



Vortex wake models with application to yawed rotor

Branlard, Emmanuel Simon Pierre

Publication date:
2013

[Link back to DTU Orbit](#)

Citation (APA):

Branlard, E. S. P. (2013). Vortex wake models with application to yawed rotor [Sound/Visual production (digital)]. North American Wind Energy Academy Symposium, Boulder, United States, 06/08/2013

DTU Library Technical Information Center of Denmark

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

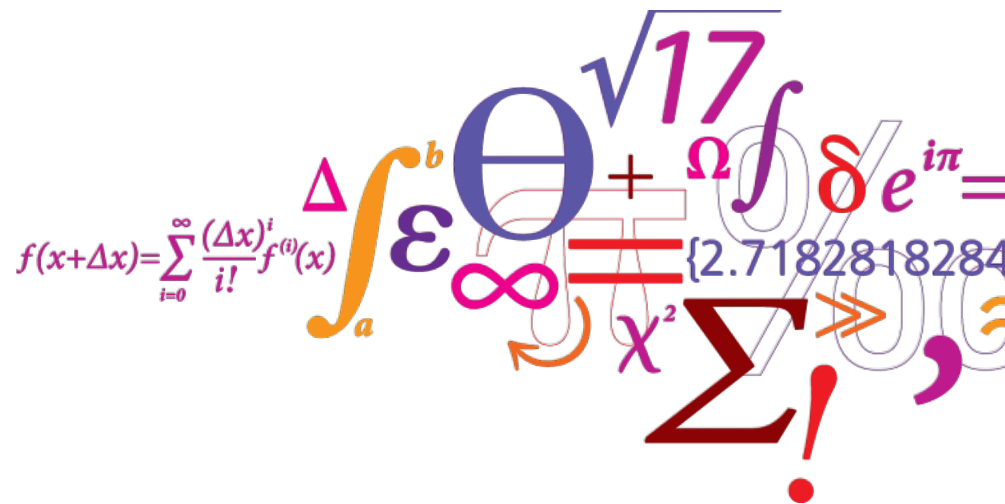
- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Vortex wake models with application to yawed rotor

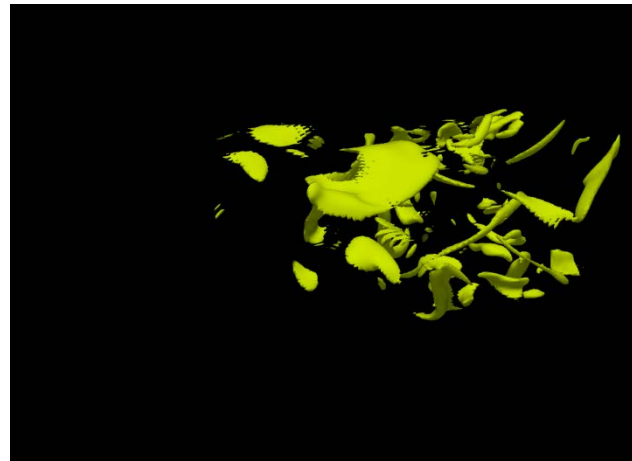
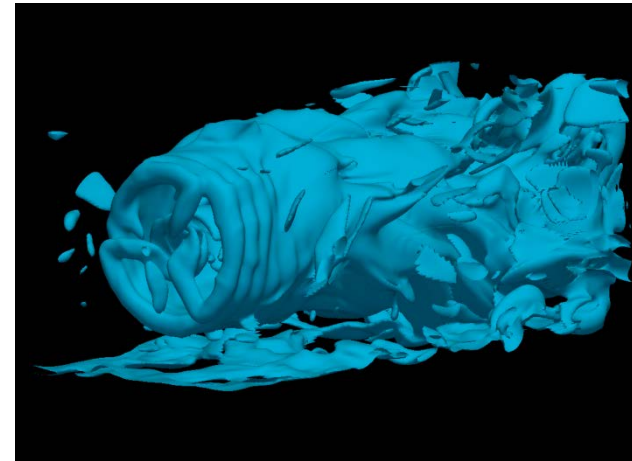
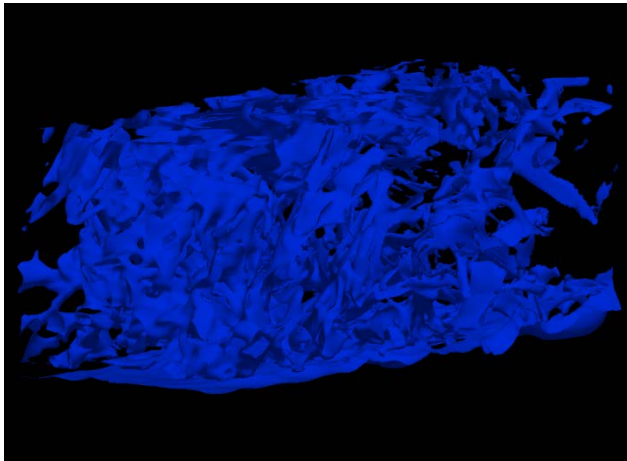
Emmanuel Branlard (PhD student, supervisor: Mac Gaunaa)

NAWEA, Boulder, August 2013

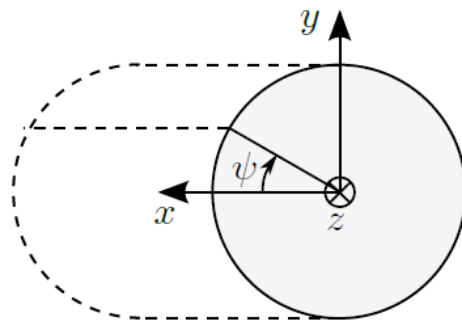
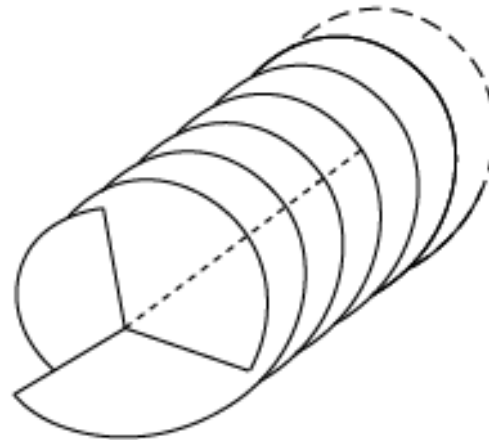


Introduction – Vorticity

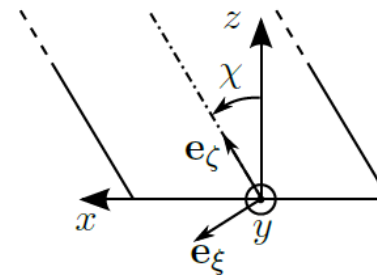
Iso-vorticity contours



1. Presentation of the model



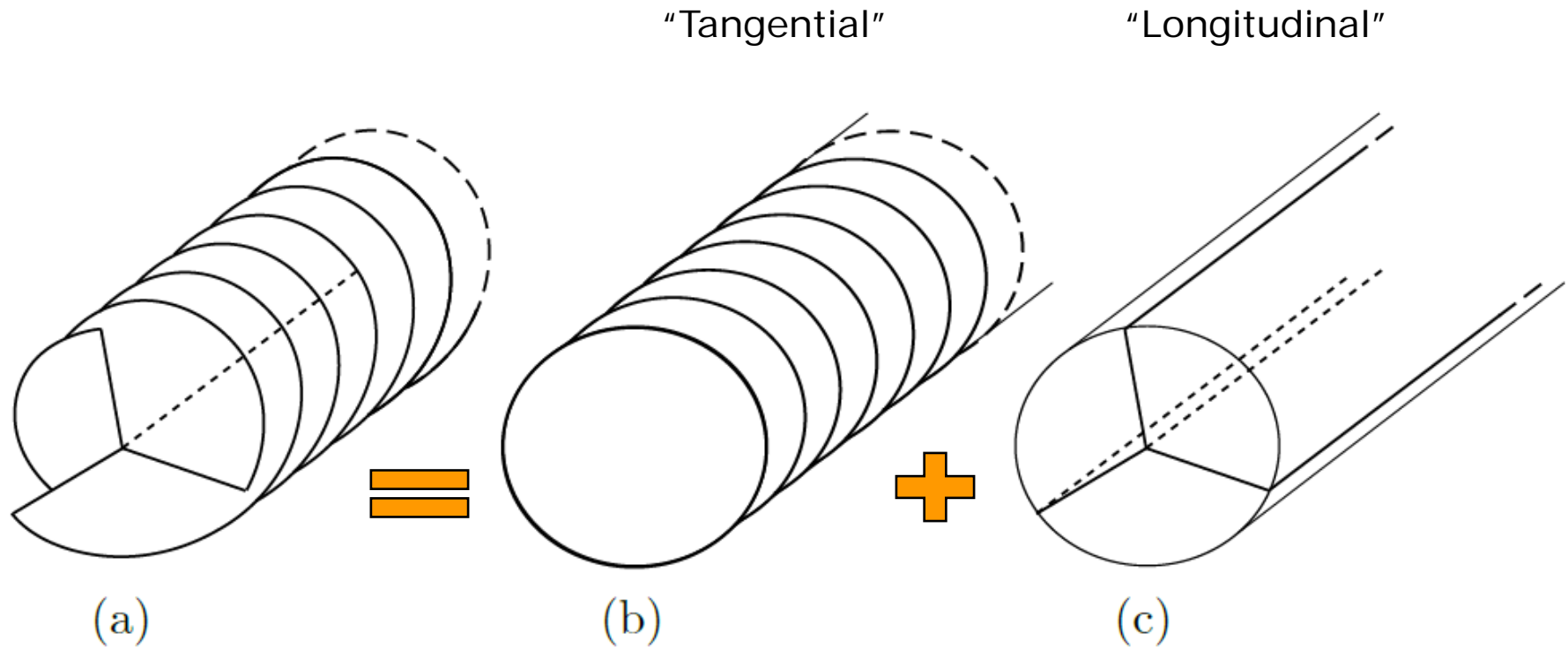
Rotor plane/Front view



Wake/Top view

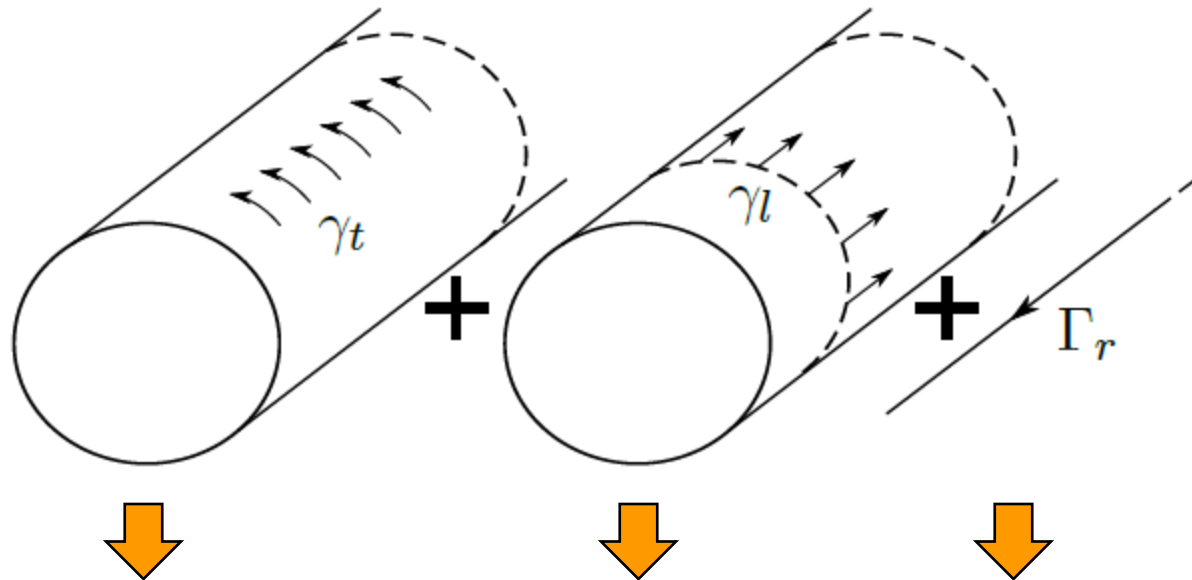
1. Presentation of the model

Decomposition of (skewed) helical wake



1. Presentation of the model

Viewed with infinite number of blades




↓
 Coleman(1945)
 Castles (1956)
 Current study

+ ↓
 ECN models(1995)
 Current Study

+ ↓
 ECN models (1995)
 WEH
 Current Study

2. Methodology for further investigations

Biot-savart law – Integration over z

$$I = C \int_0^{2\pi} \int_0^{\infty} \frac{a' + b'z}{(a + bz + cz^2)^{3/2}} dz d\theta = C \int_0^{2\pi} I_z d\theta \quad (\text{Pierce, 1899})$$


$$I_z = \frac{1}{\sqrt{c}} + \frac{2(2ab' - a'b)}{\sqrt{a}(4ac - b^2)} + \frac{4c(a' - a) + b(b - 2b')}{\sqrt{c}(4ac - b^2)} \quad (\text{Suitable for analytical expressions - Work of Coleman})$$

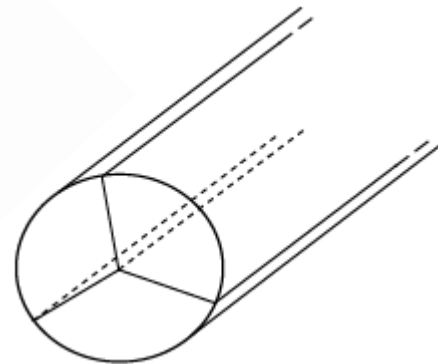
$$I_z = \frac{2(a'\sqrt{c} + b'\sqrt{a})}{\sqrt{ac}(2\sqrt{ac} + b)} \quad (\text{"Suitable" for numerical integration - Work of Castles})$$

2. Methodology for further investigations

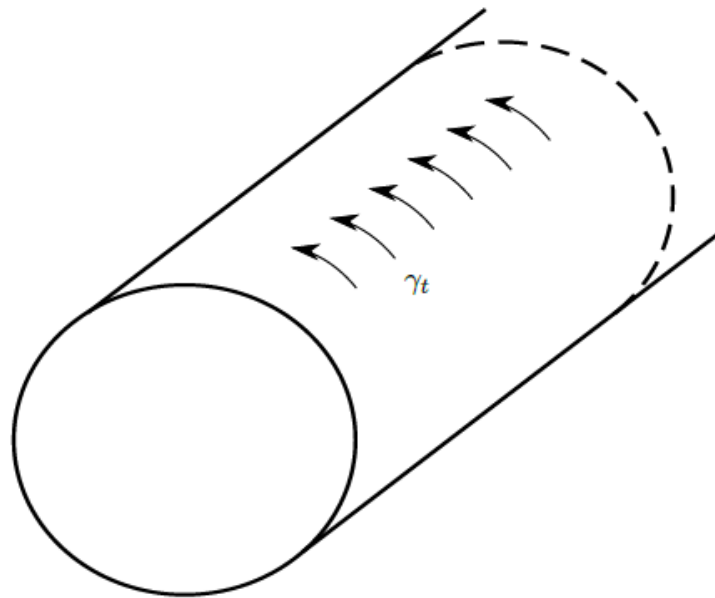
Longitudinal vorticity – semi-infinite lines

Biot-Savart law for semi-infinite line:

$$\mathbf{u}(\mathbf{x}) = \frac{\Gamma}{4\pi r_1} \frac{\mathbf{e} \times \mathbf{r}_1}{(r_1 - \mathbf{e} \cdot \mathbf{r}_1)}$$



3. Tangential vorticity

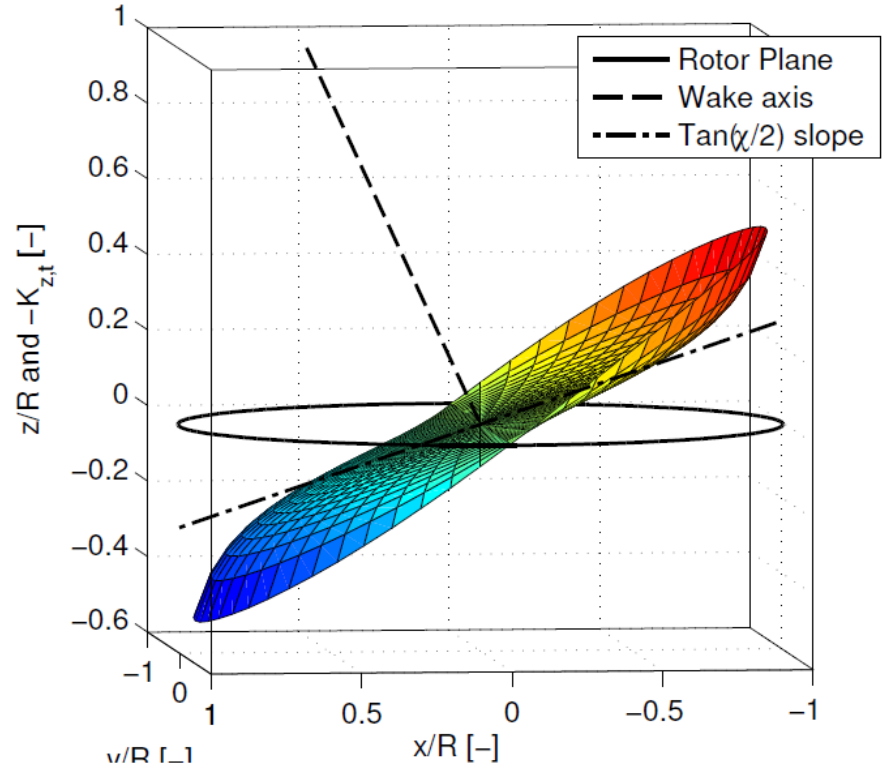


3. Tangential vorticity

Axial component

$$u_{z,t}(r, \psi, \chi) = u_{z,0} (1 + K_{z,t}(r, \chi) \sin \psi)$$

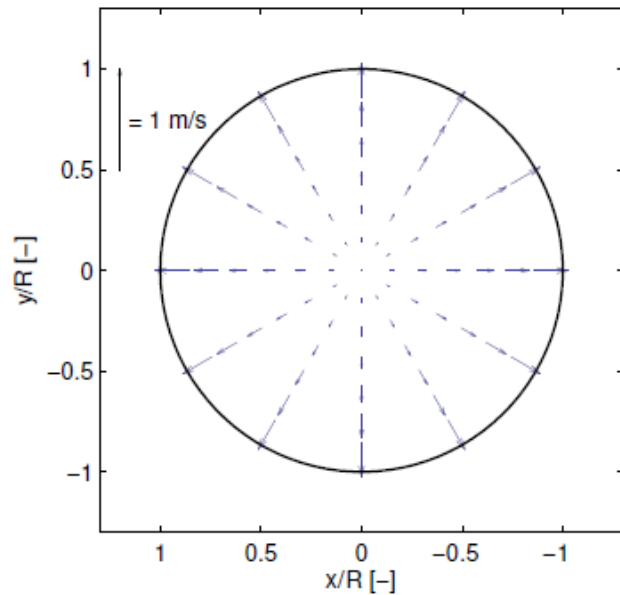
$$K_{z,t,approx} \approx \left. \frac{\partial K_{z,t}}{\partial r} \right|_{r=0} = \frac{r}{R} \tan \frac{\chi}{2}$$



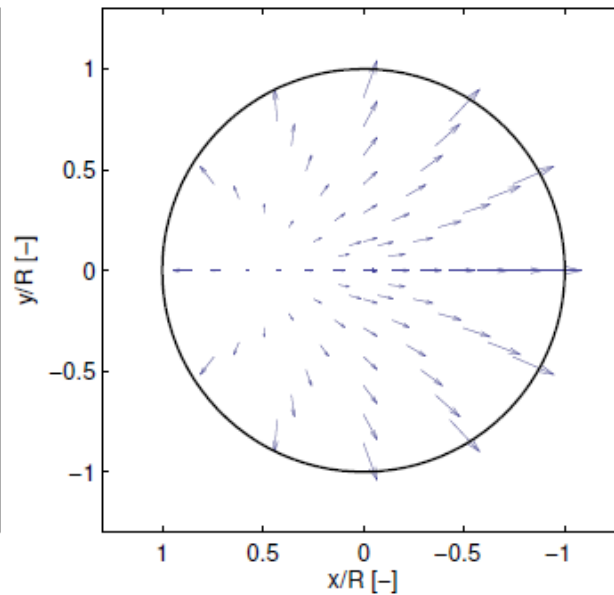
Incoming wind

3. Tangential vorticity

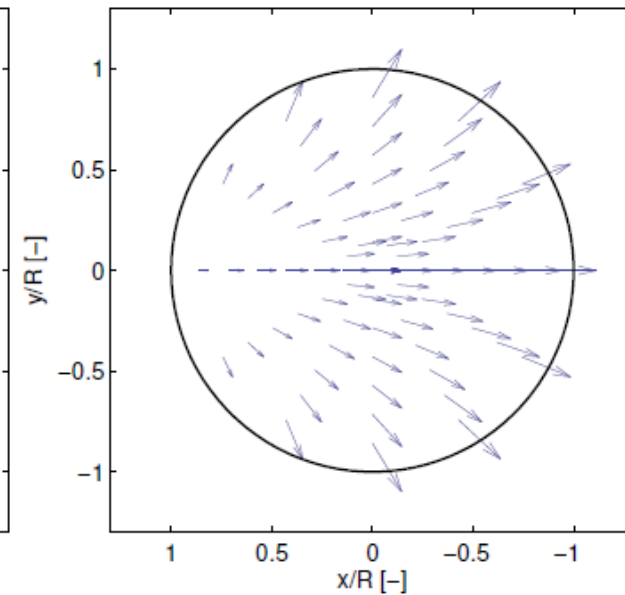
In-plane component for various skew angles



(a) $\chi = 0^\circ$



(b) $\chi = 30^\circ$



(c) $\chi = 60^\circ$

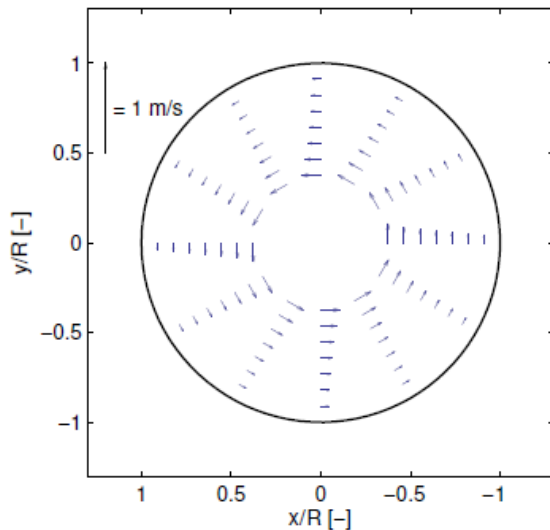
$$u_{\psi,t,approx} = -u_{z,0} \tan \frac{\chi}{2} \sin \psi$$

4. Longitudinal vorticity

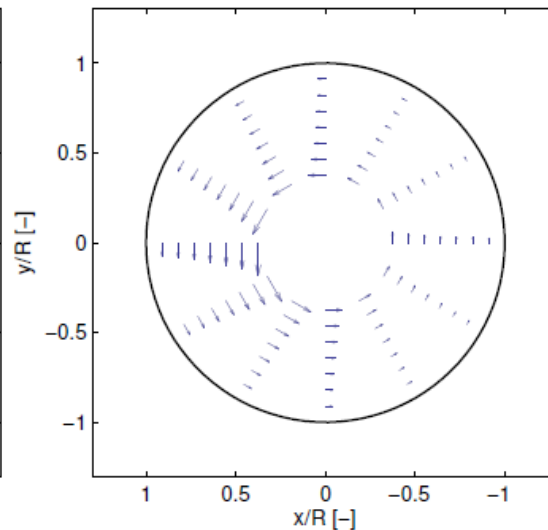
Root vortex

$$u_{z,r} = \frac{\Gamma_r}{4\pi r(1 - \cos \psi \sin \chi)} \sin \psi \sin \chi \quad (\text{See also WEH})$$

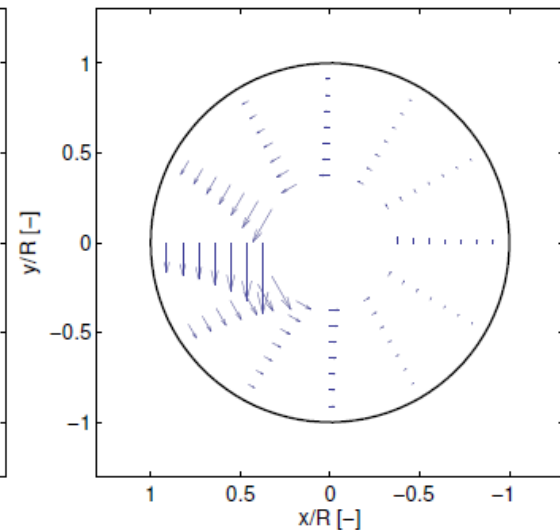
$$u_{\psi,r} = \frac{\Gamma_r}{4\pi r(1 - \cos \psi \sin \chi)} \cos \chi = u_{\psi,r}(r = 0, \chi = 0) K_{\psi,r}(\psi, \chi)$$



(a) $\chi = 0^\circ$



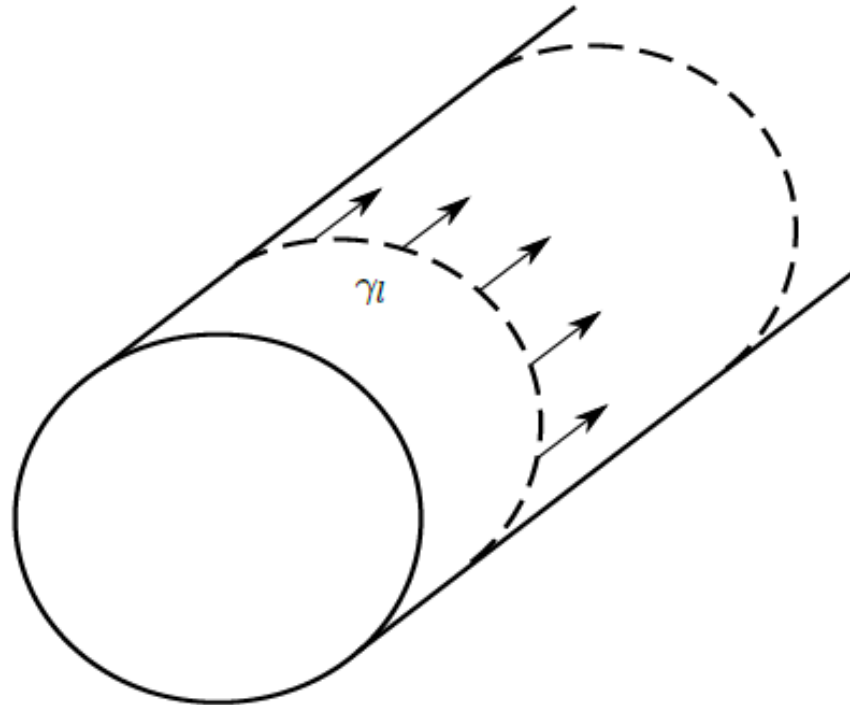
(b) $\chi = 30^\circ$



(c) $\chi = 60^\circ$

4. Longitudinal vorticity

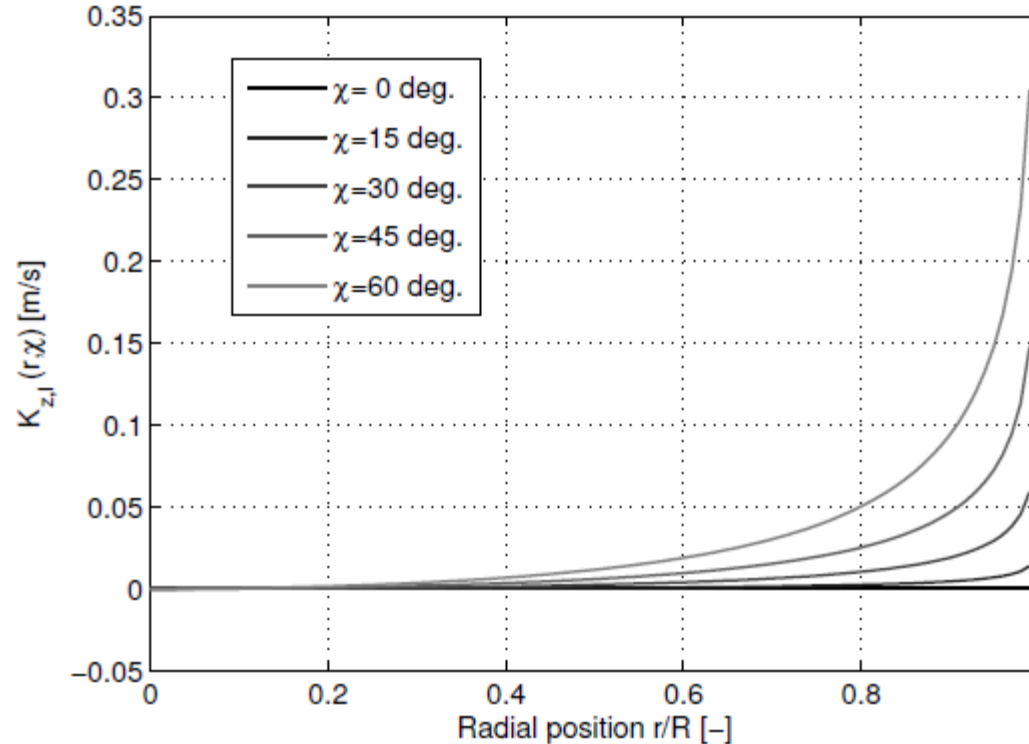
Tip-vortices



4. Longitudinal vorticity

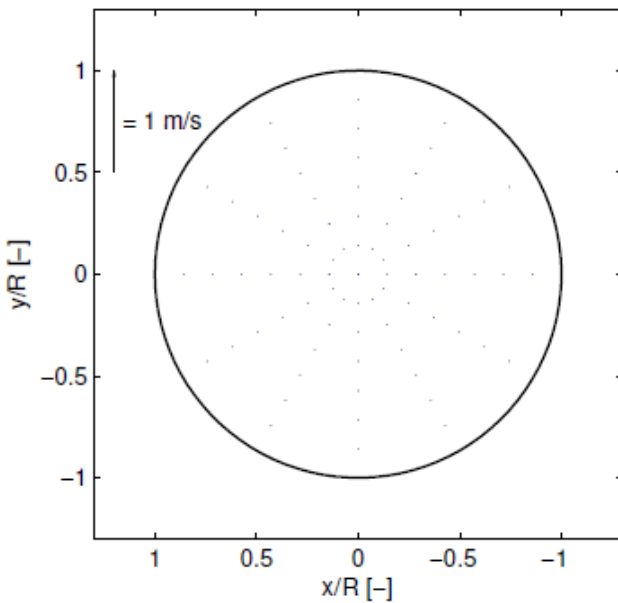
Tip-vortices – Axial component

$$u_{z,l}(r, \psi, \chi) = -\gamma_l K_{z,l}(r, \chi) \sin(2\psi)$$

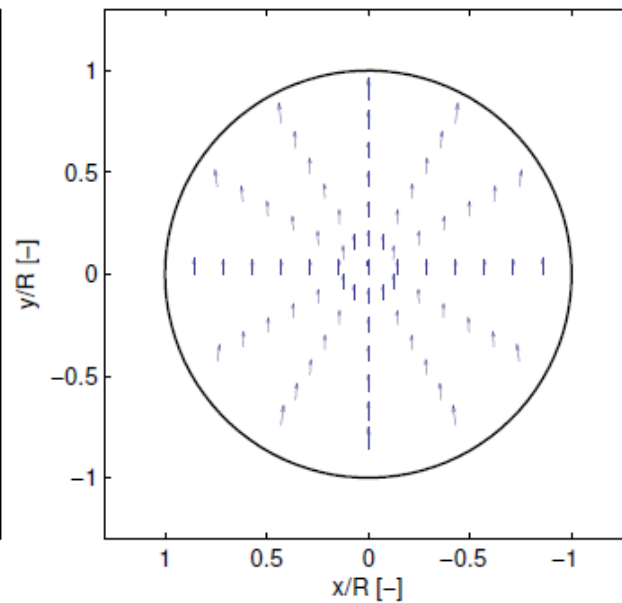


4. Longitudinal vorticity

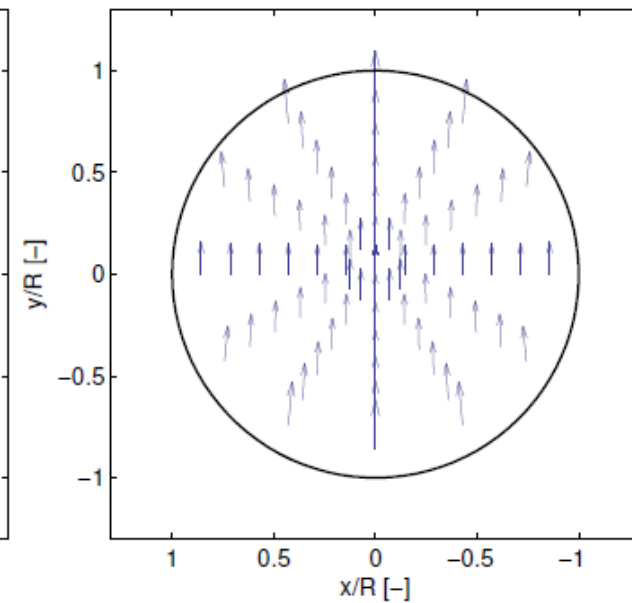
Tip-vortices – In-plane components



(a) $\chi = 0^\circ$



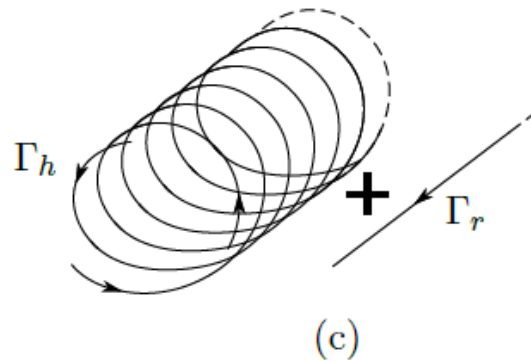
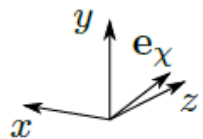
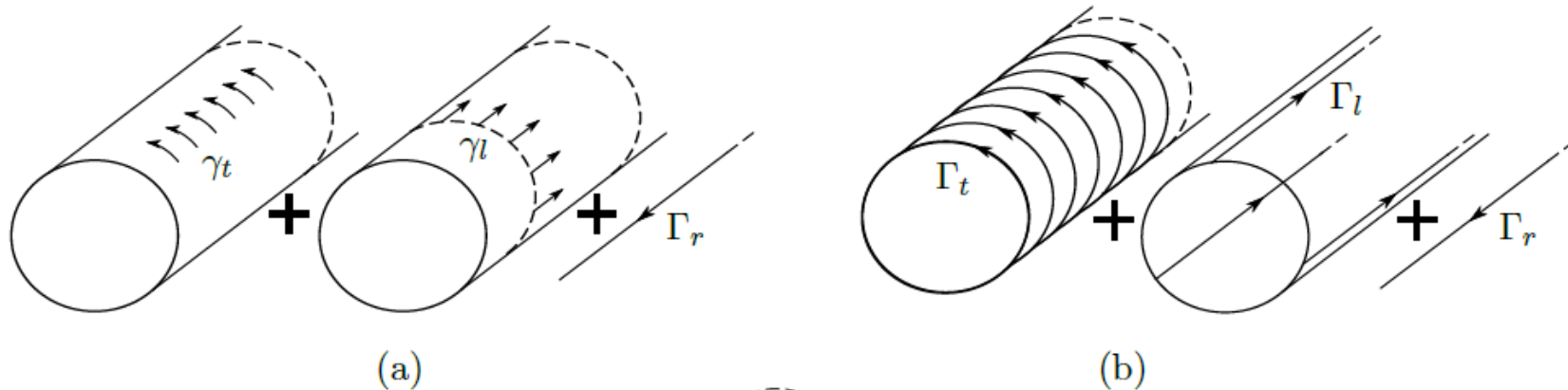
(b) $\chi = 30^\circ$



(c) $\chi = 60^\circ$

5. Putting pieces together

Relating vorticity intensity



$$\Gamma_b = \frac{\Gamma_{tot}}{n_B}$$

$$\Gamma_r = -\Gamma_{tot} \mathbf{e}_\zeta$$

$$\Gamma_h = \Gamma_b$$

$$\Gamma_l = \Gamma_b \mathbf{e}_\zeta$$

$$\Gamma_t = -\Gamma_b \mathbf{e}_\psi$$

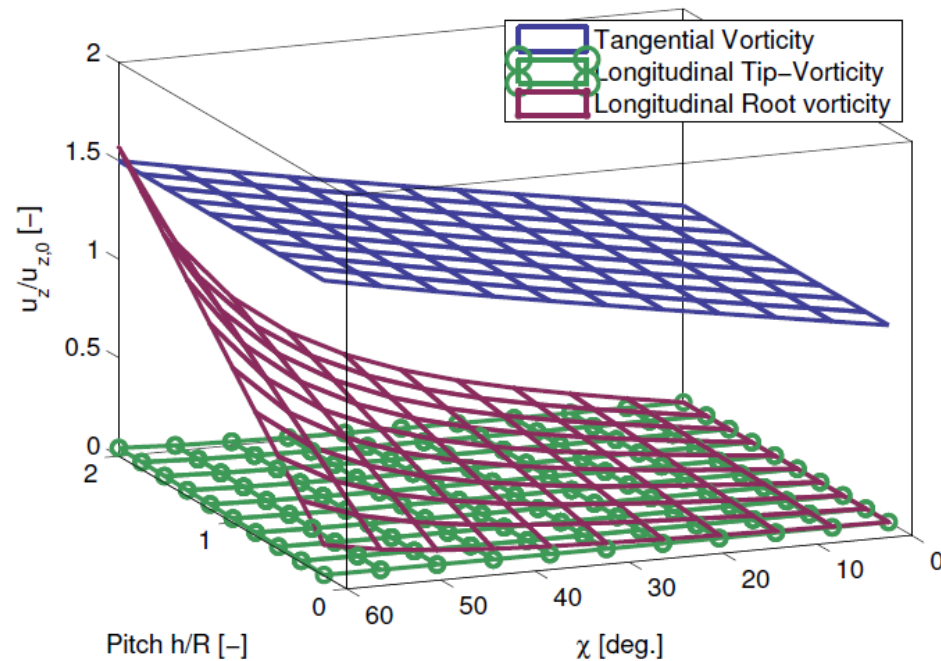
$$\gamma_t = -\frac{\Gamma_{tot}}{h / \cos \chi} \mathbf{e}_\psi$$

$$\gamma_l = \frac{\Gamma_{tot}}{2\pi R} \mathbf{e}_\zeta$$

5. Putting pieces together

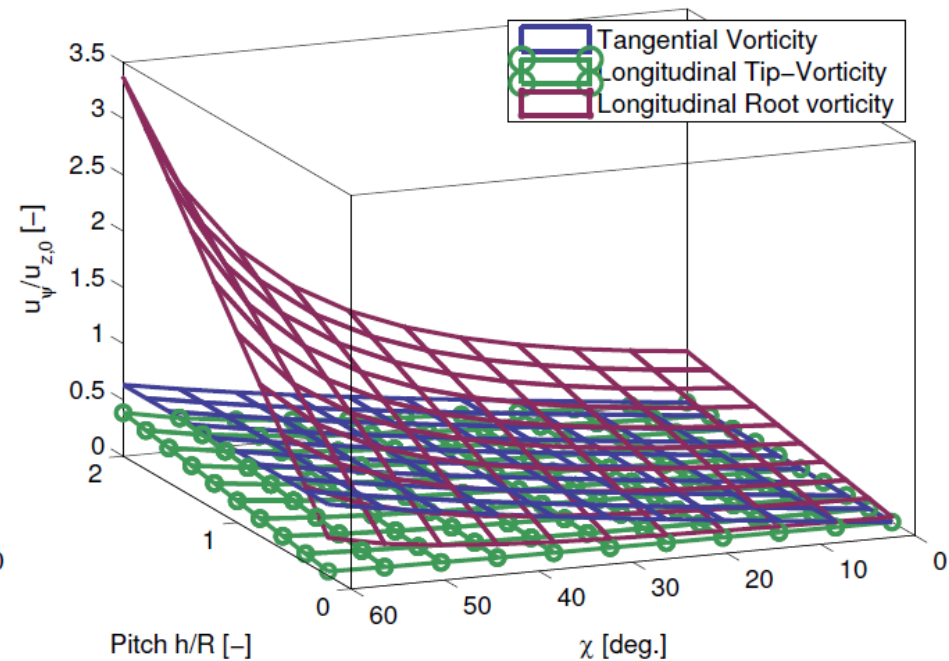
Amplitude comparison – Small pitch

Axial velocity



$$u_{z,l} / u_{z,0} = 0.5\%$$

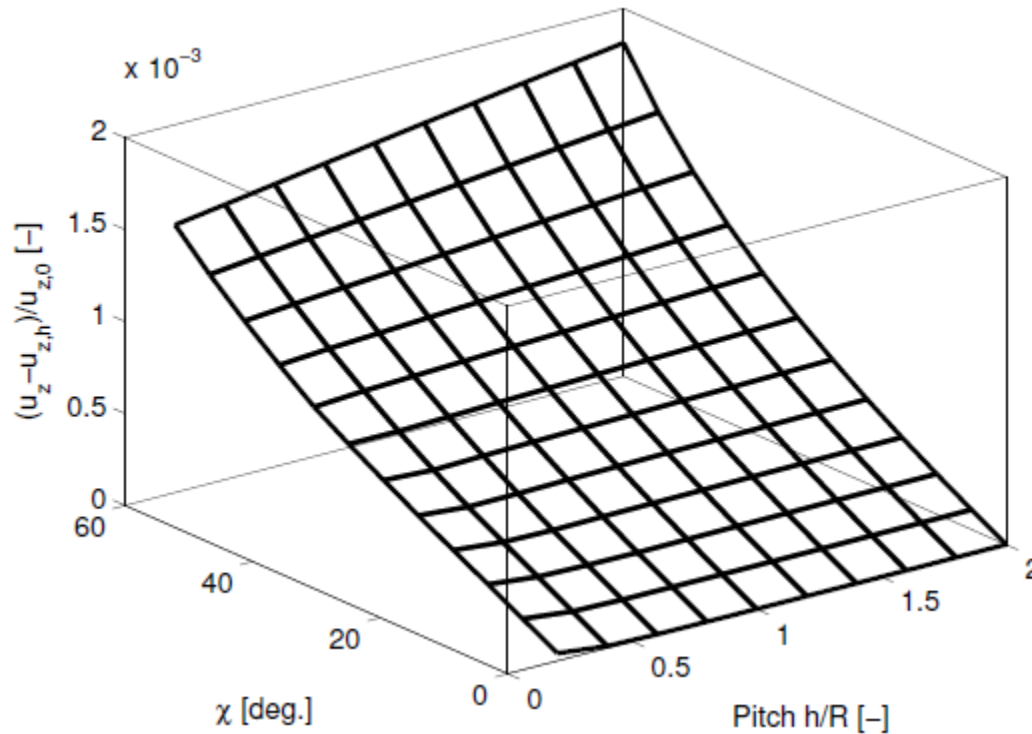
Tangential velocity



$$u_{\psi,l} / u_{z,0} = 4\%$$

5. Putting pieces together

How good is this projection ?



Conclusions

- Full velocity field from longitudinal and tangential vorticity obtained with combined analytical and numerical integration
- Simple approximations or empirical formulae can be derived for implementation in BEM codes
- Influence of longitudinal tip-vorticity is small compared to other components

Future work

- Implementation in BEM
- Comparison with free-wake vortex code and experiments
- Relaxing infinite number of blade assumption (tip-losses)
- Relaxing the constant circulation hypothesis

Thank you for your attention