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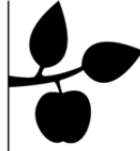
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An Indicator for Ecosystem Externalities in Fishing

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AN INDICATOR FOR ECOSYSTEM EXTERNALITIES IN FISHING

Lars Ravn-Jonsen^{a,c}, Ken H. Andersen^b, Niels Vestergaard^a

ABSTRACT

Ecosystem externalities arise when one use of an ecosystem affects its other uses through the production functions of the ecosystem. We use simulations from a size-spectrum ecosystem model to investigate the ecosystem externality created by fishing of multiple species. The model is based upon general ecological principles and is calibrated to the North Sea. Two fleets are considered: a “forage fish” fleet targeting species that mature at small sizes and a “large fish” fleet targeting large piscivorous species. Based on the marginal analysis of the present value of the rent, we develop a benefit indicator that explicitly divides the consequences of fishing into internal and external benefits. This analysis demonstrates that the forage fish fleet has a notable economic impact on the large fish fleet, but the reverse is not true. The impact can be either negative or positive, which entails that for optimal economic exploitation, the forage fishery has to be adjusted according to the large fish fishery. With the present large fish fishery in the North Sea, the two fisheries are well adjusted; however, the present combined exploration level is too high to achieve optimal economic rents.

Keywords: Ecosystem Externalities, Forage Fish, Benefit Indicator, Marine Ecosystems, Fisheries management, Size-based, North Sea

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1. INTRODUCTION

When a fish stock is fished, the impact of that fishing will also affect the other stocks in the ecosystem. For example, fishing piscivorous species should have a beneficial effect on their prey, whereas fishing forage species should have a detrimental effect on their predators. Fishing on a stock therefore incur an opportunity cost on other fisheries in terms of possible smaller outcome. In economic terms this means that a fishing fleet not only affects its own outcome but also, through the ecosystem, imposes externalities on other fleets and ecosystem users.

If fishery management have to move beyond the traditional single stock approach, the opportunity cost of catch from different stocks has to be apparent and perceptual. Our aim is to evaluate how one use of an ecosystem, here one fleet, indirectly affects others users, here other fleets, and develop an indicator that demonstrates this opportunity cost. By dividing the economic consequences into internal and external benefits we elucidate how the fishing fleets affect each other and thereby provides guidance into possible trade offs between fishing fleets in the exploitation of the ecosystem.

Accounting for the interaction between fleets requires an ecosystem model that captures the multi-species nature of the ecosystem. Purely data-driven approaches, e.g., analysis of catch data, are unable to provide an understanding of the drivers and dynamics within an ecosystem. One approach in the fishery economics literature is to use simple conceptual models to obtain qualitative insights on ecosystems (Hannesson 1983, 2002). The most common approach is to investigate the interaction of two or more trophic levels using Lotka-Volterra-type predator-prey models, e.g., Wilen and Wilen (2012). Such models capture the predator-prey interactions of different species, however, by characterizing each species by its biomass only, they fail to account for the large variation in size within each species. Individuals within a fish species varies in size from about 0.001 g to their asymptotic size of between about 10 g from forage fish to between 10 and 100 kg for the largest predatory fish. The size of individuals characterizes their interaction with other individuals (big individuals eat smaller ones), their bioenergetics, fisheries gear selectivity and, more important in this context, their economic value. To adequately resolve the ecological and the economic reality of the ecosystem (Tschirhart 2009) we therefore use a size-based model of the ecosystem.

There is a growing literature describing different types of size-spectrum models (Benoît and Rochet 2004; Pope et al. 2006; Hall et al. 2006; Hartvig, Andersen, and Beyer 2011) and their application to understanding how

marine ecosystems respond to fishing (Pope et al. 2006; Hall et al. 2006; Andersen and Pedersen 2010; Blanchard et al. 2014). These models are based on a few simple and generally accepted assumptions at the level of the individual organisms and their dynamics are explicitly driven by predation and individual growth. Individuals in the model are characterized by their size (weight). As fishing gear is size-selective and the prices of landed fish also depend upon size, these models are ideally suited for economic reasoning and calculations. We use a previously developed and calibrated size-spectrum model represent the ecological reality of the ecosystem (Hartvig, Andersen, and Beyer 2011; Andersen, Brander, and Ravn-Jonsen 2015). We use the model to calculate how a change in fishing on one fleet affects the abundance and sizes of fish target by other fleets.

To illustrate the applicability of the benefit indicator we use the North Sea fishery as a case. For this system data for the value of the different fish species broken down into size groups is available. To simplify the description we focus on the internal and external benefits of two fleets: the forage fleet targeting small species and the consumer fleet targeting large species. We use the benefit indicator to show that the forage fish fleet has a notable economic impact on the large fish fleet, but that the reverse is not true. The impact can be either negative or positive, depending upon the level of exploitation in the system, i.e. forage fishing may even have positive externality on the large fish fleet. For the North Sea we find, that at the current level of large fish fishery, the level of the forage fish fishery leads to an adequate externality. To achieve the optimal total benefit from fishing the ecosystem, however, the present combined exploitation level must be reduced.

The structure of this paper is as follows. The biological model is briefly explained in section 2, the benefit indicator is derived in section 3, the economic model is developed in section 4, and our results are presented in section 5 and discussed in section 6. In the supporting material (Ravn-Jonsen, Andersen, and Vestergaard 2015) there is additional information on: the estimation of the cost parameters (Chap. A), the estimation of the price model (Chap. B), the details of the biological model (Chap. C), the sensitivity analysis of some of the cost parameters (Chap. D), and the sensitivity of the model to the choice of the control variable (Chap. E).

2. ECOSYSTEM MODEL

The size- and trait-based model is well described elsewhere (Hartvig, Andersen, and Beyer 2011; Andersen, Brander, and Ravn-Jonsen 2015), and

we will only provide a general description of the basic principles here. For a complete description, see Ravn-Jonsen, Andersen, and Vestergaard (2015, Chap. C). The aim of a trait-based, size-spectrum model is to calculate the abundance of individuals, $N(w, W)$, as a function of the size of the individuals, w , and the asymptotic (maximum) size that the individual may reach, W (Fig. 1 panel A). The representation of the trait W as a continuous variable makes it possible to circumvent the need to represent specific species; the diversity of the fish community is instead characterized by the attribute of their asymptotic size W . The central process in the model is that large individuals eat smaller individuals. This predator-prey interaction leads to a decrease in prey abundances and to somatic growth (production) of large predator individuals. All of the parameters in the size-spectrum model are related to individual weight, which makes it possible to formulate the model with a small set of general parameters, prompting the labeling of the model as “charmingly simple” (Pope et al. 2006). The equations and parameters of the models are described in Ravn-Jonsen, Andersen, and Vestergaard (2015, Chap. C).

To disclose the opportunity costs of fishing at different trophic levels, the fishery is divided into two fleets: one targeting small forage fish and one targeting large piscivorous fish. In this context, forage fish refer to fish that are prey all of their life. The *forage fish fleet* is then characterized by catching small fish from fish species that mature at small sizes, and their harvest is used for industrial reduction into fishmeal and oil. The *large fish fleet* catches piscivorous fish, i.e., fish species that are relatively large when mature, and are sold for direct human consumption.

Fishing in the model is represented by the product of the overall fishing mortality, \mathcal{F} , and the selectivity as a function of size and trait, $\omega(w, W)$. The two fleets are characterized by the range of asymptotic sizes they target; the forage fish fleet targets species with $W < 512$ g (solid lines Fig. 1A), and the large fish fleet targets $W \geq 512$ g (dashed lines Fig. 1A). Hence the two fleets fishery are non-overlapping with respect to species. The species are only interacting through the predator-prey relation. The overall fishing mortality rate of the two fleets, $(\mathcal{F}_F, \mathcal{F}_L)$, is the control variable in the model. The size-selectivity is modeled as a trawl selectivity curve with an S-shaped function (Fig. 1B). The output of the ecological model is the harvest with respect to fish size (Fig. 1C), which gives the revenue when multiplied by price and integrated over all sizes.

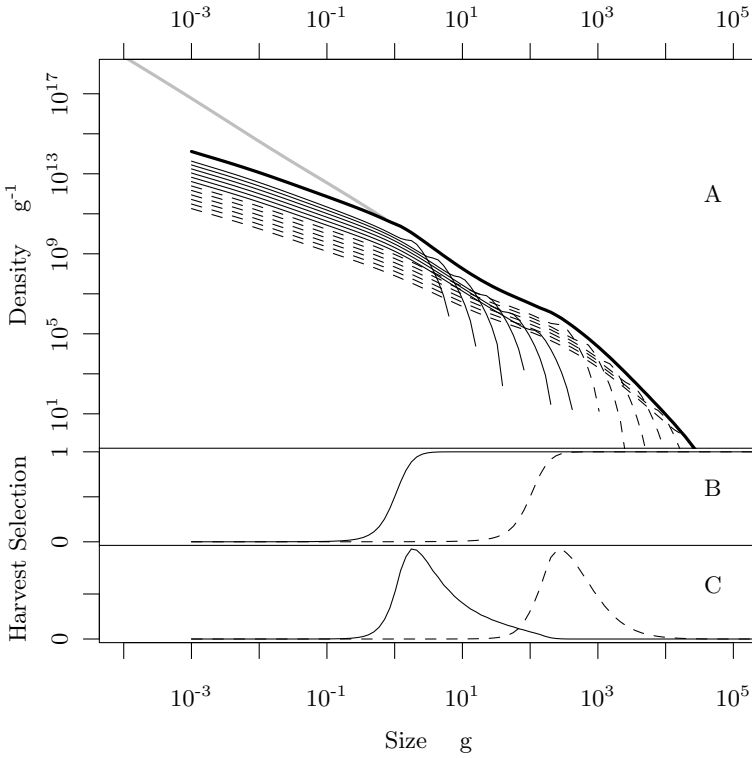


FIGURE 1: Run of the model with the fishing effort of the two fleets set to correspond with the current exploitation of the North Sea. Solid lines represent the forage fish fleet, dashed lines the large fish fleet. A: Density of fish, $N_i(w)$, as a function of individual weight, w . Each thin line represents a population that is characterized by the maximum size, W , of individuals in the population. The thick black line is the sum of all of the populations. The grey line is the background spectrum that represents the plankton community that provides food for the smallest individuals. B: Fishing size-selection function, $\omega(w)$. C: The density of the harvest. The total harvest is the integral under the curves; however, as the abscissa represents the size on the logarithmic scale, the areas under the curves are scaled by the logarithm of their size to be visual comparable: the plotted line is $\mathcal{F}\omega Nw \log(w)$.

3. BENEFIT INDICATOR

To evaluate the ecosystem wide effects of fishing, we develop a benefit indicator that is the marginal change in the present value of the rent when the system is brought from one steady state to another, taking the dynamic effects of the ecosystem during the change explicitly into consideration. A change of state is prompted through a change in the fishing pressure of one of the fishing fleets. The consequences of a change are characterized by the *internal benefit* of the fleet that imposes the change and the *external benefit* experienced by the other fleet.

Each fleet has one control variable, the overall fishing mortality rate $(\mathcal{F}_F, \mathcal{F}_L)$, where \mathcal{F}_F is for the fleet targeting forage fish and \mathcal{F}_L is for the fleet targeting larger fish. We define *continue as usual* as keeping a constant \mathcal{F} and *an action* as changing \mathcal{F} . To generalize the method, the two fleets are called i and j , where (i, j) can be either (F, L) or (L, F) .

The ecosystem services generated by the fleets i and j are the harvests y_i and y_j —appraised by the rents (net values) π_i and π_j . Harvests and the rents vary through time; to include the time component, the benefit of fleet i is summarized by Y_i and V_i , which are the present value of the harvest and the rent, respectively, using the social discount rate ρ (equivalent for fleet j):

$$Y_i := \int_0^{\infty} y_i(t) e^{-\rho t} dt \quad (3.1)$$

$$V_i := \int_0^{\infty} \pi_i(t) e^{-\rho t} dt \quad (3.2)$$

We consider a baseline situation where the ecosystem is in equilibrium with its fleets and the outputs are constant. We consider a change in the harvest of fleet i prompted by a change in \mathcal{F}_i ; the system will then no longer be in equilibrium. Because of the restriction imposed by the ecosystem functions, the change in fleet i 's harvest will lead to changes in the harvest and rent for fleet j as well. As fleet j is *continue as usual*, the changes in this fleet is an externality.

Because the change in V_j will depend upon $\Delta y_i(t)$, we use a concept from cost-effectiveness analysis (Garber and Phelps 1997; Kronbak and Vestergaard 2013) and evaluate $\Delta V_j / \Delta Y_i$. We define the benefit indicator of fleet j 's rent per unit of fleet i 's harvest at the limit, $\Delta Y_i \rightarrow 0$, as:

$$B_{j/i} := \frac{\partial V_j}{\partial Y_i} \quad (3.3)$$

The definition in (3.3) is incomplete in itself. To make it rigorous it has to be combined with the previous definition of *continue as usual* and *an action*. We depart from $\dot{N}^*(\mathcal{F}_i, \mathcal{F}_j)$ an ecosystem in equilibrium with $(\mathcal{F}_i, \mathcal{F}_j)$

$$\text{Depart from: } N_{t=0} = \dot{N}^*(\mathcal{F}_i, \mathcal{F}_j) \quad (3.4)$$

At $t = 0$ fleet i change the fishing mortality while fleet j *continue as usual*. To measure the indicator we expand the definition:

$$B_{j/i} = \frac{\partial V_j}{\partial Y_i} \quad (3.5)$$

$$= \frac{\partial V_j}{\partial \mathcal{F}_i} \left(\frac{\partial Y_i}{\partial \mathcal{F}_i} \right)^{-1} \quad (3.6)$$

$$= \left(\lim_{\epsilon \rightarrow 0} \frac{V_j(\mathcal{F}_i + \epsilon, \mathcal{F}_j) - V_j(\mathcal{F}_i - \epsilon, \mathcal{F}_j)}{2\epsilon} \right) \cdot \left(\lim_{\epsilon \rightarrow 0} \frac{Y_i(\mathcal{F}_i + \epsilon, \mathcal{F}_j) - Y_i(\mathcal{F}_i - \epsilon, \mathcal{F}_j)}{2\epsilon} \right)^{-1} \quad (3.7)$$

$$= \lim_{\epsilon \rightarrow 0} \frac{V_j(\mathcal{F}_i + \epsilon, \mathcal{F}_j) - V_j(\mathcal{F}_i - \epsilon, \mathcal{F}_j)}{Y_i(\mathcal{F}_i + \epsilon, \mathcal{F}_j) - Y_i(\mathcal{F}_i - \epsilon, \mathcal{F}_j)} \quad (3.8)$$

As only i is doing *an action* only \mathcal{F}_i is changed in line (3.6). The partial derivatives are expanded into limits from both sides in line (3.7) to allow for more precision in the numerical implementation. The benefit indicator of the fleet itself, $B_{i/i}$, can be calculated in a similar fashion. $B_{i/i}$ does not measure an externality, but it is the net-benefit to the fleet of removing one more fish, ignoring the externality of the fishing on the other fleet.

The total benefit indicator per unit of fleet i 's harvest is:

$$B_{\bullet/i} := B_{i/i} + B_{j/i} \quad (3.9)$$

where the \bullet indicates 'summed over all fleets'. This benefit is the total net benefit of removing one more fish, including opportunity costs. Here, only two fleets are considered, but the expression can be generalized to an arbitrary number of fleets by summing over all of the fleets. The benefit will be a function of the current state of the ecosystem, and if the total benefit is positive (negative), then it will, from an economic point of view, be beneficial to increase (decrease) the harvest. If $B_{\bullet/i} = 0$ for both fleets, then a marginal change in the harvest will leave the present value of the rent flow unchanged. Such a point is a candidate for a situation with optimum ecosystem services.

3.1. MODEL SIMULATIONS

The deployed ecosystem model do not allow for analytical solution for finding the benefit indicator. This section describes how the limit (3.8) is estimated with numerical experiments on the model, and how the continuous formulation (3.1) and (3.2) are approximated with equivalent discreet formulation.

The estimation is performed by allowing the model to run with mortality rates \mathcal{F}_i and \mathcal{F}_j until it converges to equilibrium $\dot{N}^*(\mathcal{F}_i, \mathcal{F}_j)$.¹ Two experiments, A and B , that departures from the equilibrium are performed; in both experiments, the fishing mortality of fleet j is fixed while the fishing mortality of fleet i is changed: $\mathcal{F}_i(A) = (1 - \epsilon)\mathcal{F}_i$ and $\mathcal{F}_i(B) = (1 + \epsilon)\mathcal{F}_i$. The change in fishing mortality leads to a dynamic response of the ecosystem model. The experiment is run for $T = 50$ years, and the system has converge to a new equilibrium. We use $\epsilon = 10^{-6}$ as a suitable compromise between precision (close to the limit $\epsilon \rightarrow 0$) and numerical noise (the signal is large relative to rounding errors in computation).

The harvest flows, $\mathbf{y}_i(A)$ and $\mathbf{y}_i(B)$, and the rent flows, $\boldsymbol{\pi}_j(A)$ and $\boldsymbol{\pi}_j(B)$, are recorded (the bold symbols indicates that the flows are discreet in time and represented as vectors $\boldsymbol{\pi} = (\pi_0, \pi_{\Delta t}, \dots, \pi_T)$). All of the vectors are of length $(T/\Delta t + 1)$, where Δt is the time step in the model. The changes in the present values are then calculated as:

$$\Delta V_j = PV(\boldsymbol{\pi}_j(B) - \boldsymbol{\pi}_j(A)) \quad (3.10)$$

$$\Delta Y_i = PV(\mathbf{y}_i(B) - \mathbf{y}_i(A)) \quad (3.11)$$

The integrals involved in the present values are estimated as:

$$PV(\boldsymbol{\pi}) = \sum_{t \in \{0, \Delta t, 2\Delta t, \dots, T - \Delta t\}} \rho^{-1} \left(e^{-t\rho} - e^{-(t+\Delta t)\rho} \right) \pi_t + e^{-T\rho} \frac{\pi_{T-\Delta t}}{\rho} \quad (3.12)$$

Here, $\sum(\dots)$ calculates the present value from $t = 0$ to $t = T$, and $e^{-T\rho} \pi_{T-\Delta t} \rho^{-1}$ estimates the present value from $t = T$ to $t = \infty$.

Finally the benefit indicator is estimated as

¹In the model, the state variables, which are the density of the fish with respect to their size, exhibit the form of travelling waves with diminishing amplitudes when the controls are constant. The system is considered to converge to equilibrium when the coefficient of variation over 25 years in all points is less than 10^{-8}

$$B_{j/i} = \frac{\Delta V_j}{\Delta Y_i} \quad (3.13)$$

4. ECONOMIC MODEL

4.1. TWO VIEWS ON PRODUCTION IN FISHERIES

In a traditional fisheries model (e.g. Getz and Haight 1989, Chap. 4), the harvest is calculated by summing the contributions from all of the differently sized groups that are fished. In the size-spectrum model, this is an integral over the abundance distribution wrt. size w (weight) of the fish, $N(w)$, weighted by the size-selectivity of the fishing gear, $\omega(w)$:

$$y = \mathcal{F} \int_0^{\infty} \omega(w)N(w)w \, dw \quad (4.1)$$

where \mathcal{F} is the overall fishing mortality.

Fisheries economists tend to instead use a production model, where the harvest, y , is the production of a fishing vessel with the factor inputs of effort, E , and stock, S , where the fish stock is an environmental variable.² The traditional approximation is to apply a Cobb-Douglas production function (e.g., Clark 1990, eq (2.8)):

$$y = qE^\alpha S^\gamma \quad (4.2)$$

The total production of the fleet is the sum of the productions of each vessel. Assuming identical vessel and effort levels, total production will have the same form as individual production:

$$\begin{aligned} y &= n q \left(\frac{E_{total}}{n} \right)^\alpha S^\gamma \\ &= q' E_{total}^\alpha S^\gamma \end{aligned} \quad (4.3)$$

²Stock is not a traditional production factor for the individual fisher as it is not under his control. It is more of an exogenous environmental variable. However, in aggregate, that is from a social viewpoint, the stock is endogenous and can be seen as a traditional production factor.

where n is the number of vessels, and $q' = n^{1-\alpha}q$. Hence, the total harvest function will be a scaled version of (4.2).

The two views on production can be unified by defining the stocks as:

$$S := \int_0^\infty \omega(w)N(w)w \, dw \quad (4.4)$$

and the overall fishing mortality rate as:

$$\mathcal{F} := q' E_{total}^\alpha S^{\gamma-1} \quad (4.5)$$

In this manner, the economic production view (4.3) and the model (4.1) will give the same production, y .

The function $S^{\gamma-1}$ in the overall fishing mortality rate (4.5) will, with the expectation of $\gamma \in [0, 1)$, be a convex decreasing function, indicating declining productivity with respect to increasing stock.

4.2. COST MODEL

Effort is an ambiguous concept; economists prefer to work with physical input factors such as labour, fuel, and provision (see e.g. Squires 1988):

$$y = q_1 x_1^{\alpha_1} x_2^{\alpha_2} \dots k^\beta S^\gamma \quad (4.6)$$

where x_i is variable input factors, and k is capital. Given (4.6), with corresponding prices p_i and a fisher assumed to minimize cost, the input factors are applied such that

$$\frac{x_i}{x_j} = \frac{\alpha_i p_j}{\alpha_j p_i} \quad (4.7)$$

Equations (4.6) and (4.7) give a production cost relationship:

$$y = q_2 G^\alpha k^\beta S^\gamma \quad (4.8)$$

where

$$G = \mathbf{p} \cdot \mathbf{x} \quad (4.9)$$

$$\alpha = \sum \alpha_i \quad (4.10)$$

$$q_2 = q_1 \left(\sum \frac{p_i}{\alpha_i} \right)^\alpha \prod_i \left(\frac{\alpha_i}{p_i} \right)^{\alpha_i} \quad (4.11)$$

The production function (4.8) can replace (4.2) to allow for the estimation of a production function based on the accounting statistics for the individual vessel, which avoids the introduction of effort. It may seem equivalent to have a fixed price on effort; however, by assuming cost minimization, we allow for substitution when relative prices between factors change (see Ravn-Jonsen, Andersen, and Vestergaard 2015, Chap. A).

Our objective is to analyse the ecosystem model from a long-run perspective. Therefore, we will derive a cost function under the assumption that all of the factor inputs of the fishing fleet are completely variable, which allows us to minimize both the operational and the capital costs.

With p_k as the price of capital, the total cost per vessel is $C = G + kp_k$. If we assume that the ecosystem is in a steady state with a total harvest of y , then the cost minimization problem is to find the number of vessels, n , the operation cost, G , and the capital, k , such that:

$$(n, G, k) = \underset{n, G, k}{\operatorname{argmin}} n(G + kp_k) \Bigg|_{y=nqG^\alpha k^\beta S^\gamma} \quad (4.12)$$

By substituting n

$$(n, G, k) = \underset{n, G, k}{\operatorname{argmin}} y \frac{(G + kp_k)}{qG^\alpha k^\beta S^\gamma} \Bigg|_{n=\frac{y}{qG^\alpha k^\beta S^\gamma}} \quad (4.13)$$

the cost minimization problem can be solved by first finding the operation cost and capital level where the unit cost is minimized, and then the number of vessels. Minimizing the unit cost implies that the input factors are applied in the ratio:

$$\frac{G}{kp_k} = \frac{\alpha}{\beta} \quad (4.14)$$

and, because we are looking for the long-run optimal level of capital, $\alpha + \beta = 1$. The total cost per vessel is then:

$$C = G + p_k k = G \left(1 + \frac{\beta}{\alpha}\right) = \frac{G}{\alpha} \quad (4.15)$$

The unit costs are:

$$\frac{C}{y} = \frac{\frac{G}{\alpha}}{q \left(\frac{\beta}{\alpha p_k}\right)^\beta G^{\alpha+\beta} S^\gamma} = AS^{-\gamma} \quad (4.16)$$

Where

$$A = q^{-1} \beta^{-\beta} \alpha^{\beta-1} p_k^\beta \quad (4.17)$$

If we do the analysis from a long-run perspective and assume an ideal cost minimizing fleet, we expect that the unit cost of harvesting will be of the form (4.16) if changes in the harvest in the short-run are small. Because section 4.1 established a relationship between the biological production function (4.1) and the economic production function (4.8), with the definition of stock given by (4.4), \mathcal{F} can be used as the control variable in the model, and the cost can be calculated using (4.16).

4.3. COST MODEL PARAMETERS

The parameters for the cost model for the two fleets (Tab. 1) are estimated for the North Sea on the basis of the accounting statistics, the landing statistics and the ICES (International Council for Exploration of the Sea) stock assessment summaries (ICES 2010b) (see Ravn-Jonsen, Andersen, and Vestergaard 2015, Chap. A). The unit cost model (4.16) has two parameters A, γ and one variable S . The value of γ is independent of how S is measured as long as it is proportional to the density of the fish in the sea. However, the value of A will depend upon the way S is measured, and there is no way to get from the spawning stock biomass, the metric of ICES, to the density of fish per m^3 , the metric of the model. The approach taken is to calibrate the model to give a unit cost that is similar to the one observed in the data. However, the rent in today's fishery is zero (Chap. A Tab. 2 and 3). The fishery where transferable quotas were first introduced in Denmark was the herring and mackerel fishery (as a test in 2003, permanent since 2007); this is likely the most cost-efficient fishery in Denmark, and we assume that the other sectors would be as efficient if properly managed. Hence, the constant A is found by setting the rent to 15.57% of the revenue in a fishery that resembles today's fishery in the North Sea.

4.4. PRICE MODEL

The price model is estimated using data from the Danish Landing Statistics (estimation details in Chap. b). Two price models are needed: p_F for the forage fish fleet, and p_L for the large fish fleet. The forage fish fleet lands fish for reduction into fishmeal. Because there is no size sorting in the landings, we assume a flat price with respect to the size of the landed fish:

$$p_F(w) = P \quad (4.18)$$

TABLE 1: Parameter estimates for the price and cost models

Parameter	Estimate	Std Error	Units
P	0.1610	0.0053	€ kg ⁻¹
ϱ	4.830	0.51	€ kg ⁻¹
b	0.0295	0.00096	kg
a	5.38	0.25	kg
ι	0.5230	0.0039	
γ_F	0.175	0.037	
γ_L	0.280	0.016	
A_F	0.05748	Calibrated	€ kg ⁻¹
A_L	0.2759	Calibrated	€ kg ⁻¹

The large fish fleet lands fish for human consumption. The prices depend upon the size, grade and species. In the model, the size is presented as a dimension; therefore, it is appropriate to give the price as a function of the size:

$$p_L(w) = \begin{cases} \varrho (1 - \exp(-((w-b)/a)^\iota)) & w \geq b \\ 0 & \text{else} \end{cases} \quad (4.19)$$

The function (4.19) is a scaled Weibull distribution function. Standard errors (Tab. 1) are based on re-sampling (Efron and Tibshirani 1993) leading to a coefficient of variation below 0.004.

Revenue for fleet i are found as:

$$R_i = \mathcal{F}_i \int_0^\infty p_i(w) \omega_i(w) N(w) w \, dw \quad (4.20)$$

and rent:

$$\pi_i = R_i - y_i A_i S_i^{-\gamma_i} \quad (4.21)$$

with y_i and S_i as respectively (4.1) and (4.4).

5. RESULTS

To illustrate the calculation of the benefit indicator, the state of the current North Sea fishery is examined. We assume that the mean landings over

the period from 2001–2009 represent the sustainable harvest the North Sea can deliver in its present state. The mean landing is 1 990 304 ton year⁻¹ (ICES 2010a); half is assumed to be from the forage fish fleet and half from the large fish fleet. Our simulations depart from a model system in equilibrium with these services and the benefit indicators are calculated with a social discount rate of $\rho = 3\%$ pa.

A change in the fishing mortality of the forage fish fleet impacts the production and rent of both fleets (Fig. 2). Initially, the harvest of the forage fish fleet shows a big increase, followed by a reduction that levels out at approximately half of the initial increase. The rent of the forage fish fleet increases initially, but eventually it levels out close to zero. The reason that the rent approaches zero, despite the increase in the harvest, is due to a slight decrease in the density of fish; even though the elasticity of the unit cost with respect to the density is only $\gamma = 0.175$, the result is a slight increase in the unit cost that affects the harvest of the entire fleet. For the large fish fleet change in fishing mortality of the forage fish fleet results in a slight drop in the harvest followed by a sustained increase in harvest volume. Despite the increase in the harvest of the large fleet, the rent decreases. This decrease is due to a decrease in the size of the fish in the large fish fleet's harvest that leads to a lower market value. The benefit indicator for the two fleets per forage fish is calculated according to (3.13). The internal benefit to the forage fleet is a slight increase, but it is offset by the much larger external benefit decrease to the large fish fleet. The total benefit of an increase in forage fishing in the North Sea today is, therefore, clearly negative.

The consequences of a change in the large fish fleet's fishing mortality rate can be evaluated in a similar manner (Fig. 3). The production of the large fish fleet shows the same pattern as the forage fish fleet, with an initial high extra harvest followed by oscillations and settling at approximately half of the initial amount. The harvest of the forage fish fleet increases initially due to the decreased predation pressure, but later it approaches zero as the predators again increase in number, although to a slightly smaller number than before. The change in the rent of the forage fish fleet is negligible, while the rent of the large fish fleet shows an initial increase followed by a drastic decrease. This decrease in the rent, despite the increase in harvest, is caused by two things: a slight decrease in the fish density, which increases the unit cost, and a decrease in size of the harvested fish, which decreases the market value. The total benefit of an increase in the large fish fleet in the North Sea today is negative and, in magnitude, higher than the externality imposed by the forage fish fleet. Both benefit indicators of the present use of the North Sea are negative

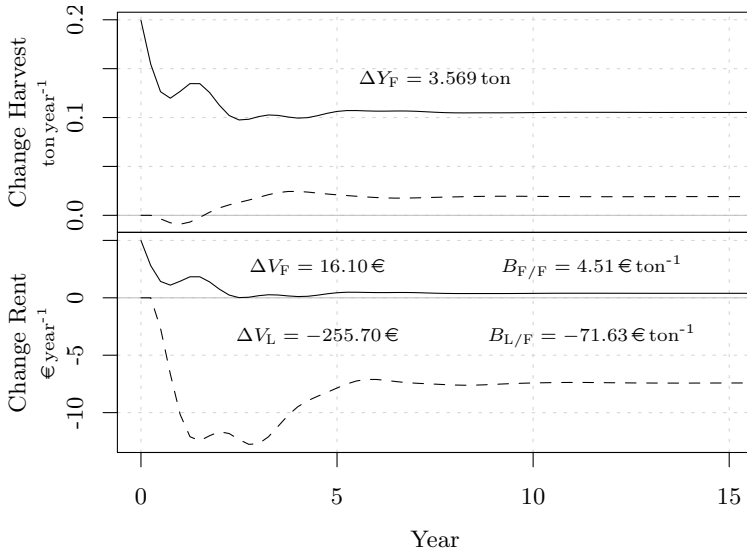


FIGURE 2: Calculation of the change in the benefit caused by a change in the forage fish fleet. The starting point of the calculation is a steady state. At time $t = 0$, the forage fish fleet fishing mortality is slightly changed, while the fishing mortality of the large fish fleet is unchanged. The change in the fishing mortality leads to a change in the production (upper panel) and the rent (lower panel) of the forage fish fleet (solid lines) and the large fish fleet (dashed lines).

TABLE 2: The benefit indicators of the present use of the North Sea.

With respect to	Forage fish fleet		Large fish fleet	
Internal Benefit	$B_{F/F}$	4.5 € ton^{-1}	$B_{L/L}$	$-1\,093 \text{ € ton}^{-1}$
External Benefit	$B_{L/F}$	-71.6 € ton^{-1}	$B_{F/L}$	-3 € ton^{-1}
Total Benefit	$B_{\bullet/F}$	-67.1 € ton^{-1}	$B_{\bullet/L}$	$-1\,096 \text{ € ton}^{-1}$

(Tab. 2), the benefit of the ecosystem services from the North Sea could be improved by reducing both fleets' harvests.

Fig. 4 presents the internal and external benefit indicators calculated with a discount rate of $\rho = 3\%$ pa. The zero contour lines in the two internal panels cross one another at point A. At this point society optimizes the benefit from the two services but ignores the externality. The negative externalities amount to -132 € ton^{-1} inflicted on the large fish fleet for the marginal fish caught by the forage fish fleet and -12 € ton^{-1} on the forage fish fleet for the marginal fish caught by the large fish fleet.

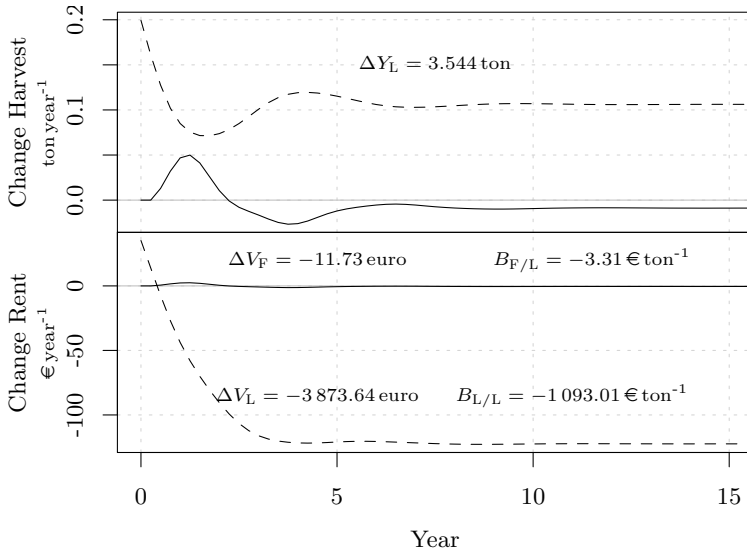


FIGURE 3: Illustration of the benefit indicator method caused by a change in the large fish fleet. The change in fishing mortality leads to a change in the production (upper panel) and the rent (lower panel) of the forage fish fleet (solid lines) and the large fish fleet (dashed lines).

To find a global optimum, the total benefit indicator must be considered (Fig. 5). The optimum is where the total benefit of the two fleets is zero (point B). This point may be reached by approximately halving the harvests of the two fleets.

The external benefit from the forage fish fleet ($0-150 \text{ € ton}^{-1}$) generally far exceeds the internal benefit ($0-30 \text{ € ton}^{-1}$). This phenomenon implies that the management of the forage fish fleet should consider the large fish fleet. The zero contour line of the total benefit indicator for the forage fish fleet follows diagonals up left and right from point B (Fig 5). This result indicates that the optimal forage fish harvest is dependent upon the volume of the large fish fleet's harvest.

In contrast, in absolute values, the internal benefit of the large fish fleet ($0-1000 \text{ € ton}^{-1}$) generally dwarfs the external benefit ($0-12 \text{ € ton}^{-1}$). Thus, the influence of the large fish fleet on the forage fish fleet is rather small, and it can, for practical purposes, be ignored. The zero contour line for the large fish fleet in Fig 5 is vertical, indicating that the optimal harvest level of the large fish fleet is independent of the forage fish fleet.

A striking result of Fig. 4 is that the forage fish fleet can create a positive externality for the large fish fleet. To understand the mechanism

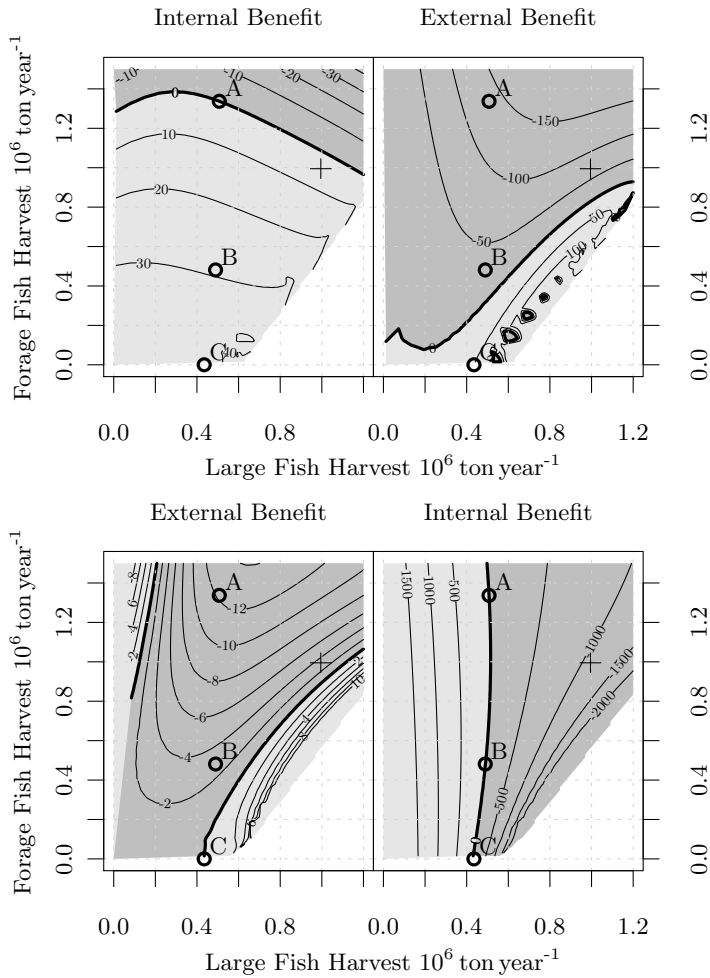


FIGURE 4: The benefit indicators (€ ton^{-1}) for the North Sea forage fish fleet (top) and large fish fleet (bottom) divided into internal benefit, $B_{i/i}$, and external benefit, $B_{i/j}$. Four points of special interest are marked: the plus sign is the current state of the North Sea, A is where the internal benefits of the two fleets cross, B is where the total benefits of the two fleets cross (Fig. 5), and C is an arbitrarily chosen point where the externality on the large fish fleet from the forage fish fleet is positive

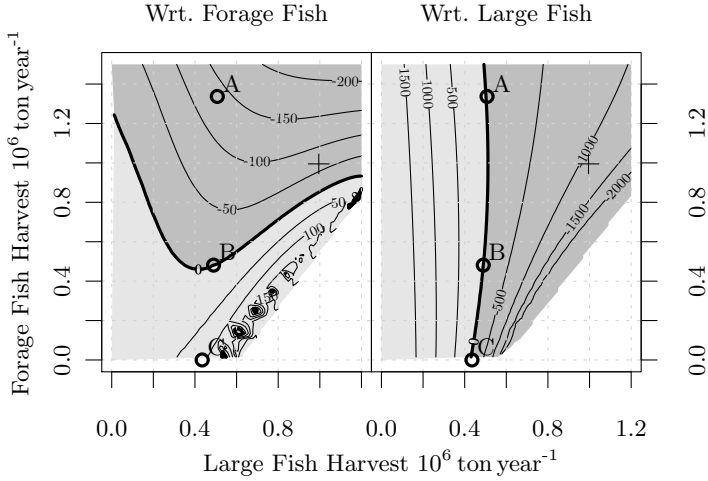


FIGURE 5: Total benefit indicator (€ ton^{-1}) for the forage fish fleet, $B_{\bullet/F}$ (left), and the large fish fleet, $B_{\bullet/L}$ (right). The points marked are the same as in Fig. 4

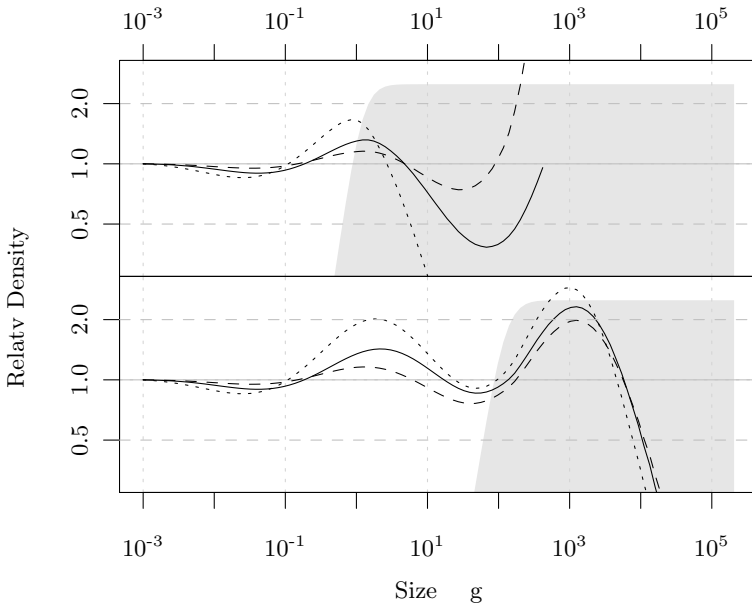


FIGURE 6: Abundance of fish as a function of individual size (both axes logarithmic) at the points A-C in Figs. 4 and 5; A is dotted, B is solid, and C is dashed. The abundance of forage fish (top) and large fish (bottom) is scaled relative to the unfished situation. The grey regions illustrate each fleet's selection function.

behind this result, the population level at three points, marked A, B, and C in Fig. 4 and 5 are examined (Fig. 6). As the harvest of the forage fish is increased, i.e., moving from point C to B, the abundance of large forage fish within the size range targeted by the fishery decreases. The decreased abundance of the forage fish releases the predation pressure on smaller individuals in the size range of 1-10 g. The decreased abundance of large forage fish affects the large fish in two opposite ways: 1) it removes some of the food for the largest fish (> 1 kg), and 2) it reduces the competition for food for the juvenile individuals of the large fish. Moving from C to B, the effect of the reduced competition appears most important because the large fish generally increase in abundance. Only when moving from B to A are the very large fish (> 5 kg) negatively affected by the lower abundance of food from the forage fish. The impact on abundance is modest, but because the price of the large fish is high, this reduction is responsible for the negative externality at high harvest rates.

6. DISCUSSION AND CONCLUSION

We have developed a general methodology to analyze the internal and external consequences of fishing an ecosystem in terms of the benefit indicator. The method has been applied to quantify the externalities that a forage fish fleet and a large fish fleet in the North Sea generate for one another. The generalization of the methodology to more than two ecosystem services is straightforward. Even though the model is calibrated to resemble the North Sea, it builds on properties that are generally found across ecosystems. The results, therefore, have general value and may be applied to other systems, at least in qualitative terms.

Economic analyses often look at the first order derivative, known as the margin. This gives easy interpretative indicator, as for example Weitzman (2003)'s stationery rate of return³. The Weitzman's stationery rate of return give a easy interpretable number to compared with the discount rate. We have here developed a indicator to inform about the trade-off

³The stationary rate of return is minus the ration between the marginal cash flow wrt. capital and the marginal cash flow wrt. investment, all measured for a stationary capital. With symbols:

$$R(k) = -\frac{\frac{\partial g}{\partial k}}{\frac{\partial g}{\partial \dot{k}}} \quad (6.1)$$

where $g(k, \dot{k})$ is the net cash flow as a function of capital k and investment \dot{k} (Weitzman 2003)

when exploiting an marine ecosystem based on marginal analysis, equation (3.3). As we define our indicator based on an equilibrium situation, our indicator resembles Weitzman's' stationary approach. However the marine ecosystem is dynamic, and can not instantly go from one equilibrium situation to an other; there is a dynamic path that have economic implications. The indicator is designed to capture this dynamic, by discounting both the economic and physical changes.

While using the method of discounting seems to be widely accepted for aggregating the economic outcome for a (infinite) time flow, equation (3.2), using the method of discounting of the physical effect, equation (3.1), seems to encounter some skepticism. This is, however, a method widely used in cost effectiveness analysis in health technology assessment, recommended for example by National Institute for Health and Care Excellence (NICE 2012). Here the effect is evaluated by discount the benefit in form of quality adjusted life years.

An alternative to our benefit indicator would be the discounted value of the cash flow with a change in fishing mortality rate or fishing effort. Both fishing mortality rate and fishing effort are model variables with no tangible physical realization. An indicator based on non tangible model properties is less informative than an indicator based on well known physical entity. We therefore choose to use the quantity of removed fish as denominator in our indicator. An other approach would be to find a implicit discount rate similar to the Weitzman (2003)'s stationery rate of return; similarly to the method applied in Ravn-Jensen (2011). However, that will not give an indicator divided into external and internal effect, and it will only be computable for the part of the ecosystem states where there is an intertemporal choice of exploitation (Ravn-Jensen 2011).

As defined in section 3, the benefit indicator is interlinked to the choice of control variable, as the control variable defines what *continue as usual* and *a change* signify. To test how sensitive the benefit indicator is to the choice of the control variable, the model was reformulated with the harvest as the control variable. The results are presented in Ravn-Jensen, Andersen, and Vestergaard (2015, chap. E) and show consistency with the found benefit indicator, with the exception of the external benefit from the forage fish fleet. For the external benefit of the forage fleet, the zero contour line moves up so that the value for today's fishery changes from -71 € ton^{-1} to 100 € ton^{-1} . Nevertheless, the general picture and the optimal point are convergent, which shows that the benefit indicator is a proper indicator of the net benefit, though the values for the external benefit at the present exploitation rate must be interpreted cautiously.

The intersection of the zero contour lines of the total benefit indicator

in Fig. 5 indicates the economic optimum. For simplicity, the decision variables have been reduced to two dimensions. In the real world, there are many more possibilities for decision variables such as a change in the size selectivity, a change in the selectivity with respect to the traits, or a change in the fishing mortality over time. Therefore, it may be possible to increase the benefit by exploring other dimensions of the control variable.

In current fishery management, securing the reproduction of the fish stock is considered to be an important goal. The ecological model has fixed reproduction, that is, there is no feedback from the abundance of the adult fish onto the abundance of the offspring. This phenomenon is in line with the classic yield-per-recruit analysis in fisheries science (Beverton and Holt 1957). Thus, the shadow price of reproduction is not part of the benefits calculated in Fig. 4 and 5. This approach is taken to highlight only the trophic system, such that all of the effects stem from predation and growth of the individual. Therefore, our analysis cannot stand alone; reproduction must also be considered. However, the model indicates the benefit from a substantial reduction in the harvest, which would simultaneously reduce the probability of reproduction failure.

The economic aspects of the model consist of a price model and a cost model. Both of these models are based on data from Denmark; however, because Danish fisheries are part of the global market, the models are generic. The price model is divided into two parts: one for forage fish, and one for large fish. Large fish are regarded as landed for direct human consumption, and we find in the Ravn-Jonsen, Andersen, and Vestergaard (2015, Chap. B) that price increases with size, with minor variations from year to year. We are confident that the price model reflects the willingness of the industry to pay with respect to size. However we notice that the price analyse is static, without the dynamic caused by supply and demand. We leave to future work to incorporate an economic marked in the model.

The cost model is described as a power function of the biomass in the sea. The model needs two parameters for each fleet, the exponents γ_i , and the coefficients A_i . The value of the exponents differ between studies; e.g., Sandberg (2006) found values in the range of 0.18 to 0.48 for different herring and cod fleets, and Eide et al. (2003) found 0.42 for cod. Compared to those studies, the values found in the present study (0.18 and 0.28) are on the low end. In Ravn-Jonsen, Andersen, and Vestergaard (2015, Chap. D), the sensitivity of this parameter is tested by increasing the exponents by a factor of 1.5. The change in γ does not qualitatively change figures 4 or 5. The value of the benefit to the forage fish fleet is slightly sensitive to a change in the exponent, while the large fish fleet is almost insensitive.

As our model does not use the same stock concepts as the ICES stock assessments, we have calibrated the coefficients in the cost model so that the present fishery gives 15.57% rent—the same as the best-managed fishery in Denmark. This is a rough estimate, as the present fishery yields zero rent (Chap. A, Tab. 2 and 3). In Ravn-Jonsen, Andersen, and Vestergaard (2015, Chap. D), the sensitivity of this calibration is tested by calibrating under the assumption of zero rent in the present fishery. Again, the figures do not change qualitatively; however, the value of the benefit to the forage fish fleet is more sensitive to this parameter than is the value to the large fish fleet. The reason for this difference in sensitivity between the two fleets stems from the underlying price structures: while the large fish fleet benefits from an increase in the price caused by the increase in the size of the fish as a result of the relieved exploitation level, the forage fish fleet has a flat price relative to the size of the fish. Since both fleets benefit from an increase in density as a result of the relieved exploitation levels in the form of a decreased unit cost. However, for large fish, the change in price dominates the change in cost. The economics of the forage fish fleet are then dominated by the density effect on the cost, and for the large fish fleet, the economics are dominated by the price response to the size of the fish.

The size-spectrum model has been used for ecological impact assessments of fishing on the ecosystem (Andersen and Pedersen 2010; Houle et al. 2013; Jacobsen, Gislason, and Andersen 2014). One common impression from those analyses was that a fishery on the large fish imposed a positive influence on the forage fish, due to the reduced predation pressure on the forage fish when the consumer fish were removed. In contrast, the results from present model, as presented in Fig. 4, uncovers that the externality on the forage fish fleet generated by the large fish fleet is ignorable. The externality is ignorable because it is dwarfed by the intertemporal cost in the large fish fleet's own fishery if it is not managed close to the optimum. The contrast in interpretation between present model and the others highlights the importance of economic analyses over purely ecological analyses of the impact of fishing on an ecosystem.

There are situations where the forage fish fleet generates a positive externality on the large fish fleet, and there are situations where they generate a negative externality. The explanation for this phenomenon must be observed in the different functions the species fill for one another during their lifespan. If we focus on a mature forage fish, that is approximately 100 g, it will fill three different functions with respect to the large fish species: 1) the function of a predator on larvae and juveniles, 2) the function of a competitor to similarly sized fish and 3) the function of prey

(forage fish) for larger fish. The first two have a negative influence, while the last has a positive influence. The economic analysis shows that this triple functionality leads to a requirement of accommodating the harvest of forage fish and to the exploration of the large fish. Traditional bio-economic models have been centred on the mature fish. The multi-species models that include interaction with the mature level that do exist, such as the Lotka-Volterra type model, are restricted to modelling predator-prey, competition or mutualism, but not all three at once. Consequently, these models disregard an important part of the ecological functionality.

Concluding remarks

Overall, this model shows that the exploitation of the forage species has a notable economic impact on the large species fishery, but the reverse is not true. The analysis shows that the naïve perception, where the forage species is only viewed as food for the large species, is too simple. The predation of the forage fish species on the juveniles of the large species and the competition between the forage fish species and the juveniles of the large species can, if the density of the forage fish is too high, dominate over the function of the forage fish as prey. Thus, the harvest of the forage fish must be adjusted to the harvest of the large fish. The present management of the North Sea is, given the current exploitation rate of the large fish, not far from having the right forage fish harvest. However, the model's optimal point ($481 \cdot 10^3 \text{ ton year}^{-1}$, $489 \cdot 10^3 \text{ ton year}^{-1}$) is approximately half of the current harvest in the North Sea, which indicates that the present exploitation is too high. To improve the utilization of the ecosystem, management must acknowledge the externalities that the fisheries impose upon one another.

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