From Canards of Folded Singularities to Torus Canards in a Forced van der Pol Equation - DTU Orbit (08/11/2017)

From Canards of Folded Singularities to Torus Canards in a Forced van der Pol Equation

In this article, we study canard solutions of the forced van der Pol equation in the relaxation limit for low-, intermediate-, and high-frequency periodic forcing. A central numerical observation made herein is that there are two branches of canards in parameter space which extend across all positive forcing frequencies. In the low-frequency forcing regime, we demonstrate the existence of primary maximal canards induced by folded saddle nodes of type I and establish explicit formulas for the parameter values at which the primary maximal canards and their folds exist. Then, we turn to the intermediate- and high-frequency forcing regimes and show that the forced van der Pol possesses torus canards instead. These torus canards consist of long segments near families of attracting and repelling limit cycles of the fast system, in alternation. We also derive explicit formulas for the parameter values at which the maximal torus canards and their folds exist. Primary maximal canards and maximal torus canards correspond geometrically to the situation in which the persistent manifolds near the family of attracting limit cycles coincide to all orders with the persistent manifolds that lie near the family of repelling limit cycles. The formulas derived for the folds of maximal canards in all three frequency regimes turn out to be representations of a single formula in the appropriate parameter regimes, and this unification confirms the central numerical observation that the folds of the maximal canards created in the low-frequency regime continue directly into the folds of the maximal torus canards that exist in the intermediate- and high-frequency regimes. In addition, we study the secondary canards induced by the folded singularities in the low-frequency regime and find that the fold curves of the secondary canards turn around in the intermediate-frequency regime, instead of continuing into the high-frequency regime. Also, we identify the mechanism responsible for this turning. Finally, we show that the forced van der Pol equation is a normal form-type equation for a class of single-frequency periodically driven slow/fast systems with two fast variables and one slow variable which possess a non-degenerate fold of limit cycles. The analytic techniques used herein rely on geometric desingularisation, invariant manifold theory, Melnikov theory, and normal form methods. The numerical methods used herein were developed in Desroches et al. (SIAM J Appl Dyn Syst 7:1131–1162, 2008, Nonlinearity 23:739–765 2010).

General information

State: Published
Organisations: Department of Applied Mathematics and Computer Science , Mathematics , Boston University, Inria Paris-Rocquencourt Research Centre
Authors: Burke, J. (Ekstern), Desroches, M. (Ekstern), Granados, A. (Intern), Kaper, T. J. (Ekstern), Krupa, M. (Ekstern), Vo, T. (Ekstern)
Number of pages: 47
Pages: 405-451
Publication date: 2016
Main Research Area: Technical/natural sciences

Publication information

Journal: Journal of Nonlinear Science
Volume: 26
Issue number: 2
ISSN (Print): 0938-8974
Ratings:
BFI (2017): BFI-level 2

Web of Science (2017): Indexed Yes

BFI (2016): BFI-level 2

Scopus rating (2016): SJR 1.461 SNIP 1.367 CiteScore 2.15

Web of Science (2016): Indexed yes

BFI (2015): BFI-level 2

Scopus rating (2015): SJR 1.759 SNIP 1.569 CiteScore 2.09

BFI (2014): BFI-level 2

Scopus rating (2014): SJR 1.648 SNIP 1.738 CiteScore 1.92

BFI (2013): BFI-level 2

Scopus rating (2013): SJR 1.78 SNIP 1.811 CiteScore 2.18

ISI indexed (2013): ISI indexed yes

BFI (2012): BFI-level 2

Scopus rating (2012): SJR 1.443 SNIP 1.897 CiteScore 1.91

ISI indexed (2012): ISI indexed yes

Web of Science (2012): Indexed yes

BFI (2011): BFI-level 2

Scopus rating (2011): SJR 0.827 SNIP 1.427 CiteScore 0.93