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Mitigation of airspace congestion impact on airline networks

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ABSTRACT

In recent years European airspace has become increasingly congested and airlines can now observe that en-route capacity constraints are the fastest growing source of flight delays. In 2010 this source of delay accounted for 19% of all flight delays in Europe and has been increasing with an average yearly rate of 17% from 2005 to 2010. This paper suggests and evaluates an approach to how disruption management can be combined with flight planning in order to create more proactive handling of the kind of disruptions, which are caused by congested airspace. The approach is evaluated using data from a medium size European carrier and estimates a lower bound saving of several million USD.

1. Introduction

Running an airline is a complex business where hundreds of aircraft need to be scheduled and maintained. Thousands of flights need to be dispatched every day. Tens of thousands of crew members need to be rostered and millions of passengers need to be transported from one location to another every year. To accomplish this enormous task airlines have for several decades relied on Operations Research (OR) to stay competitive and conduct careful and efficient planning of every single activity in their operation. Unfortunately these efficient plans are hardly ever being executed as originally intended.

In 2010 24% of all flights in Europe and 18% of all flights in the US were delayed more than 15 min and consequently experienced some sort of disruption (Eurocontrol and FAA, 2012). Bad weather, technical problems, crew reporting sick and in recent years to an increasing extent also airspace being congested are all examples of uncertainty elements.

To manage these deviations there has during the last couple of decades been a move in airline related OR research to an increased focus on the real-time execution of the airline. In this paper we take OR based disruption management one step further in the direction toward the actual flight operation as we combine disruption management and flight planning.

The paper initially gives a short introduction to disruption management and the main work processes, which exists in an *Operational Control Center (OCC)* in an airline. The paper provides a literature review on disruption management with a special focus on integrated disruption management as well as flight planning. The paper goes into further detail with Air Traffic Flow Management (ATFM). In this paper we suggest a network representation and a model, which handles integrated recovery decisions with flexible flight trajectories. We describe a framework for using the integrated decision approach and use this to evaluate our suggested approach. Finally we present our findings in terms of a lower bound for the annual saving, which can be obtained by using the approach.

A contribution of this paper is to suggest and evaluate an approach to how disruption management can be combined with flight planning in order to create more proactive handling of the kind of disruptions, which are caused by congested airspace.

The paper suggests a method for increased interaction between Ops Controllers and flight planners in order to make sure that the network effects of any trajectory selection is properly incorporated in the decisions.

The paper introduces a flight planning based aircraft recovery model, which takes into account both passenger misconnections

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and congested airspace constraints.

2. Disruption management

Whenever an event occurs, which makes an airline deviate from its planned schedule or its planned crew rosters, the airline is disrupted. Most larger airlines operate a hub and spoke network, where efficient use of aircraft and crews are causing the airline not to have crew following the aircraft. This is due to the fact that crew work rules are much more restrictive than the rules which can be applied to aircraft. The tight planning of aircraft and crew is causing an airline to become very vulnerable to disruptions, as a delay of a single inbound flight to a hub quickly can propagate to other flights.

Most airlines have an Operational Control Center (OCC). In the OCC *Ops Controllers* monitor the operation of the airline and manage disruptions to the schedule and are responsible for a well-functioning network of flights, crew and passengers on the day of operation.

The organizational setup of an OCC varies from airline to airline and does to a large extent depend on the size of the airline. There are, however, some typical organizational entities, which are present in virtually any OCC. These are:

- Airline Operations Controllers: These are responsible for the overall operation of the airline's schedule on the day of operation.
- Aircraft Controllers: This group of people are responsible for maintaining a feasible schedule and aircraft routing, including that each aircraft is routed back to their scheduled and unscheduled maintenance activities at one of the maintenance stations.
- Crew Controllers: When the recovery of the schedule and aircraft routings inflict changes to the schedule, these changes need to be verified for feasibility with the Crew Controllers.
- Customer Service Representatives: The Customer Service Representatives in the OCC are responsible for maintaining a proper level of service to the airline's passengers, which is especially important to keep in focus during times of irregular operations.
- Maintenance Controllers: This group of people are in contact with the maintenance department of the airline and communicates to the Aircraft Controllers in case a maintenance activity will not be finished on time.
- Flight Dispatchers: A dispatcher is responsible for a number of individual flights and does on a flight-by-flight basis take care of everything from collecting relevant weather information for a flight to calculating the *flight plan* and monitoring the status and potential risks related to the flight while it is en-route.

2.1. Previous work on disruption management

In order to find good recovery solutions in a limited amount of time OR techniques have been applied to the problem. The full problem of recovering all 3 resource areas of aircraft, crew and passengers is, however, so complex that no work has been published so far, which cover all 3 areas in one single integrated model. The published models are typically inspired by how the airlines do their manual problem solving, and the models usually address one single resource area each. A good introduction to disruption management in the airline industry can be found in Belobaba et al. (2009). Kohl et al. (2007) describes a large scale EU-funded project, called Descartes, which addresses various aspects of disruption management. The reader is also referred to an extensive survey of operations research used for disruption management in the airline industry by Clausen et al. (2010).

Of the 3 resource areas mentioned above, aircraft recovery was the first area to be addressed through the application of OR by Teodorović and Guberinić (1984). This work was merely academic in its scope and only considered flight delays. Jarrah et al. (1993) were the first to publish 2 models, which in combination were capable of producing solutions, which were useful in practice. The drawback of Jarrah et al. (1993) was that cancellations and delays could not be traded off against each other within one single model. This drawback was later on resolved in the work by Yan and Yang (1996). Thengvall et al. (2001) later on extended this model to also include so-called protection arcs, which serve the purpose of keeping the proposed solutions somewhat similar to the original schedule. Rosenberger et al. (2003) present a model based on the set packing problem. Andersson (2006) proposes two meta-heuristics based on simulated annealing and tabu search. Results show that the tabu search heuristic is best and can find high quality solutions in less than a minute. Recently Eggenberg et al. (2010) proposed a generalized recovery framework using a timeband network, where the same model can be used to solve either an aircraft recovery problem, a passenger recovery problem or a crew recovery problem.

The second problem, which has been addressed by the OR community is the crew recovery problem, which was initially addressed in the work by Johnson et al. (1994). Later work include Wei et al. (1997), Stojković et al. (1998), Lettovsky (2000) and Medard and Sawhney (2007).

The third area, passenger recovery, has only been addressed by a very limited amount of published research. The main contribution in this area is done by Bratu and Barnhart (2006), who present a Passenger Delay Model. Vaaben and Alves (2009) does a comparison of sequential passenger re-accommodation with re-accommodation based on an IP model.

3. Air Traffic Control (ATC) and flight planning

The airspace of a country is regulated by the authorities of the country. In the US it is the Federal Aviation Administration (FAA). While the different countries in Europe regulate their own airspace, they have to a large extent agreed on common rules and have also established a common control entity called Eurocontrol. Both Europe and the US have established an overarching control layer for their Flight Information Regions (FIRs) called Air Traffic Flow Management (ATFM). In Eurocontrol ATFM is performed by the Central Flow Management Unit (CFMU).

To coordinate traffic and ensure safety a number of additional elements are defined for the airspace. Among these are *waypoints* and *airways*. Together with waypoints the airways create a directed graph, where waypoints represent nodes and airways represent arcs.

In order to fly from one airport to another it is necessary to calculate a path through the airspace graph. This process is called *Flight Planning*. For further reading regarding airspace and ATC, the reader is referred to Belobaba et al. (2009) and Cook (2007).

A flight plan describes how the aircraft is going to fly from a Point Of Departure (POD) to a Point Of Arrival (POA) and has to be filed with Air Traffic Control (ATC) before the flight is allowed to take off. The route is specified as a sequence of waypoints and altitudes.

Calculating a flight plan is a complex optimization problem in itself. It has, however only been addressed by academia to a rather limited extent compared to other airline related problems. Altus (2012) gives an overview of flight planning related literature and the complexities associated to the problem.

3.1. Cost index

An important concept to decide about speed is the *cost index*. All modern aircraft in commercial aviation use cost index as an input to their on-board computer, which is also know as a Flight Management System (FMS). The pilot enters a cost index into the FMS, which basically tells the computer what the value of time is compared to the value of fuel as given in the following definition of Cost Index, where fuel in this definition is measured in kilograms.

$$\text{Cost Index} = \frac{dollars/min}{dollars/kg} \tag{3.1}$$

The definition of the Cost Index consequently expresses the number of kilos of fuel, which the FMS should be willing to burn, in order to save one minute of time. As seen from the definition, a cost index of 0 will minimize the fuel burn by indicating that cost of time is seen as having zero value. The problem with the cost index definition is that it assumes that the cost of time is linear, which is far from the case in normal airline operation. A number of factors contribute to the fact that cost of time is not linear. In Altus (2010) sources like subsequent flights, operational flexibility, crew connections, passenger connections and goodwill are listed as examples that make cost not linear but rather piecewise linear in time.

4. Air Traffic Flow Management (ATFM)

As previously mentioned the airspace is divided into FIRs, where each FIR has a control center for the area, ACC. In regions with a high density of air traffic an additional coordination layer on top of the ACCs have been established to coordinate the flow of traffic between the FIRs and in this way ensure that air traffic in specific areas do not exceed capacity. The practice of coordinating air traffic across various FIRs from a system perspective is referred to as Air Traffic Flow Management (ATFM). ATFM is not carried out in the same way in the US and in Europe.

ATFM in the US is taken care of by the Air Traffic Control Systems Command Center (ATCSCC) located in Northern Virginia. Under nominal operating conditions the ATCSCC does not put special regulation in place in order to restrict the flow of air traffic as the US National Airspace System (NAS) can handle the demand under these conditions. However, when the NAS becomes disrupted due to adverse weather, equipment outages, runway closures or demand surges, the ATCSCC applies special regulations in order to restrict the flow of traffic through the system. One such type of regulation is the Ground Delay Program (GDP) which was initiated in 1998.

The GDP initiative has been very successful and has according to Metron Aviation avoided 50,000 h of assigned ground holding since it was initiated (Vossen et al., 2012). Building on this success FAA did in the summer of 2006 implement the *Airspace Flow Program (AFP)* initiative, which extends the GDP procedures to the en-route environment. With the AFP the ATCSCC can enforce a flow restriction across a predefined borderline referred to as a Flow Constrained Area (FCA) and thus restrict the flow of flights in one direction across the FCA. Each airline is granted a number of slot times according to the Ration By Schedule scheme also used for GDPs. An AFP related slot time is a small time window where the airline is granted the right to pass through the FCA with one flight. The airline is allowed to decide which flight should use the slot time and also which time to depart. In order to help dispatchers find good candidates for slot swaps in case of an AFP, Abdelghany et al. (2007) presented a heuristic to do this. For some flights the carrier may choose to completely avoid this constraint by filing a flight plan, which takes the flight around the FCA.

While the present paper addresses how the OCC of an airline can respond to ATFM restrictions in a way which affects the network of the airline to the least extent, the paper of Bertsimas et al. (2011) proposes how ATFM with rerouting possibilities should be handled from a central ATM point of view. This paper has its off-set in the seminal paper of Bertsimas and Patterson (Bertsimas et al., 2000).

Cook and Tanner (2012) explore flight prioritization principles and argues that trajectories and departure times should increasingly be decided through Collaborative Decision Making (CDM) in order for the aircraft operators to achieve the “best business outcome”. The present paper contributes by providing the aircraft operator's perspective of this interaction given specific airspace congestion constraints. This includes the evaluation of trade-offs between passenger delays, fuel burn costs and trajectory selection when providing the aircraft operator's suggestion to the “best business outcome”.

ATFM in Europe is taken care of by the Central Flow Management Unit (CFMU), which is a part of Eurocontrol and located in Brussels. When a flight in Europe flies from point A to point B the pilot – or dispatcher, if the flight belongs to an airline – files a flight plan with the local airspace authorities of point of departure (POD). CFMU receives the flight plan and calculates when the flight will pass through a number of different air sectors on its way. In case any of these sectors have reached their capacity limit, CFMU will issue a *Calculated Take-Off Time (CTOT)*, which is later than the originally intended departure time in the flight plan filed by the carrier. CFMU grants access through the congested air sector on a first-come-first-serve basis in the order of time when flight plans were filed. Based on this policy CFMU issues CTOT-delays to the flights, which have filed flight plans through the congested sector.

In case the dispatcher of an airline determines that the CTOT-delay is too large, he may choose to cancel the flight plan and file another flight plan, which takes the flight around the congested airspace. By doing so he frees up a bit of capacity in the congested air sector.

When looking to the sky, airspace may seem plentiful compared to the amount of aircraft manoeuvring in it. Airspace does, however, get congested in areas with a high flight density such as some parts of Europe and the US. Combined with flight density there are two main reasons why airspace gets congested. Both are due to safety regulations (Belobaba et al., 2009) as ATC needs to keep a large separation between aircraft in their area, and because ATC is currently based on human controllers, which implies a limitation to how many aircraft a controller can safely monitor at any given point in time.

Congested areas over Europe can be followed using CFMU's *Network Operations Portal (NOP)*. Not a day passes by without the NOP portal showing various areas in Europe, where en-route and airport delays must be expected.

Whenever a disruption occurs it typically results in some form of flight delay. A flight delay could for instance be caused by one or more checked-in passengers not boarding the flight and their bags will consequently have to be off-loaded for security reasons, which often results in a delay. This is referred to as a *primary delay*. This delay may have a knock-on effect on a subsequent flight in which case this second flight delay is reported as a *reactionary delay*. The International Airline Travel Association (IATA) have defined a set of delay codes for both primary and reactionary delays. Airlines use these codes for reporting their delays to Eurocontrol and the delay causes among all airlines are roughly split fifty–fifty between primary and reactionary delays (Eurocontrol, 2010).

It is especially interesting to look at primary delay causes due to the fact that if these are reduced the corresponding reactionary delays will also be reduced. In their yearly reports Eurocontrol has published the distribution of primary delays causes for flights in Europe. For 2010 the distribution is shown in Fig. 1.

In Fig. 1 it is noted that the majority of the primary delays (41.8%) are caused by factors related to the airline itself, such as technical problems, baggage delays, checked-in passengers not showing up, etc. The second largest portion (32.5%) of primary delays are caused by factors related to Air Traffic Flow Control Management (ATFCM), which is basically the part of Eurocontrol taking care of the flow of flights through different sectors in Europe. The largest subset of the ATFCM-delays are so-called *en-route delays* and correspond to 19.09% of all flight delays.

While en-route delays is not the biggest source of primary delays, it is, however, the fastest growing source of delays (Network Operations Report, 2010). This source of delays in Europe has increased with an average yearly rate of 17% from 2005 to 2010, which is a good reason to address exactly this kind of delays. That en-route delays have been rising so sharply in recent years is due to the fact that European airspace is close to reaching its capacity limit. A similar development has also been seen in some areas of the US, especially in the densely populated North East. This is the main reason why both the US and Europe have initiated huge programs called Next Generation Air Traffic Control (NextGen) in the US and SESAR in Europe. Both programs aim at increasing airspace capacity by e.g. enabling more direct flight paths and reduced aircraft separation requirements.

5. Combining flight planning and disruption management

In the work flow in the OCC there is a high degree of interaction between Ops Controllers and people in the related areas of aircraft, crew and customer service. Based on the experience of the first author and his 15 years in the airline industry, there is little interaction between Ops Controllers and dispatchers, who at some airlines are not even located in the same room.

Ops Controllers take care of the overall network of flights and use a combination of swaps, delays and cancellations in order to recover from a disruption. Dispatchers on the other hand, look at individual flights and make local decisions about trajectory and speed.

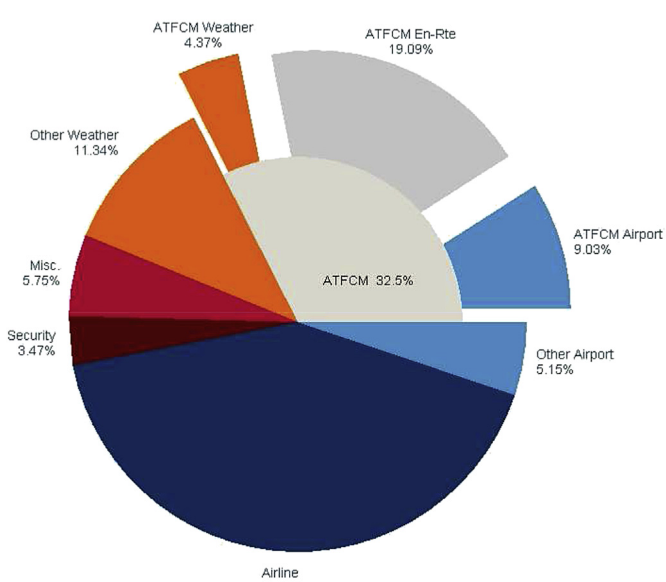


Fig. 1. Primary delay causes in 2010.
Source: Network Operations Report for 2010, Eurocontrol.

There is little focus on the flexibility, which flight planning can provide when searching for good recovery solutions. The proposal of this paper is a model, which can do exactly this. It includes various flight trajectories in disruption management decisions in order to change flight planning decisions from being local decisions for individual flights to being decisions, which serve the entire airline network in the best possible way in terms of both fuel burn and passenger connections. The model formulation allows for both rerouting trajectories and speed change trajectories to be handled. The computational experiments do, however, only consider rerouting trajectories as only these are relevant for the short haul European flight schedule, which are used for the experiments.

6. Modelling

In this section we describe the network representation of the problem as well as the mathematical model, which is based on the network. The model is based on a *time-space network* representation of the airline's schedule and planned maintenance activities. The nodes in a time-space network represent both time and location. In the current application the locations are airports.

6.1. Network representation

In the aircraft recovery literature the modelling is generally based on different variants of three network representations as surveyed in Clausen et al. (2010). 1) A *connection network*, where the flight activities are represented by nodes in the network. 2) A *time-line network* where each node represents a point in time and a location, while flights are represented as arcs in the network. 3) Finally a *time-band network* has been used as a variant hereof, where points in time are aggregated into so-called time-bands. The latter two representations both belong to the *time-space* class of representations, where nodes represent both a point in time and space. As the purpose of this paper is to alleviate the problems of congested airspace by combining flight planning and disruption management, we have made the choice of a time-line network. This representation has the advantage of an exact representation of time and location at an airport together with an intuitive and logical way of representing flight plans as arcs in this network.

The basic layout of the network is shown in Fig. 2. In this network time is increasing from left to right and each horizontal line represents an airport location. A white square node represents a source node for a specific aircraft. This is the current location of this aircraft at the start of the *recovery window*. The recovery window is a time window where the algorithm is allowed to make changes to the aircraft schedule. A black square node is a sink node for a specific aircraft and represents a time where this aircraft must be present at the specified airport.

The small network example in Fig. 2 could be operated by two aircraft. One starting in airport A, visiting airport B and C before returning back to airport A; and another aircraft starting in airport B, travelling to airport D and back to B before ending at airport D.

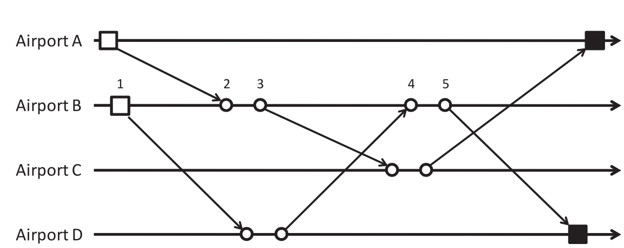


Fig. 2. Basic layout of time-space network.

The network also contains *ground arcs* (not shown) that represent the time spend on the ground, in-between flights. As an example, the network in Fig. 2 would contain four ground arcs for airport B (an arc between node 1 and 2, one between 2 and 3 and so on). The network also contains *maintenance arcs* which, like ground arcs, connect two nodes associated with the same airport. However, maintenance arcs represent planned repairs and/or inspections and are usually mandatory, where ground arcs are optional.

In order to produce a valid recovery solution where no additional post-processing is required it is important to have complete control over which specific aircraft will perform each activity. This is obvious for maintenance activities, where a mandatory maintenance of an aircraft requires that this specific aircraft arrives at the maintenance hangar.

Additional arcs (*delay arcs*) are introduced to allow delays (shown in Fig. 3). Here each dashed arc represents a delay of the original flight (drawn with a solid line). Each dashed arc represents a particular amount of delay.

In more traditional aircraft recovery, arcs would have represented flights. The arcs introduced so far allow traditional recovery, where a multi-commodity network flow model can decide how to best recover the schedule using a combination of the three traditional recovery techniques: Swapping flights, delaying flights and maybe cancelling some flights.

In the current network representation the arcs do not just represent flights but rather *flight plans*. These flight plans are calculated using a flight planning system and include more detailed information regarding how the flight will be conducted. This includes trajectory, speed and fuel burn.

6.1.1. Speed change arcs

By providing a different *cost index* as input to the flight plan calculation the cruise speed and consequently the fuel burn will change. Compared to the normal cost index for the airline and aircraft type, a lower cost index will result in increased flying time and a lower fuel burn, while a higher cost index results in shorter flying time and increased fuel burn. The network representation of speed change arcs are illustrated in Fig. 4. The solid arcs indicate flight plans where a flight is flown at the standard cost index of the airline, while the dashed lines indicate flight plans with either lower or higher cost index setting. The figure illustrates in a very simplified example the additional flexibility, which the speed change arcs provide with respect to recovery. For flight f1, which departs with a delay, the schedule can be recovered by either selecting a faster flight plan for flight f1 or by maintaining flight f1 at standard speed and delaying the departure of flight f2, while at the same time selecting a faster flight plan for f2. In Marla et al. (2011) the speed change arcs are analysed in detail. The paper concludes that speed change arcs are mainly of benefit to long haul flight as these spend significant amount of time at cruise speed.

6.1.2. Congestion related arcs

While speed change arcs are mainly interesting for long haul flights, another kind of arcs are relevant for short haul flights. A large amount of short haul flights in Europe and the North East of the US operate in congested airspace. For this reason it is

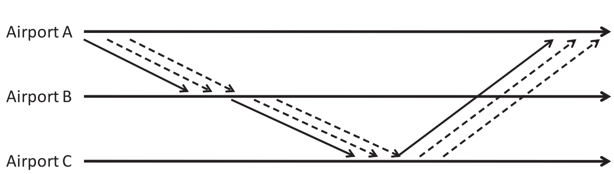


Fig. 3. Delay arcs.

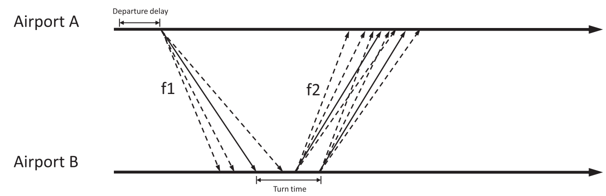


Fig. 4. Speed change arcs.

interesting to extend the network with flight plan arcs, which alter the trajectory of flights in order to avoid congested airspace. This is illustrated in Fig. 5, where the first part of the figure (a) shows two flight plans. One flight plan traversing a congested volume of airspace, which will result in an estimated departure delay, and another flight plan following a sequence of waypoints taking the flight around the congested volume of airspace. Part (b) of the figure shows the corresponding arcs for flight f1. The leftmost solid arc represents the flight plan taking the flight around the congested airspace. The dashed arcs illustrate that it may be relevant to speed up the flight for this trajectory due to the longer route. The second solid arc for flight f1 represents the direct flight plan, where the route traverses the congested airspace with the consequence that the flight will depart with a calculated departure delay (CTOT). The figure also illustrates that the en-route delay can lead to propagation of the entire delay or parts of the delay to subsequent flights.

6.1.3. Arc reduction techniques

As previously mentioned we solve the aircraft specific recovery problem, which results in a multi-commodity network flow problem where each aircraft is modelled as a commodity. This results in a large number of flight plan arcs, which are candidates for being present in the network. In order to reduce the solution time of the resulting MIP problem, an arc reduction technique is applied. This technique is inspired by the constraint programming world, as combining methods from Constraint Programming (CP) and Linear Programming (LP) can often lead to improved solution times (Vaaben, 1998).

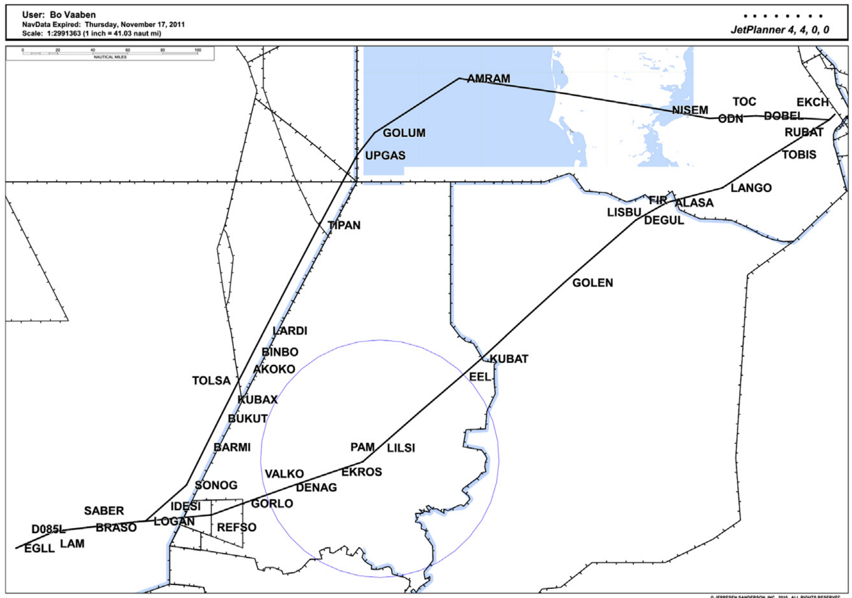
In CP a variable describing a set is referred to as a *Constrained Set Variable*. We denote the Constrained Set Variable X. The domain of X is defined by two sets. (1) A *possible set* of elements P and (2) a *required set* of elements R where $R \subseteq P$. Given this definition of X it is possible to reduce the domain of X by either expanding R or reducing P. X is determined when $R = P$.

Inspired by the constraint propagation technique used in CP we distinguish between *departure nodes* and *arrival nodes* in the time-space network and let all the departure nodes have a Constrained Set Variable of aircraft. When building the network we apply forward domain propagation of possible aircraft from departure node to arrival node. For a real life size airline network covering for instance the US this propagation technique eliminates the construction of arcs for e.g. aircraft situated on the US West coast in the morning, which do not need to be represented for flights departing in the morning on the US East coast.

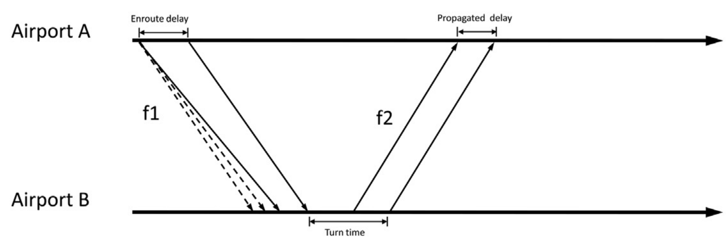
Propagation of the required sets R are done backwards in the network starting at the sink nodes and from the departure nodes of the maintenance activities where a specific aircraft is required. This can help predetermining that certain aircraft are required on certain arcs and reduce the need for arc and consequently flight plan generation. It can also pre-determine certain infeasibilities in case $R \not\subseteq P$ for a departure node.

6.2. Passenger misconnection protection

When airspace congestions occur the airline is typically faced



(a) Flight plans through and around congested airspace



(b) Arc representation of congestion related flight plans

Fig. 5. Network representation of congested airspace arcs.

with the decision whether it should accept a departure delay or file a flight plan with a trajectory, which takes a flight around the volume of congested airspace. In order to have the airline make the right trade-off between the additional fuel burn cost, which the longer route incurs, and the cost of having some passengers loose their connection due to the delayed departure we introduce passenger misconnection constraints as also used in the work by Marla et al. (2011). It is noted that equivalent misconnection constraints can be used for crew connections, which would be introduced with a higher violation penalty.

6.3. AFP slot constraints

When an AFP is imposed, airlines will receive a number of slot times where they are allowed to pass through an FCA. The airlines decide themselves, which flights will make use of these slot times. This gives the US airlines increased control over their flights in a congestion situation compared to their European counterparts, but does also introduce some additional complexity as they need to prioritize, which flights should use which time slots through the FCA. To help with this prioritization we propose the AFP Slot constraints as illustrated in Fig. 6.

Consider an example with possible trajectories for two flights to Newark airport (EWR) departing from Chicago (ORD) and Detroit (DTW) respectively. Before the departure of these flights ATCSCC has issued an AFP. When the AFP is issued, the airline in question is granted a number of time slots for passing through the FCA. In this example we assume that two such time slots are granted. So the

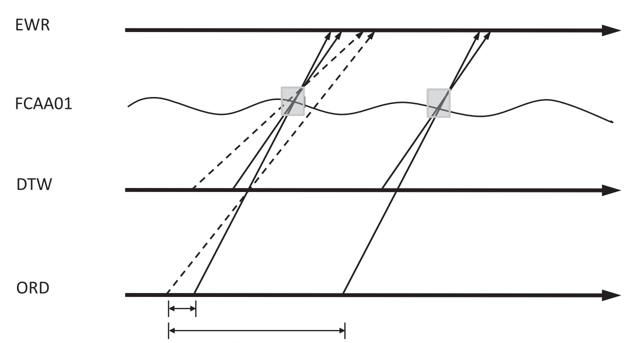


Fig. 6. Illustration of AFP constraint.

trajectories through the FCA need to respect the time slots granted by ATCSCC. Alternative trajectories, which are taking flights around the FCA are, however, not restricted by the time slots for passing through the FCA.

Fig. 6 shows the corresponding flight plan representation as arcs in the time-space network. The FCA area is here represented as a wave line to indicate that the FCA is not one single point in space but rather a borderline or an area. The two time slots granted for passing through the FCA are marked as grey shades on the FCA. The solid trajectories from the Fig. 6 are represented as solid arcs. It is noted that the same direct trajectory taking a flight through the first time slot can also be used in a later flight plan taking the flight through the second time slot. The dashed arcs on the graph indicate flight plans using trajectories taking the flight around the FCA. The dashed arcs are consequently not restricted by the FCA.

With the example provided above it is noted that there are 3 possible decisions for each flight, which is affected by an AFP on its most direct trajectory:

- *Direct trajectory*. Choose a direct trajectory through the FCA at one of time slots provided. The usage of these trajectories will typically also result in a departure delay. Combined with the possibility of changing speed it can result in some additional speed change arcs through the time slots. These speed change arcs are not shown on the figure.
- *Alternate trajectory*. Choose a trajectory around the FCA. This is represented by a single dashed arc for each affected flight. Again, combined with the speed change possibility it can result in some additional speed change arcs, which are not depicted on the figure.
- *Flight cancellation*. Choose to cancel a flight. This would typically also result in another flight cancellation in order to cancel a complete round trip from the hub to a destination and back again.

7. Mathematical model

Let $G = (\mathcal{N}, \mathcal{A})$ be a graph representing the network described in section 6. Let $\mathcal{N}$ be the set of nodes in the network, where these are divided into departure nodes $N_d \in \mathcal{N}$ and arrival nodes $N_a \in \mathcal{N}$. Let A be the set of available aircraft and F the set of flights to be carried out in the recovery period T.

Arcs in the network are either flight plan arcs $C \in \mathcal{A}$ or ground arcs $G \in \mathcal{A}$. Every flight f has a set of possible flight plans denoted $C_f \in C$. Each flight plan $k \in C_f$ connects a possible departure node $n \in N_d$ with an arrival node in N_a. From each arrival node a ground arc is created to the first subsequent departure node respecting the turn time between the two corresponding flights. From each departure node for a flight plan there exists additional outgoing ground arcs to the subsequent departure node on that airport location in order to ensure cancellation capability of the model.

To control the creation of feasible aircraft paths through the network we define N_n^- as the set of incoming arcs to each node $n \in \mathcal{N}$ and N_n^+ is the set of outgoing arcs from each node $n \in \mathcal{N}$. For each aircraft $a \in A$, a supply $s^n = 1$ is associated with the node n where the aircraft is known to start at the beginning of time window T, and a demand of $s^n = -1$, where it starts the next flight just outside the time window T.

Let x_f^k be a binary variable, which takes the value 1 if flight plan $k \in C_f$ of flight $f \in F$ is used in the recovery solution and 0 otherwise. Similarly the binary variable y_g takes on the value 1 if ground arc $g \in G$ is present in the solution and 0 otherwise. We let the binary variable z_f denote if flight $f \in F$ is cancelled. In that case it takes the value 1 and 0 otherwise.

Let $\mathcal{P}$ be the set of passenger itineraries operated within the recovery period, and n_p represent the number of passengers on itinerary $p \in \mathcal{P}$. Let $IT(p)$ be the set of flight legs in itinerary p, $IT(p,l)$ the l^{th} flight leg in itinerary p. Let $MC(p,f,k)$ denote the set of onward flight plans following flight f in passenger itinerary p to which there is insufficient time to connect from flight plan k of flight f. This set consequently corresponds to the set of flight plans, which in combination with the selection of flight plan k will cause a misconnection for the passengers on itinerary p. Let λ_p be a binary variable, which takes the value 1 if the passengers on itinerary p are disrupted and 0 otherwise.

To control the usage of flight plans traversing an AFP and consequently consuming the slot time resource in the AFP we define the binary constant d_{fb}^k, which takes the value 1 if flight plan k of flight f makes use of time slot b and 0 otherwise. In case the airline network is affected by various AFPs we let the enumeration sequence of AFP2 continue from the end of the enumeration sequence of AFP1 etc.

The problem can now be formulated as follows:

Minimize:

$$\sum_{f \in F} \sum_{k \in C_f} c_f^k x_c^k + \sum_{f \in F} c_f z_f + \sum_{p \in \mathcal{P}} \tilde{c}_p n_p \lambda_p \tag{7.1}$$

Subject to:

$$\sum_{k \in C_f} x_f^k + z_f = 1 \qquad \forall f \in F \tag{7.2}$$

$$\sum_{g \in N_n^-} y_g + \sum_{(f,k) \in N_n^-} x_f^k + s^n = \sum_{g \in N_n^+} y_g + \sum_{(f,k) \in N_n^+} x_f^k \qquad \forall n \in \mathcal{N}, \forall a \in A \tag{7.3}$$

$$x_{IT(p,l)}^k + \sum_{m \in MC(p,IT(p,l),k)} x_{IT(p,l+1)}^m - \lambda_p \le 1 \qquad \forall k \in C_{IT(p,l)},$$

$$\forall l \in 1, \ldots, |IT(p)| - 1, \ \forall p \in \mathcal{P} \tag{7.4}$$

$$\lambda_p \ge z_f \qquad \forall f \in IT(p), \forall p \in \mathcal{P} \tag{7.5}$$

$$\sum_{f \in F} \sum_{k \in C_f} d_{fb}^k x_f^k \le 1 \qquad \forall b \in B \tag{7.6}$$

$$x_f^k \in 0,1 \qquad \forall k \in C_f, \forall f \in F \tag{7.7}$$

$$z_f \in 0,1 \qquad \forall f \in F \tag{7.8}$$

$$\lambda_p \in 0,1 \qquad \forall p \in \mathcal{P} \tag{7.9}$$

$$y_g \ge 0 \qquad \forall g \in G \tag{7.10}$$

Here constraints (7.2) ensure that every flight is either carried out and thus assigned a flight plan or cancelled.

Constraints (7.3) are referred to as either *flow conservation constraints* or *aircraft balance constraints*. It requires that if an aircraft flows into a node, it must also leave it again except for the source and sink nodes in the network where we in the source node have a supply of the aircraft, $s^n = 1$, while the sink node has a demand of an aircraft $s^n = -1$. For all other nodes we have $s^n = 0$. The constraints thus ensures that for every aircraft a path is found from source to sink in the network.

Constraints (7.4) enforce λ_p to be 1 for every combination of flight plan arcs, which will result in one or more passenger itineraries misconnecting. λ is penalized in the objective function proportionally to the number of passengers on this itinerary, who will lose their connection when this combination of arcs are in basis.

Constraints (7.5) ensure that if a flight is cancelled then passengers onboard that flight will also be counted and penalized as misconnecting.

Constraints (7.6) ensure for every AFP time slot that only a single flight plan is allowed to traverse the corresponding FCA in the time slot.

Constraints (7.7)–(7.10) are all integrality constraints for respectively: Flight plan selection, flight cancellations, passenger misconnections and the usage of ground arcs.

To respect maintenance activities, which are aircraft specific, we model these as special “flights” where the possible set of aircraft, which can carry out this activity, is only one single aircraft.

Regarding the objective function (7.1) this is divided into 3 parts:

- *Flight plan cost*. The cost parameter c_f^k is a sum of the following cost elements: Incremental fuel cost, flight delay cost and aircraft swap cost.
- *Cancellation cost*. The cost parameter c_f specifies the cost penalty for cancelling a flight or a maintenance activity, which is modelled as a special kind of “flight”. The cost for cancelling a flight is also penalized as the passengers on that flight will be counted as misconnecting. The flight cancellation cost is consequently set rather low. The cancellation cost for maintenance activities is set very high to make the cancellation of a maintenance activity correspond to an infeasible solution. We refer to this practice as a *soft constraint*.
- *Passenger misconnection cost*. The parameter $\tilde{c}_p$ is an approximate cost of re-accommodation for each disrupted itinerary $p \in P$, because we assume that if a passenger itinerary p is disrupted, the passengers are re-accommodated on the next available itinerary to the destination in the next flight bank.

As we do not include a complete passenger re-accommodation in this model, but only an approximation of the passenger impact, we measure the full impact of the flight plan selection by subsequently running the resulting solutions from this model through a commercially available re-accommodation tool called the Jeppesen Passenger Re-accommodation Solver (Vaaben and Alves, 2009), which takes the full passenger itineraries and aircraft capacities into account and calculates the passenger re-accommodation cost.

8. Experimental framework

In this section we describe the data and experimental framework used to evaluate the proposed solution approach. The airline data, which is used for the experiments, have generously been made available to us by a medium sized European carrier. The carrier operates a hub-and-spoke network with approximately 250 daily flights serving 60 cities on multiple continents. The airline is consequently severely impacted by airspace congestions in the European region. In our experiments we focus on fleets covering short haul flying within Europe, which are the flights mainly exposed to airspace congestions. The data received from the airline contains its historic flight schedule covering 3 months and including both planned and actual times. Along with this we have also received matching passenger reservations with complete itineraries for a period of 2 weeks. Data concerning airspace congestions are collected from the Eurocontrol Network Operations Portal (NOP).

The framework of software modules and data used for the experiments is illustrated in Fig. 7. The Flight Plan Manager reads the planned schedule and disrupted state from a database along with passenger loads for each flight. Based on this information the Flight Plan Manager calls the flight planning engine, which is a commercial software tool from Jeppesen and ensures that flight plans are continuously updated for all flights in the time period observed. For flight plans, which are affected by congested airspace, an alternative flight plan avoiding the congestion is also calculated. All flight plans are stored and continuously updated in the flight plan cache to enable fast retrieval, when a disruption needs to be solved.

The Integrated Flight Planning and Disruption Management module, contains the implementation of the optimization model formulated in section 6. When a disruption needs to be solved, this module retrieves schedule, disruption state, fleet information and passenger itineraries along with relevant and updated flight plans from the Flight Plan Cache. The optimization run leads to a simultaneous decision on: Delays, Swaps, Cancellations, Trajectory and fuel burn.

The recovery solution from this process is subsequently evaluated by a commercially available *passenger re-accommodation solver*, which calculates the actual passenger re-accommodation cost. This final evaluation step is carried out due to the fact the proposed recovery model in section 6 does not take the full passenger itineraries into account, but only passenger connections.

8.1. Parameter assumptions

For input parameters in the model we assume the following values. The airline's own cost for a delayed passenger is assumed to be \$1.09 per minute. This input is based on the airline's own internal calculations of this cost for year 2008 and includes passenger re-accommodation and loss of goodwill.

Fuel is assumed to be \$0.478 per lb, which is equivalent to \$3.65 per gallon. This is based on the airlines own reported cost of approximately 750 € per metric ton in February 2010. This price has been converted to 2008 numbers using a conversion rate of 1 € = \$1.27 in November 2008 according to the European central bank and according to IATA charts indicating that the fuel cost in February 2010 was 0.903 times the cost in November 2008.

The normal Cost Index for the airline is assumed to be 30. All flight plans are calculated at this speed since no speed changes are considered in this experiment. The cost per disrupted passenger c_p is based on the assumption that misconnecting passengers will be re-accommodated in the next bank of flights, which gives an average delay of 7 h for the flight schedule of this airline. Using the cost per passenger delay minute of \$1.09, this gives a misconnection cost of \$457.8. A swap cost of \$500 is assumed for swaps within the same fleet. Swaps between fleets are not used in these experiments. The cost is based on parameter calibration with airline Ops Controllers.

A flight cancellation cost c_f of \$20,000 is assumed and is also based on parameter calibration with Ops Controllers. For the purpose of flight plan calculations an average passenger weight, including luggage, of 100 kg has been used.

9. Computational experiments

Our models are implemented in C++ with a direct interface to the MIP solver Xpress version 19.00. The experiments are conducted on a server running Linux and equipped with a 64 bit Intel Xeon E5440 processor with 4 cores and 16 GB of RAM.

The cases used to evaluate the model are based on 3 months of historical disruption data combined with a subset of airspace congestions. It has unfortunately not been possible to replicate all airspace congestion to the flight planning engine for the purpose of the evaluation. For this reason the results should be seen as a conservative lower bound for the savings which can be achieved by applying the approach.

The evaluation is based on 28 scenarios distributed over the seven days of the week in order to capture the varying flight schedule and passenger flows during the course of a week. The seven days have, however not been selected from the same week, but have been evenly distributed over the three months in order to even out some of the traffic variations from month to month.

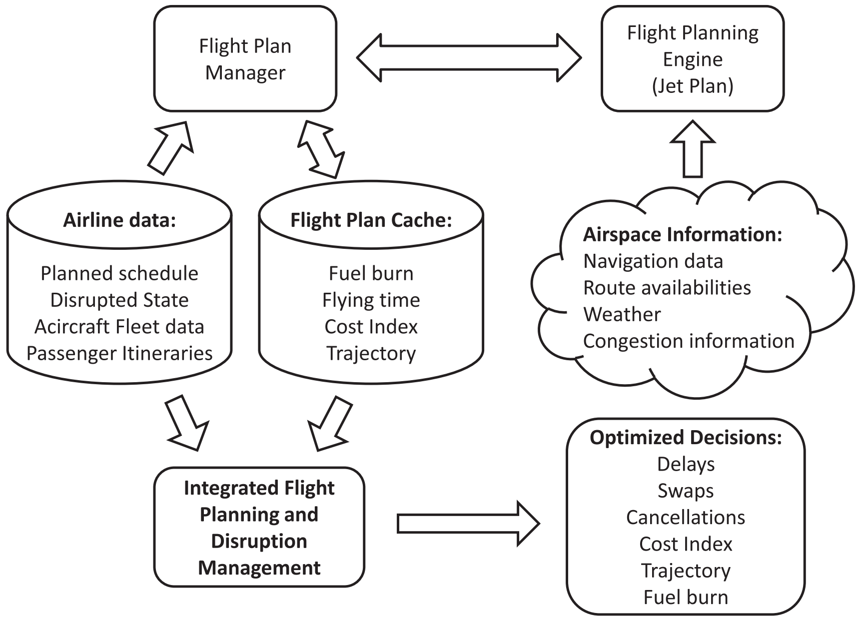


Fig. 7. Framework for integrated flight planning and disruption management.

9.1. Example: heavy fog over the Netherlands

In this example heavy fog over the Netherlands is causing increased separation requirements, which reduces the ATC capacity in the area. This is resulting in a volume of airspace experiencing congestions during a course of 5 h. The area has an extension of approximately 15,400 square nautical miles (nm^2), where the congestion in average have caused a 15 min estimated departure delay for flights traversing the congested area. The recovery time window has been set to 48 h, which leaves 93 flights belonging to the Airbus 320 fleet, consisting of 12 aircraft, in the window. Of the 93 flights, 14 pass through the congested airspace during the presence of the congestion. Fig. 8 shows the recovery solution for the example. It is noted that flights marked with a blue dot in the lower left corner have been assigned a flight plan, which takes the flight around the congested area, which would otherwise have caused a departure delay of approximately 15 min. These are the 5 flights: 875, 876, 892, 891 and 811. These flights have been assigned a flight plan, which deviates from the lowest cost flight plan, in order to avoid the congested area.

Fig. 9 shows a trajectory view of the same solution as displayed in the Gantt view in Fig. 8. It is notable that the flights going to and from Amsterdam (AMS) airport are forced to enter the congested area and depart with a 15 min delay. For the flights to and from London it is less obvious that it is cheapest to select a trajectory through the congested area and accept the departure delay. For the flight to Brussels (BRU) a trajectory around the congestion is selected, while the flight back from Brussels should go through the congestion. It is worth noting that the hub of the airline is located in the periphery of Europe and in spite the fact that the congestion occurs far from the hub, it does have a significant impact on the network operation of the airline.

Table 1 shows the variations in results over the different days of the week, where the week day selection has been distributed evenly over the course of the 3 months of schedule data, which has been available. It can be observed that the same congestion results in a large difference in the number of congestion-affected flights, which are diverted around the congestion. This is due to three factors:

- Day-to-day variations in the schedule
- Differences in passenger flows
- Differences in historical disruptions

The day to day variations in the schedule is estimated to have less impact on the variations as there is a rather high re-occurrence

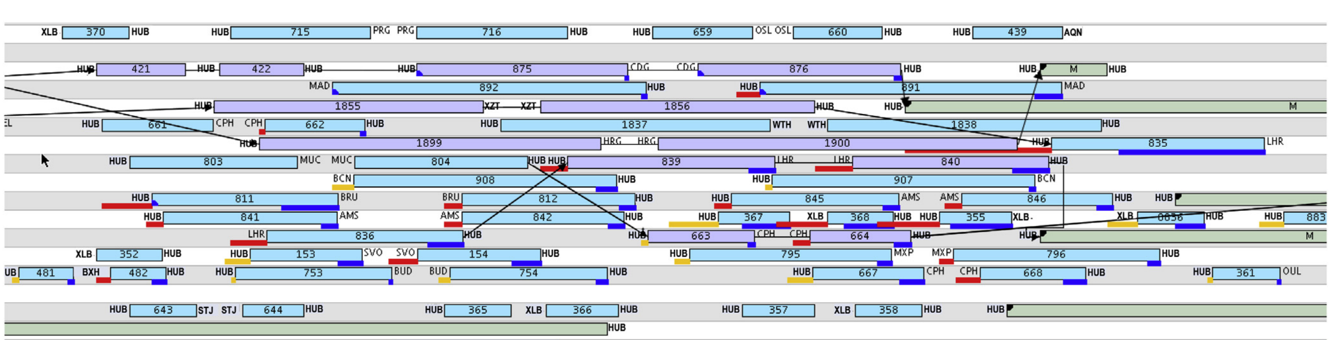


Fig. 8. Gantt display showing recovery solution for the example.

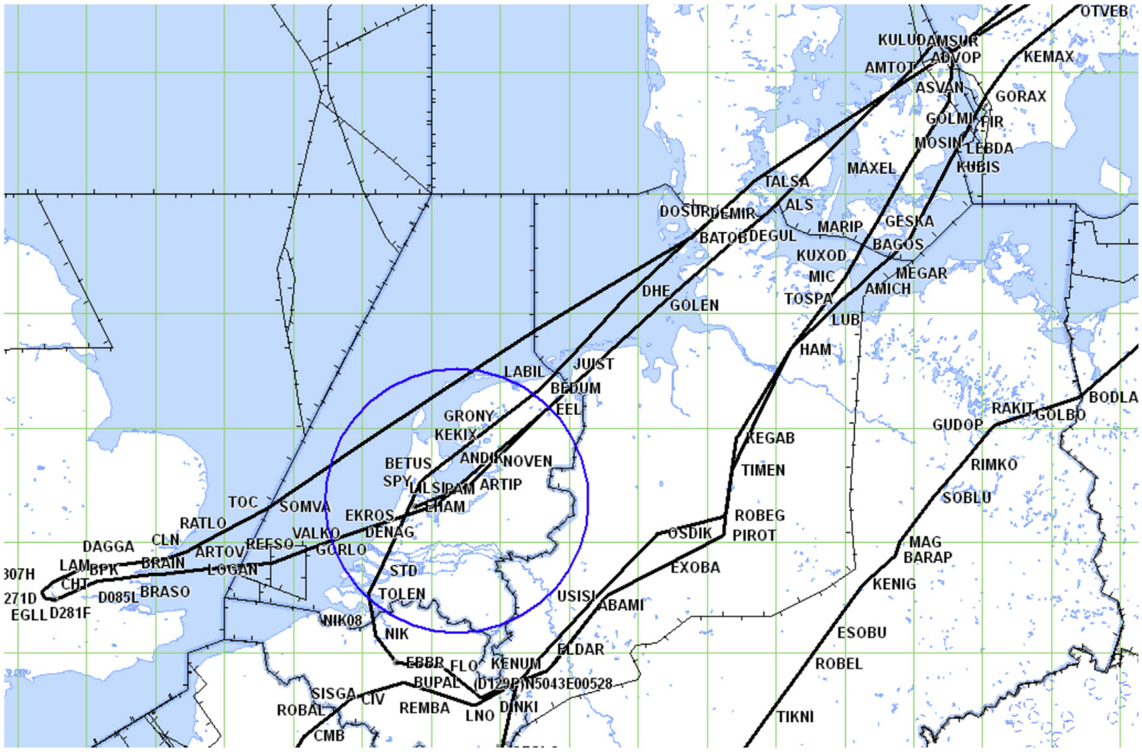


Fig. 9. Trajectory view showing recovery solution for case 1.

Table 1
Result of approach for same congestion on different days.

Weekday	Flights in recovery window	Congestion affected flights (%)	Congestion affected flights diverting around congestion (%)	Additional fuel pr. diverted flight (%)	Re-accommodation cost saving (%)	Total cost saving (%)	Total cost saving (USD)
Monday	93	15.1	35.7	3.87	23.61	22.90	30,135
Tuesday	100	17.0	64.7	3.89	2.94	1.89	3131
Wednesday	102	14.7	40.0	3.96	12.35	10.41	5251
Thursday	78	16.7	46.2	4.15	1.24	0.56	853
Friday	79	17.7	42.9	4.33	3.12	1.79	1297
Saturday	64	14.1	55.6	3.48	20.72	19.11	8493
Sunday	82	19.5	56.3	3.32	23.61	22.54	29,662
Average	85.4	16.4	48.7	3.86	12.51	11.31	11,260

of the same flights each day. The majority of the variation stems from the differences in passenger flows from day to day and the differences in level and type of disruption each day. The differences in passenger flows affect the extent to which passenger connections influence the solution due to constraints (7.9). Similarly, an input disruption with various larger flight delays will already have used up the aircraft turn time and passenger Minimum Connection Time (MCT) buffers in the schedule. These disruptions are consequently more likely to render solutions, where flights are diverted around congested airspace as an additional departure delay will immediately lead to delay propagation through constraints (7.3) and (7.9).

The interesting observation from Table 1 is the large variation in the percentage of flights, for which it is cost beneficial to select a trajectory around a congested area. There is furthermore an even larger variation in how cost beneficial it is to divert these flights around the congestion. This emphasizes the fact that it is difficult for a dispatcher, who does not have the entire network overview and does not have individual passenger connection information, to decide when he should select a trajectory around a congested area of airspace and when he should rather accept a departure delay and take the most fuel efficient trajectory.

On the day of operation fast decisions are one of the key factors to avoiding that a disruption propagates to other parts of the network. Solution times for a disruption management system should consequently be kept low and ideally below 2 min in running time as various reruns may need to be carried out by the ops controller (Marla et al., 2011). The solutions presented for a single Airbus 320 fleet of a medium size European carrier do in average involve 89 flights in the recovery time window and solves to optimality in less than a second. The average problem sizes contain 38,000 constraints and 59,000 variables. The fast runtimes are mainly due to the fact that additional flight plan arcs only need to the generated for the few of the 89 trajectories, which are actually affected by the congested airspace.

Table 1 shows an average saving of $11,260 per day when this airspace congestion occurs. In order to obtain a lower bound estimate of a yearly saving by applying the approach, the statistics department of Eurocontrol were kind to provide us with a list of their most frequently congested areas in year 2008, combined with

Table 2
Selection of en-route congested areas of Northern European airspace with savings estimate for flexible trajectories.

Airspace area	Average regulation duration (minutes)	Number of days with en-route regulation above 15 min	Average daily saving with flexible trajectories (USD)	Yearly saving with flexible trajectories (kUSD)
North West of Poland	235	333	2309	769
Holland	301	318	11,260	3581
South Baltic Sea	106	81	6254	507
East of Denmark	247	79	3109	246
Lower bound saving	–	–	–	5103

the number of days where each of these locations were imposed an en-route regulation with more than 15 min of departure delays. The approach has been evaluated on a selection of some of the most frequently congested areas in Northern Europe, which provides a lower bound for the saving, which can be achieved by applying the approach of using flexible flight plans in the recovery decisions, when congested areas of airspace are involved. Each of the four evaluated areas have been evaluated over seven days as for the case with the Netherlands in the previous Table 1. The lower bound estimate is consequently based on 28 scenarios, which all solve to optimality in less than 1 s.

As mentioned previously, the airline, which has contributed data to this research, does not have its hub in a central part of Europe and is somewhat retracted from the main congested areas of the continent. Despite that fact, it is notable that airspace congestions over the Netherlands are one of the most contributing areas to the savings potential of the approach. Based on that observation it is assumed that airlines, which are more centrally located in Europe, would be able to benefit considerably more from the approach.

The results in Table 2 show an estimated lower bound of yearly savings of 5.1 million USD for the airline's Airbus 320 fleet consisting of 12 aircraft.

10. Conclusions and future work

The main conclusion from this work is that it is possible to integrate dispatch decisions regarding flight trajectories in the recovery decisions. An optimization based recovery system, which integrates traditional recovery with flexible flight trajectories, can in an environment, which is severely impacted by airspace congestions, contribute with a yearly saving of several million USD. For a medium size European carrier, with a hub located outside of central Europe, a lower bound yearly savings potential of 5.1 million USD is estimated compared to traditional recovery without flexible trajectories.

For future work it would be relevant to apply the method to a larger fleet in order to evaluate the feasibility of the approach for a larger scale operation. Here it should be mentioned that the current results already show quite some room for scaling up the problem size, as the tested problems currently solve to optimality in less than a second.

For additional future work it would be relevant to apply the approach to a US-based airline, preferable one with a significant part of its operation in the North East of the US, where most US airspace congestions occur.

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