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Analytical velocity field in just a sec*

*Using a cylindrical vortex model to assess the induction zone in front of aligned and yawed rotors

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Abstract

Analytical formulae for the velocity field induced by a cylindrical vortex wake model are applied to assess the induction zone in front of aligned and yawed rotors. The results are compared to actuator disk (AD) simulations for different operating conditions, including finite tip-speed ratios.

Introduction

- Joukowski derived the cylindrical vortex wake model of a rotor [1]
- Coleman et al. derived the axial induction for yawed rotors at infinite λ [2]
- Castles et al. used a superposition of such models at infinite λ [3]
- Branlard et al. derived the three components of inductions at finite λ [4]
- A solution for the superposition of such models at finite λ was obtained [5]
- These recent developments are here applied to the induction zone

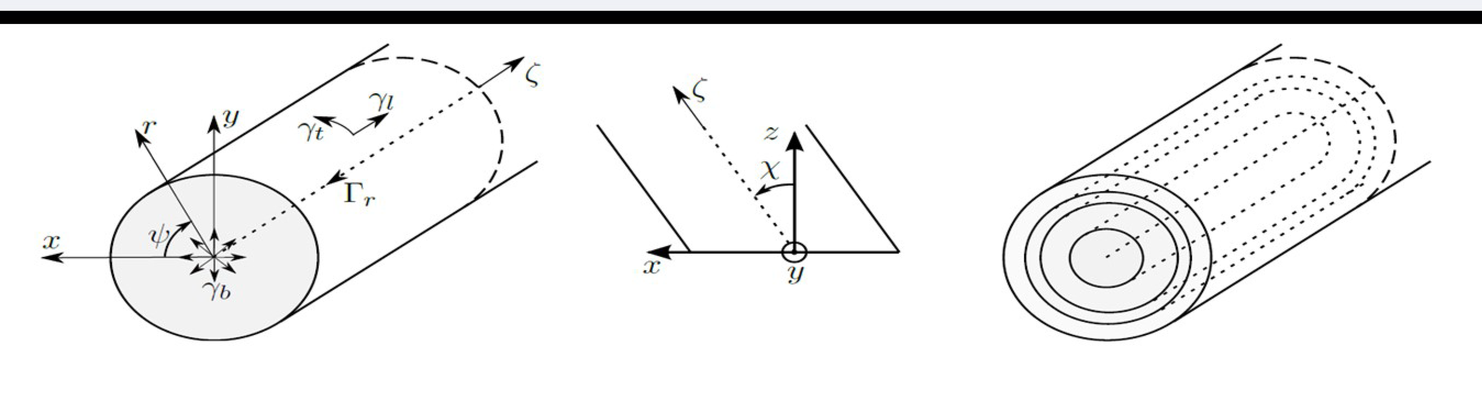

Figure 1: (left:) Elementary system and components of vorticity (middle:) Skew angle, (right:) Superposition of elementary systems.

Results

Results for aligned flows

Comparisons of the vortex cylinder model (VC) with Actuator Disk (AD) simulations at different operating conditions (CT, λ).

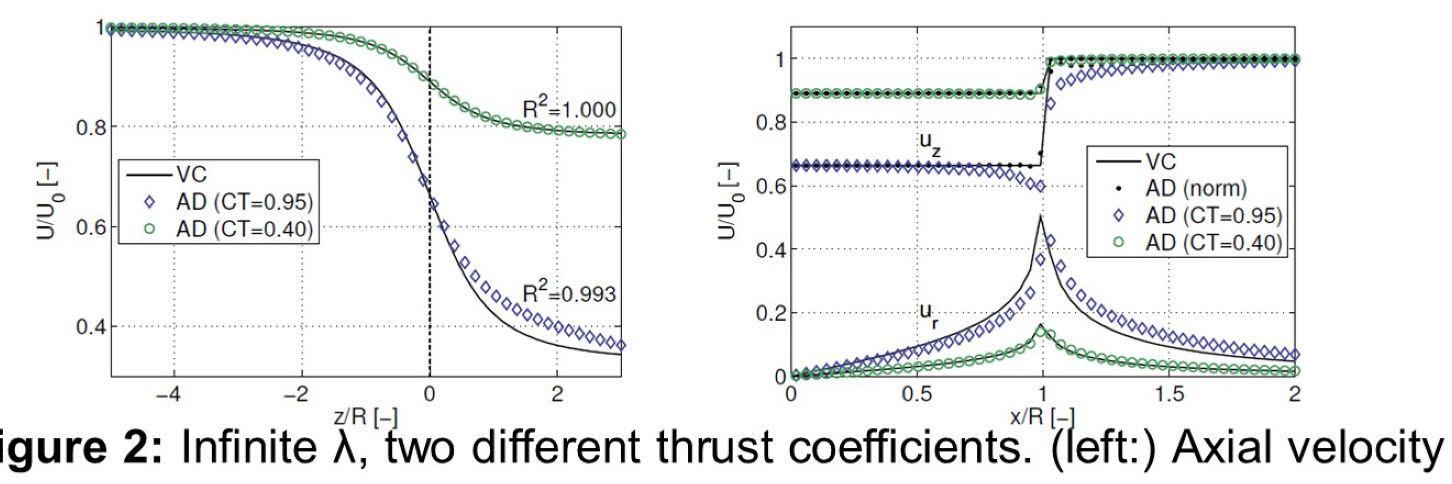

Figure 2: Infinite λ, two different thrust coefficients. (left:) Axial velocity on the rotor axis. (right:) Axial and radial velocity on the rotor disk. The absence of wake expansion of the model is more critical at high CT.

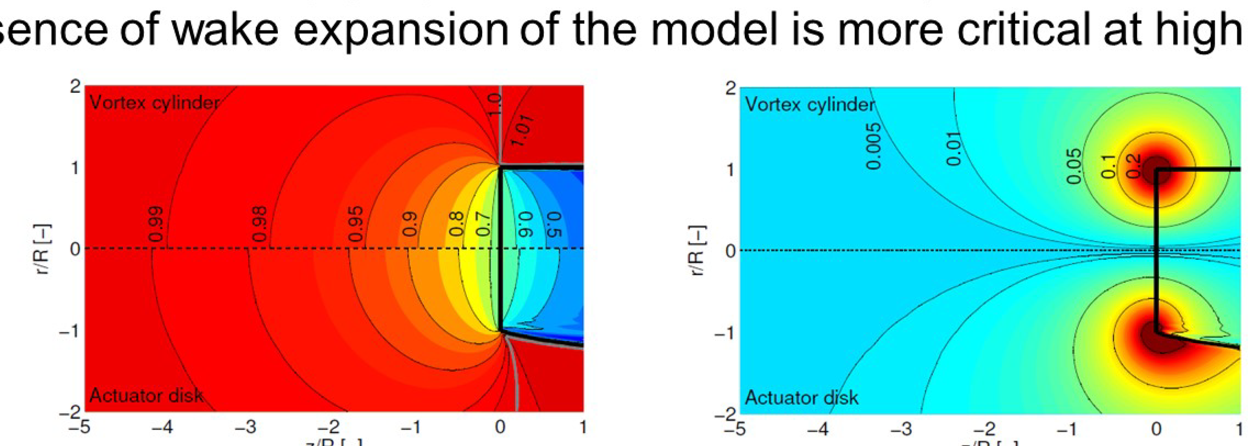

Figure 3: Infinite λ. Axial (left) and radial (right) velocity contours at CT=0.95. Despite the challenging high CT the induction zone is well described by the model.

	$C_T = 0.4$				$C_T = 0.95$			
	$\lambda = 2$	$\lambda = 6$	$\lambda = 10$	$\lambda = \infty$	$\lambda = 2$	$\lambda = 6$	$\lambda = 10$	$\lambda = \infty$
Mean	0.1%	0.1%	0.1%	0.0%	0.1%	0.2%	0.3%	0.4%
Max	0.2%	0.2%	0.2%	0.2%	1.8%	2.5%	2.7%	3.3%

Table 1: Finite λ. Parametric study for different operating conditions (CT, λ). Mean and maximum relative error in the induction zone ( )

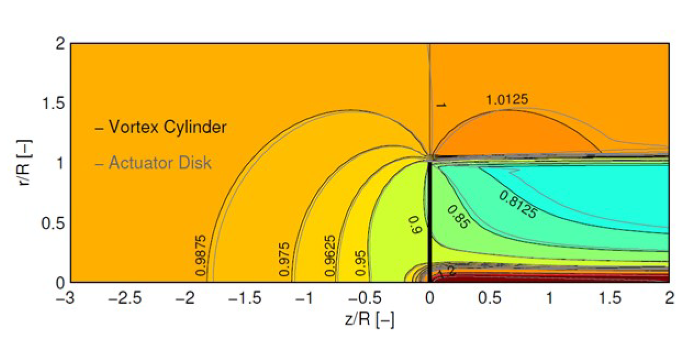

Figure 4: Finite λ. Axial velocity contours at CT=0.40 and λ=2. The presence of a high velocity core is captured by both models.

Results for yawed flows (χ=30 deg)

Comparisons of the vortex cylinder model (VC) with Actuator Disk (AD) simulations at different operating conditions (CT, λ).

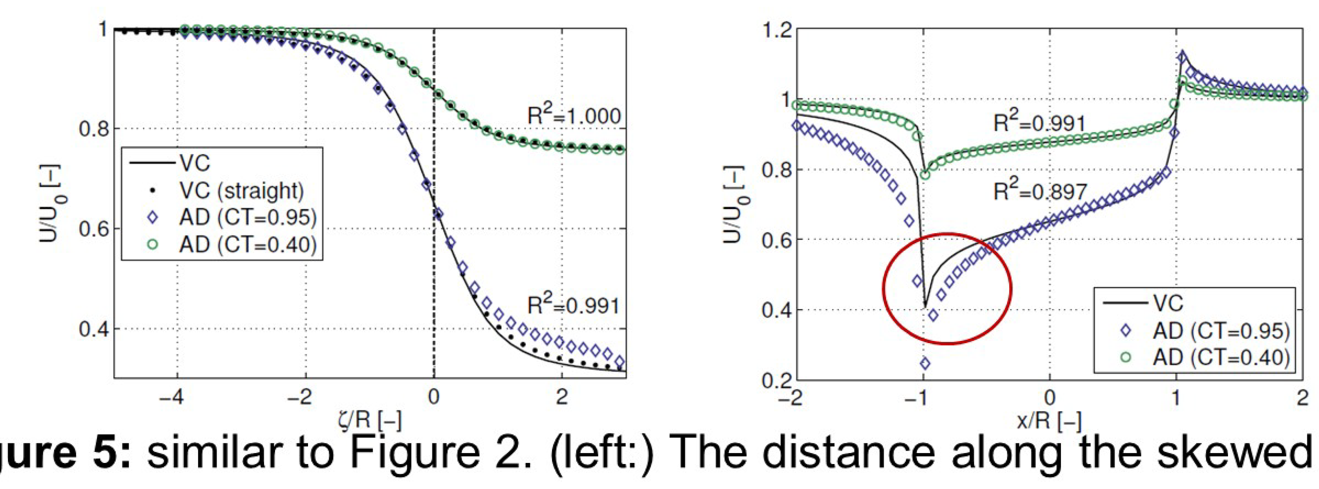

Figure 5: similar to Figure 2. (left:) The distance along the skewed axis is used, results are close to the straight cylinder on this axis. (right:) Wake expansion "far from the wind" is challenging at high CT(◯).

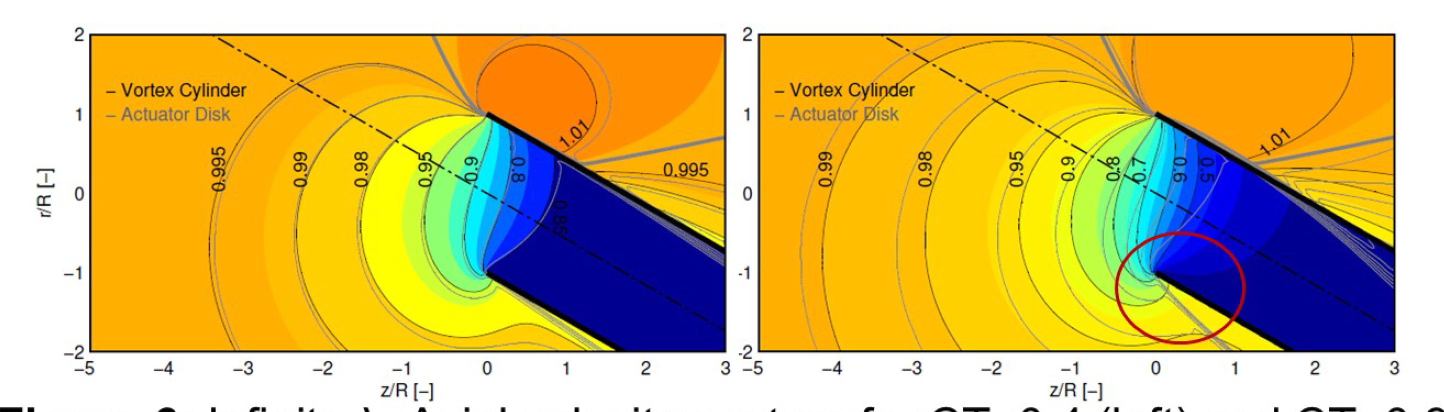

Figure 6: Infinite λ. Axial velocity contour for CT=0.4 (left) and CT=0.95 (right). The difference of wake expansion "far from the wind" is seen on the right. It clearly affects the induction on this side of the rotor.

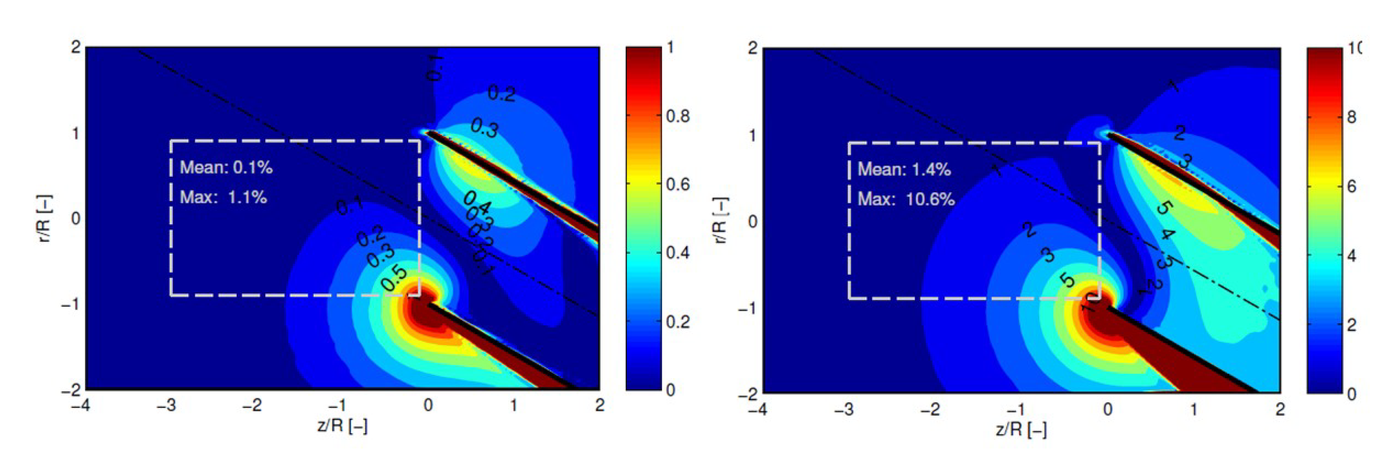

Figure 7: Infinite λ. Relative error in the induction zone ( ) for CT=0.4 (left) and CT=0.95 (right).

Conclusions

- The velocity field from the analytical formulae of the cylindrical vortex wake model agreed to a high degree with the ones obtained from Actuator disk simulations. A mean relative error of 1.4% was obtained in the induction zone for the challenging case of CT=0.95 and χ=30 deg.
- The model can be used for rapid (less than a second) estimates of the induction zone.

References

[1] Joukowski N.E., Vortex theory of screw propeller, I., Théorie tourbillonnaire de l'hélice propulsive, 1929
[2] Coleman, R. P., et al. Evaluation of the induced velocity field of an idealized helicopter rotor. *NACA L5E10, 1945*
[3] Castles, W., et al. The normal component of the induced velocity in the vicinity of a lifting rotor and some examples of its application. Tech. rep., NACA 1184, 1954.
[4] Branlard E., et al. Cylindrical wake model: *, Wind Energy, 2014, Wind Energy 2015
[5] Branlard E., et al. Superposition of vortex cylinders for steady and unsteady simulation of rotors of finite tip-speed ratio, Wind Energy, 2015.

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