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# Nonlinear Adaptive Control of Exhaust Gas Recirculation for Large Diesel Engines

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**Abstract:** A nonlinear adaptive controller is proposed for the exhaust gas recirculation system on large two-stroke diesel engines. The control design is based on a control oriented model of the nonlinear dynamics at hand that incorporates fuel flow and turbocharger speed changes as known disturbances to the exhaust gas recirculation. The paper provides proof of exponential stability for closed loop control of the model given. Difficulties in the system include that certain disturbance levels will make a desired setpoint in  $O_2$  unreachable, for reasons of the physics of the system, and it is proven that the proposed control will make the system converge exponentially to the best achievable state. Simulation examples confirm convergence and good disturbance rejection over relevant operational ranges of the engine.

*Keywords:* Exhaust gas recirculation, diesel engines, nonlinear control, adaptive control, modelling for control, parameter estimation.

## 1. INTRODUCTION

Emissions from diesel engines are subject to restriction due to awareness of environmental effects of the emissions. The Tier III restrictions, limiting the emission of  $NO_x$  from marine diesels in selected areas, as was presented by the International Maritime Organization, IMO (2013) will be introduced in 2016. The IMO Tier III rules for environmental protection specifies a reduction of 76% of  $NO_x$  emission compared to the Tier II standard in specified Emission Control Areas, including most of the North American coastal areas, among others. This motivates the ship industry to develop technologies that reduce the emissions of  $NO_x$ .

One of such technologies is Exhaust Gas Recirculation (EGR), which has been applied to four-stroke engines in the automotive industry for several decades. The principle is to recirculate part of the exhaust gas into the engine intake. This decreases the oxygen content and increases the heat capacity of the scavenging gas. In turn the peak temperatures during combustion are decreased, resulting in a decrease in the formation of  $NO_x$  during combustion. Unfortunately, lowering the oxygen content of the scavenging gas also affects the combustion efficiency. At excessively low scavenge air oxygen levels, the engine will produce visible smoke. Thus the optimal scavenging oxygen level is a compromise between fuel economy, smoke formation and  $NO_x$  emissions.

To prepare for the Tier III restrictions, engine designer MAN Diesel & Turbo (MDT) has introduced EGR technology on their large two-stroke marine diesel engines. Other technologies for  $NO_x$  reduction are also being used,

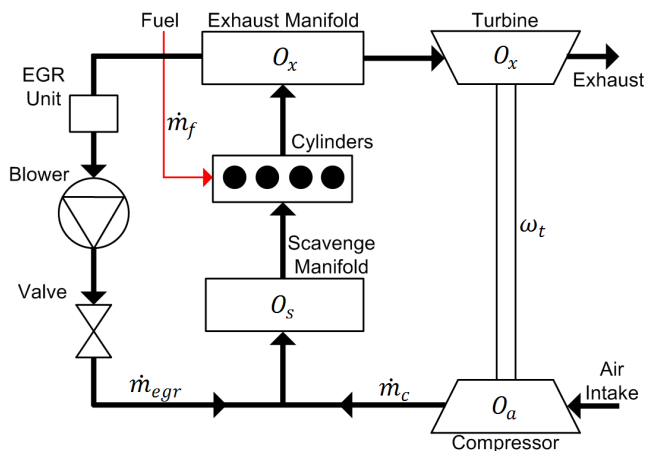


Fig. 1. Simplified overview of engine gas flows.

but the scope of this paper is the control of the EGR system. As the scavenge pressure of a two-stroke engine is higher than the exhaust pressure, a blower is used in the EGR string to provide a pressure increase. The blower speed must be carefully controlled to obtain an EGR flow that leads to the appropriate amount of oxygen in the scavenging gas. A simplified schematic of the engine air path is shown in Figure 1. Some components that are not essential to the paper have been omitted from the Figure. The EGR unit shown in the Figure removes corrosive  $SO_x$  and cools the recirculated gas.

The overall control objective is to obtain feedback control of the oxygen concentration  $O_s$  in the scavenge manifold using either the speed setpoint of the EGR blower or

the opening of the EGR valve as actuator input. This method has been applied to several engine setups. During stationary running conditions existing fixed gain control has shown ability to keep  $O_s$  adequately close to a setpoint. However, this feedback control, being based on an  $O_s$  measurement with inherent sensor dynamics and measurement delay, is an essential limitation for performance. This becomes an issue when handling hard acceleration of the ship and in high sea conditions where waves have significant impact as a fluctuating load torque on the propeller shaft (Hansen et al., 2013a). In both these conditions the engine RPM controller adjusts the flow of fuel into the cylinders and thus changes the appropriate EGR flow. The slow nature of the system and difficulties inherent to measuring oxygen concentration in the scavenge manifold makes the control system react slowly to such disturbances. To avoid smoke formation from too low oxygen, it is currently necessary to limit the possible ship acceleration when the EGR system is running. Such a limitation is undesirable and far from possible in all operating situations. Therefore, an alternative control concept is needed that can cope with pressure dependent sensor measurement delay, sensor dynamics and the nonlinear dynamics of the gas recirculation system, and yet provide a high performance closed loop control.

A clear difference between the EGR control system developed by MDT and the EGR systems in the automotive industry is the effort available for commissioning an EGR controller for an engine configuration. Each automotive engine design is thoroughly tested on a test bench before releasing for large scale production. In opposition to this the specific large two-stroke engine designs are produced in very low numbers, they are sometimes not tested until the first engine is produced and even then very limited test time is available due to very high test running cost. It is furthermore possible that a large two-stroke engine will be reconfigured during its time of operation. The consequences of these practical issues are that manual tuning for the individual design is not applicable and that observer design based on a priori data is impractical. This means that the control design must be robust not only towards changes in system behaviour but also towards imprecise design data.

Numerous examples of modelling and control of EGR systems for automotive engines exist in literature. Notable examples are Wahlström and Eriksson (2011a), Wahlström and Eriksson (2011b) and van Nieuwstadt et al. (2000). Jankovic et al. (2000) proposed nonlinear control of automotive EGR systems using a control Lyapunov function. Modelling of large two-stroke engines have been treated in both classical literature, e.g. Blanke and Andersen (1984), Hendricks (1986) and more recently in Theotokatos (2010) and Hansen et al. (2013b) though only the latter includes an EGR system. Hansen et al. (2013a) presented EGR control design with SISO methods and feed forward of the fuel index. The main issues were found to be parameter sensitivity and the dead time of the oxygen sensor.

This paper introduces an adaptive nonlinear controller for the EGR system, based on a system model that is significantly simpler than traditional mean value models. The control law incorporates known disturbances for faster rejection of these. Exponential stability of the simplified

closed loop system is proven by Lyapunov's direct method. Simulation examples confirm convergence and disturbance rejection properties of the controller.

The control oriented model of the EGR system behavior is briefly introduced in Section 2. Control design and stability proofs are found in Section 3. The closed loop system of the simple EGR model and the controller is simulated in Section 5 followed by a discussion of the validity of the results in Section 6.

## 2. SYSTEM MODEL

This section introduces a model of the scavenge oxygen dynamics in the EGR system. The model is intended as a simplification that is useful for controller design as opposed to conventional mean value approaches that represent a more sophisticated replication of physical processes. In the simple model, the nonlinearities of the stationary system response is used as an input nonlinearity to a first order system. The end result is a first order Hammerstein system with multiple inputs and one output.

### 2.1 Static model

The static model of the scavenge manifold oxygen fraction assumes that the ambient oxygen fraction  $O_a$ , compressor mass flow  $\dot{m}_c$ , recirculated mass flow  $\dot{m}_{egr}$  and fuel mass flow  $\dot{m}_f$  are known.

During stationary conditions, the oxygen fraction in the exhaust  $O_x$  is a function of compressor flow, ambient oxygen fraction, fuel flow  $\dot{m}_f$  and stoichiometric oxygen-to-fuel ratio  $k_f$ . Assuming a complete, lean combustion,  $O_x$  is modelled as in Hansen et al. (2013b).

$$O_x = \frac{\dot{m}_c O_a - \dot{m}_f k_f}{\dot{m}_c + \dot{m}_f} \quad (1)$$

The oxygen fraction in the scavenge manifold  $O_s$  at stationary state is the average of ambient and exhaust oxygen weighted by compressor flow and recirculated flow  $\dot{m}_{egr}$ , respectively.

$$O_s = \frac{\dot{m}_c O_a + \dot{m}_{egr} O_x}{\dot{m}_c + \dot{m}_{egr}} \quad (2)$$

Combining (1) and (2) leads to a static model of  $O_s$ , based on the 3 major flows.

$$O_s = O_a - (O_a + k_f) \frac{\dot{m}_f}{\dot{m}_c + \dot{m}_f} \cdot \frac{\dot{m}_{egr}}{\dot{m}_c + \dot{m}_{egr}} \quad (3)$$

Isolating the recirculated flow in (3) leads to an expression that is useful for the control design.

$$\dot{m}_{egr} = \frac{\dot{m}_c (O_a - O_s)}{O_s - \frac{\dot{m}_c O_a - \dot{m}_f k_f}{\dot{m}_c + \dot{m}_f}} = \frac{\dot{m}_c (O_a - O_s)}{O_s - O_x} \quad (4)$$

The recirculated flow and the fuel flow are both assumed to be available to the controller, but the compressor flow is not. Estimation from a compressor map is ruled out as maps that covers all operating points are not practically available for each engine. Instead the flow is approximated as a simple function of compressor speed  $\omega_t$

$$\dot{m}_c = \omega_t^a \cdot \theta \quad , \quad a \in [1 : 2] \quad , \quad \theta > 0 \quad (5)$$

where  $a$  and  $\theta$  are constants. A similar approximation was done by Hendricks (1986) where the compressor flow

was approximated as a function of the scavenge pressure. Introduction of EGR adds to the inaccuracy of (5) and  $\theta$  is expected to change slightly depending on the operating point. The constant  $a$  depends on the specific engine.

## 2.2 Dynamic model

In traditional models the turbocharger dynamics receive great emphasis due to their significant contribution to the system behaviour as was shown by Blanke and Andersen (1984). In the present paper the turbocharger speed  $\omega_t$  is treated as a known disturbance rather than a state, thus avoiding the interdependency between fuel flow and turbocharger speed. The focus of this model is the oxygen fractions, thus the main dynamics are the mixing of gas in the manifolds. Furthermore the scavenge oxygen sensor is expected to contribute with varying time delay and first order dynamics.

Neglecting the pure time delay, the mixing and sensor dynamics are lumped together as a single first order system in this approach to obtain the simplest model. A known time constant  $\tau$  is assumed. The nonlinearity expressed in the static model is treated as an input nonlinearity and the result is a first order Hammerstein system with multiple inputs and one output.

The recirculated flow is treated as an actuated input  $u$ , whereas fuel flow and turbine speed are gathered in the vector signal  $d$  as known disturbances.

$$u = \dot{m}_{egr} \quad , \quad d(t) = \begin{bmatrix} \dot{m}_f \\ \omega_{tc} \end{bmatrix} \quad (6)$$

The measured scavenge oxygen fraction is the state variable and a reference value  $r$  between zero and ambient oxygen fraction is also defined.

$$x = O_s \quad , \quad r = O_{s,ref} \quad , \quad 0 < r < O_a \quad (7)$$

Combining (3) and (5) the static expression of the scavenge oxygen fraction is a function  $g$  of the input, known disturbances and unknown parameter  $\theta$ .

$$g(\theta, d, u) = O_a - (O_a + k_f) \frac{\dot{m}_f}{\omega_t^a \theta + \dot{m}_f} \cdot \frac{\dot{m}_{egr}}{\omega_t^a \theta + \dot{m}_{egr}} \quad (8)$$

The Hammerstein system with the static expression as input nonlinearity and known time constant  $\tau$  is then

$$\tau \dot{x} = g(\theta, d, u) - x \quad (9)$$

## 3. CONTROLLER

This paper proposes a nonlinear adaptive controller. The control law is based on Equation (4) which is an inversion of the input nonlinearity of the Hammerstein system. The inversion is defined as

$$h(\theta, d, r) = \frac{\omega_t^a \theta (O_a - O_{s,ref})}{O_{s,ref} - \frac{\omega_t^a \theta \cdot O_a - \dot{m}_f k_f}{\omega_t^a \theta + \dot{m}_f}} \quad (10)$$

As the parameter  $\theta$  is expected to vary slightly depending on the operating point a nonlinear parameter estimator continuously provides an estimate  $\hat{\theta}$  for use in the control law. The estimator is similar to the ones proposed by Tyukin (2003) in the way a direct term makes the time derivative of the estimate depend on the time derivative

of a measurement without explicitly having to differentiate any signals. The proposed controller is

$$\hat{\theta} = k \cdot \left( \tau \dot{x} + \int x - g(\hat{\theta}, d, u) dt \right) \quad (11)$$

$$u = \begin{cases} h(\hat{\theta}, d, r) & \text{if } h(\hat{\theta}, d, r) \in [0; u_{max}] \\ u_{max} & \text{otherwise} \end{cases} \quad (12)$$

where  $k$  is an observer gain and  $u_{max}$  is the highest possible EGR flow.

The conditional form of the control law is necessary in the case where it is not physically possible to invert the static model, based on the known disturbances and the estimated parameter.

The proposed controller specifies a setpoint of the EGR flow and assumes that the current EGR flow is known. Thus an inner loop that controls the blower speed and valve opening based on a measurement or an estimate of the flow is required. This inner loop is not treated further in this paper.

## 4. STABILITY ANALYSIS

This section investigates the stability properties of the closed loop system. The parameter estimator (11) and control law (12) are assumed to act on the Hammerstein system (9). The reference value  $r$  is constant.

The analysis considers the convergence of the control error  $\tilde{x} = x - r$  and the parameter estimation error  $\tilde{\theta} = \hat{\theta} - \theta$ .

The stability analysis is divided into two parts, each dealing with one of the two cases of the control law. Before the analysis it is necessary to introduce two positive limits  $\gamma_g$  and  $\gamma_h$  regarding the sensitivity of the functions  $g$  and  $h$

$$\frac{\partial g(\hat{\theta}, d, u)}{\partial \hat{\theta}} \geq \gamma_g \quad , \quad \left| \frac{\partial g(\theta, d, h(\hat{\theta}, d, r))}{\partial \hat{\theta}} \right| \leq \gamma_h \quad (13)$$

The validity of these limits will be revisited in the last part of the analysis.

### 4.1 First case

Lyapunov's direct method is used to prove exponential stability when

$$h(\hat{\theta}, d, r) \in [0; u_{max}] \quad (14)$$

The dynamics of the system state, given the control law are

$$\tau \dot{x} = g(\theta, d, h(\hat{\theta}, d, r)) - x \quad (15)$$

Note that  $h$  inverts  $g$  in the actuated input

$$r = g(\theta, d, h(\theta, d, r)) \quad (16)$$

From (15) and (16), with constant  $r$

$$\tau \dot{\tilde{x}} = g(\theta, d, h(\hat{\theta}, d, r)) - g(\theta, d, h(\theta, d, r)) + r - x \quad (17)$$

$$\Leftrightarrow \tau \dot{\tilde{x}} = \eta(\tilde{\theta}, d) - \tilde{x} \quad (18)$$

where

$$\eta(t, \tilde{\theta}) = g(\theta, d, h(\theta + \tilde{\theta}, d, r)) - g(\theta, d, h(\theta, d, r)) \quad (19)$$

From (9),  $\tau \dot{x} + x = g(\theta, d, u)$ , hence the dynamics of the parameter estimator error are

$$\begin{aligned}\dot{\hat{\theta}} &= \hat{\theta} = k \cdot (\tau \dot{x} + x - g(\hat{\theta}, d, u)) \\ &= k \cdot (g(\theta, d, u) - g(\theta + \tilde{\theta}, d, u)) = -k\tilde{g}(\tilde{\theta}, d)\end{aligned}\quad (20)$$

where

$$\tilde{g}(t, \tilde{\theta}) = g(\theta + \tilde{\theta}, d, h(\theta + \tilde{\theta}, d, r)) - g(\theta, d, h(\theta + \tilde{\theta}, d, r)) \quad (21)$$

The time derivative of the observer error  $e = [\tilde{x} \ \tilde{\theta}]^T$  is defined as  $f(t, e)$

$$\dot{e} = \begin{bmatrix} \frac{1}{\tau} (\eta(t, \tilde{\theta}) - \tilde{x}) \\ -k\tilde{g}(t, \tilde{\theta}) \end{bmatrix} = f(t, e) \quad (22)$$

A Lyapunov function  $V$  is chosen, where  $c$  is a constant

$$V = \frac{1}{2}\tilde{x}^2 + \left( \frac{\gamma_\eta^2}{8k\gamma_g\tau(1-\tau c)} + \frac{c}{2k\gamma_g} \right) \tilde{\theta}^2, \quad 0 < c < \frac{1}{\tau} \quad (23)$$

The derivative of  $V$  is

$$\begin{aligned}\frac{\partial V}{\partial t} + \frac{\partial V}{\partial e} f(t, e) &= \tilde{x} \frac{1}{\tau} (\eta(t, \tilde{\theta}) - \tilde{x}) \\ &\quad - \left( \frac{\gamma_\eta^2}{4k\gamma_g\tau(1-\tau c)} + \frac{c}{k\gamma_g} \right) \tilde{\theta} \cdot k\tilde{g}(t, \tilde{\theta}) \\ &= - \left( \frac{1}{\tau} - c \right) \tilde{x}^2 - c\tilde{x}^2 + \frac{1}{\tau} \tilde{x} \eta(t, \tilde{\theta}) \\ &\quad - \left( \frac{\gamma_\eta^2}{4\gamma_g\tau(1-\tau c)} + \frac{c}{\gamma_g} \right) \tilde{\theta} \tilde{g}(t, \tilde{\theta})\end{aligned}\quad (24)$$

The contributions from  $\tilde{g}(t, \tilde{\theta})$  and  $\eta(t, \tilde{\theta})$  are limited by use of the conditions (13)

$$\tilde{\theta} \cdot \tilde{g}(t, \tilde{\theta}) = \tilde{\theta} \cdot \int_{\theta}^{\theta + \tilde{\theta}} \frac{\partial g(s, d, u)}{\partial s} ds \geq \gamma_g \tilde{\theta}^2 \quad (25)$$

$$|\eta(t, \tilde{\theta})| = \left| \int_{\theta}^{\theta + \tilde{\theta}} \frac{\partial g(\theta, d, h(s, d, y))}{\partial s} ds \right| \leq \gamma_\eta |\tilde{\theta}| \quad (26)$$

Thus

$$\begin{aligned}\frac{\partial V}{\partial t} + \frac{\partial V}{\partial e} f(t, e) &\leq - \left( \frac{1}{\tau} - c \right) \cdot \\ &\quad \left( \tilde{x}^2 + \frac{\gamma_\eta^2}{4\tau^2(\frac{1}{\tau} - c)^2} \tilde{\theta}^2 - \frac{\gamma_\eta}{\tau} |\tilde{x}| \cdot |\tilde{\theta}| \right) - c(\tilde{x}^2 + \tilde{\theta}^2) \\ &= - \left( \frac{1}{\tau} - c \right) \left( \tilde{x} - \frac{\gamma_\eta}{2\tau(\frac{1}{\tau} - c)} \tilde{\theta} \right)^2 - c\|e\|^2 \leq -c\|e\|^2\end{aligned}\quad (27)$$

Theorem 4.10 in Khalil (2002) implies exponential stability of  $e = 0$  when both conditions (13) apply.

#### 4.2 Second case

This part of the stability analysis considers the case where

$$h(\hat{\theta}, d, r) \notin [0; u_{max}] \quad (28)$$

that is, when the static system is not invertible within the actuator limits. More insight into when this occurs can be gained by reviewing the equations defining the system. By (1),  $O_x < O_a$  when all signals and parameters are positive.  $O_s$  is a weighted average of  $O_a$  and  $O_x$ , hence  $O_s \in ]O_x; O_a]$ . Small EGR flows are required for  $O_s$  close

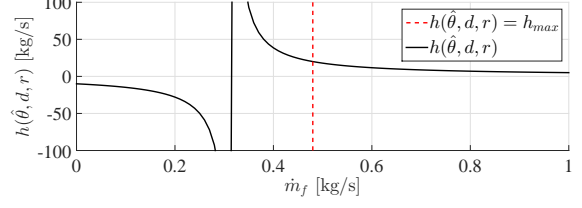


Fig. 2. Example of  $h(\hat{\theta}, d, r)$  when varying  $\dot{m}_f$ .

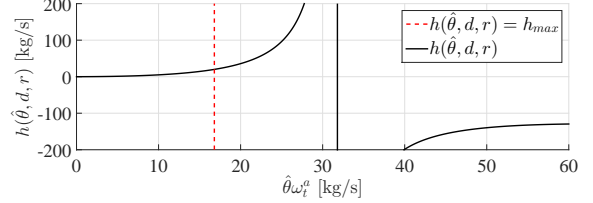


Fig. 3. Example of  $h(\hat{\theta}, d, r)$  when varying  $\hat{\theta}\omega_t^a$ .

to  $O_a$  and large EGR flows are required for  $O_s$  close to  $O_x$ . No physically possible values of EGR flow result in  $O_s$  equal to or lower than  $O_x$ . As  $O_{s,ref} < O_a$ , problems with inverting the system only occurs when  $O_{s,ref}$  is low compared to  $O_x$ .

Figures 2 and 3 illustrates the issue of mathematically inverting the system with examples of  $h(\hat{\theta}, d, r)$ , when varying  $\dot{m}_f$  and  $\hat{\theta}\omega_t^a$ , respectively. The reference value is fixed at 17% in both cases.

Normal operation occurs at the rightmost part of Figure 2. Lowering the fuel flow increases the exhaust oxygen fraction and thus calls for a higher EGR flow to reach the reference value  $r$ . The dashed line indicates the value of  $\dot{m}_f$  for which the maximum EGR flow (in this case 20 kg/s) is reached. The required EGR flow approaches infinity as the estimated exhaust oxygen fraction approaches the reference value. The result is a vertical asymptote in Figure 2. For all values of  $\dot{m}_f$  below the dashed line, the best option available to the controller is the maximum EGR flow. Beyond the asymptote, the values of  $h$  are negative. Care must be taken when implementing the control law as the asymptote represents an undefined value of  $h$ . This is solved by evaluating whether the denominator of (10) is close to 0.

Figure 3 depicts  $h(\hat{\theta}, d, r)$  when varying the estimated compressor flow  $\omega_t^a \hat{\theta}$ . Normal operation occurs at the leftmost part of the figure. Higher estimated compressor flow result in higher exhaust oxygen fraction and thus requires a higher EGR flow. As above, the point where the maximum EGR flow is reached is marked with a dashed line and the vertical asymptote indicates the point where the estimated exhaust oxygen fraction equals the reference value. Again, for all values of  $\omega_t^a \hat{\theta}$  beyond the dashed line, the best option available to the controller is the maximum EGR flow.

It is important to distinguish between the case where the actual static system is non-invertible and the case where only the estimated static system is non-invertible. In the first case, the maximum EGR flow is the optimal choice. For both cases it is important that  $\hat{\theta}$  converges to 0 such

that the control law converges to either the correct system inversion or the maximum flow. The isolated convergence of  $\tilde{\theta}$  is proven using Lyapunov's direct method.

The following Lyapunov function is chosen

$$V = \frac{1}{2}\tilde{\theta}^2 \quad (29)$$

The first sensitivity condition implies

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial e} f(t, e) = -k\tilde{\theta}\tilde{g}(t, \tilde{\theta}) \leq -k\gamma_g\tilde{\theta}^2 \quad (30)$$

From Theorem 4.10 in Khalil (2002)  $\tilde{\theta}$  will converge exponentially toward 0. Thus, the convergence depends on the first sensitivity condition rather than the control law.

#### 4.3 Sensitivity conditions revisited

The lower limit of the sensitivity of  $g(\theta, d, u)$  to  $\theta$  is used in both cases of the stability analysis.

$$\frac{\partial g(\theta, d, u)}{\partial \theta} = (O_a + k_f) \frac{(2\omega_t^a \theta + \dot{m}_f + \dot{m}_{egr}) \dot{m}_f \dot{m}_{egr} \omega_t^a}{(\omega_t^a \theta + \dot{m}_f)^2 (\omega_t^a \theta + \dot{m}_{egr})^2} \quad (31)$$

If positive lower and upper limits are defined for all parameters and signals, a lower limit ( $\gamma_g$ ) of the sensitivity exists. Thus first sensitivity condition is only satisfied if the EGR flow has a positive lower limit. Considering (10), the commanded EGR flow is positive, unless either the estimated compressor flow is zero or if  $O_{s,ref}$  equals  $O_a$ . Thus the estimated parameter must be initialised with positive value and will not converge when  $r = O_a$ .

The second sensitivity condition is only used for the first part of the stability analysis. It specifies an upper bound to the absolute value of the sensitivity of  $g(\theta, d, h(\hat{\theta}, d, r))$  to  $\hat{\theta}$ . With the chain rule

$$\left| \frac{\partial g(\theta, d, h(\hat{\theta}, d, r))}{\partial \hat{\theta}} \right| \leq \left| \frac{\partial g}{\partial u}(\theta, d, h(\hat{\theta}, d, r)) \right| \cdot \left| \frac{\partial h}{\partial \hat{\theta}}(\hat{\theta}, d, r) \right| \quad (32)$$

The first term on the right side is

$$\left| \frac{\partial g}{\partial u}(\theta, d, h(\hat{\theta}, d, r)) \right| = \left| \frac{\dot{m}_f (O_a + k_f) \omega_t^a \theta}{(\omega_t^a \theta + \dot{m}_f)(\omega_t^a \theta + \dot{m}_{egr})^2} \right| \quad (33)$$

As all signals and parameters have positive lower and upper bounds, an upper bound to the expression exists.

The second term on the right hand side of (32) is

$$\left| \frac{\partial h}{\partial \hat{\theta}}(\hat{\theta}, d, r) \right| = \left| \frac{(\hat{\theta}^2 \omega_t^{2a} (O_a - r) - (k_f + r)(2\dot{m}_f \hat{\theta} \omega_t^a + \dot{m}_f^2)) (O_a - r) \omega_t^a}{((\hat{\theta} \omega_t^a + \dot{m}_f)r - \hat{\theta} \omega_t^a O_a + \dot{m}_f k_f)^2} \right| \quad (34)$$

All signals and parameters have positive bounds, except for  $\hat{\theta}$ . The denominator is only zero when

$$r = \frac{\hat{\theta} \omega_t^a O_a - \dot{m}_f k_f}{\hat{\theta} \omega_t^a + \dot{m}_f} \quad (35)$$

From (1), the equation above applies when the reference value is equal to the estimated exhaust oxygen fraction.

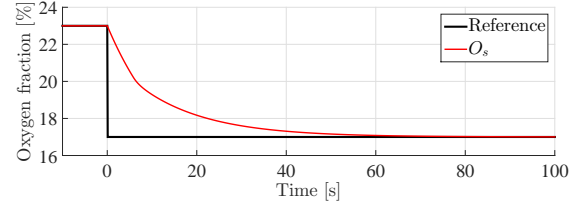


Fig. 4. Simulation of  $O_s$  during EGR system start-up.

This corresponds to the vertical asymptotes in Figures 2 and 3 and therefore does not apply in the first case of the stability analysis. Thus a positive lower limit must exist for the denominator of (34). Furthermore, as both the numerator and the denominator of (34) are second order polynomials in  $\hat{\theta}$  an upper limit of (34) exists. Having bounded both terms on the right hand side of (32), an upper ( $\gamma_\eta$ ) limit to the sensitivity exists and the second sensitivity condition applies in the first case of the stability analysis.

## 5. SIMULATION

The convergence of the state and parameter errors are illustrated by two simulation examples. The disturbance signals and model parameters are within the range of values of a real system. Table 1 shows the values along with the gain  $k$  of the parameter estimator.

Table 1. Parameters used for simulation:

$O_a$	23 %	$r$	17 %
$\tau$	15 s	$k_f$	3.4
$\dot{m}_f$	1-3 kg/s	$\theta$	2 g/RPM
$\dot{m}_{egr,max}$	20 kg/s	$\omega_t$	10 kRPM
$a$	1	$k$	20 (g/RPM)/s

In a real engine the turbocharger speed is affected by the fuel and EGR flows. This effect is not present in the simulation here as both fuel flow and turbocharger speed are kept constant except for a single step in fuel flow in the second simulation.

### 5.1 Convergence during start-up

The first example illustrates the convergence of state and parameter errors during start-up of the EGR system. The initial parameter estimate is 5 times the actual parameter to show convergence under the second case of the control law. Simulated scavenge oxygen fraction, EGR flow and the estimated parameter are shown in Figures 4, 5 and 6.

An EGR system start-up is simulated as a step response in the reference  $O_{s,ref}$  from 23% (ambient) to 17% at time 0 seconds. After the step,  $O_s$  converges to the new reference value without overshoot. EGR flow is zero before the step as this keeps  $O_s$  at the ambient level no matter what positive value the compressor flow has. Without EGR flow, the model loses sensitivity to  $\hat{\theta}$  so the parameter does not converge. Immediately after the step, the erroneous parameter estimate causes maximum EGR flow. The parameter estimate and thus the EGR flow converges after about 10 seconds to their final values without any overshoot.

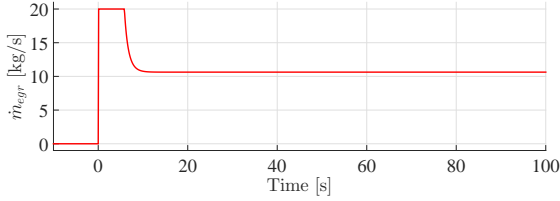


Fig. 5. Simulation of  $\dot{m}_{egr}$  during EGR system start-up.

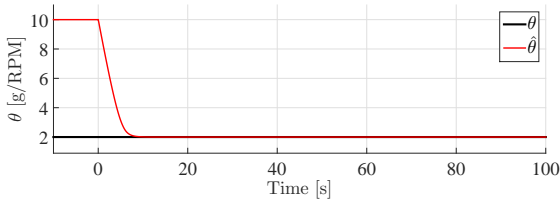


Fig. 6. Simulation of  $\hat{\theta}$  during EGR system start-up.

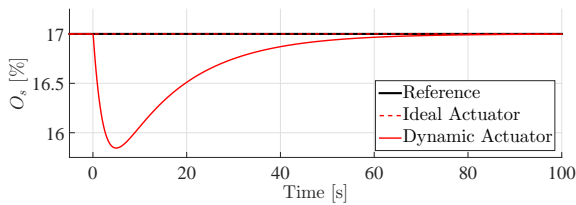


Fig. 7. Simulation of  $O_s$  during fuel flow step.

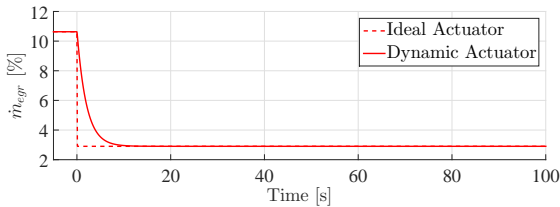


Fig. 8. Simulation of  $\dot{m}_{egr}$  during fuel flow step.

### 5.2 Disturbance step

The second simulation example illustrates how the controller handles a fuel flow step from 1 to 3 kg/s when the parameter estimate has converged. Figures 7 and 8 show simulated  $O_s$  and  $\dot{m}_{egr}$ , respectively (dashed lines). The combination of perfect input inversion and no actuator dynamics makes the controller compensate perfectly for the step. A simulation that includes first order actuator dynamics of the form

$$\tau_{act} \frac{d\dot{m}_{egr}}{dt} = u - \dot{m}_{egr} \quad (36)$$

with  $\tau_{act} = 2$  s, is also shown. The actuator dynamics make  $O_s$  deviate to just below 16% before converging to the reference value without overshoot. The parameter estimate (not shown) is not affected by the step in any of the cases as the estimate will converge as long as the EGR flow is positive.

## 6. DISCUSSION OF VALIDITY

The intention of the presented design approach is to model the essential system behaviour and develop a controller that is robust toward the remaining unmodelled dynamics. The most significant simplifications are the compressor flow model and the assumption of first order dynamics. The approximate time constant might vary slightly depending on the operating point. Also, as the dynamics depend on the specific  $O_s$  sensor, this assumption should be revisited with an analysis of the behaviour of specific sensor types. Compensation for the time delay of the  $O_s$  measurement is also an issue. Exponential convergence of the control error is a positive indication of robustness of the proposed controller towards unmodelled dynamics. However, a thorough study of the control performance when simulating control of a more sophisticated model is regarded as a necessary step before introducing the method in practice.

Estimation and control of the EGR mass flow is a prerequisite for the proposed controller. Although this increases the controller complexity further, it also facilitates a control structure where the overall  $O_s$  controller is not dependent on whether the EGR flow is actuated by varying the blower speed or the valve opening.

## 7. CONCLUSION

A Hammerstein model was developed of the scavenge oxygen fraction of an EGR system. The model is intended for control design rather than accurate simulation. A nonlinear adaptive controller was proposed based on the simple model of the scavenge oxygen fraction. A controller was developed that inverts the input nonlinearity of the Hammerstein model and continuously estimates a parameter that change with the operating point of the turbocharger. The parameter estimator includes a tuning parameter whereas the control law requires no tuning and can be parameterized purely on overall engine metadata. Exponential convergence of control and parameter errors where proven with Lyapunov's direct method. Certain disturbance values were shown to make the  $O_2$  setpoint unreachable and it was proven that the system converges to the optimal state when using the proposed controller. Simulations confirmed convergence and good compensation of known disturbances also when actuator dynamics was included.

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