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Comparing superconducting and permanent magnets for magnetic refrigeration

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We compare the cost of a high temperature superconducting (SC) tape-based solenoid with a permanent magnet (PM) Halbach cylinder for magnetic refrigeration. Assuming a five liter active magnetic regenerator volume, the price of each type of magnet is determined as a function of aspect ratio of the regenerator and desired internal magnetic field. It is shown that to produce a 1 T internal field in the regenerator a permanent magnet of hundreds of kilograms is needed or an area of superconducting tape of tens of square meters. The cost of cooling the SC solenoid is shown to be a small fraction of the cost of the SC tape. Assuming a cost of the SC tape of 6000 \$/m^2 and a price of the permanent magnet of 100 \$/kg, the superconducting solenoid is shown to be a factor of 0.3-3 times more expensive than the permanent magnet, for a desired field from 0.5-1.75 T and the geometrical aspect ratio of the regenerator. This factor decreases for increasing field strength, indicating that the superconducting solenoid could be suitable for high field, large cooling power applications. © *2016 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution 3.0 Unported License.* [http://dx.doi.org/10.1063/1.4943305]

I. INTRODUCTION

From the onset of room temperature magnetic refrigeration devices in 1976 low temperature superconducting (SC) magnets have been used as field sources.[1] Initially the lack of strong permanent magnet (PM) materials did not leave any other options than electromagnets and SC magnets. But even after the widespread use of strong PMs became possible in the late 1980s, the use of SC magnets for research devices continued.[2,3] Using SC magnets have allowed for wide temperature spans to be achieved and reported. The benchmark magnetocaloric material Gd has an adiabatic temperature increase upon magnetisation of about 3 K in a 1 T field, rising to 8 K and 14 K in 3 T and 7 T fields, respectively.[4] However, most SC magnetic refrigeration devices have used existing cryostats with SC magnets not originally designed for operating magnetocaloric regenerators. Thus they are far from optimised with respect to the full utilisation of the magnetised volume. A few reports of the efficiency of devices operating with SC magnets have been given. One of the most cited ones is the work by Blumenfeld et al. (2002)[5] where a coefficient of performance (COP) of above 11 is reported using a high temperature SC magnet. However, this value did not include the power consumed to operate the SC, which was cooled below 40 K. Lowering the operating temperature of the SC increases the current capacity but also increases the cryocooler load. It has been claimed that a SC magnet would be best suited for large scale applications,[3] but a full comparison between SC magnets and permanent magnet assemblies, aimed at large devices operated at different temperatures, remains to be shown. In this paper we explore the cost needed to purchase and operate a high temperature SC magnet at 77 K versus the cost of an equivalently performing permanent magnet assembly, with the view of using the magnets in magnetic refrigeration devices.

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II. SYSTEM CONSIDERED

We consider an active magnetic regenerator (AMR) with a cylindrical geometry. The AMR is the heart of a magnetic refrigerator. The volume, V, and the aspect ratio, a, are the two parameters that specify the geometry of the regenerator. These are given as

$$V = L\pi R^2$$
$$a = \frac{L}{R} \tag{1}$$

as a function of the length, L, and radius, R, of the regenerator. We consider a regenerator made of commercial grade gadolinium and with a magnetization as reported in Bjørk et al. (2010),[6] albeit here measured up to an external field of 5 T. We assume an average temperature of the Gd of 293 K. This is needed for demagnetization considerations, as will be discussed subsequently. We consider an application that requires 3 kW of cooling at a 20 K temperature span. This could for example be cooling for a supermarket application. Scaling the regenerator mass obtained from Bjørk *et al.* (2016)[7] for the lowest cost magnetic refrigerator to the desired 3 kW, a needed regenerator volume of 5 L is obtained, assuming a regenerator porosity of 33%.

A. Superconducting magnet

We consider a SC solenoid that is able to generate a field throughout the regenerator. Since it will be of finite length, there will be flux leakage through the ends of the cylinder. While the complete field from a finite length solenoid can be described by elliptic integrals of the first, second and third kind, we here approximate the complete magnetic field as the magnetic field along the cylinder length axis, z, which for a finite length solenoid is given as

$$B_z = \frac{\mu_0 N_{\text{wind}} I_{\text{safety}} I_c}{2} \left(\frac{\frac{L}{2} - z}{\sqrt{\left(z - \frac{L}{2}\right)^2 + R^2}} + \frac{\frac{L}{2} + z}{\sqrt{\left(z + \frac{L}{2}\right)^2 + R^2}} \right) \tag{2}$$

where N_{wind} is the number of windings, I_c is the critical current in ampere, which is a function of applied magnetic field and temperature. I_{safety} is a dimensionless safety factor, which has a range from 0-1, that determines how close to the critical current the superconductor is operated. This equation can be spatially averaged to find the number of windings that make the average B_z in the regenerator volume equal to a desired specified magnetic field, B_{des}.

The SC tape used in the analysis is a second generation (2G) high temperature superconducting tape, i.e. a $REBa_2Cu_3O_{7-\delta}$ coated conductor, where RE = rare earth, from Superpower Inc. Specifically, we consider the SCS4050 tape, for which the critical current for the superconductor is given in Zhang et al. (2014)[8] at different temperatures and magnetic fields. For every single magnet configuration the appropriate critical field is found by interpolation in temperature and magnetic field for the quoted values. The field experienced by the solenoid windings is assumed to be the average field generated by the solenoid. The interpolated values for I_c per width are in the range of 100-1000 A cm^{-1}, depending on the generated magnetic field.

Note that the critical current scales linearly with the width of the SC tape as does the cost of the tape. As the generated field is linear in the critical current and the number of windings, it does not matter if a solenoid is chosen with a thin tape and a correspondingly large number of windings or a wide tape with fewer windings. Therefore the price of the tape only depends on the area of the tape used. The price of the tape is given as

$$\text{Cost}_{\text{SC}} = A_{\text{SC tape}} \text{Cost}_{\text{SC per area}} \tag{3}$$

where $A_{\text{SC tape}}$ is the area of tape needed and $\text{Cost}_{\text{SC per area}}$ is the cost of the SC tape per area.

The cost of the superconducting solenoid is not only given by the cost of purchasing the tape alone. As the tape must be cooled to a specific operating temperature to achieve a desired I_c, there is an operational cost of the solenoid in addition to the cost of the cryo-system providing the cooling. In order to estimate this cost, we approximate the SC solenoid as a thin cylindrical shell with the

same dimensions as the AMR regenerator. Assuming that the SC solenoid is kept in a vacuum, the only absorbed heat will be through radiation heating of the superconducting tape, on both the inner and outer surfaces of the cylinder. This heat is given by

$$Q_{\text{Rad}} = 4\pi RL\epsilon\sigma_{\text{SB}}((293\ K)^4 - T_{\text{SC}}^4) \tag{4}$$

where ϵ is the emissivity of the SC tape, here taken to be $\epsilon = 0.1$ and σ_{SB} is the Stefan-Boltzmann constant. In addition to this heat rate, heat is also absorbed from the current leads in a high temperature superconductor, with a magnitude of $Q_{\text{Leads}} = 90$ W/kA.[9] We assume that both of these heat gains are removed by a cryocooler. Such a cryocooler generally consume an electrical power of approximately 15 times the cooling capacity provided at 77 K.[10,11] We assume a capital cost of the cryocooler of $200 per Watt cooling power at 77 K. In total this means that the cost of cooling the SC solenoid is given by

$$\text{Cost}_{\text{cooling}} = 15(Q_{\text{Rad}} + Q_{\text{Leads}})\$_{\text{Wh}}t + \$200(Q_{\text{Rad}} + Q_{\text{Leads}}) \tag{5}$$

where the first term is the operational cost of cooling the SC tape, i.e. the cost of electricity, and the last term is the capital cost of the cryocooler. Here $\$_{\text{Wh}}$ is the price of electricity, which is taken to be 10 cents per kilowatt hour (kWh), and t is the time of operation.

B. Permanent magnet

We wish to compare the price of the superconducting solenoid with that of a permanent magnet. As we consider a cylindrical regenerator, the PM must accommodate this geometry. The Halbach cylinder[12,13] is the cylindrical structure with constant remanence, B_{rem}, that most efficiently generates a homogeneous field in a cylindrical bore.[14,15] The Halbach cylinder has a remanence in cylindrical coordinates given by $B_{\text{rem,r}} = B_{\text{rem}}\cos(\phi)\hat{r}$ and $B_{\text{rem},\phi} = B_{\text{rem}}\sin(\phi)\hat{\phi}$. In the following we take the remanence of the magnet to be $B_{\text{rem}} = 1.2$ T, and the density of the permanent magnet to be $\rho_{\text{mag}} = 7800$ kg m^{-3}.

In order to consider a Halbach cylinder of finite length, a set of 38,457 numerical simulations of a finite length Halbach cylinder previously published[7] were used to determine the minimum mass of magnet needed to produce a desired magnetic field for a given regenerator volume and aspect ratio.

III. RESULTS

We wish to compare the initial cost of the two magnet technologies. We consider a case where the SC magnet is operated at $T = 77$ K and we assume a safety factor for the superconducting magnet of $I_{\text{safety}} = 0.75$, i.e. it is operated at 75% of the critical current density to avoid instabilities and quench.[16]

The two types of magnets can of course only be compared in the regime where the magnetic field they each are able to generate, are identical. The superconducting solenoid will be able to generate a magnetic field substantially stronger than the permanent Halbach cylinder. The latter has experimentally been shown to be able to generate a flux density of 4 T,[17] but due to the finite coercivity of the permanent magnets a field much larger than 2 T usually cannot be generated.[18–20] By comparing the superconducting solenoid and the Halbach cylinder at a field up to 1.75 T, an indication of the feasibility of using solenoids with even higher field can be gained. In order to truly assess the feasibility of using a superconducting solenoid, a full active magnetic regeneration modeling study, similarly to a recent study done using the Halbach cylinder where 38,800 AMR simulations were performed,[7] would have to be conducted. This is outside the scope of the present study.

A. Demagnetization effects

The magnetic field generated by the solenoid is along the cylinder axis, while the field generated by the permanent Halbach magnet is along the *x*-axis, i.e. across the cylindrical regenerator. Since the regenerator is a soft magnet it will influence the generated magnetic field. For these
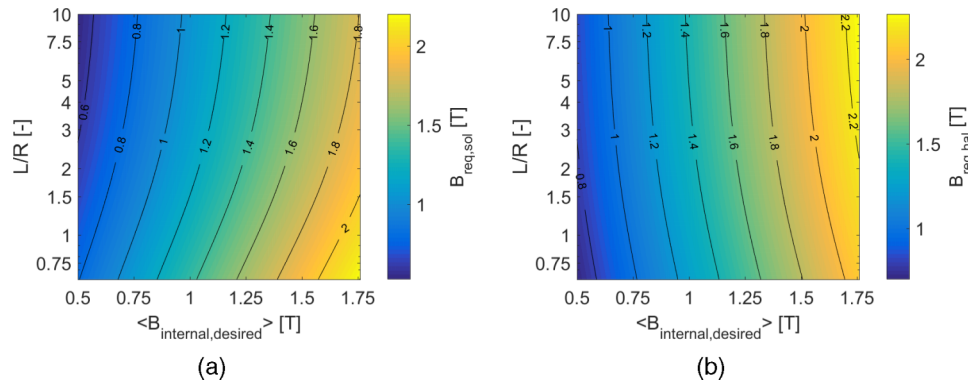
FIG. 1. The magnetic flux density that must be generated in order to produce a desired internal field for a given aspect ratio of the regenerator cylinder, for both a) a superconducting solenoid and b) a Halbach cylinder. The white area is where sufficient magnetization data is not available.

two geometries the demagnetization factor is not the same, and thus the internal magnetic field in the regenerator will not be the same for the same generated magnetic field. However, we wish to compare magnets that generate the same internal field in the regenerator, i.e. such that a refrigeration device with a superconducting magnet and a device with a Halbach magnet would perform identically. This is accomplished by choosing a desired internal average field in the regenerator and iterating the demagnetization equation, Eq. (6), until the desired internal field is obtained, and the applied field, H_{appl}, which is subsequently used to calculate the cost of the magnet, is known.

$$H_{int} = H_{appl} - N_{dem}M(H_{int}) \tag{6}$$

Here N_{dem} is the geometrical demagnetization factor, which depends on the aspect ratio of the sample.[21] In Eq. (6) the magnetization $M(H_{int})$ as a function of internal field must be known. As mentioned above, we assume a regenerator made of commercial grade gadolinium and with a magnetization as a function of internal field as reported in Bjørk et al. (2010)[6] but here measured up to 5 T. For this commercial grade gadolinium the adiabatic temperature change in a 1 T internal field is 3.3 K at the Curie temperature, $T_C = 295.1$ K.

Shown in Fig. 1 is the field that the SC solenoid and the Halbach cylinder must generate in order to provide a required internal field as a function of the aspect ratio of the regenerator cylinder. It is clear from the figure that the Halbach cylinder must provide a much larger magnetic field than the solenoid in order to generate the same internal field. For a 1 T internal flux density, the Halbach cylinder must generate a flux density between 1.2-1.4 T while the corresponding numbers are 1.05-1.4 T for the solenoid, depending on aspect ratio. The demagnetization factor is largest for low aspect ratios for the solenoid and vice versa for the Halbach cylinder.

B. Cost of the different magnets

Knowing the field that the solenoid and the Halbach cylinder must each generate to provide a desired internal field, the size and subsequent cost can be computed for each type of magnet. Shown in Fig. 2 is the area of superconducting tape needed and the mass of permanent magnet material needed as a function of desired average internal field and aspect ratio of the regenerator. As can be seen from the figures, a permanent magnet of some hundreds of kilograms is needed to produce the desired field. Similarly an area of superconducting tape of tens of square meters is needed.

The price of each type of magnet is trivially calculated from Fig. 2 by simply multiplying with the cost of the superconducting tape per area or the cost of the permanent magnet per kg. In order to more easily compare the two designs, Fig. 3 shows the factor between the cost of the superconducting magnet and the cost of the permanent magnet.

The above factors do not take into account the cost of providing cooling to the SC solenoid. Shown in Fig. 4 is the total cost of cooling the SC solenoid to 77 K as a function of the aspect ratio
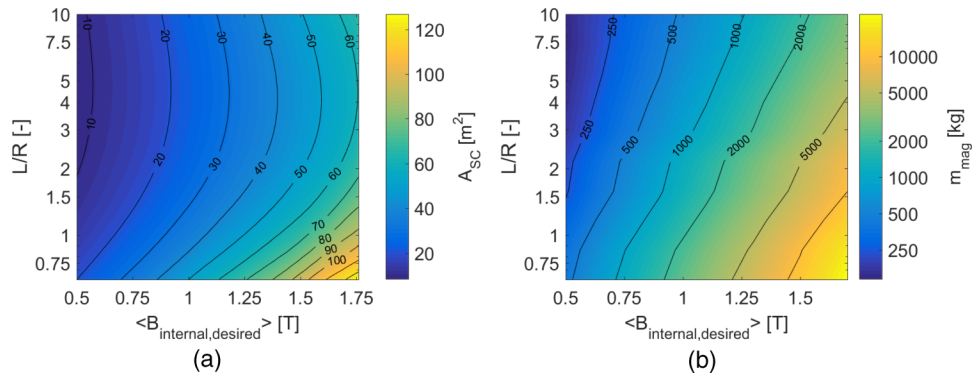
FIG. 2. a) The area of the superconducting tape needed and b) the mass of the permanent magnet material needed to produce a desired internal field for a given aspect ratio of the regenerator cylinder. The white area comes from the limited magnetization data in Fig. 1.

and the lifetime of the device. The cost of cooling is a very weak function of the desired magnetic field and therefore the cost as a function of this parameter has been averaged. At a device lifetime of $t = 15$ years, the capital cost of the cryocooler is approximately equal to the cost of electricity consumed. Even after 15 years, the total cost of cooling is $\approx \$10.000$, which is no more than a few percent of the price of the SC tape. Thus the cost of cooling has no significant impact on the results presented in Fig. 3. It is also of interest to consider the impact of the electricity consumption of the cryocooler on the COP of the AMR. As previously argued, optimizing the COP is beyond the scope of this article. However, the COP of the cryocooler alone can be estimated. As previously argued, the 5 L AMR considered should provide approximately 3 kW of cooling. To generate this the cooling power of the cryocooler varies between 8-25 W at 77 K, equivalent to 120-375 W electricity consumption of the cryocooler, multiplying with the efficiency factor of 15. This is a COP of 8-25, much larger than the expected COP of the AMR, and thus this will not have a significant influence on AMR performance.

From Fig. 3 it is seen that in general for low aspect ratios and high fields, the SC is cheaper than the PM, and vice versa. Assuming a cost of the SC tape of 6 \$/m for a 1 mm wide tape, i.e. 6000 \$ per m^2 and a price of the permanent magnet of 100 \$/kg, the superconducting solenoid will be a factor of 0.3-3 times more expensive than the permanent magnet. As can be seen from Fig. 3 this factor continues to decreases at higher fields, which indicate a clear prospect of using superconducting solenoids for high field, large cooling power refrigeration devices.
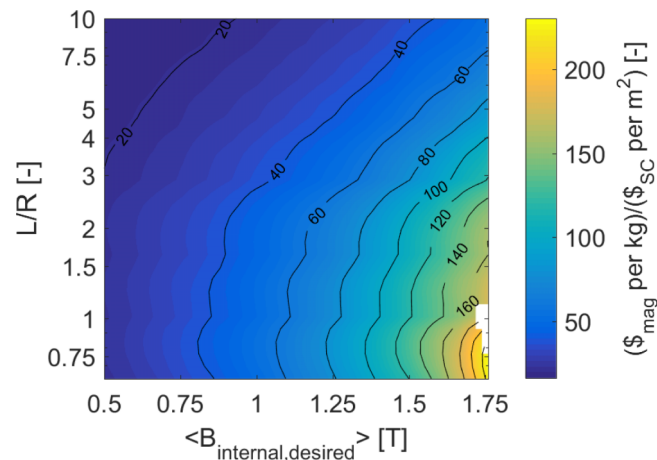

FIG. 3. The factor between the price of the SC tape per m^2 and the price of the magnet per kg. The white area comes from the limited magnetization data in Fig. 1.
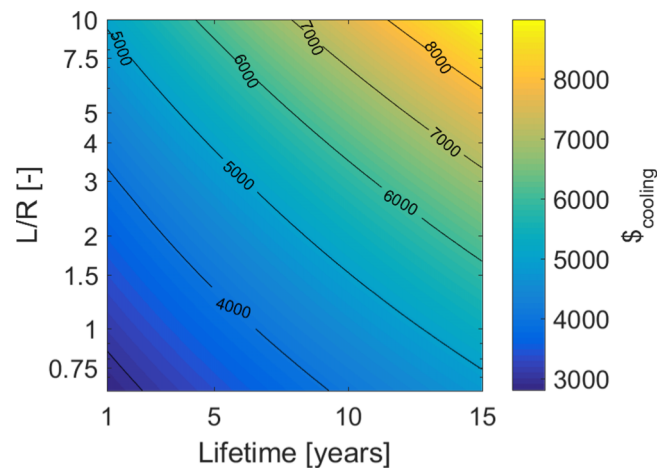
FIG. 4. The total cost of cooling the SC solenoid to 77 K, as a function of the aspect ratio of the solenoid and the lifetime of the device.

IV. CONCLUSION

The cost of a high temperature superconducting solenoid and a Halbach cylinder for use in magnetic refrigeration was determined. A five liter active magnetic regenerator volume was considered and the price of each type of magnet was determined as a function of aspect ratio of the regenerator and desired internal magnetic field. It was shown that to produce a 1 T internal field in the regenerator a permanent magnet of some hundreds of kilograms was needed or an area of superconducting tape of tens of square meters. The cooling cost of the superconducting solenoid was shown not to be significant. Finally, assuming a cost of the SC tape of 6000 \$/m^2 and a price of the permanent magnet of 100 \$/kg, the superconducting solenoid was shown to be a factor of 0.3-3 times as expensive as the permanent magnet in the present range of field considered, but with the factor decreasing for increasing field strength, indicating that the superconducting solenoid could be suitable for high field, large cooling power applications.

[1] G. V. Brown, J. Appl. Phys. **47**(8), 3673–3680 (1976).
[2] R. Bjørk, C. R. H. Bahl, A. Smith, and N. Pryds, Int. J. Refrig. **33**, 437–448 (2010).
[3] A. Kitanovski, J. Tusek, U. Tomc, U. Plaznik, M. Ozbolt, and A. Poredos, *Magnetocaloric Energy Conversion - From Theory to Applications* (Springer, 2015).
[4] S. M. Benford and G. V. Brown, J. Appl. Phys. **52**, 2110–2112 (1981).
[5] P. E. Blumenfeld, F. C. Prenger, A. Sternberg, and C. Zimm, Adv. Cryo. Eng. **47**(A), 1019 (2002).
[6] R. Bjørk, C. R. H. Bahl, and M. Katter, J. Magn. Magn. Mater. **322**, 3882–3888 (2010).
[7] R. Bjørk, C. R. H. Bahl, and K. K. Nielsen, Int. J. Refrig. Accepted for publication (2016).
[8] Y. Zhang, T. F. Lehner, T. Fukushima, H. Sakamoto, and D. W. Hazelton, IEEE Trans. Appl. Supercond. **24**, 1–5 (2014).
[9] C. Rey, *Superconductors in the Power Grid - Materials and Applications* (Woodhead Publishing, 2015).
[10] Sumitomo Heavy Industries, Croycooler Product Catalogue (2015).
[11] Sunpower Inc., Cryotel Cryocoolers Product Catalogue (2015).
[12] J. C. Mallinson, IEEE Trans. Magn. **9**, 678–682 (1973).
[13] K. Halbach, Nucl. Instrum. Methods 169 (1980).
[14] J. H. Jensen and M. G. Abele, J. Appl. Phys. **79**, 1157–1163 (1996).
[15] R. Bjørk, "The magnetic properties of the ideal hollow cylindrical magnet," J. Magn. Magn. Mater. (Submitted).
[16] C. Senatore, M. Alessandrini, A. Lucarelli, R. Tediosi, D. Uglietti, and Y. Iwasa, Supercond. Sci. Technol. **27**, 103001 (2014).
[17] M. Kumada, T. Fujisawa, and Y. Hirao, in *Proc. Second Asian Part. Accel. Conf.* (2001), pp. 840–842.
[18] F. Bloch, O. Cugat, G. Meunier, and J. C. Toussaint, IEEE Trans. Magn. **34**, 2465–2468 (1998).
[19] R. Bjørk, C. R. H. Bahl, A. Smith, and N. Pryds, J. Appl. Phys. **104**, 13910 (2008).
[20] R. Bjørk, A. Smith, and C. R. H. Bahl, J. Magn. Magn. Mater. **384**, 128–132 (2015).
[21] R. I. Joseph, J. Appl. Phys. **37**, 4639–4643 (1966).