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Publication date:
2015

Document Version
Publisher's PDF, also known as Version of record

DTU Compute
Department of Applied Mathematics and Computer Science

Bifurcation Analysis of Structures in a Convection Model

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This study investigates the convection driven evolution of isolated structures in a magnetized plasma. The analysis is based on numerical simulations of the evolution of two-dimensional plasma filaments in a nonuniform magnetic field. A minimal model for interchange motions describes the coupling between a thermodynamic variable and the plasma vorticity. The analysis focuses on classification of topological changes of the structures as they develop in time and as the Rayleigh number varies.

INTRODUCTION

At the edge region of a magnetically confined toroidal plasma the transport of particles and energy is dominated by recurring outbreaks of coherent plasma structures called blobs[1]. In fluid dynamics similarly shaped structures known as plumes[2] arise when a fluid in a gravitational field is driven by thermal convection. This work focuses on the evolution of the blob topology during propagation.

MINIMAL MODEL FOR INTERCHANGE MOTIONS

We consider a minimal interchange model[1]:

$$\left(\frac{\partial}{\partial t} + \hat{\mathbf{z}} \times \nabla\phi \cdot \nabla\right)\theta = \kappa \nabla_\perp^2 \theta,$$
$$\left(\frac{\partial}{\partial t} + \hat{\mathbf{z}} \times \nabla\phi \cdot \nabla\right)\Omega + \frac{\partial \theta}{\partial y} = \nu \nabla_\perp^2 \Omega,$$

and $\nabla_\perp^2 \phi = \Omega$. Here $\theta(x, y, t)$ is the thermodynamic variable (e.g. density, pressure or temperature), $\Omega(x, y, t)$ the vorticity, and $\phi(x, y, t)$ the electrostatic potential. κ and ν are parameters related to the Rayleigh and Prandtl numbers by $\mathrm{Ra} = 1/(\kappa\nu)$ and $\mathrm{Pr} = \nu/\kappa$. $x \in [-20, 30]$ and $y \in [-25, 25]$ are the spatial coordinates and $t \in [0, 20]$ is time. Dirichlet boundary conditions were applied on all boundaries.

All simulations were initialized as

$$\theta(x, y, t = 0) = e^{-\frac{1}{2}(x^2+y^2)},$$
$$\Omega(x, y, t = 0) = 0,$$
$$\phi(x, y, t = 0) = 0.$$

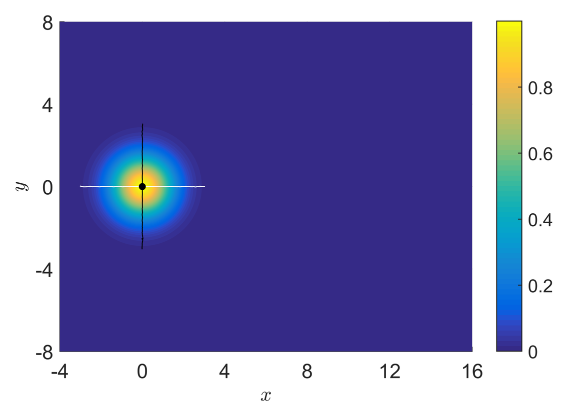

Figure: Plot of $\theta(x, y, t = 0)$ and the two initial nullclines.

A DYNAMICAL SYSTEMS APPROACH

Streamlines of the thermodynamic variable θ are determined from

$$\begin{pmatrix} \partial_s x \\ \partial_s y \end{pmatrix} = \begin{pmatrix} -\partial_y \theta(x, y, t_0; \mathrm{Ra}) \\ \partial_x \theta(x, y, t_0; \mathrm{Ra}) \end{pmatrix} \qquad (1)$$

Since $d\theta/ds = 0$ these streamlines are level curves of θ. We shall investigate bifurcations of the streamline topology as t and Ra varies. Equilibrium points of the system (1) are stationary points of θ. Intersections of the nullclines of the system (1) determines the stationary points. The eigenvalues of the Hessian of θ evaluated at a stationary point determines the type (local maximum, local minimum, or saddle) of the stationary point [3].

BIFURCATION ANALYSIS

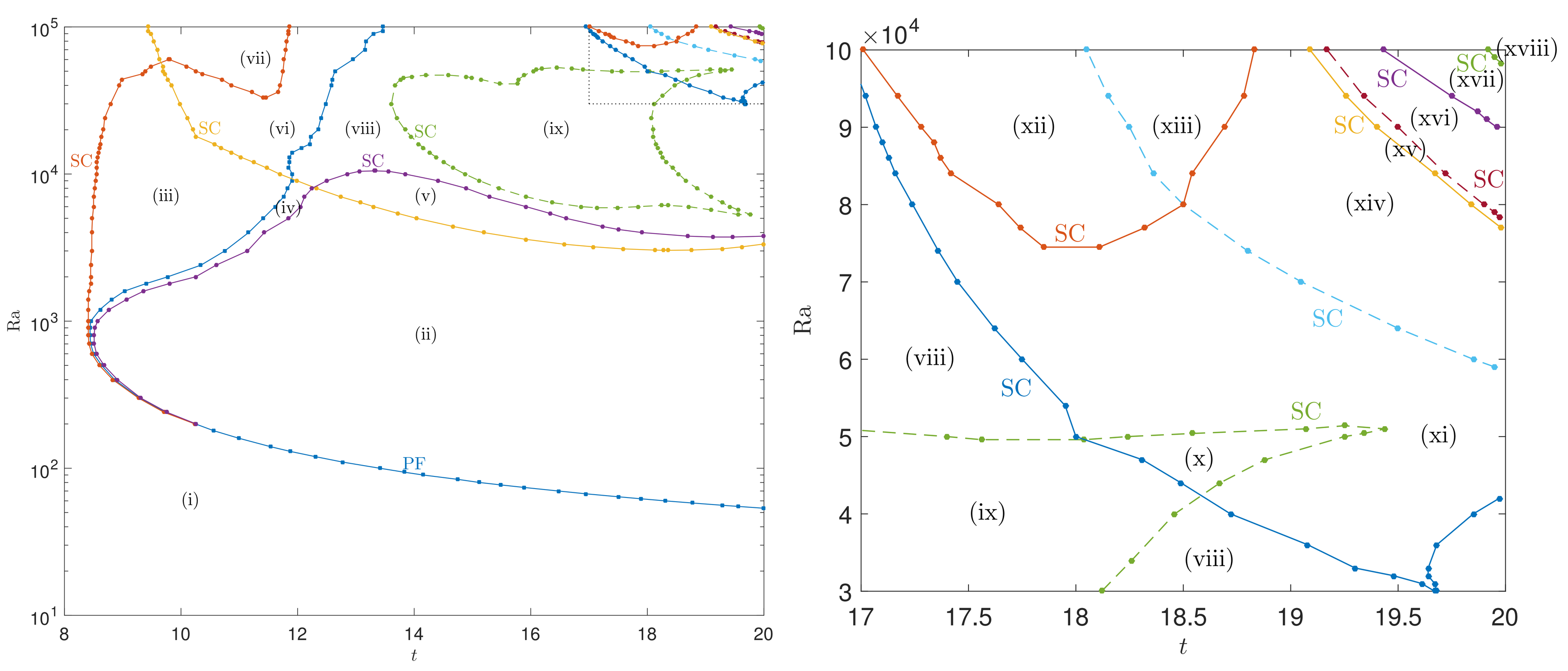

Figure: Bifurcation diagram with t and Ra as bifurcation parameters. "PF" indicates a pitchfork bifurcation and "SC" a saddle-center bifurcation. Solid and dashed lines indicate bifurcations involving local maxima and local minima, respectively.

THE DIFFERENT STRUCTURES

The bifurcation curves in the bifurcation diagram separates the (t, Ra)-parameter space into 18 regions. Within each region any set of streamline patterns of θ are topologically equivalent.

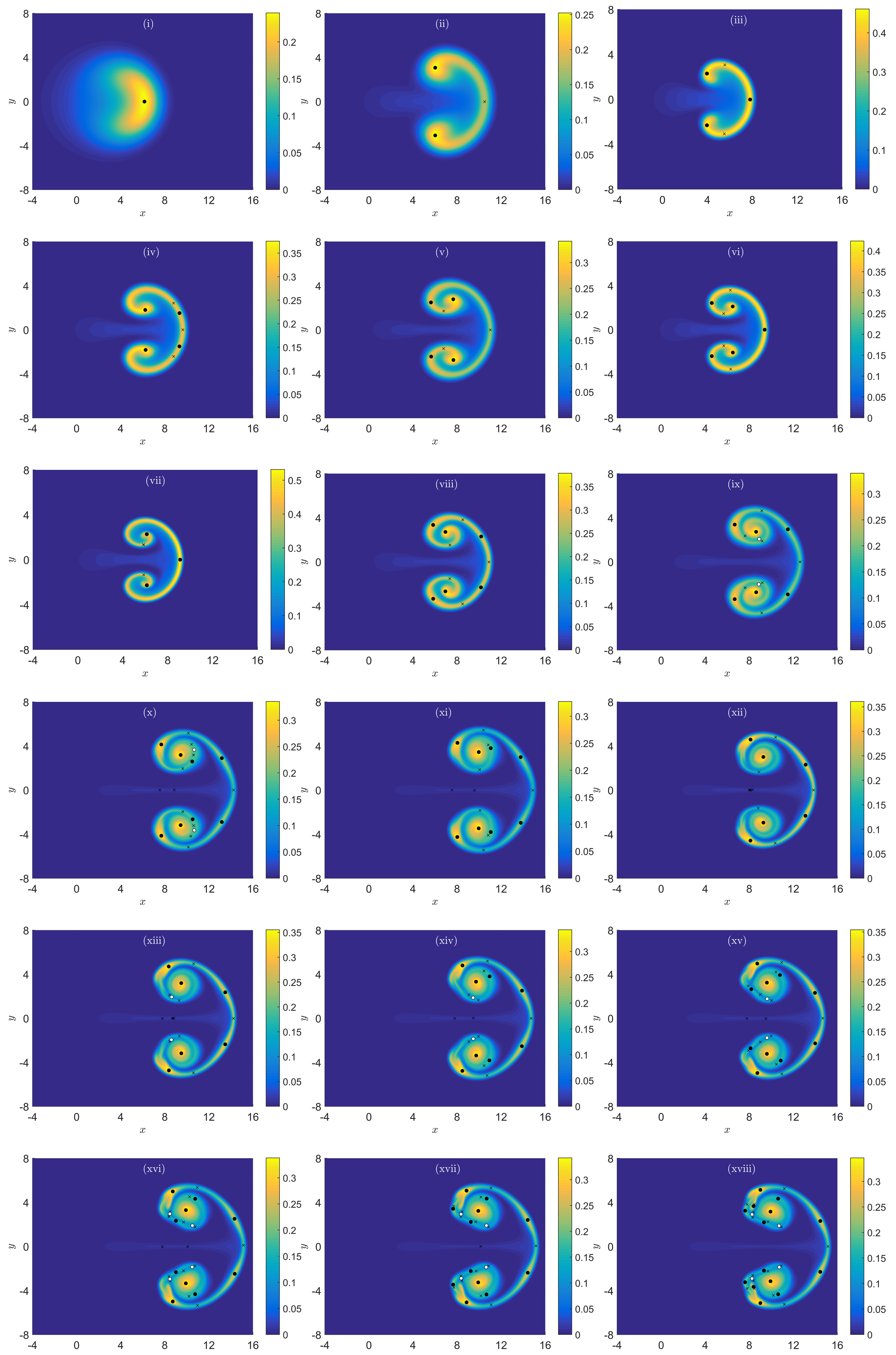

Figure: A representative solution $\theta(x, y, t_0)$ for each of the 18 regions in the bifurcation diagram. Black dots indicate local maxima, white dots indicate local minima and crosses indicate saddles. Local maxima and minima of θ correspond to centers of the streamlines of θ and saddles of θ correspond to topological saddles of the streamlines of θ.

CONCLUSION AND OUTLOOK

We have carried out a bifurcation analysis of the topological changes of seeded blobs using time and Rayleigh number as parameters. A larger Rayleigh number and time give rise to a more complicated topology with more stationary points. Future work includes:

- Investigate dependence on the Prandtl number.
- Compare with a similar bifurcation diagram for the vorticity.
- Determine measurable characteristics of the different topological structures such as propagation speed.

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