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Improvement of a near wake model for trailing vorticity

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Abstract. A near wake model, originally proposed by Beddoes, is further developed. The purpose of the model is to account for the radially dependent time constants of the fast aerodynamic response and to provide a tip loss correction. It is based on lifting line theory and models the downwash due to roughly the first 90 degrees of rotation. This restriction of the model to the near wake allows for using a computationally efficient indicial function algorithm. The aim of this study is to improve the accuracy of the downwash close to the root and tip of the blade and to decrease the sensitivity of the model to temporal discretization, both regarding numerical stability and quality of the results. The modified near wake model is coupled to an aerodynamics model, which consists of a blade element momentum model with dynamic inflow for the far wake and a 2D shed vorticity model that simulates the unsteady buildup of both lift and circulation in the attached flow region. The near wake model is validated against the test case of a finite wing with constant elliptical bound circulation. An unsteady simulation of the NREL 5 MW rotor shows the functionality of the coupled model.

1. Introduction

The dynamic effects of trailed vorticity behind a wind turbine blade on the induced velocities at the blade are considered with a focus on the influence on the aeroelastic behavior. In many state of the art codes for wind turbine aeroelasticity, the unsteady aerodynamics are computed using a blade element momentum (BEM) model with several additions, such as tip loss correction and dynamic stall model, cf. Madsen et al. [1]. In a BEM model, the momentum equation is solved at different radial sections of the rotor independently, ensuring that the induced velocities are in balance with the forces at the blades. Unsteady effects are usually also modeled for each section independently, such as dynamic inflow, which takes into account that the turbine wake development delays the response to changes in wind speed or pitch, or unsteady airfoil aerodynamics, which model the faster time lags in the change of lift due to airfoil motion, turbulence and flow separation.

In reality, the flow at different radial sections is coupled through the wake, which can be modeled as trailed and shed vorticity. According to Leishman [2], the effects of the shed wake are mostly local and the overall aerodynamics along the blade are mainly depending on the trailed wake, where the most important contribution comes from the tip vortex. The influence of the trailed vortices is often computed with a BEM model combined with a tip loss model to account for the increased induction at the tip due to the finite number of blades. Due to the assumption of radial independence in the BEM formulation and because the dynamic inflow time constants are a function only of radius, the present modeling does not predict accurately the time varying trailed vorticity along the blade due to turbulence or blade vibrations.

Beddoes [3] has developed a near wake model that accounts for the unsteady trailed vorticity. It is based on a lifting line model, which is restricted to the first quarter revolution behind a single blade. This restriction makes it possible to use exponential functions to model the decreasing induction from trailed vortex filaments as the blade moves away from them. Madsen and Rasmussen [6] implemented the model for use on wind turbine aeroelasticity. and demonstrated the models basic capability to compute the aerodynamic damping as function of mode shape and not only as function of radius. The original model did not include downwind convection of the vortex filaments away from the rotor plane, but Wang and Coton [4] included the influence tilt angle of the trailed vortices in the axial induction. Andersen [5] added an optimization method for the exponential functions to reduce errors due to the approximation of the induction, compared to the exact evaluation of the Biot-Savart law.

Madsen and Rasmussen [6] suggested to couple the near wake model with a BEM model for the far wake. This BEM model would not include a tip loss correction, because that is implicitly included in the near wake model. The thrust coefficient, on which the computation of the induction using a BEM model is based on, is reduced by a coupling factor. In their work, the bound vorticity for the near wake model has been determined using Joukowski's law, which states that the steady circulation is proportional to the steady lift.

In this work, the core algorithm of the near wake model has been altered to ensure that the downwash due to trailed vortex elements is calculated with the same precision independent of their size and distance from the blade section. Also the influence of the spatial discretization of the blade on the results is investigated, using three different point distributions. An iterative solution scheme with a relaxation factor is introduced to ensure the stable behavior of the near wake model, especially as part of a coupled aerodynamics model. Joukowski's law, the proportionality of lift and circulation, does not hold for unsteady calculations. Therefore the unsteady circulation is determined separately in the attached flow region, analogue to the unsteady lift in the dynamic stall model by Hansen et al. [7].

With these additions, the modeling of the development of the downwash on a wing with a constant elliptical circulation becomes independent of the time step and converges consistently to the analytical steady state solution. It is shown that the altered trailing algorithm is also faster than the original version. To show the capabilities of the coupled model to handle the unsteady case, it has been used to simulate the aerodynamics of the NREL 5 MW reference turbine. The steady induction agrees well with results from a code comparison [1].

This paper is structured as follows: First, the near wake model and its modifications are described. Then the coupled model is introduced. Finally, results from both near wake model and coupled model are presented.

2. Description of the near wake model

In this section the near wake model is introduced, starting with the original model by Beddoes and followed by a modified version of the vortex trailing algorithm, which is less time step dependent. Furthermore, different point distribution methods and a stabilization of the model through an iterative computation of the downwash are presented. In the last part of the section, a brief outline of the coupling to a far wake model is given.

2.1. Original model by Beddoes, modified by Madsen and Rasmussen

Based on the Biot-Savart law, the induced downwash of a vortex filament with length $\mathrm{d}s$ and vortex strength $\Delta\Gamma$, which is trailed from radius r at a point on the blade and stays in the rotor blade, cf. Figure 1, can be evaluated as

$$\mathrm{d}w = \frac{\Delta\Gamma\,\mathrm{d}s}{4\pi r^2}\frac{1-\left(1-\frac{h}{r}\right)\cos(\beta)}{\left[1+\left(1-\frac{h}{r}\right)^2-2\left(1-\frac{h}{r}\right)\cos(\beta)\right]^{3/2}}, \tag{1}$$

where h is the distance between the vortex trailing point and the calculation point where the downwash is evaluated. The value of h is negative when the vortex is closer to the blade root than the section. The angle $\beta = \Omega t$ determines the position of the infinitesimal vortex element on the circular arc, the angle the rotor has rotated with the constant angular velocity Ω since the vortex filament has been trailed from the blade. The downwash from the circular arc could be evaluated by numerically integrating Equation (1) from 0 to 90 degrees.

To avoid these time consuming integrals, Beddoes derived an equation that gives the decrease of the induction by a vortex filament $\mathrm{d}w$ compared to its original induction $\mathrm{d}w_0$, when it has just left the lifting line. This equation is then approximated using two exponential functions:

$$\frac{\mathrm{d}w}{\mathrm{d}w_0} = \frac{\left(\frac{h}{r}\right)^2\left[1-\left(1-\frac{h}{r}\right)\cos(\beta)\right]}{\left[1+\left(1-\frac{h}{r}\right)^2-2\left(1-\frac{h}{r}\right)\cos(\beta)\right]^{3/2}} \approx 1.359e^{-\beta/\Phi} - 0.359e^{-4\beta/\Phi}, \tag{2}$$

where Φ is a geometric factor depending on the positions of vortex trailing point and calculation point:

$$\Phi = \frac{\pi}{4}\left|\left(1+\frac{h}{2r}\right)\ln\left(1-\frac{h}{r}\right)\right|. \tag{3}$$

Madsen and Rasmussen [6] replaced the term 1 + h/(2r) by 0.75 for cases where h/(2r) is smaller than -0.25, which increases the accuracy of the exponential approximation when the vortex trailing point lies further inboard than the calculation point for the induced downwash. This modification is used in the calculations presented here.

The computational effort can be dramatically reduced by using the exponential functions. The downwash W can then be split into two contributions [3]:

$$W^i = X_w^i + Y_w^i, \tag{4}$$

where the index i denotes the time step and X_w and Y_w are state variables that represent the slowly and quickly decreasing components of the induction from the near wake according to Equation (2):

$$X_w^i = X_w^{i-1}e^{-\Delta\beta/\Phi} + 1.359D_w e^{-\Delta\beta/2\Phi} \tag{5a}$$

$$Y_w^i = Y_w^{i-1}e^{-4\Delta\beta/\Phi} - 0.359D_w e^{-2\Delta\beta/\Phi}, \tag{5b}$$

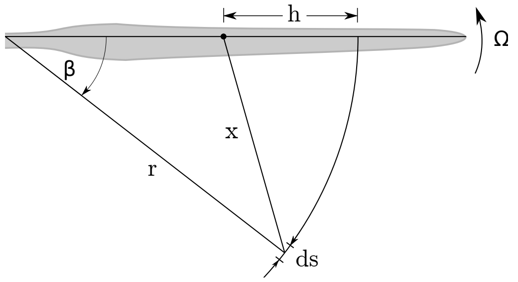

Figure 1. Geometry at a blade rotating with the constant angular velocity Ω. The downwash at a distance h from the vortex trailing point shall be computed. Since the infinitesimal vortex element with length ds has left the lifting line, the blade has been rotated by the angle β.

where $\Delta\beta = \Omega\Delta t$ is the azimuthal angle traveled in one time step Δt. The components X_w^{i-1} and Y_w^{i-1} from the last time step contain the complete induction from the old circular arc. These values are then multiplied by exponential factors, which depend on $\Delta\beta$, and the positions of vortex trailing point and calculation point, through their influence on the geometric factor Φ. The second terms on the right hand side of Equation (5) contain the induction due to the new finite length vortex filament D_w, which has been trailed during the time step. This induction D_w is computed using the Biot-Savart-law, assuming the newest element is a straight vortex with length Δs and perpendicular to the lifting line [3]:

$$D_w = \frac{\Delta\Gamma\left(\frac{\Delta s}{|h|}\right)}{4\pi r\frac{h}{r}\left[1+\left(\frac{\Delta s}{h}\right)^2\right]^{1/2}}, \tag{6}$$

where $\Delta\Gamma$ is the strength of the vortex. The contributions from X_w and Y_w to the induction D_w from the newest element are depending on its length, because Y_w decreases four times faster with increasing angle β than X_w, cf. Figure 2. In Equation (5), D_w is multiplied not only by 1.359 and -0.359, but also by the respective exponential factors corresponding to the middle of the element to take this length dependence into account. As shown in Figure 2, this gives not only the desired approximation of the contributions from X_w and Y_w, but also leads to an underestimation of the induction due to the newest element, because the exponential factors do not add up to 1 for a finite element length. The error due to this underestimation is growing with increasing time step and decreasing distance between calculation point and vortex trailing point.

2.2. New formulation of the trailing algorithm

The purpose of the modification explained in the following is to ensure a time step independent behavior of the trailing algorithm in case of a prescribed, constant circulation. Because the circulation is constant, this time step independence means that trailed vortex elements are evaluated correctly independent of their size, which varies with the time step. The decrease of induction from the old part of the wake, the first terms on the right hand side of Equation (5), is already time step independent, because $e^x e^x = e^{2x}$. Therefore, to make the whole algorithm time step independent, both the value of the initial downwash D_w and the way it is split into X_w and Y_w have to be corrected.

Instead of calculating a D_w for the whole first time step, the $\Delta s = \Delta\beta r$ in equation (6) is replaced by $\tilde{\beta} r$:

$$\tilde{D}_w = \frac{\Delta\Gamma\left(\frac{\tilde{\beta} r}{|h|}\right)}{4\pi r\frac{h}{r}\left[1+\left(\frac{\tilde{\beta} r}{h}\right)^2\right]^{1/2}}, \tag{7}$$

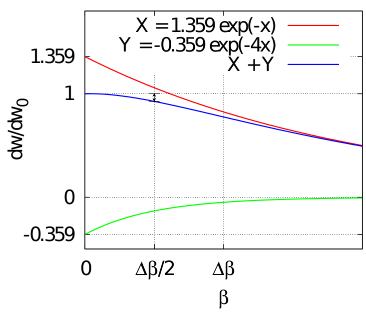

Figure 2. Illustration of the exponential trailing functions. The black mark indicates the underestimation of the induction due to the multiplication of D_w by the exponential factors in Equation (5) for an element with length $\Delta\beta$.

where $\tilde{\beta}$ is a constant, very small angle, for which the induction is approximately constant along the vortex filament: $\mathrm{d}w/\mathrm{d}w_0 \approx 1$ for $\beta \in [0;\tilde{\beta}]$. In the simulations presented here $\tilde{\beta} = 10^{-10}$ rad has been used.
The induction D_w for the first time step Δt can then be approximated as:

$$D_w = \tilde{D}_w \frac{\Delta\beta}{\tilde{\beta}} \left\langle \frac{\mathrm{d}w}{\mathrm{d}w_0} \right\rangle, \tag{8}$$

where $< \mathrm{d}w/\mathrm{d}w_0 >$ denotes the average value of $\mathrm{d}w/\mathrm{d}w_0$ given in Equation (2) over the whole length of the newest element. This average can be obtained by integrating:

$$\begin{aligned} D_w &= \tilde{D}_w \frac{\Delta\beta}{\tilde{\beta}} \frac{1}{\Delta\beta} \int_0^{\Delta\beta} \frac{\mathrm{d}w}{\mathrm{d}w_0}\, d\beta \\ &= \frac{\tilde{D}_w}{\tilde{\beta}} \int_0^{\Delta\beta} (1.359 e^{-\beta/\Phi} - 0.359 e^{-4\beta/\Phi})\, d\beta \\ &= \frac{\tilde{D}_w \Phi}{\tilde{\beta}} [1.359(1 - e^{-\beta/\Phi}) - \frac{0.359}{4}(1 - e^{-4\beta/\Phi})] \end{aligned} \tag{9}$$

For small values of $\tilde{\beta}$, which can be chosen independent of the time step, this is a good approximation of the downwash induced by the first vortex filament. The error due to calculating D_w based on a straight vortex filament is replaced by the error caused by using Beddoes' functions. As opposed to the way D_w is obtained before, it can be consistently split in X_w and Y_w, which leads to a modified version of Equation (5):

$$X_w^i = X_w^{i-1} e^{-\Delta\beta/\Phi} + 1.359 \frac{\tilde{D}_w \Phi}{\tilde{\beta}} (1 - e^{-\Delta\beta/\Phi}) = X_w^{i-1} e^{-\Delta\beta/\Phi} + D_X \Delta\Gamma (1 - e^{-\Delta\beta/\Phi}) \tag{10a}$$

$$Y_w^i = Y_w^{i-1} e^{-4\Delta\beta/\Phi} - 0.359 \frac{\tilde{D}_w \Phi}{4\tilde{\beta}} (1 - e^{-4\Delta\beta/\Phi}) = Y_w^{i-1} e^{-4\Delta\beta/\Phi} + D_Y \Delta\Gamma (1 - e^{-4\Delta\beta/\Phi}). \tag{10b}$$

The implementation of the new algorithm shows another advantage: In addition to ϕ and h also the factors D_X and D_Y for the induction from the new vortex elements are constant for each combination of calculation point and vortex trailing point. Therefore they can be computed once, at the initialization of the model,

$$D_X = 1.359 \frac{\tilde{D}_w \Phi}{\tilde{\beta} \Delta\Gamma} = 1.359 \left(\frac{\Phi r}{4\pi h|h|} \right) \left[1 + \left(\frac{\tilde{\beta} r}{h} \right)^2 \right]^{-1/2} \tag{11a}$$

$$D_Y = -0.359 \frac{\tilde{D}_w \Phi}{4\tilde{\beta} \Delta\Gamma} = -0.359 \left(\frac{\Phi r}{16\pi h|h|} \right) \left[1 + \left(\frac{\tilde{\beta} r}{h} \right)^2 \right]^{-1/2}, \tag{11b}$$

which was not possible in the original algorithm, as Δs in Equation (6) for D_w is not a constant. Furthermore, only two instead of four evaluations of exponential functions are necessary.

2.3. Influence of helical pitch angle, Wang and Coton
So far it has been assumed by Beddoes that the vortices are trailed in the rotor plane. In reality the vortices move in the helical wake. Wang and Coton [2] took this into account by

including the pitch angle φ of the vortex path in the calculation of the axial induction. For an inflow perpendicular to the rotor plane and assuming a constant helical pitch, that angle can be defined as

$$\tan(\varphi) = \frac{V_\infty - w_a}{\Omega r + w_t}, \tag{12}$$

where V_∞ is the free stream velocity, w_a and w_t the axial and tangential induction, assumed to be constant in the annular tube, and Ω the rotational speed. The axial part of D_w is:

$$D_{w,a} = D_w \cos(\varphi). \tag{13}$$

This calculation of $D_{w,a}$ does not contain the increasing distance of the vortex filaments from the blade sections as they move downwind.

2.4. Discretization of the blade

The algorithm used in this paper distinguishes between two kinds of points: vortex trailing points and calculation points. At the calculation points, the downwash, lift and circulation are determined. The circulation difference of two neighboring calculation points is then trailed at the vortex trailing point between them. At root and tip of the blade the vortex strength is the complete circulation of the nearest calculation point. Three different ways of distributing the points along the blade have been investigated:

- The **equidistant** distribution gives a constant resolution along the blades. The vortices are trailed from the root and tip of the blade, which is discretized in n sections, each with a vortex trailing point at both ends and a calculation point in the middle.
- The **cosine** distribution places the vortex trailing points at equal angle of a half circle over the blade, cf. Figure 3. The calculation points of the sections are placed in the middle of two trailed vortices.
- The **full cosine** distribution, which is used in the AWSM code [8] places the calculation points and vortex positions at equi-angle increments, also shown in Figure 3.

Figure 4 shows an example of how a continuous elliptical circulation is represented by constant values at the calculation points for the different point distributions.

2.5. Numerical stability

The near wake model can become numerically unstable if the downwash induced by the vortex filaments that have been trailed in one time step is so big that the predicted induction starts to diverge. The downwash will then lead to a negative lift of a bigger absolute value than the positive lift in the previous time step. The resulting trailed filaments with a vortex strength of the opposite sign will induce an even bigger negative downwash. If this is the case, the circulation and downwash can reach unphysical values in a few time steps.

The instability occurs especially for bigger time steps and close positions of calculation point and vortex trailing point, for example close to the tip when a cosine or full cosine distribution is used. Then the influence of the wake of the previous time step, that would stabilize the algorithm, decays quickly. The 2D shed vorticity effects on lift and circulation also stabilize the model. The problem of instability can be solved by running an iterative version of the NWM with the relaxation factor r:

$$W_r^j = W^{j-1} r + W^j (1 - r), \tag{14}$$

where j denotes the iteration. This iterative process is used in the following coupled model.

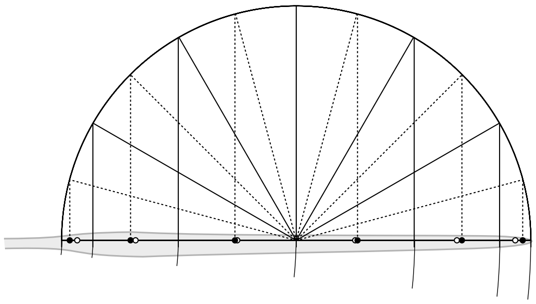

Figure 3. Sketch of the cosine and full cosine point distribution. The calculation points are placed at the positions marked by white (cosine) or black (full cosine) dots.

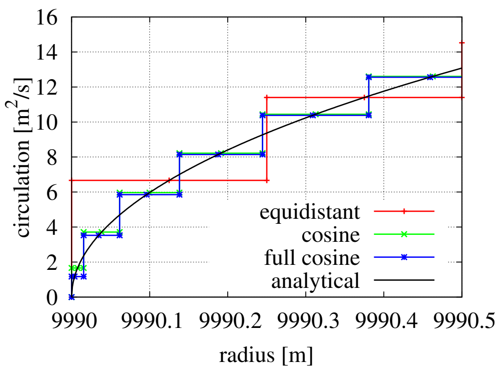

Figure 4. Resolution of an elliptical circulation close to the edge of the wing investigated in section 3 for the different point distributions.

Algorithm 1 One time step of the coupled model with near wake and far wake induction

```
while NOT convergence do
    AOA=AOA(W_iter), vrel=vrel(W_iter)
    calculate quasisteady lift and circulation
    apply unsteady airfoil aerodynamics model to lift and circulation
    call BEM(CT*k_FW)
    call NWM(unsteady circulation)
    W_iter=W_lastiter_NW*r+W_iter_NW*(1-r)+W_iter_FW
    if abs(W_iter-W_lastiter)< ε then convergence = true
end while
```

2.6. Coupling to a far wake model

Because the near wake model only takes a fraction of one rotor revolution into account, it has to be coupled with a far wake model that calculates the induction from the missing part of the wake. A BEM model is used for this purpose. To consider the fraction of the induction that is computed by the near wake model, the thrust coefficient from the BEM model is multiplied by the coupling factor k_{FW} [5, 6], which depends on the operating conditions of the turbine. It is defined as:

$$C_{T,FW} = C_{T,BEM} k_{FW}, \qquad k_{FW} < 1 \tag{15}$$

where $C_{T,BEM}$ denotes the thrust coefficient obtained from the momentum balance of induced velocities without the near wake model and $C_{T,FW}$ is the reduced thrust coefficient used when the far wake BEM is coupled to the near wake model. The coupling factors used in this work are the result of simulations where the integral thrust coefficient from a coupled model has been matched with a BEM model with tip loss correction for the investigated operating conditions, as suggested by Andersen [5]. This matching ensures that steady state results from the coupled model agree with the classical BEM model for different combinations of wind speed, rotational speed and pitch angle. The structure of the coupled model is shown in algorithm 1. It includes the unsteady effects on circulation and on lift, the NWM and the BEM model with reduced thrust coefficient for the far wake.

3. Results

The modifications of the NWM are investigated for a wing with elliptical circulation. Simulations of the NREL 5 MW reference turbine [9] show the capabilities of the coupled model.

3.1. Near wake model: elliptical wing with prescribed circulation

A wing with a prescribed elliptical circulation has been used to investigate the influence of the spatial and temporal discretization. It is modeled as a 10 m long section at the end of a 10 km long blade to approximate a parallel free stream, similar to the case presented by Madsen and Rasmussen, [6]. The circulation at radius r is given as

$$\Gamma = 30\,\frac{\mathrm{m}^2}{\mathrm{s}}\sqrt{(1-((r-9995\,\mathrm{m})/5\,\mathrm{m})^2)}, \tag{16}$$

which results in a constant downwash of 1.5 m/s along the wing according to lifting line theory. The blade rotates with 0.03359 rpm, which is equivalent to a free stream velocity at the wing of about 35 m/s.

To investigate the effect of the different spatial discretizations introduced in Section 2.4, Figure 5 shows the downwash of the near wake model in steady state for a 175 m long wake. The blade is discretized with 40 (left plot) or 80 (right plot) calculation points, corresponding to 41 and 81 vortex trailing points. While all spatial distributions perform well in the middle sections, the equidistant and cosine distribution lack accuracy close to the edges of the wing. They even lead to negative downwash in the outer sections of the blade.

To show the time step independence of the proposed method to obtain the downwash of the newest trailed filament described in Section 2.2, Figure 6 shows plots of the buildup of the downwash induced by the wake trailed from the wing with prescribed elliptical circulation. At the beginning of the simulation, $t = 0$, there is no trailed wake and therefore no downwash. The plots of each color represent the distribution of the downwash after $t = 0.01$ s, $t = 0.05$ s and $t = 5$ s. A full cosine distribution with 40 sections has been used to discretize the wing. The new formulation proposed in Section 2.2 is compared to the original algorithm, Equation (5), for time steps of 0.01 s and 0.001 s. For all computations the downwash in the middle of the wing is increasing over time. The model based on the new formulation reaches a constant downwash of 1.4949 m/s along the wing after 5 s simulated time, which is close to the analytical steady

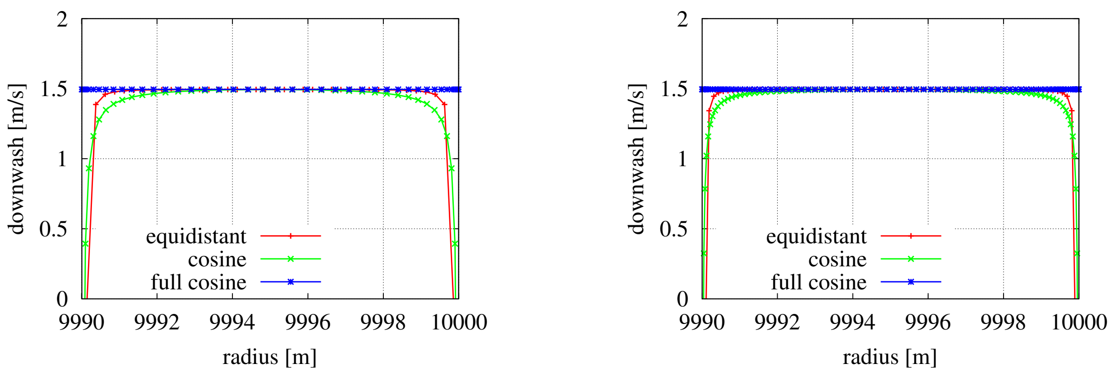

Figure 5. Comparison of the steady downwash at an elliptical wing with a wake of 175 m length. The calculation points and vortex trailing points have been distributed using the distribution methods described in Section 2.4. The blade has been discretized using 40 (left plots) or 80 (right plots) calculation points.

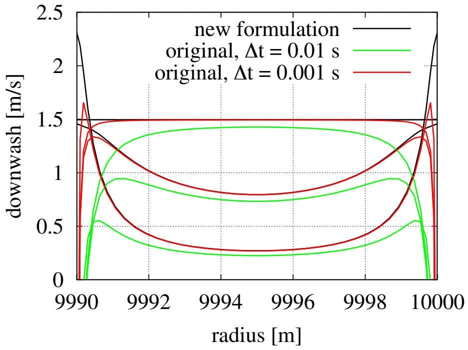

Figure 6. Buildup of induced velocities at the elliptical wing with a prescribed circulation at t=0.01 s, t=0.05 s and t=5.0 s, corresponding to 35 cm, 1.75 m and 175 m long trailed wakes. The new formulation, Equation (10), leads to the same results, independent of the time step.

solution of 1.5 m/s. Close to the middle of the wing Beddoes' algorithm agrees well with the new formulation for a time step of 0.001 s, but the accuracy gets worse towards both ends of the wing. If the original algorithm is used with a time step of 0.01 s, the downwash is significantly underestimated on the whole wing, again with the biggest errors at the ends.

3.2. Coupled model: NREL 5 MW reference turbine blade

Steady results of the coupled model for the NREL 5 MW reference turbine [9] are shown in Figure 7. The wind speed in the computation is 8 m/s, perpendicular to the rotor plane, and the rotational speed 9.21 rpm. The blade pitch angle is 0°. At the first 10 meters of radius, where there are no aerodynamic profiles, all the induction due to the root vortex is accounted for by the near wake model. Along large parts of the blade, the relatively constant induction due to the BEM model dominates, but closer to the tip the ability of the near wake model to capture the tip vortex becomes apparent. The combined axial induction from near and far wake agrees well with the code comparison results by Madsen et al. [1].

The thrust coefficients due to near wake, far wake and complete wake for an unsteady simulation are shown in Figure 8. The wind speed and rotational speed are identical as in the steady simulation. For simplicity, the dynamic inflow model used for the far wake had a dimensionless time constant of 1, independent of the radial position. After 100 seconds, the blades start to perform synchronous prescribed vibrations with an amplitude of 0.25 m and a frequency of 1 Hz perpendicular to the rotor plane. The shape of these vibrations is assumed to be the mode shape of a clamped-free prismatic beam. It can be seen that the unsteady aerodynamic effects due to these vibrations are mostly modeled by the near wake model. At 110 s, a pitch step of 4° within 0.5 seconds is performed, where the coupled model captures both fast and slow parts of the response.

4. Conclusions

The sensitivity to the spatial discretization of a near wake model, originally proposed by Beddoes, has been investigated. It was found that the case of a prescribed elliptic circulation could be modeled with better accuracy using a distribution based on equi-angle increments between calculation points and vortex trailing points as opposed to an equidistant distribution.

To overcome the high sensitivity of the model on the time step a new formulation of the trailing wake algorithm based on integration of the trailing functions has been developed. This formulation makes the calculation of the downwash time step independent for a constant trailed vorticity, which means that trailed vortex elements are evaluated with the same accuracy, independent of their size. Therefore the modified algorithm can be used with bigger time steps. In addition to that, each time step is computed faster because larger parts of the algorithm can be computed once in the initialization of the program.

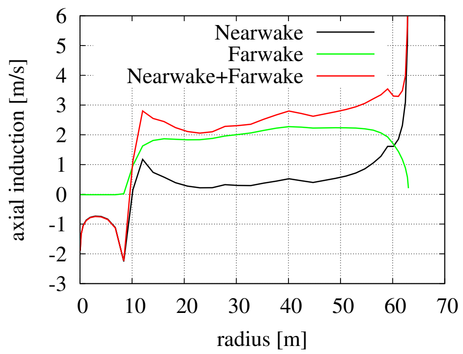

Figure 7. Induction due to near and far wake for the NREL 5MW rotor.

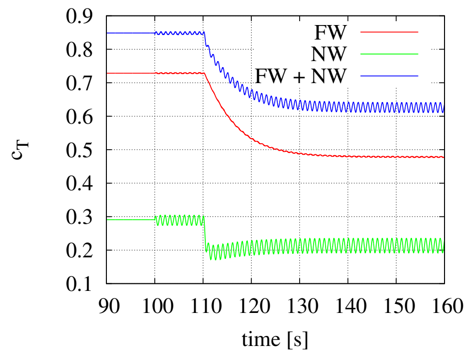

Figure 8. Integral C_T of the NREL rotor with a pitch step and blade vibrations.

The model has been stabilized by introducing an iterative solution of the downwash from the near wake at each time step. The near wake model is coupled to the traditional BEM model of the far wake induction by sharing of the total induction through a coupling factor. The coupled model includes unsteady shed vorticity effects for both lift and circulation in the region of attached flow and a blade element momentum model for the far wake. The steady and unsteady behavior of the model has been illustrated based on the NREL 5 MW reference turbine. The coupled model agrees well with established models with regard to the distribution of axial induction and is capable of modeling the unsteady aerodynamic effects at different time scales.

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