Technical University of Denmark

Real-time Distributed Economic Dispatch forDistributed Generation Based on Multi-Agent System

Luo, Kui; Wu, Qiuwei; Nielsen, Arne Hejde; Østergaard, Jacob

Link back to DTU Orbit

DTU Library
Technical Information Center of Denmark

Real-time Distributed Economic Dispatch for Distributed Generation Based on Multi-Agent System

Kui Luo
China Electric Power Research Institute (CEPRI)
Beijing, China
luokui@epri.sgcc.com.cn

Jacob Østergaard
Centre for Electric Power and Energy
Department of Electrical Engineering
Technical University of Denmark
Lyngby, Denmark
joe@elektro.dtu.dk

Qiuwei Wu
Centre for Electric Power and Energy
Department of Electrical Engineering
Technical University of Denmark
Lyngby, Denmark
qw@elektro.dtu.dk

Arne Hejde Nielsen
Centre for Electric Power and Energy
Department of Electrical Engineering
Technical University of Denmark
Lyngby, Denmark
ahn@elektro.dtu.dk

Abstract—The distributed economic dispatch for distributed generation is formulated as a optimization problem with equality and inequality constraints. An effective distributed approach based on multi-agent system is proposed for solving the economic dispatch problem in this paper. The proposed approach consists of two stages. In the first stage, an adjacency average allocation algorithm is proposed to ensure the generation-demand equality. In the second stage, a local replicator dynamics algorithm is applied to achieve nash equilibrium for the power dispatch game. The approach is implemented in a fully distributed manner with local computation and communication among neighboring agent. The feasibility and effectiveness of this approach is demonstrated by a numerical test system.

Keywords—Multi-Agent System; Replicator dynamic; Distributed economic dispatch; Distributed generation

I. INTRODUCTION

The integration of distributed generation and advanced communication technology will make the electric grid more efficient, flexible and reliable. Real time and optimal operations of the power system are required. Economic dispatch(ED) is one of the fundamental optimization challenges that allocates the total power demand among multiple generation units in the most economical way, while satisfying both the unit- and system-level constraints [1]. The existing economic dispatch algorithms can be classified into two categories: the analytical algorithms (such as lambda iteration [2] and gradient search) and heuristic algorithms (such as genetic algorithm [3], particle swarm optimization [4]). Despite the excellent performance, most of existing economic dispatch approaches are centralized. Because of the large amounts of data to process and the communication network subject to topology variations, it is difficult for a centralized method to provide optimal control setting in real-time. In addition, centralized schemes are usually expensive to implement, inflexible, and susceptible to single-point failures.

A recent trend is to solve the ED problem in a distributed manner and incorporate distributed intelligence. As distributed strategies can be more suitable to handle topological variations and accommodate plug-and-play features. Moreover, they are more robust, scalable, and can better accommodate a large number of units compared to centralized approaches. Finally, distributed strategies can effectively exploit sparse communication network with limited message passing among participating units to aid the cooperation of dispersedly located components in the system.

Existing research on decentralized and distributed economic dispatch solutions can be found in some literatures [5-14]. In [7] frequency deviation-based method base on local available information is proposed, however, due to the lack of communication, it may not be effective to utilize all available resources in the network. Distributed algorithm, such as consensus algorithms [8-9] and auction algorithms[10], are used to solve the ED problem, which requires only local computation and information exchange among some neighbor units through a local communication network. In [11] a distributed dynamic programing algorithm is proposed to optimally allocate the total power demand among different generation units, while the global load demand need to acquire before executing the algorithm. An improved distributed gradient algorithm is proposed in [12], but it needs feedback on the power mismatch from the shift in steady state frequency due to primary control. In [13] a fully decentralized approach consisting of three stages is proposed to solve the economic dispatch problem, while the global generation information is needed for every agent to solve the ED problem alone, too much data flowing over the communication network may degrade the effectiveness of the approach. In [14] replicator dynamics algorithm is applied in distributed optimization, it assumes the algorithm to be initialized with a feasible power allocation, but the inequality constraints for generation units are not well considered.

Moreover, with the increasing penetration of renewable resources and distributed generation (DG), the interests and characteristics of renewable resources and distributed generation are not yet represented in existing methods. This paper proposes a distributed replicator dynamics approach based on multi-agent system (MAS) to solve the ED problem for distributed generation. According to the proposed solution, each generator and load has an associated agent that communicates with its neighboring agents only. No centralized or specialized agent is used to coordinate the operation of the autonomous agents. The proposed approach is based on two stages: 1) Initialize generation power using adjacency average allocation algorithm and 2) solve the ED by local replicator dynamics in a distributed manner. The salient features of the proposed approach are the following:
  • The economic dispatch problem is solved in a distributed fashion, no global load and generation information is needed;
  • Proposed adjacency average allocation algorithm guarantees the equality constraints, and provides fast and effective initialization for replicator dynamics;
  • Designed local replicator dynamics algorithm can handle inequality constraints, not be sensitive to the initial state;
  • Renewable resources can be integrated into the power dispatch properly considering their interests.

II. FORMULATION OF THE ECONOMIC DISPATCH PROBLEM

The ED minimizes the total generation cost $C(P)$ given by the sum of the generation costs for each unit i , $C_i(P_i)$, as follows:

$$min \quad C(P) = \sum_{i=1}^{n} C_i(P_i) \tag{1}$$

Subject to

$$\sum_{i=1}^{n} P_i = P_d \tag{2}$$

$$P_i^{min} \le P_i \le P_i^{max} \tag{3}$$

where $C_i(P_i)$ is the cost function of generation unit i ; P_i is the power output of generation unit i ; n is the number of generators; P_d is the total system load; P_i^{min} , P_i^{max} are the lower and upper limits of generation unit i , respectively.

The generation cost function is modelled with the following quadratic formula:

$$C_i(P_i) = a_i + b_i P_i + c_i P_i^2 \tag{4}$$

where a_i , b_i and c_i are the fuel cost coefficients for unit i .

In this paper, small distribution generation units operating close to the load points in the power distribution system are considered, thus some technical constraints such as losses in long transmission lines, valve-point effects, prohibited zones, and ramp-rate limits[13] for the large generation units are not included.

III. PRELIMINARY

In this section, notations of graph theory are presented to model the networked environment, and the replicator dynamics algorithm is introduced.

A. Graph Theory

Denote $G = (H, E)$ as a graph consisting of a set of nodes $H = \{1, 2, \ldots, n\}$ and a set of edges $E \subseteq H \times H$. An undirected edge from i to j is denoted by an unordered and distinct pair $(i, j) \in E$, the neighbor set of node i is denoted by $N_i = \{j \in H \mid (i, j) \in E\}$. A path is referred to a sequence of edges which connect a sequence of nodes. Two nodes are called connected if there contains a path from i to j. A graph is connected if and only if there exists a path between any two nodes.

B. Replicator dynamics

Replicator dynamic [15], [16]models the interaction of an homogeneous population, where fractions of individuals play a symmetric game, the one that has a better playoff will increase its population in the habitat.

A particular choice of replicator dynamics is given by

$$\dot{P_i} = P_i(f_i(P_i) - \overline{f}(P)) \tag{5}$$

where $\dot{P_i}$ denotes the increase rate of individuals in the population playing strategy i; P_i is the amount of individuals in the population playing strategy i; $f_i(P_i)$ is the fitness function of the individuals that play strategy i; $\overline{f}(P)$ is the average fitness function in population.

As the population size P_d is constant, $\overline{f}(P)$ can be defined as

$$\overline{f} = \frac{1}{P_d} \sum_{j=1}^{n} P_j f(P_j) \tag{6}$$

then the constraint set Δ_P would be invariant, as

$$\Delta_P = \left\{ P_i \in R_+^n : \sum_{i=1}^{n} P_i = P_d \right\} \tag{7}$$

which means $P_i(t) \in \Delta_P$ for all $t \ge 0$.

The proportions of individuals in each habitat evolve until each individual reaches the same fitness. If $\dot{P_i} = 0$, for all $i = 1, 2...n$, that is

$$f_i(P_i^*) = \overline{f}^* \tag{8}$$

the evolution process reaches an equilibrium.

IV. LOCAL REPLICATOR DYNAMICS

A. Local Replicator Dynamics Algorithm

For the distributed system, communication networks are constrained, and nodes can only have information about their neighbors, thus the ordinary replicator dynamics cannot be

used in this environment. A local version of the original replicator dynamics in (5) is proposed in [15] to account for local interactions of fractions of the population over a graph G.

The local replicator dynamics(LRD) is given by

$$\dot{P}_i = \frac{P_i}{P_d}(f_i(P_i)\sum_{j\in N_i}P_j - \sum_{j\in N_i}P_jf_j(P_j)) \quad (9)$$

for all $i \in H$, where $f_i(P_i)$ is still the fitness function that describes the payoff for the i th node, while the average fitness defined in (6) is changed, it takes the summations only over the neighborhood of node i instead of all node in the system. Therefore, the full information constraint for the calculation of the original average fitness in (6) is relaxed and the LRD can be used to handle the network constraints in the ED problem (1) given by the topology of the graph G.

The steady state is achieved when

$$f_i(P_i^*)\sum_{j\in N_i}P_j^* = \sum_{j\in N_i}P_j^*f_j(P_j^*) \qquad \text{for all } i \in H \quad (10)$$

which is satisfied if $f_i(P_i^*) = f_j(P_j^*)$ for all $j \in N_i$ and all $i \in H$. Assuming that the graph G is connected, thus at steady state all fitnesses are equal, that is

$$f_i(P_i^*) = f_j(P_j^*) = \overline{f}(P^*) \qquad \text{for all } i,j \in H \quad (11)$$

where $\overline{f}(P^*)$ is the equilibrium average fitness. Hence, LRD and RD have the same equilibrium point $P^* \in \Delta_p$. Besides, Δ_p is also invariant under the LRD, which has been proved in [15].

Taking the load demand as the population, and dispatched DG as a habitat to be chosen by the individuals, the amount of individuals in each DG will be the power P_i assigned to the ith DG. Thus LRD can be used for the power dispatch. Taking into account the equilibrium condition of the replicator dynamics, an appropriate choice for the fitness of each DG should be related to its marginal cost. Let $f_i(P_i) = \partial C_i(P_i)/\partial P_i$, using (1) and (4), it is obtained that

$$f_i(P_i) = -2c_iP_i - b_i \quad (12)$$

Given that fitness functions are a measure of the playoff, a big enough positive constant B can be added to all fitness functions to ensure $f_i(P_i) > 0$, for $P_i^{\min} \le P_i \le P_i^{\max}$, $i \in H$. Notice that this constant displacement does not affect the optimization process. Thus the final fitness function is defined as

$$f_i(P_i) = B - b_i - 2c_iP_i \quad (13)$$

In order to approximate the continuous-time local replicator dynamics, one option is the discrete-time version of the model, given by

$$P_i[k+1] = P_i[k](1+\alpha(f_i[k]\sum_{j\in N_i}P_j[k] - \sum_{j\in N_i}P_j[k]f_j[k]))$$
$$+ \sum_{j\in N_i}l_j[k] \qquad k=0,1,2... \quad (14)$$

$$f_i[k+1] = B - b_i - 2c_iP_i[k+1] \quad (15)$$

where α is a step size, $l_j[k+1]$ is the unbalance power of the jth DG, $j \in N_i$.

B. Renewables Consideration

As the cost functions of renewable resources are not the same as DG, and renewable generators are mostly owned by private sectors, how to incorporate renewable resources into ED is a problem. Assuming that integrating as much renewable energy as possible into the system is the desired objective and renewable sources are regulated to have priorities compared with fossil sources. In this case, the LRD formulation of the ED problem (14) remains the same, while the fitness will be configured to reflect the interest of renewable sources, as

$$f_i^r(P_i) = \begin{cases} m_iP_i^{\max} & do \quad want \\ m_iP_i^{\min} & don't \quad want \end{cases} \quad (16)$$

where m_i is an objective factor that reflect the objective of renewable sources, it can be tuned according to desired objective. If renewable sources want to join the power dispatch, it will determine a large fitness, which guarantee the maximum power dispatched to renewable sources in the LRD evolve process. On the contrary, a small fitness will be determined. Notice that the fitness will keep fixed in the evolve process.

C. Inequality Constraints Handling

To avoid violations of inequality constraints, we can update the equation (14) and (15) as follows:

$$P_i^u[k+1] = \begin{cases} P_i^{\max} & P_i[k+1] > P_i^{\max} \\ P_i[k+1] & P_i^{\min} \le P_i[k+1] \le P_i^{\max} \\ P_i^{\min} & P_i[k+1] < P_i^{\min} \end{cases} \quad (17)$$

$$f_i^u[k+1] = B - b_i - 2c_iP_i^u[k+1] \quad (18)$$

If the calculated generation power $P_i[k+1]$ of a DG lies within the bound, the power will be updated as usual; otherwise, the unit will be fixed to its violated limit, and the unbalance power produced by drawing back to its violated limit will be compensated by the neighbor DGs/agents. The unbalance power compensated by each neighbor is defined as

$$l_i[k+1] = (P_i[k+1] - P_i^u[k+1])/Z_i \quad (19)$$

where Z_i is the number of neighbor DGs for ith DG.

After DG power violating limit, the DG will be excluded from next LRD evolve process until it find the suitable conditions to join the process again. However, lack of node in the strong connected network will cause some node isolated from network, which may bring the problem that evolve process cannot continue. For handling this problem, the concept of transfer agent has been used. When the DG power violating limit or quitting the LRD evolve process, the respect DG Agent will act as transfer agent building virtual communication channels to make the neighbor agents connected.

Fig. 1 illustrates the idea of building virtual communication channels. Assuming the case of bound violation with DG 1. The affected communications links are the 4 red links associated with agent 1 in Fig. 1(a). After the bound violation, generation power of agent 1 will be fixed and excluded from next LRD evolve process, however, agent 1 acting as transfer agent still participates in the information exchange. As shown in Fig. 1(b), agent 1 will take the incoming data from a neighbor agent and send it directly to all other agents that are originally connected to the agent through agent 1, as long as there is no direct communication link between the two neighbor agents. In this way, it is like the topology of the communication network is reconfigured as the one shown in Fig. 1(c). It should be noted that agent 1 still use its current fitness at bound point to calculate the power in the evolve process left to see if the calculated power lies within the bound. If the expected condition appears (the calculated power lies within the bound), the original communication topology will be restored and agent 1 will rejoin the evolve process again.

It should be noted that during the process of bound violation there are some unbalanced power flowing in the network, but the power will be dispatched legitimately by LRD eventually.
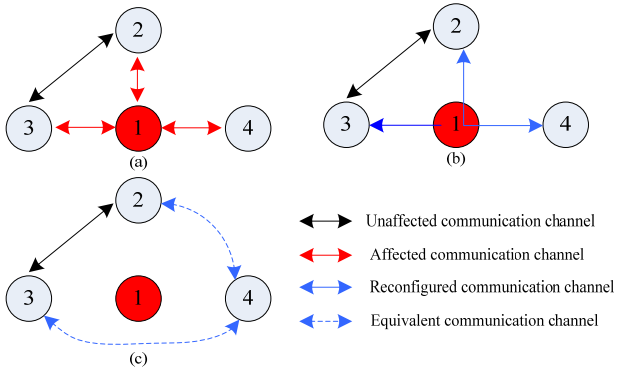

Fig. 1. Construction of virtual communication channel by transfer agent

V. IMPLEMENTATION OF PROPOSED APPROACH

The approach presented in this section is proposed primarily for solving the ED in a fully distributed manner by MAS. It is assumed that each DG and load in the system has an agent responsible for and the graph is strong connected, which means there must an channel connected with an agent. The proposed approach consists of two stages. The first stage aims at determining the initial generation power for DG agents, while the second stage is power dispatch using LRD.

A. Adjacency Average Allocation Algorithm

In most previously proposed decentralized and distributed algorithms, the total demand in the network are needed to execute the algorithm, which requires the global information. In this paper, an adjacency average allocation algorithm is proposed to initialize the generation power of DG by the interaction between neighbor load agents and DG agents. The adjacency average allocation algorithm is a fully distributed

algorithm, which allocate the distributed load power to their adjacency node equally as the initialization for the second stage.

Adjacency average allocation algorithm makes it a good initialization since load power are almost distributed in the generation equally, which can increase the LRD convergence speed.Notice that the load agent is only activated at the first stage for initialization, since the power balance is preserved over time by (7).

B. Implementation of LRD

For the second stage, the proposed LRD is implemented based on a MAS framework, which is developed using Java Agent Development (JADE) platform. Each generation agent has two main functions: local information update (LIU) and information exchange (IE) with neighboring agents, as shown in Fig. 2. The MAS framework only include generation agents since load agents are not working in the second stage.

LIU is in charge of updating the fitness, generation power and unbalance power in the evolve process according to (17), (18) and (19), based on its own parameters and information obtained from neighboring agents.

IE is responsible for exchanging information with the neighboring agents. The supporting communication network topology can be designed to be either the same or independent to the topology of the power network, as long as the designed communication graph is strongly connected.
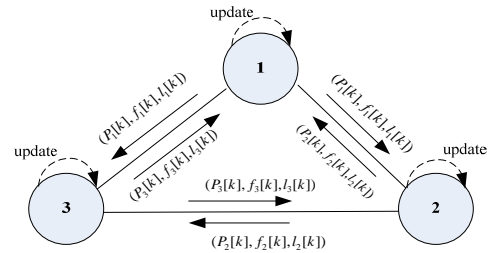

Fig. 2. The LRD in MAS framework

VI. CASE STUDIES

In this section, several case studies are discussed to show the effectiveness of the proposed approach.

A. Adjacency Average Allocation Algorithm

The system with 3 DGs and 3 loads are considered. The generation cost coefficients and the generation limits are summarized in Table I. The supporting communication topology is as shown in Fig. 3.

TABLE I. DG DATA

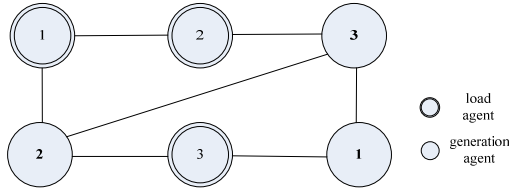| Unit	$P_i^{\min}$	$P_i^{\max}$	c_i	b_i	a_i
1	0	80	0.0156	7.92	561
2	0	80	0.0194	7.85	310
3	0	60	0.0482	7.8	78

Fig. 3.   System topology

In this case study, different distributed load demand are applied to observe the impact of adjacency average allocation algorithm to the DG initial generation power. The total load demand is 159 kW, Table II shows two scenarios with different load demand for load agent.

TABLE II.   DISTRIBUTED LOAD DEMAND

Scenario	Load 1(kW)	Load 2(kW)	Load 3(kW)
1	20	49	90
2	58	59	42
3	46	65	48

With adjacency average allocation algorithm，DG agents can determine the initial power for LRD. The results with different scenarios are shown in Table III. For scenario1, the initial power is located in the normal range, while for scenario 2, the initial power of DG2 and DG3 is nearly violate the bound. Scenario 3 is the worst situation, in which DG3 violate the upper limit. Thus with distributed load demand initial power for DGs can be different, but for all the initial conditions, the designed LRD with transfer agent can get an optimal solution no matter where the initial power locates, which will be discussed in the next cases.

TABLE III.   LOAD DEMAND ALLOCATION IN DGS

Scenario	DG 1(kW)	DG 2(kW)	DG 3(kW)
1	45	65	49
2	21	79	59
3	24	70	65

Compare to the flooding-based consensus algorithm [13] and discovery algorithm [17], adjacency average allocation algorithm is really fast for DGs to perceive the load demand and only local information is needed, avoided getting the global information.

With more load agents and more communication channels, the load demand will be more distributed, and the initial power for each generation agent will be more equal, which will provide a good start point for the LRD next stage.

B. Transfer Agent for Bound vialation

In order to investigate the effectiveness of inequality constraints handling by transfer agent, Three cases are compared. The first two cases for scenario 2 show the performance with and without generation constraints, and   the third case considers transient bound violation handling by transfer agent. The last case for scenario 3 demonstrates the effectiveness of LRD with different initial states.

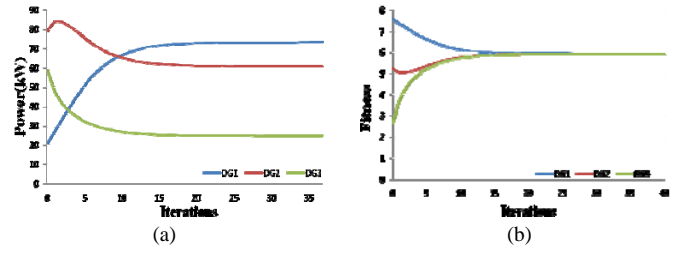(a)   (b)

Fig. 4.   Without generation constraint: (a) power, (b) fitness

As without generation constraints, the DGs' constraints are not imposed. The evolve process of DG output power and fitness is shown in Fig. 4, it can be observed the fitness asymptotically converge to a common value after 35 iterations, which means the equilibrium point is achieved. Hence, the optimization goal is fulfilled. The generator output powers are $P_1=73.24$kW , $P_2=60.78$kW , $P_3=24.98$kW and the average fitness is 5.94. Although all the final outputs are within the generators' operational ranges, careful examination on the plots shows that DG2 power goes beyond 80 kW during the transient response since the generation constraints are not imposed. This is not desirable due to there will be chances for the final dispatched power exceeding limit.

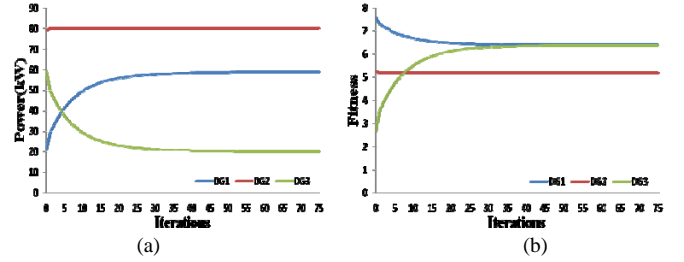(a)   (b)

Fig. 5.   With generation constraint: (a) power, (b) fitness

Case with generation constraints is considered, the results are shown in Fig. 5. We notice that DG2 gets saturated after the second iteration due to the generation constraints and it is excluded from the next evolve process. Thus DG2 output power remains at the maximum point, and the fitness stay unchanged in the next evolve process. DG1 and DG3 will continue the evolve process, and converge to a new equilibrium point after 75 iterations, which is different from the one without DG constraints. Based on the results, $P_1=58.71$kW , $P_2=80$kW MW, $P_3=20.29$kW , no DG power exceed the operation ranges even in the transient responses, but the final solution is not optimal due to DG 2 converge to the power limit early, which makes the solution pointless.
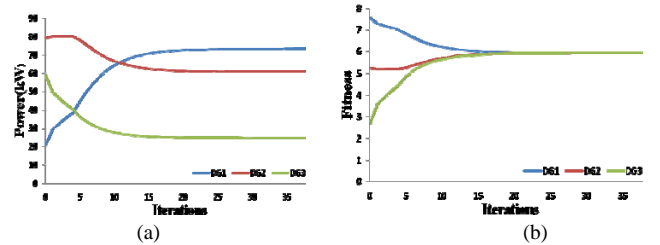

(a)   (b)

Fig. 6.   Transfer agent with generation constraint: (a) power, (b)fitness

For the case with generation constraints, transfer agent is used to solve the transient bound violation problem. Shown in Fig. 6, DG2 gets saturated after the second iteration, at the same time agent 2 will act as the transfer agent and quit the next evolve process until the rejoin condition appears. On one hand agent 2 will build a virtual communication channel between DG1 and DG3, on the other hand, it will still calculate its generation power with the information from neighbors in every iteration. It can be seen that at the fourth iteration, DG2's fitness at the maximum point is less than the average fitness of DG1 and DG3, so it will rejoin the evolve process automatically according to the characteristic of LRD. Finally, the fitness of all DGs converge to the same value 5.94, and the optimal solution is $P_1=73.24$kW , $P_2=60.78$kW , $P_3=24.98$kW , which is the same as the case without DG constraints.

By the method of transferring the unbalance power to the neighbor agents when transfer agent draws back its dispatched power to normal range, bound violation can be solved legitimately.

C. Priority to Renewable Generation

For the Scenario1, adding a 20kW renewable generation unit, connecting with DG3, in the network. Assumed that the renewable source is a photovoltaic(PV) generator which wants to join the power dispatch because of the daytime. Since there is no load connected with, there will no load agent assigning power to it in the first stage, thus the initial power for PV will be zero, and the other DGs' initial condition will be the same as Table III.

Shown in the Fig. 7, bigger fitness is selected by PV Agent, and remains fixed in the evolve process. When the other 3 DGs converge to a steady condition the LRD will be end, at the same time PV fulfill its maximum power, thus the interest of PV is highly considered. The final dispatched power is $P_1=63.85$kW , $P_2=53.22$kW , $P_3=21.93$kW , and the average fitness for 3 DGs is 6.24. Compared to the case without generation constraint, the dispatched power is less for DGs due to PV joins the dispatch, and the average fitness is increased, which means the marginal cost for DGs is decreased.
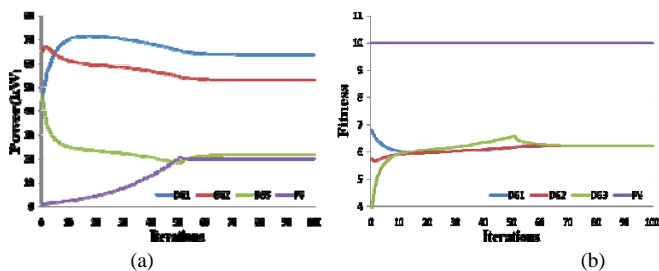

Fig. 7.   Adding renewable generation (a) power, (b) fitness

Moreover, at the 50th iteration, significant fluctuations can be observed for DG3 power and fitness. This is because DG3, the only neighbor of PV, has to compensate the unbalance power when PV draws back to the maximum power after violation bound.

This case also can be seen as the plug and play test, due to no initial power assigned to PV, PV as a new generator can be plugged at any time during the evolve process. Finally the PV is well adapted into the system from the result obtained previous.

D. Computational Efficiency and Convergence Analysis

The distributed nature of the proposed approach affects both computational and convergence times. It should be noted that the proposed approach has a two-stage architecture. For the first stage, the number of iterations does not really depend on the topology of the communication graph, while the number of second stage iterations is related to the graph diameter, such as a line and a fully-connected topologies are different.

Moreover, for the proposed approach no global information is needed in the initialization stage and the LRD implementation stage, which make it a fully distributed implementation.

VII. CONCLUSION

This paper proposes a fully distributed economic dispatch approach for distributed generation based on MAS framework. Each generation and load is modelled as an agent and strongly connected communication network is sufficient for the information exchange. By the proposed approach, adjacency average allocation algorithm enables to address the equality constraints, and guarantees a good initialization for the local replicator dynamics. Designed local replicator dynamics algorithm is well adapt to different initial condition, and renewable resources can be integrated properly. With transfer agent constructing the virtual communication channel, some restrictions on the LRD can be released. Moreover, due to the fast speed of approach, real time economic dispatch can be guaranteed.

REFERENCES

[1] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*. New York, NY, USA: Wiley, 1996.

[2] C. E. Lin and G. L. Viviani, "Hierarchical economic dispatch for piecewise quadratic cost functions," *IEEE Trans. Power App. Syst.*, vol.PAS-103, no. 6, pp. 1170–1175, Jun. 1984.

[3] C.-L. Chiang, "Improved genetic algorithm for power economic dispatch of units with valve-point effects and multiple fuels," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1690–1699, Nov. 2005.

[4] J. Sun, V. Palade, X.-J. Wu, W. Fang, and Z. Wang, "Solving the power economic dispatch problem with generator constraints by random drift particle swarm optimization," IEEE Trans. Ind. Inf., vol. 10, no. 1, pp. 222–232, Feb. 2014.

[5] L.Du, S.Grijalva, R.G. Harley, "Game-Theoretic formulation of power dispatch with guaranteed convergence and prioritized best response," *IEEE Trans. Sustain. Energy*, vol. 6, no. 1, pp. 51–59, Jan. 2015.

[6] V. Loia and A. Vaccaro, "Decentralized economic dispatch in smart grids by self-organizing dynamic agents," *IEEE Trans. Syst., Man, Cybern.: Syst.*, vol. 44, no. 4, pp. 397–408, 2014.

[7] R. Mudumbai, S. Dasgupta, and B. B. Cho, "Distributed control for optimal economic dispatch of a network of heterogeneous power generators,"*IEEE Trans. Power Syst.*, vol. 27, no. 4, pp. 1750–1760, Nov. 2012.

[8] G. Binetti, A. Davoudi, F. L. Lewis, D. Naso, and B. Turchiano, "Distributed consensus-based economic dispatch with transmission

losses," *IEEE Trans. Power Syst.*, vol. 29, no. 4, pp. 1711–1720, Jul. 2014.

[9] S. Yang, S. Tan, and J.-X. Xu, "Consensus based approach for economic dispatch problem in a smart grid," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4416–4426, Nov. 2013.

[10] G. Binetti, A. Davoudi, D. Naso, B. Turchiano, and F. Lewis, "A distributed auction-based algorithm for the non-convex economic dispatch problem," *IEEE Trans. Ind. Informat.*, vol. 10, no. 2, pp.1124–1132, 2014.

[11] Y.Xu, W. Zhang, W. Liu"Distributed dynamic programming-based spproach for economic dispatch in smart grids," IEEE Transactions on Industrial Informatics, vol. 11, no. 1, pp.166–174, 2015.

[12] W. Zhang, W. Liu, X. Wang, L. Liu, and F. Ferrese, "Online optimal generation control based on constrained distributed gradient algorithm," IEEE Transactions on Power Systems, vol. 30, no. 1, pp. 35–45, Jan. 2015.

[13] W.T. Elsayed, E. F. El-Saadany, " Fully decentralized approach for solving the economic dispatch problem," *IEEE Trans. Power Syst.*, vol. PP, no. 99, pp. 1–11, Oct. 2014.

[14] A. Pantoja, N. Quijano, and K. M. Passino, "Dispatch of distributed generators under local-information constraints," in American Control Conference, (Portland, OR), pp. 2682–2687, June 2014.

[15] Weibull J. Evolutionary game theory[M]. USA: The MIT press, 1997: 68-108.

[16] A. Pantoja and N. Quijano, "Distributed optimization using population dynamics with a local replicator equation," in Proceedings of the 51st IEEE Conference in Decision and Control, pp. 3790–3795, 2012.

[17] X.S. Yang, S.S.S. Hosseini, A.H Gandomi, "Firefly algorithm for solving non-convex economic dispatch problems with valve loading effect," Applied Soft Computing, vol. 12, no. 2, pp. 1180–1186, Mar. 2012.