 10 to 20 elements per wave length is used as good approximation for the frequency range of interest, including up to the 2nd eigenfrequency of the soil layer $\alpha_0=3/2\eta\pi$, where $\eta=\sqrt{2(1-\nu)/(1-2\nu)}$. Note that α_0 is a dimensionless frequency related to the eigenfrequency of the soil layer, since it is given as the product of the wave number and the height of the soil layer. The 3D model comprising the mesh refinement is shown in Fig. 1.

NUMERICAL STUDY

Two layered soil profile characterized by high stiffness contrast is analyzed. In Fig. 2 the two types of 3D numerical models developed to account for different depths of the surface soil layer with respect to the length of the foundation are shown. In the study the soil profile with height equal to the length of the pile is defined as Profile 1, while the one with increased height as Profile 2.

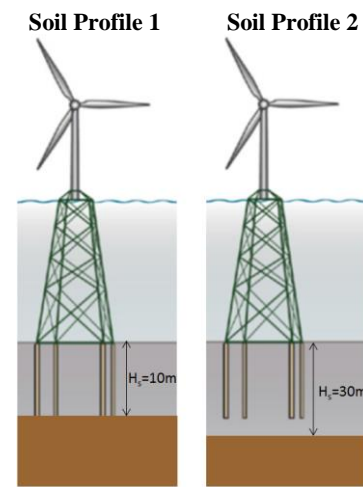


Fig. 2 Illustration of the two soil profiles investigated in this study.

First, the results only for the end bearing pile foundation (Profile 1) are compared with several analytical formulations available in the literature. The different pile modelling procedures with continuum elements and shell elements are implemented in order to achieve a direct comparison with the analytical solutions and consistency with the respective assumptions. As concerning floating piles (Profile 2), the effect of the height and stiffness of the soil stratum on the soil-foundation system response are further examined. A parametric study clarifies the role of the slenderness ratio and the foundation diameter on the vertical dynamic behavior of floating foundations.

VALIDATION OF THE NUMERICAL MODEL

The numerical results for the end bearing pile case are compared respectively with the different analytical solutions formulated by Nogami & Novak (1976), Hu et al. (2004), Wu et al. (2013) and Zheng et al. (2014). The reference case analyzed only for the validation of the numerical model consists of a solid concrete pile with diameter $d=1\text{m}$ and length $H_p=10\text{m}$, embedded in a soil layer with constant shear wave velocity $V_s=68\text{m/s}$, hysteretic material damping $\zeta=1.0\%$ and Poisson's ratio $\nu=0.40$. The two normalized dynamic components (real part of the complex valued stiffness term divided by the corresponding static component K^0 and imaginary part of the complex valued stiffness terms divided by the corresponding dynamic component K_v) of the vertical

stiffness is presented with respect to the non-dimensional frequency α_0 . In Fig. 3 the real (K_v) and the imaginary ($2\zeta_v$) part of the dynamic vertical impedance are shown. Both the analytical solutions and the numerical model exhibit a drop of stiffness at the 1st eigenfrequency of the soil layer ($\alpha_0=1/2\eta\pi$). However, the analytical formulation developed by Zheng et al. (2014) and Wu et al. (2013) results in an additional cut-off frequency around $\alpha_0=2$. Zheng et al. (2014) motivated it by the fact that the radial displacements were accounted for in the solution. Nevertheless, the trend of the abovementioned analytical formulations does not resemble the numerical model pattern, where there are not any limitations on the dynamic strains induced in the soil. In addition, the dynamic vertical stiffness is overestimated by the outcome of the implementation of the analytical solutions by Wu et al. (2013) and Zheng et al. (2014). After the 1st resonance of the soil layer, it is observed a linear decrease of the dynamic stiffness. The imaginary part of the dynamic component of the vertical impedance is combined with the generated damping due to soil-foundation interaction. The radiation damping is produced for frequencies higher than the 1st eigenfrequency of the soil layer. And after the 1st resonance of the soil medium all the analytical studies converge to the same linear trend of the viscous type radiation damping. The numerical model compares well with the analytical studies by Novak and Nogami (1976) and Hu et al. (2004). Hence this provides a validation of the numerical modelling methodology which is hereafter applied to a parametric study.

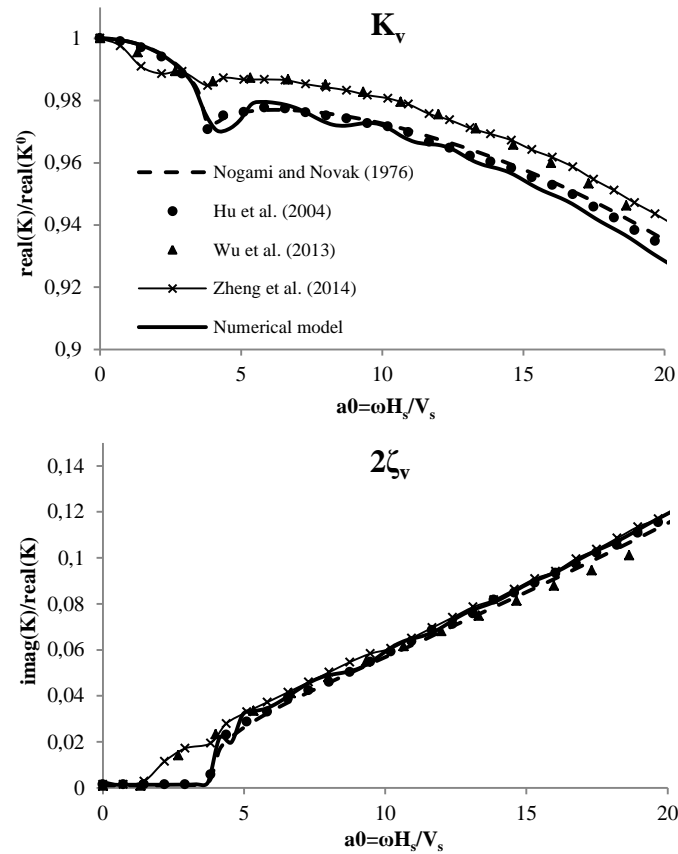


Fig. 3 Variation of the vertical stiffness and damping with respect to the dimensionless frequency for Profile 1.

PARAMETRIC STUDY

The dynamic response of floating piles (Profile 2) is analyzed by employing finite element analysis described in the previous section. In the current study the effects of the pile diameter, the height and the stiffness of the soil layer on the soil-floating pile response are

investigated. This makes it possible to discuss the role of some popular dimensionless parameters such as the stiffness ratio E_p/E_s and the slenderness ratio H_p/d on the dynamic behavior of the foundation. The cases selected in this study including also the dimensionless parameters are listed in Table 1, while the rationale for their selection was to examine foundations (piles and suction caisson) with different slenderness ratio ($H_p/d=20, 10, 2$ and 1 – case 8, 1, 6 and 7, respectively) embedded in a homogenous soil layer with various constant profiles of shear wave velocity ($V_s=250, 400$ and 500m/s – case 1, 4 and 5, respectively), thickness ($t=r_0/50$), hysteretic material damping ($\zeta=5.0\%$) and Poisson's ratio ($\nu=0.35$).

Table 1. Dimensionless parameters and cases selected in the parametric analysis.

Case Nr.	H_s [m]	H_p [m]	d [m]	V_s [m/s]	H_p/d	E_p/E_s
1 (Ref.)	30	10	1	250	10	60
2	15	10	1	250	10	60
3	20	10	1	250	10	60
4	30	10	1	400	10	23
5	30	10	1	500	10	15
6	30	10	5	250	2	60
7	30	5	5	250	1	60
8	30	20	1	250	20	60

The reference case analyzed is $d=1\text{m}$, $V_s=250\text{m/s}$, $H_p=10\text{m}$ and $H_s=30\text{m}$. Three different caisson modellings were deployed for case 6: 1) equivalent solid pile, for which equivalent material properties are applied to match the axial stiffness; 2) shell pile, where the foundation is modelled by its shell and 3) caisson with cap, as illustrated in Fig. 4. Concerning the caisson model with cap, the foundation skirt and the cap had respectively thickness of $t_{\text{skirt}}=d/100$ and $t_{\text{cap}}=5t_{\text{skirt}}$.

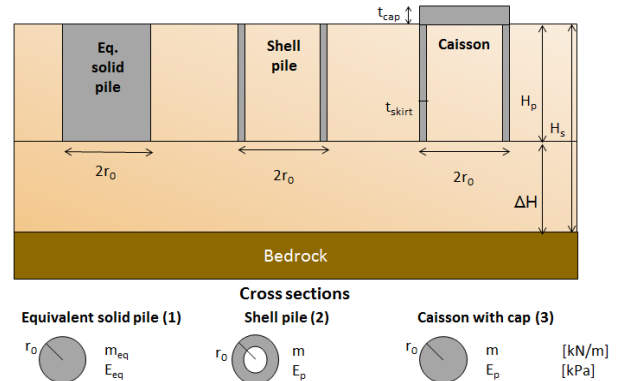


Fig. 4 Foundation geometries investigated for the case of caissons.

Effect of the height of the soil layer

In Fig. 5 the real (K_v) and the imaginary ($2\zeta_v$) components of the vertical stiffness are shown for different heights of the soil layer (case 1, 2 and 3). The drop of stiffness at the 1st eigenfrequency of the soil layer ($\alpha_0=1/2\eta\pi$) becomes more marked in the case of floating piles with $H_s/H_p=3$. In addition, it is observed a constant linear increase in the dynamic stiffness pattern for frequency higher than the 1st eigenfrequency of the soil layer. In the frequency range $\alpha_0=6-12$ it is

noticed that the vertical dynamic stiffness has higher values than the corresponding static component. The radiation damping (viscous type) is generated for frequencies higher than the 1st eigenfrequency of the soil layer. The three cases investigated exhibit identical slope, while the offset recorded approximately at the 1st resonance of the soil medium increases with H_s , implying higher radiation damping for deeper soil deposits.

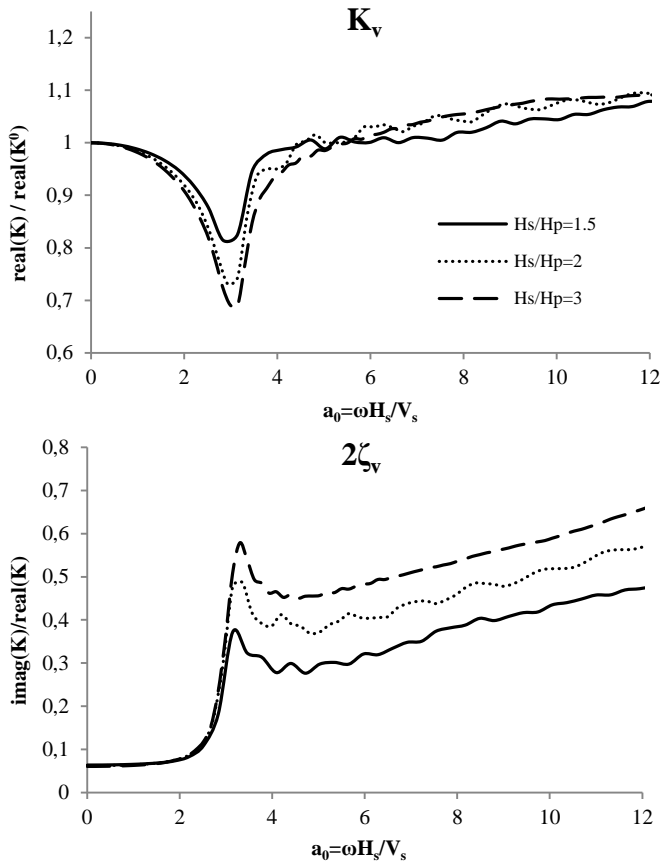


Fig. 5: Variation of the vertical dynamic stiffness and damping coefficients with respect to the non-dimensional frequency for Profile 2. Effect of the height of the soil layer on the real component and the imaginary component for cases 1, 2 and 3.

Effect of the stiffness of the soil layer

In Fig. 6 the real (K_v) and the imaginary ($2\zeta_v$) parts of the dynamic vertical impedance are presented for different values of the shear wave velocity of the soil layer ($V_s=250, 400$ and 500m/s - case 1, 4 and 5, respectively). The same values as in the reference case are used for the height of the foundation and the soil layer. Slightly scattered results are recorded by increasing the shear wave velocity of the soil layer.

The reduction of stiffness observed at the 1st eigenfrequency of the soil layer is to some extent less marked for stiff soil profiles ($V_s=500\text{m/s}$). The fact that the oscillation at the 1st resonance becomes more distinct when the soil stiffness decreases, is concurrent with the outcomes of Liingaard (2006). In the intermediate frequency interval ($\alpha_0=\pi/2-7$) the vertical dynamic impedance does not seem to be substantially affected by the increase of the shear wave velocity of the soil medium. When the soil is very stiff, the real component of the stiffness tends to be considerably independent of the frequency after 1st resonance. These findings are in agreement with the work of Nogami et al. (1976) for the case of end bearing piles. In addition, a quite linear increase of the pattern is recorded in the high frequency range.

The analysis shows that increasing the shear wave velocity of the soil layer or decreasing E_p/E_s the damping decreases. In addition, the radiation damping generated after the 1st eigenfrequency is characterized by a linear trend which can be characterized as viscous.

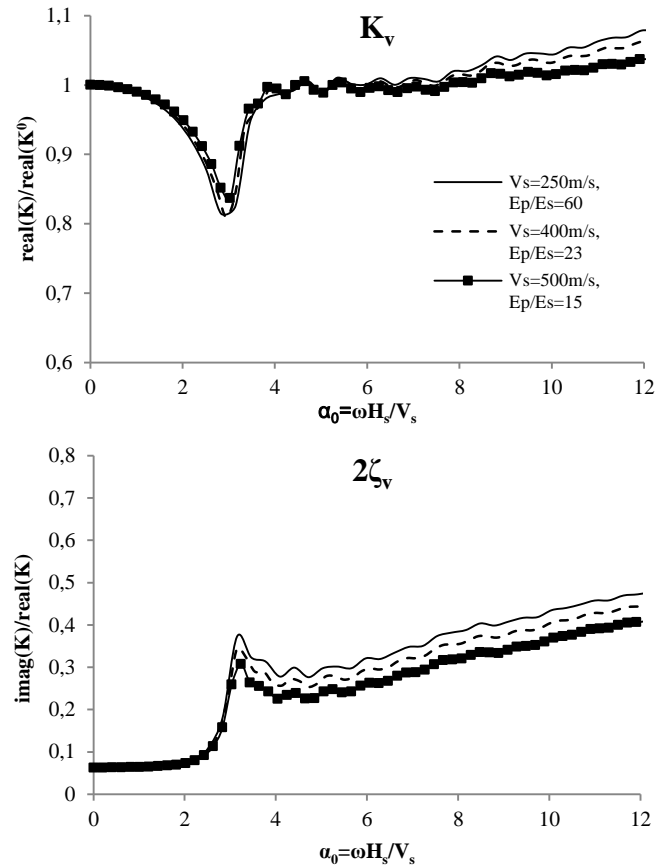


Fig. 6: Variation of the vertical dynamic stiffness and damping coefficients with respect to the non-dimensional frequency. Effect of the stiffness of the soil layer on the real component and the imaginary component for cases 1, 4 and 5.

Effect of the foundation geometry

Several numerical models were established to investigate the effect of the foundation geometry on the dynamic impedances particularly for suction caissons (case 6). In Fig. 7 the real (K_v) and the imaginary ($2\zeta_v$) part of the vertical dynamic impedances are shown for the case of the suction caisson modelled as 1) shell pile, 2) caisson with the cap and 3) equivalent solid pile. It was observed that all the three models attained the same reduction in stiffness at the 1st eigenfrequency of the soil medium. The numerical outcomes of the caisson with cap, solid equivalent pile and the shell pile modellings match almost perfectly up to $\alpha_0=8$. This indicates that the presence of the lid does affect slightly the vertical dynamic response of the foundation in the high frequency range. Moreover, the discrepancy in the numerical outcomes between the caisson with cap, equivalent solid and shell pile model observed in the high frequency interval might also be due to the fact that viscous damping is applied to the soil within the skirts of the caisson with cap and shell pile model. However, it might be concluded that the geometry of the foundation influenced slightly the vertical dynamic response for frequencies higher than $\alpha_0=8$. The numerical models displayed similar results concerning the radiation damping associated to the vertical component of the stiffness.

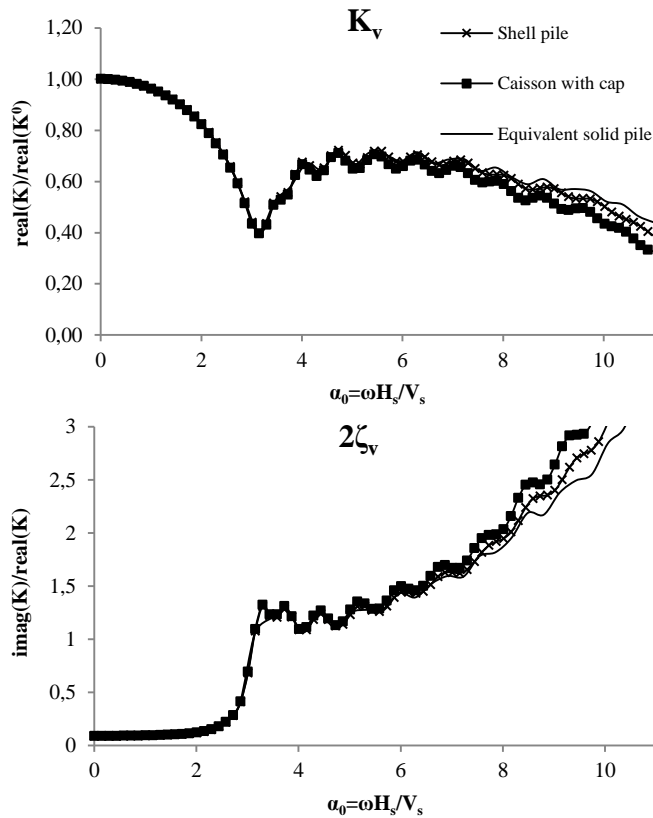


Fig. 7: Variation of the vertical dynamic stiffness and damping coefficients with respect to the non-dimensional frequency. Effect of the foundation geometry on the real component and the imaginary component for case 6.

Effect of the slenderness ratio

In Fig. 8 the real (K_v) and the imaginary ($2\zeta_v$) parts of the vertical dynamic impedance are displayed, varying the slenderness ratio H_p/d (cases 1, 6, 7 and 8). The parametric study is conducted keeping the same height and shear wave velocity of the soil layer as in the reference case. It is evident that two types of dynamic trend can be distinguished adopting the lateral rigidity criteria proposed by Randolph (1981). In the case of long piles ($H_p/d \geq 10$, case 1 and 8) the reduction in stiffness at the 1st eigenfrequency of the soil layer ($\eta\pi/2$) becomes more marked by decreasing the slenderness ratio. The dynamic impedance is moderately sensitive to the variation of H_p/d and it is characterized with some extent by an almost constant pattern for frequencies higher than the 1st resonance. Indeed, any drop of stiffness at the 2nd eigenfrequency of the soil medium is recorded. This might be attributed to the fact that response of the system is controlled to large extent by the dissipative soil medium as observed in Novak (1977) for end bearing piles. The corresponding results for shallow foundations ($H_p/d < 10$, case 6 and 7) are also plotted in Fig.8. The dynamic stiffness coefficient is substantially reduced in these cases. Note that the diameter of the caisson is larger hence this might be associated with the drop in the stiffness and increase of damping compared to the cases 1 and 8.

Furthermore, it is observed that the dynamic stiffness increases up to the 2nd horizontal eigenfrequency of the soil medium ($3\pi/2$), while just after $\alpha_0 = 3\pi/2$ this is reversed by a sudden decrease. This trend could be explained by recalling that the dynamic response is controlled by the caisson than the soil. In addition, the outcomes show that only a small change in the numerical results is detected from $H_p/d=1$ to $H_p/d=2$. It

can be noticed that the magnitude of the dynamic vertical impedance overall increases with the skirt length at higher frequencies ($\alpha_0 > 6$) as reported by Liingaard (2006).

Note that the difference between cases 1 and 8, and cases 6 and 7 can be also due to the effect of H_s/H_p derived by comparing impedances that refer to the same diameter.

The radiation damping presents an increased step variation on the frequency interval for $H_p/d < 10$. When referring to long piles (case 1 and 8) the imaginary part exhibits values lower than the damping ratio in the frequency range $\alpha_0 = 0 - \eta\pi/2$.

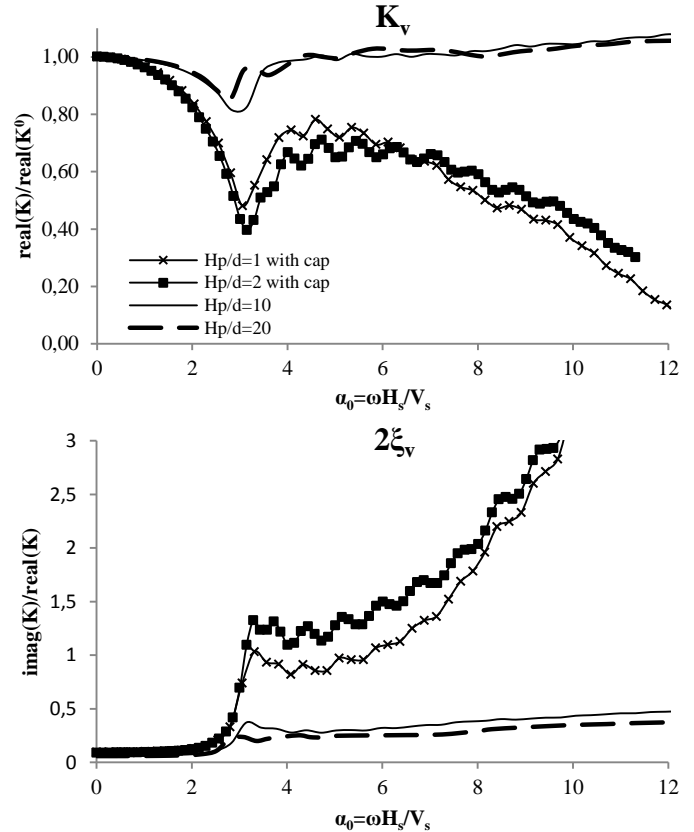


Fig. 8: Variation of the vertical dynamic stiffness with respect to the non-dimensional frequency. Effect of the slenderness ratio on the real component and the imaginary component for cases 1, 6, 7 and 8.

In Fig. 9 the vertical displacement of the foundation is plotted as a function of the depth at the 1st vertical resonance, highlighting the difference observed on the dynamic behavior between shallow foundations (case 6) and flexible piles (case 1).

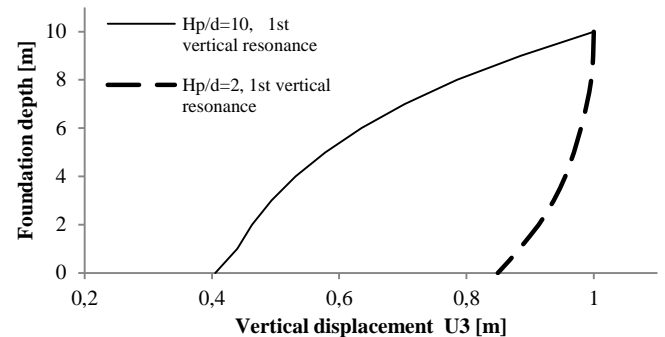


Fig. 9: Distribution of the foundation vertical displacement along the depth at the 1st vertical eigenfrequency of the soil layer for the case 1 and 6.

The deflected shape obtains a different curvature for the two cases, while the flexible pile seems to result in less displacement at the tip compared to the rigid caisson.

CONCLUSIONS

In this study numerical analysis is performed to investigate the vertical dynamic response of piles and suction caissons embedded in viscoelastic soil. Predictions from the numerical models have been found to be in good agreement with existing rigorous analytical solutions. A parametric study has been conducted to analyze the vibration characteristics and the effects of main parameters of floating foundations.

The dynamic soil-pile interaction analysis of floating piles has shown that the dynamic impedances are slightly affected by increasing E_p/E_s . On the other hand an increase of the height of the soil layer on the vertical dynamic impedance determines a more evident reduction of stiffness at the 1st resonance and consequently the damping generated is higher.

Moreover, the foundation diameter d has been found quite substantial parameter to determine the behavior of the foundation.

The proposed numerical model establishes a versatile practical tool that provides the soil-foundation vertical impedance coefficient. This might be applied in the frame of the substructure approach, to perform complete dynamic soil-structure interaction analyses of structures on such kind of foundations.

However, the suggested model is limited by the assumptions of linearity in the soil layer and foundation materials, and the perfect contact at the soil-foundation interface.

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