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Low Impedance Voice Coils for Improved Loudspeaker Efficiency

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ABSTRACT

In modern audio systems, utilizing switch-mode amplifiers, the total efficiency is dominated by the rather poor efficiency of the loudspeaker. For decades voice coils have been designed so that nominal resistances of 4 to 8 Ω is obtained, despite modern audio amplifiers, using switch-mode technology, can be designed to much lower loads. A thorough analysis of the loudspeaker efficiency is presented and its relation to the voice coil fill factor is described. A new parameter, the drivers mass ratio, is introduced and it indicates how much a fill factor optimization will improve a driver's efficiency. Different voice coil winding layouts are described and their fill factors analysed. It is found that by lowering the nominal resistance of a voice coil, using rectangular wire, one can increase the fill factor. Three voice coils are designed for a standard 10" woofer and corresponding frequency responses are estimated. For this woofer it is shown that the sensitivity can be improved approximately 1 dB, corresponding to a 30% efficiency improvement, just by increasing the fill factor using a low impedance voice coil with rectangular wire.

1. INTRODUCTION

A system designed for high quality audio reproduction involves many different blocks. These often include power supply, Digital Signal Processing (DSP), pre-amplifier, amplifier, cross-over network, loudspeaker enclosure and transducer. However all these blocks can be boiled down to three essential blocks which are:

- Power supply

- Amplifier

- Transducer

Modern power supplies and amplifiers are widely utilizing switch-mode power technology which is described in [1]. This is due to the high efficiency this technology offers, which theoretically can reach 100%. In practice the theoretical efficiency cannot be reached but it is possible to reach efficiencies above 90% for audio applications [2] and [3]. Moreover switch-mode power audio amplifiers delivers excellent audio performance with Total Har-

monic Distortion (THD) beneath 0.005% [4]. The transducer of the sound system is the loudspeaker driver which normally is mounted in a carefully designed enclosure. Loudspeaker drivers and their enclosures are well described in literature [5], [6], [7]. The impedance of the driver consist of an acoustical-, a mechanical and an electrical-part and is carefully described in [8] and [9]. The nominal resistance of the driver is characterized by the voice coil resistance. This resistance is also know as the DC resistance, corresponding to the resistance at 0 Hz. For decades it has been widely accepted to aim for voice coil DC resistances between 3 and 8 Ω. However modern switch-mode power audio amplifiers can easily be designed to deliver power to lower load resistances. A reduction of the load resistance leads to a downgrade of the amplifier's voltage rail requirement for a given output power as discussed in [10]. In [11] it is shown that a low rail voltage can benefit the efficiency of the switch-mode power audio amplifier due to lower switching losses. In addition to this the switch-mode power supply can be spared in battery driven systems, such as portable and automotive sound systems, when the DC resistance becomes so low, that the battery voltage is sufficient for generating a desired sound pressure level. On the transducer side a lower voice coil DC resistance can be obtained using rectangular or foil windings resulting in higher fill factors which can benefit the efficiency of the transducer, as discussed in [12].

This paper will focus on the design of low impedance voice coils for loudspeaker drivers and how these designs can benefit the efficiency of the driver and thereby the whole sound system.

2. LOUDSPEAKER EFFICIENCY

Conventional loudspeaker drivers are known to have very poor efficiency, typically in the range 0.2-2% depending, among others, on the magnet system, diaphragm size and weight and driver type. The overall efficiency for a system, from electrical input to acoustical output, is simply the product of the efficiencies of the subsystems. For a system consisting of a power supply, an audio amplifier and a loudspeaker the total efficiency becomes:

$$\eta_{tot} = \eta_{supply} \cdot \eta_{amp} \cdot \eta_{spk} \qquad (1)$$

Since the efficiency of the amplifier and the power supply is much higher than the efficiency of the loud-

speaker, it is clear that the overall efficiency for such systems normally will be dominated by the poor efficiency of the loudspeaker. One method to compensate for the poor efficiency of the driver is to improve the impedance match between the driver diaphragm and the air load. This can be done by using various types of acoustical horns [7]. For low frequency drivers, horn speakers tend to increase the overall size of the speaker significantly and therefore they will not be discussed further in this paper.

The efficiency of a loudspeaker driver is simply the ratio between the acoustic power emitted from the diaphragm, P_A, and the electrical power flowing into the voice coil, P_E.

$$\eta_0 = \frac{P_A}{P_E} \qquad (2)$$

A more common measure is the loudspeaker driver's sensitivity, where the Sound Pressure Level (SPL) is measured in dB at a given distance with a given input power. The SPL can be expressed as:

$$SPL = 10 \log \left(\frac{P_E \frac{\eta_0}{A}}{P_o} \right) \qquad (3)$$

Where $P_o = 10^{-12}$ W/m^2 is the reference acoustic sound power and A is the area of the sound surface at a given distance from the driver. Normal sensitivity measurements are measured at 1W@1m with the driver mounted in an infinite baffle, thus playing into a half sphere environment. A half sphere environment corresponds to $A = 2\pi r^2$, where r is the radius of the half sphere. With $r = 1$ meter we get a 8 dB reduction in sensitivity. The Sound pressure level at 1W@1m can then be expressed as:

$$SPL_{1W@1m} = 10 \log \left(\frac{\eta_0}{P_o} \right) - 8\text{dB} \qquad (4)$$

The sensitivity is normally in the range 85-95 dB. In literature [7] the efficiency is expressed as a function of the force factor, Bl, the diaphragm area, S_D, the DC resistance of the voice coil, R_E, and the total mass of the diaphragm when attached to the voice coil, M_{ms}:

$$\eta_0 = \frac{\rho_0 \left(Bl \right)^2 S_D^2}{2\pi c R_E M_{ms}^2} \qquad (5)$$

Where $\rho_0 = 1.18$ kg/m^3 is the density of air and $c = 345$ m/s is the speed of sound in air. The force
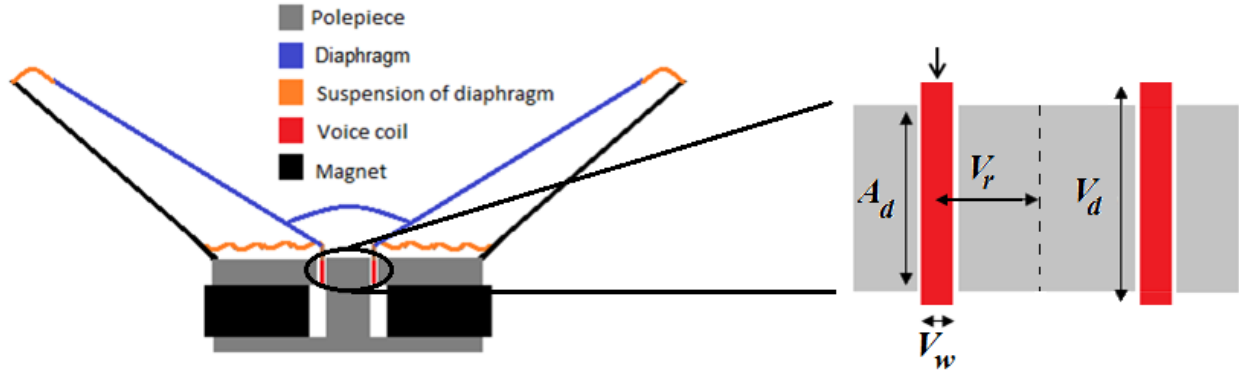
Fig. 1: Loudspeaker driver and voice coil winding area.

factor is the product between the the static magnetic field strength of the permanent magnet and the length of the wire in the air gap. From eq. 5 it seems obvious that for a given driver with a specific diaphragm area and magnet system, one can increase the efficiency by lowering the DC resistance or by increasing the force factor. However these two solutions cancels each other out. This can be shown when considering how the DC resistance and force factor are related to the winding area of the loudspeaker driver. A conceptual model of a loudspeaker driver and its voice coil is shown in fig. 1. V_d is the winding depth, V_w is the winding width, A_d is the air gap depth and V_r is the radius of the voice coil. Normally the winding depth will be bigger than the air gap depth to compensate for non linearities caused by the displacement [13] resulting in an overhung voice coil design.

Assuming that the entire winding area can be filled with wire the force factor and DC resistance can be written as follows:

$$R_E = \frac{l_w}{\sigma A_{wire}} = \frac{NV_o}{\sigma \frac{V_w V_d}{N}} = \frac{N^2 V_o}{\sigma V_w V_d}$$

$$Bl = Bl_w \frac{A_d}{V_d} = BNV_o \frac{A_d}{V_d}$$

(6)

Where l_w is the total length of the wire, A_{wire} is the cross sectional area of the wire, $V_o = 2\pi V_r$ is the circumsphere of the voice coil, N is the number of turns and σ is the conductivity of the conductor. For copper the conductivity is $\sigma_{Cu} = 5.96 \cdot 10^7 \ S/m$.

Considering eq. 5 we find the ratio between the force factor squared and the DC resistance to be constant.

$$\frac{(Bl)^2}{R_E} = \frac{B^2 N^2 V_o^2 \left(\frac{A_d}{V_d}\right)^2}{\frac{N^2 V_o}{\sigma V_w V_d}} \downarrow$$

$$\frac{(Bl)^2}{R_E} = \frac{B^2 V_o V_w A_d^2 \sigma}{V_d} = \mathbf{constant}$$

(7)

Due to this constant ratio the efficiency is theoretically also a constant for a given driver with a given magnet system. However the above expression is only valid if one assumed that the entire available winding area is filled with conducting copper, which in practice is not possible. Therefore one have to consider the fill factor of the voice coil, i.e. the ratio between the area of the conducting copper and the total winding area:

$$\kappa = \frac{A_{Cu}}{A_w}$$

(8)

Taking the fill factor into account we get:

$$R_E = \frac{N^2 V_o}{\sigma V_w V_d \kappa}$$

(9)

$$\frac{(Bl)^2}{R_E} = \frac{B^2 V_o V_w A_d^2 \sigma \kappa}{V_d} \neq \mathbf{constant}$$

Moreover the total moving mass of the driver, M_{ms}, can also be expressed as a function of the fill factor:

$$M_{ms} = M_{d+cf} + M_{vc}$$

$$M_{d+cf} = M_d + M_{cf}$$

(10)

$$M_{vc} = V_d V_w V_o \rho_{wire} \kappa$$

Where M_d is the mass of the diaphragm, M_{cf} is the mass of the coil former, M_{vc} is the mass of the voice coil wire and ρ_{wire} is the density of the wire material. Now it is possible to express the efficiency of the loudspeaker driver as a function of the fill factor by substituting eq. 9 and eq. 10 into eq. 5:

$$\eta_0(\kappa) = \frac{\rho_0 S_D^2 B^2 V_o V_w A_d^2 \sigma \kappa}{2\pi c V_d \left(M_d + M_{cf} + V_d V_w V_o \rho_{wire} \kappa \right)^2} \quad (11)$$

From eq. 11 it is seen that the efficiency, expressed as a function of the fill factor, is a ratio between a linear function and and a second order polynomial of the simplified form of:

$$\eta_0(\kappa) = \frac{\kappa}{(a + b \cdot \kappa)^2} \quad (12)$$

Where a and b are constants. Considering positive values of κ we find the efficiency to have a shape as shown in fig. 2. In this particular example it is seen that the maximum of efficiency is found at a theoretical fill factor of 50%. In general the fill fac-
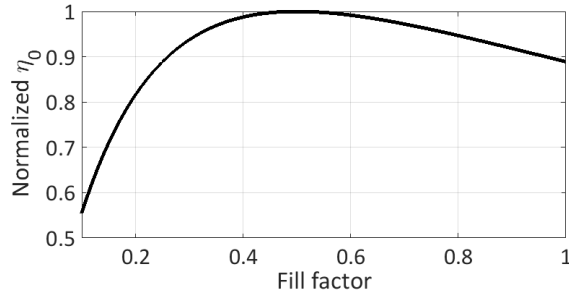

Fig. 2: The efficiency as a function of the fill factor

tor corresponding to the maximum efficiency, can be found by solving for the derivative of the efficiency function. In the simplified expression from eq. 12 we find:

$$\eta_0'(\kappa) = \frac{1}{(a+b\cdot\kappa)^2} - \frac{2b\cdot\kappa}{(a+b\cdot\kappa)^3}$$

$$\eta_0'(\kappa) = \frac{1}{(a+b\cdot\kappa)^2} - \frac{2b\cdot\kappa}{(a+b\cdot\kappa)^3} = 0 \downarrow \quad (13)$$

$$\kappa_{optimal} = \frac{a}{b}$$

Applying this solution to eq. 11 yields:

$$\eta_0'(\kappa) = 0 \downarrow$$
$$\qquad\qquad\qquad\qquad\qquad\qquad (14)$$
$$\kappa_{optimal} = \frac{M_d + M_{cf}}{V_d V_w V_o \rho_{wire}}$$

However this is only true for cases where:

$$M_d + M_{cf} \geq V_d V_w V_o \rho_{wire} \quad (15)$$

This is due to the fact that the theoretical maximum fill factor is $\kappa = 1$, i.e. 100%. Therefore we will redefine eq. 14 by introducing a new parameter: The driver's mass ratio:

$$M_{ratio} = \frac{M_d + M_{cf}}{V_d V_w V_o \rho_{wire}} \quad (16)$$

It is seen that the mass ratio is the ratio between the mass of the diaphragm plus the coil former and the maximum mass of the actual wire within the voice coil. It is quite common that typical drivers will have a mass ratio above one. For these cases the optimal fill factor is $\kappa = 1$, meaning that these drivers' efficiencies would benefit from a fill factor optimization. However the impact on the efficiency achieved by this fill factor optimization is dependent on how far above one the mass ratio is. This is shown on fig. 3 where the efficiency is normalized around a fill factor of 50%, which is not an uncommon fill factor for voice coil designs. From the figure it is seen that the impact on the efficiency becomes greater for higher mass ratios.
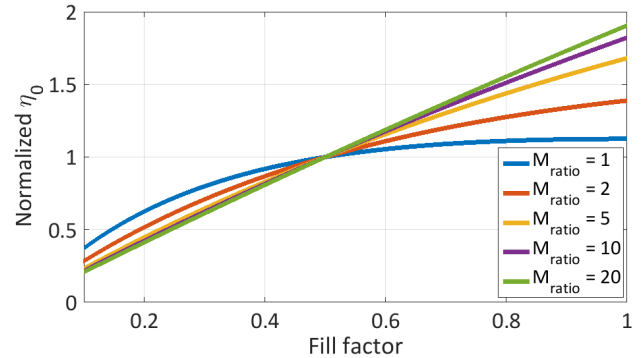
Fig. 3: Impact on efficiency for different mass ratios

3. SMALL-SIGNAL PARAMETERS

The small-signal parameters are well described in litterature, [5], [6], [7], and is also referred to as Thiele-Small parameters. This section will focus on the small-signal parameters known as the quality factors of the loudspeaker driver. That is the electrical-, the mechanical-, and the total-quality factor, Q_{ES}, Q_{MS} and Q_{TS}. These parameters determine the

frequency response of the driver and therefore it is relevant to investigate how they are affected by the fill factor of the voice coil.

3.1. Electrical quality factor

From literature, [7], the electrical quality factor can be expressed as:

$$Q_{ES} = \frac{R_E}{(Bl)^2} \sqrt{\frac{M_{ms}}{C_{ms}}} \qquad (17)$$

Where C_{ms} is the mechanical compliance of the diaphragm suspension, which normally is given in the data sheet for a given driver. In eq. 9 it was shown that the relationship between the force factor, Bl and the DC resistance, R_E, could be expressed as a function of the fill factor. Similar it was shown in eq. 10 that the total moving mass, M_{ms}, also could be expressed as a function of the fill factor. Applying these two equations to eq. 17 yields:

$$Q_{ES}(\kappa) = \frac{\frac{N^2 V_o}{\sigma V_w V_d \kappa}}{\left(BNV_o \frac{A_d}{V_d}\right)^2} \sqrt{\frac{M_{d+cf} + V_d V_w V_o \rho_{wire} \kappa}{C_{ms}}} \downarrow$$

$$Q_{ES}(\kappa) = \frac{V_d}{\sigma V_o V_w B^2 A d^2 \kappa} \sqrt{\frac{M_{d+cf} + V_d V_w V_o \rho_{wire} \kappa}{C_{ms}}} \qquad (18)$$

From eq. 18 it is seen that when the fill factor increases the electrical quality factor decreases.

3.2. Mechanical quality factor

The mechanical quality factor is given by:

$$Q_{MS} = \frac{1}{R_{ms}} \sqrt{\frac{M_{ms}}{C_{ms}}} \qquad (19)$$

Where R_{ms} is the mechanical resistance of the diaphragm suspension, normally provided in loudspeaker driver data sheets. Similar to the electrical quality factor we can express the mechanical quality factor as a function of the fill factor by substituting eq. 10 into eq. 19:

$$Q_{MS}(\kappa) = \frac{1}{R_{ms}} \sqrt{\frac{M_{d+cf} + V_d V_w V_o \rho_{wire} \kappa}{C_{ms}}} \qquad (20)$$

From eq. 20 it is seen that the mechanical quality factor increases with the fill factor.

3.3. Total quality factor

The total quality factor of a loudspeaker driver is simply the combination of the electrical- and mechanical-quality factor:

$$Q_{TS} = \frac{Q_{MS} Q_{ES}}{Q_{MS} + Q_{ES}} \qquad (21)$$

The total quality factor determines the frequency response of the driver. This is evident from the pressure transfer function of the loudspeaker driver which is given by:

$$p = \frac{\rho_0}{2\pi} \frac{Ble_g}{S_d R_E M_{as}} G(s) \qquad (22)$$

Where $\rho_0 = 1.18$ kg/m^3 is the density of air, e_g is the electrical input to the voice coil, S_d is the diaphragm area, $M_{as} = M_{ms}/S_D^2$ is the acoustic mass of the diaphragm and air load, and finally $G(s)$ is the second order high pass transfer function related to the total quality factor.

$$G(s) = \frac{(s/\omega s)^2}{(s/\omega s)^2 + (1/Q_{TS})(s/\omega s)} \qquad (23)$$

Where $\omega_s = 2\pi f_s$ and f_s is the resonance frequency of the driver given by:

$$f_S = \frac{1}{2\pi \sqrt{M_{ms} C_{ms}}} \qquad (24)$$

For an optimal flat response the total quality factor should be approximately 0.7. For lower values we get a reduction in the low frequency content and for higher values we get a ripple, boosting the lower frequencies. The transfer function from eq. 22 is only valid for low frequencies. For higher frequencies a roll off is expected due to the self inductance of the voice coil, however this effect can be minimized using a copper cap in the magnetic design. An expression for the total quality factor as a function of the fill factor can be obtained by substituting eq. 18 and 20 into eq. 21:

$$Q_{TS}(\kappa) =$$

$$\frac{(M_{d+cf} + V_d V_w V_o \rho_{wire} \kappa) V_d}{C_{ms} \sqrt{\frac{M_{d+cf} + V_d V_w V_o \rho_{wire} \kappa}{C_{ms}}} (\sigma V_o V_w B^2 A d^2 \kappa + R_{ms} V_d)} \qquad (25)$$

From eq. 25 it is seen that the total quality factor decreases with the fill factor.

4. FILL FACTOR OF VOICE COIL LAYOUTS

This section presents an analysis of the fill factor for conventional voice coil layouts using round wire and an unconventional layout using rectangular wire.

4.1. Round winding layout 1

Conventional voice coil designs utilize round wire in two layers. The concept of the winding layout is shown in fig. 4. It is evident that the round
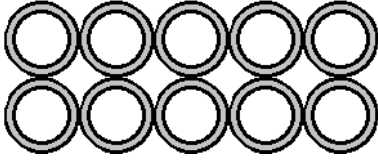

Fig. 4: Round winding layout 1.

shape causes some air gaps to exist between the wire. For this winding layout the maximum fill factor becomes:

$$\kappa_{round,1} = \frac{A_{Cu}}{A_w} = \frac{\pi r_w^2}{(2r_w)^2} = \frac{\pi}{4} \approx 0.79 \qquad (26)$$

Where r_w is the radius of the wire. Taking the insulation of the wire into account the fill factor will be even lower.

$$\kappa_{round,1} = \frac{A_{Cu}}{A_w} = \frac{\pi \left(r_w - \frac{t_{iso}}{2}\right)^2}{(2r_w)^2} \qquad (27)$$

Where t_{iso} is the total insulation thickness on both sides of the wire. In conventional voice coil designs using this winding layout DC resistances between 3 and 8 Ω are quite normal.

4.2. Round winding layout 2

During production of voice coils with round wire in two layers, the windings are often squeezed together as shown in fig. 5. It is seen that the round shape
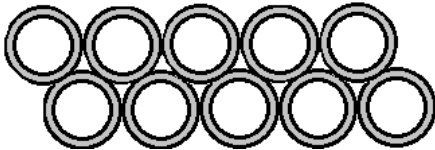

Fig. 5: Round winding layout 2.

still causes some tiny air gaps to exist. However

the size of the air gaps are reduced compared to the layout from fig. 4. This leads to an increment of the maximum fill factor:

$$\kappa_{round,2} = \frac{A_{Cu}}{A_w} = \frac{\pi r_w^2}{\left(2 + \sqrt{3}\right) r_w^2} = \frac{\pi}{\left(2 + \sqrt{3}\right)} \approx 0.84 \qquad (28)$$

With insulation the fill factor of this winding layout becomes:

$$\kappa_{round,2} = \frac{A_{Cu}}{A_w} = \frac{\pi \left(r_w - \frac{t_{iso}}{2}\right)^2}{\left(2 + \sqrt{3}\right) r_w^2} \qquad (29)$$

4.3. Rectangular winding layout

Considering an unconventional voice coil winding layout utilizing rectangular wire, as shown in fig. 6, it is possible to achieve a higher fill factor.

$$\kappa_{rec} = \frac{A_{Cu}}{A_w} = \frac{(W_w - t_{iso})(W_t - t_{iso})}{W_w W_t} \qquad (30)$$

Where W_w is the winding width and W_t is the winding thickness. In this winding layout there exist no
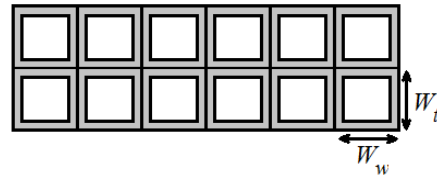

Fig. 6: Winding layout utilizing rectangular wire.

air gap between the wires and therefore the theoretical maximum fill factor is $\kappa_{rec} = 1$, i.e. 100%. In order to fill out the entire winding area, the width of the rectangular wire will change as a function of the DC resistance of the voice coil. For lower DC resistances less turns is required, resulting in a greater wire width and thereby higher fill factor. For this reason the fill factor can be increased by lowering DC resistance and thereby the number of turns. Knowing the voice coil width, V_w, the air gap depth, A_d, the voice coil circumsphere V_o and the winding depth, V_d, one can determine the exact number of turns for a given DC resistance and insulation thickness. This is done by realizing that for a given voice coil geometry the DC resistance is only dependent

Name	Parameter	Value
Voice coil width	V_w	0.45 mm
Voice coil radius	V_r	13.5 mm
Voice coil circumsphere	V_o	84.8 mm
Voice coil depth	V_d	12 mm
Air gap depth	A_d	8 mm
Magnetic field strength	B	1 T
Mass of diaphragm and coil former	M_{d+cf}	21 g
Compliance of diaphragm suspension	C_{ms}	0.7 mm/N
Resistance of diaphragm suspension	R_{ms}	0.75 Kg/s
Diaphragm Area	S_D	332 cm^2

Table 1: Parameters and geometry for 10" woofer

on the number of turns and the insulations thickness which can be assumed to be a given value.

$$R_E = \frac{lw}{\sigma A} = \frac{NV_o}{\sigma W_{t,cu}(W_w - t_{iso})} = \frac{NV_o}{\sigma W_{t,cu}\left(\frac{V_d}{0.5N} - t_{iso}\right)}$$
(31)

Where $W_{t,Cu} = W_t - t_{iso}$ is the actual thickness of the copper in the wire. By solving for the number of turns N in eq. 31 we get:

$$N = \frac{-R_E W_{t,Cu} t_{iso}\sigma + \sqrt{R_E^2 W_{t,Cu}^2 t_{iso}^2 \sigma^2 + 8 R_E V_o V_d W_{t,Cu}\sigma}}{2V_o}$$
(32)

From this the actual width of the copper in the wire can be determined as:

$$W_{w,Cu} = W_w - t_{iso} = \frac{Vd}{0.5N} - t_{iso} \qquad (33)$$

This is valid for voice coil with two layers. When the number of turns becomes equal to one, $N = 1$, only one layer exist and a theoretical filling factor of 100% is obtained. In this case the voice coil becomes a one layer solid ring of conducting material. However the DC resistance will be very small, close to zero, which will make it difficult to drive, even with modern switch-mode audio power amplifiers.

4.4. Comparison of winding layouts

Fig. 7 shows a comparison between the different voice coil winding layouts. Note that the x-axis has the relative scale t_{iso}/d_w, where d_w is the diameter of the wire or, for the rectangular wire, the thickness, W_t. It is seen that the voice coil layout using rectangular wire can achieve a higher fill factor than that of the layouts using round wire. For the rectangular wire the decrement rate of the fill factor as the

insulation thickness increases, is dependent on the shape of the wire. For high ratios of W_t/W_w there will be a fast decrement rate of the fill factor and for low ratios there will be a slower decrement rate.
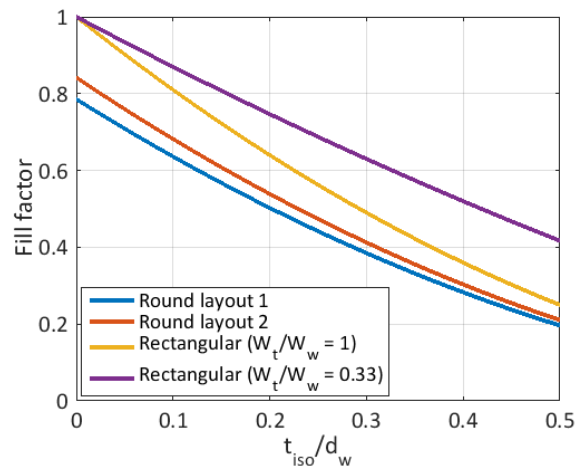

Fig. 7: Fill factor comparison of different winding layouts.

5. VOICE COIL DESIGN

This section presents the voice coil designs for a 10" woofer using the two voice coil layouts for round wire and the voice coil layout for rectangular wire with a low DC resistance of 0.5 Ω. The frequency responses of the three designs are evaluated, compared and discussed. The relevant parameters for the 10" woofer are listed in table 1. It is assumed that the insula-

Design	N	R_E	Bl	κ	η_0	Q_{ES}	Q_{MS}	Q_{TS}	$SPL\ 1W@1m$
Round layout 1	107	5.2Ω	6.04 T·m	58%	0.77%	0.82	7.71	0.74	90.9 dB
Round layout 2	100	4.1Ω	5.95 T·m	64%	0.83%	0.75	7.75	0.68	91.2 dB
Rectangular layout	39	0.5Ω	2.20 T·m	82%	1.00%	0.61	7.87	0.56	92.0 dB

Table 2: Driver parameters for each design

tion thickness is the same for all three designs and has the value of $t_{iso} = 31\ \mu$m, [14], and that all the voice coil designs utilizes regular copper wire. For this driver the mass ratio, M_{ratio}, is calculated to be:

$$M_{ratio} = \frac{M_{d+cf}}{V_d V_w V_o \rho_{wire}} \approx 5.1 \qquad (34)$$

For this mass ratio we will expect to see some improvement of the efficiency when optimizing the fill factor according to fig. 3.

5.1. Design with round winding layout 1
This design utilizes the winding layout shown in fig. 4. The diameter of the whole wire and the actual copper inside is found to be:

$$W_d = \tfrac{1}{2}V_w = \underline{0.225\text{mm}}$$
$$W_{d,Cu} = W_d - t_{iso} = \underline{0.194\text{mm}} \qquad (35)$$

From this the number of turns can be calculated:

$$N = \frac{2V_d}{W_d} \approx \underline{107} \qquad (36)$$

Now the length of the wire can be calculated:

$$l_w = V_o N \approx \underline{9.08\text{m}} \qquad (37)$$

Now we can use eq. 4, 6, 11, 18, 20, 25 and 27 to find the DC resistance, the force factor, the fill factor, the efficiency, the quality factors and the sensitivity of the driver. The results are listed in table 2.

5.2. Design with round winding layout 2
This voice coil uses the winding layout shown in fig. 5. The diameter of the wire and the conducting copper can be found using:

$$W_d = \frac{2}{2+\sqrt{3}}V_w = \underline{0.241\text{mm}}$$
$$W_{d,Cu} = W_d - t_{iso} = \underline{0.210\text{mm}} \qquad (38)$$

Then the number of turns and the length of the wire can be calculated:

$$N = \frac{2V_d}{W_d} \approx \underline{100}$$
$$l_w = V_o N \approx \underline{8.48\text{m}} \qquad (39)$$

Again we can use eq. 4, 6, 11, 18, 20, 25 and 29 to find the parameters of interest which are listed in table 2.

5.3. Design with rectangular winding layout
This design utilizes the winding layout from fig. 6. For this particular design the DC resistance is simply chosen to be:

$$R_E = 0.5\ \Omega \qquad (40)$$

The thickness of the wire and the actual conductor can now be found:

$$W_t = \tfrac{1}{2}V_w = \underline{0.225\text{mm}}$$
$$W_{t,Cu} = W_t - t_{iso} = \underline{0.194\text{mm}} \qquad (41)$$

The number of turns can be calculated using eq. 32.

$$N = \frac{-R_E W_{t,cu} t_{iso}\sigma + \sqrt{R_E^2 W_{t,cu}^2 t_{iso}^2 \sigma^2 + 8R_E V_o V_d W_{t,cu}\sigma}}{2V_o} \downarrow$$
$$N = \underline{39} \qquad (42)$$

Now the width of the wire and the actual conducting copper can be calculated using eq. 33:

$$W_w = \frac{Vd}{0.5N} = \underline{0.616\text{mm}}$$
$$W_{w,Cu} = W_w - t_{iso} = \underline{0.585\text{mm}} \qquad (43)$$

Now the length of the wire can be calculated:

$$l_w = V_o N \approx \underline{3.31\text{m}} \qquad (44)$$

Finally the parameters of interest are found using eq. 4, 6, 11, 18, 20, 25 and 30. The results are listed in table 2.
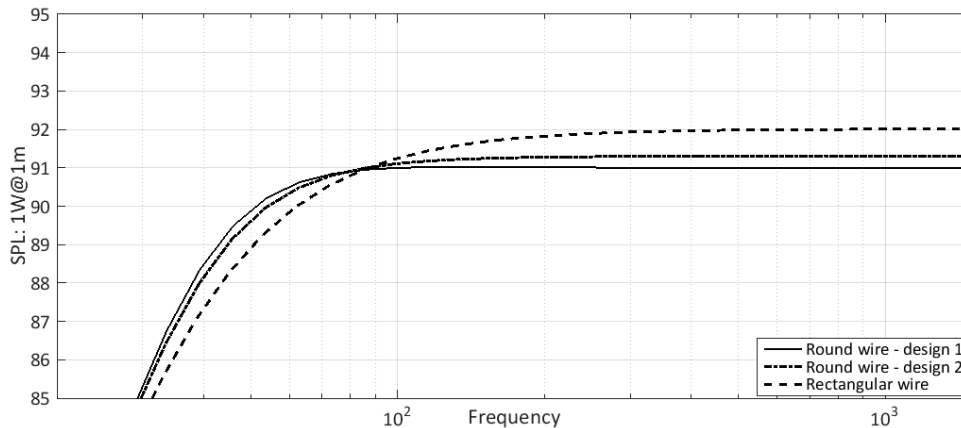
Fig. 8: Frequency response of the three woofer designs.

5.4. Frequency responses

From table 2 we can see that when the fill factor increases the efficiency/sensitivity of the woofer increases as well. Moreover it is seen that the total quality factor decreases when the fill factor is improved. From this the frequency response is expected to gain higher sound pressure levels for midrange frequencies but a slight dampening of the low frequency content when the fill factor is optimized.

We can plot the frequency of the three designs using the pressure transfer function from eq. 22. The electrical input voltage is set so that a rms power of 1 watt is obtained:

$$e_g = \sqrt{1 \text{ W} \cdot R_E} \qquad (45)$$

This ensures that the frequency responses of the different designs can be compared properly. Fig. 8 shows the frequency responses of the three different designs. It is seen that the frequency responses behave exactly as predicted. The decrement of the total quality factor for the high fill factor design causes a minor attenuation of low frequencies. Moreover the sensitivity increases and the difference between the use of round wire, with low fill factor, and rectangular wire, with high fill factor, is approximately 1 dB, corresponding to approximately 30% efficiency improvement.

6. CONCLUSION

This paper has presented a thorough analysis of the loudspeaker efficiency and its relation to the fill factor of the voice coil. It has been shown that the efficiency can be expressed as a function of the fill factor and the geometry of the magnet system. In addition to this a new parameter, the mass ratio of the driver, was introduced. This parameter indicates how much a given driver will benefit from fill factor optimization. The higher mass ratio the higher impact. Moreover the small-signal parameters related to the frequency response of the driver were analysed, e.g the quality factors. It was found that the total quality factor of driver decreases when the fill factor increases. Different voice coil winding layouts have been described and analysed with respect to their fill factors. It was found that by using rectangular wire to realize voice coils with low DC resistance the fill factor could be increased compared to that of conventional voice coils using round wire. Finally three voice coils were designed for a 10" woofer, using different voice coil winding layouts. The response of the woofer was calculated and it showed that the sensitivity for the woofer could be increased up to 1 dB just by optimizing the fill factor using low impedance voice coils realized with rectangular wire. This corresponds approximately to an astonishing 30% efficiency improvement. Since the efficiency of the driver dominates that of an entire audio system consisting of power supply, amplifier and loudspeaker, this efficiency improvement is

valid for the whole audio system. More over this improvement could be even better if the woofer was designed to have a higher mass ratio. In addition to this the total quality factor of the driver could also be designed so that a maximum flat response is obtained, meaning that the low frequencies would not be attenuated.

7. FUTURE WORK

Future work includes a practical realization of a loudspeaker driver using a fill factor optimized-, low impedance-voice coil with rectangular wire. In addition to this measurements of the actual response of a fill factor optimized driver should be compared to the theoretically calculated. Audio power amplifiers capable of handling low impedance drivers are to be developed but with the use of switch-mode technology this should be possible. The authors encourage the development of a new loudspeaker design, with high fill factors, high mass ratios and low DC resistance. This could allow for a series of high efficiency drivers.

8. ACKNOWLEDGEMENTS

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