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**Nielsen, Kaspar Kirstein; Bahl, Christian; Smith, Anders**

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# DEMAGNETIZING FIELDS IN ACTIVE MAGNETIC REGENERATORS

## K.K. Nielsen, C.R.H. Bahl and A. Smith

DTU Energy Conversion, Technical University of Denmark
Frederiksborgvej 399, DK-4000 Roskilde, Denmark
kaki@dtu.dk, chrb@dtu.dk, ansm@dtu.dk

ABSTRACT — A magnetic material in an externally applied magnetic field will in general experience a spatially varying internal magnetic field due to demagnetizing effects. When the performance of active magnetic regenerators (AMRs) is evaluated using numerical models the internal field is often assumed to be spatially constant and equal to the applied field, thus neglecting the demagnetizing field. Furthermore, the experimental magnetocaloric properties used (adiabatic temperature change, isothermal entropy change and specific heat) are often not corrected for demagnetization. The demagnetizing field in an AMR is in general both a function of the overall shape of the regenerator and its morphology (packed particles, parallel plates etc.) as well as the magnetization of the material. Due to the pronounced temperature dependence of the magnetization near the Curie temperature, the demagnetization field is also temperature dependent.
We propose a relatively straightforward method to correct sufficiently for the demagnetizing field in AMR models. We discuss how the demagnetizing field behaves in regenerators made of packed spheres under realistic operation conditions.

## 1. INTRODUCTION

A magnetic body, i.e. a body with a non-zero magnetization, generates a magnetic field. In zero applied field, a soft ferromagnetic body will contain magnetic domains, each with a unidirectional magnetization, but with varying orientation from domain to domain. It is energetically favorable for the domains to arrange themselves such that the total (vector) sum of the magnetization of the sample, i.e. the observed magnetization, will be zero. Upon application of an external magnetic field, $H_{app}$, the domains will tend to align with the applied field and thus make the total magnetization different from zero. At some relatively low field (of the order 0.1 T for the soft materials we consider) the domains will almost be aligned, and the measured magnetization will be close to the intrinsic magnetization; see Ref. [1] elsewhere in these proceedings for a phenomenological discussion of this.

The total field both inside and outside the magnetized body will differ from the applied field due to this demagnetizing field. This may be expressed in the following relation:

$$\mathbf{H} = \mathbf{H}_{app} + \mathbf{H}_{dem},$$ (1)

where the applied field is $\mathbf{H}_{app}$ and the demagnetizing field is $\mathbf{H}_{dem}$. For certain geometries it is possible to derive analytical expressions for the demagnetizing field. Examples of this are ellipsoids, infinite sheets and infinite cylinders. In these cases the demagnetizing field (inside the magnetized body) is exactly:

$$\mathbf{H}_{dem} = -\overline{\overline{\mathbf{N}}}\mathbf{M}.$$ (2)

The magnetization is denoted by $\mathbf{M}$ and the demagnetizing tensor is $\overline{\overline{\mathbf{N}}}$. For a sphere this reduces to a tensor with a value of 1/3 on the diagonal entries and zero elsewhere, which is typically formulated simply as a scalar $N$. For an infinite cylinder the tensor also reduces simply to a scalar $N$ with the value zero when magnetized along the axial direction while it is 0.5 upon magnetizing in the radial direction. Analytical formulae for $\overline{\overline{\mathbf{N}}}$ are available for the general ellipsoid in the literature [2]. Assuming a spatially constant magnetization it is possible to analytically find an average demagnetization factor in a rectangular prism [3] and a finite cylinder [4].

Outside the magnetized body the field will in general vary spatially; for a sphere, the field is that of a dipole. For a rectangular prism the field it produces may be calculated approximately by discretizing the prism into small rectangular prisms within which the magnetization is assumed to be constant; see Ref. 3 for such detailed calculations.

In an active magnetic regenerator (AMR) both the internal field inside the solid refrigerant and the external field generated by the magnetization of the solid are of considerable importance. The internal field determines the magnitude of the magnetocaloric effect while the external field contributes to the total field throughout the regenerator (and to a minor extent in the magnetic field source). Even with a perfectly homogeneous applied field the field over the regenerator will not be spatially constant. Furthermore, as the temperature of a regenerator in operation varies from the hot end to the cold end, the effect of the demagnetizing field is larger at the cold end due to the increase in magnetization as the temperature is lowered. In the following we discuss various practical methods for calculating the magnetic field distribution in an operating AMR, i.e. in cyclic steady-state.

## 2. METHODS

The framework for calculating the field in a stack of parallel plates under non-uniform conditions has been presented elsewhere [5],[6]. Essentially, the plates are discretized into small rectangular prisms inside which the magnetization is assumed to be constant. The stray field from such a prism is then found analytically and the vector sum of the stray fields of the prisms and the applied field is then the total field. By allowing for a variation of the magnetic properties with temperature through an appropriate state function it is possible to completely model the detailed 3D structure of the magnetic field in an AMR. It is assumed in such a scheme that the stray field from the regenerator does not influence the magnetic field source.

A similar approach has recently been developed for an AMR consisting of packed spheres [7]. Assuming perfect spheres the stray field of each sphere is that of dipole. The resulting magnetic field may then be found at any position **r** in space as the vector sum of each of the individual dipoles. With this approach it is even possible to model a regenerator with spheres with different diameters. The equations solved may be written as (for calculating the field at the location **r**):

$$\mathbf{H}_{\text{ext}}(\mathbf{r}) = \mathbf{H}_{\text{app}}(\mathbf{r}) + \sum_{j=1}^{n} \mathbf{H}_{\text{dip},j}(\mathbf{r} - \mathbf{r}'_j) \tag{3}$$

$$\mathbf{H}_{\text{dip},j}(\mathbf{r}) = \frac{1}{4\pi} \left( \frac{3(\mathbf{M}_j \cdot \mathbf{r})\mathbf{r}}{r^5} - \frac{\mathbf{M}_j}{r^3} \right) \tag{4}$$

$$\mathbf{H}_i(\mathbf{r}) = \mathbf{H}_{\text{ext}}(\mathbf{r}) - N\mathbf{M}_i(T_i, \mathbf{H}_i(\mathbf{r})) \tag{5}$$

Here $n$ is the number of spheres. The last equation (5) gives the field, $\mathbf{H}_i$, in the $i$'th sphere located at the position **r** with $N=1/3$. Equations (3)-(5) are solved iteratively for $\mathbf{H}_i$ and $\mathbf{M}_i$.

Combining the magnetostatic model with an AMR model, i.e. a model solving the coupled partial differential equations (PDEs) governing heat transfer in the fluid and magnetocaloric solid is in principle straight-forward. These PDEs may be expressed as:

$$\varepsilon \rho_{\text{f}} c_{\text{f}} \left( \frac{\partial T_{\text{f}}}{\partial t} + \mathbf{u} \cdot \nabla T_{\text{f}} \right) = \nabla \cdot (k \nabla T_{\text{f}}) - h a_s (T_{\text{f}} - T_s) \tag{6}$$

$$(1-\varepsilon)\rho_s c_s \frac{\partial T_s}{\partial t} = \nabla \cdot (k_s \nabla T_s) + h a_s (T_{\text{f}} - T_s) - (1-\varepsilon)\rho_s T_s \frac{\partial s}{\partial H} \frac{\partial H}{\partial t} \tag{7}$$

Here, the density is $\rho$, the specific heat is $c$, temperature is $T$, time is $t$, the velocity field is **u**, thermal conductivity is $k$, convective heat transfer coefficient $h$, specific surface area $a_s$, the total entropy is $s$ and the porosity of the regenerator is $\varepsilon$. It is here important to emphasize that Eqs. (6)-(7) are derived assuming volume averages, i.e. the individual particles / spheres are not spatially resolved here.

The equations (3)-(5), or the equivalent equations for geometries other than packed spheres, are strongly coupled with the governing equations for the AMR (Eqs. (6)-(7)): The temperature field inside the regenerator is determined by the total magnetic field, which in its turn depends on the temperature through the magnetic equation of state. The full solution to the problem may thus, in principle, be obtained by solving the above mentioned equations with an appropriate state function for the magnetization and an expression for the total entropy of the regenerator solid (magnetocaloric material).

In terms of computational time, however, there is a rather large difference in convergence times for the two sets of equations. Equations (3)-(5) have to be solved at each time step in the time evolution of Eqs. (6)-(7). For realistic regenerator geometries the solution for the magnetic field takes of the order tens of minutes whereas a single timestep in the regenerator equations is of the order milliseconds (on the same hardware). Furthermore, the modeled AMR geometry is typically simplified into 1D or 2D porous medial models, i.e. exploiting continuum averages. The reason for this is that for geometries like packed spheres and similar structures resolving the full heat transfer and fluid flow problem is simply not realistic. The level of detail in the two models is therefore not the same and it may be justifiable to simplify the spatially resolved magnetostatic calculations when applied in an AMR model.

Such a scheme has previously been suggested [8]. Assuming that in periodic steady-state operation the temperature distribution in an AMR is linear and the applied field profile is given as a function of space and time a scalar field coupling the two models may be defined as:

$$K(\mathbf{r},t) \equiv \frac{H_{app}(\mathbf{r},t) - H(\mathbf{r},t)}{M(H(\mathbf{r},t),T(\mathbf{r}))} \qquad (8)$$

$$H_{AMR} = H_{app}(\mathbf{r},t) - K(\mathbf{r},t)M(H_{AMR}(\mathbf{r}),T(\mathbf{r})) \qquad (9)$$

Note that the magnitudes of the fields and magnetization are used here. In Eq. (9) the magnetic field $H_{AMR}$ denotes the magnetic field applied in the AMR model (Eq. (7)), i.e. Eq. (9) is solved iteratively at each timestep while temporally evolving Eqs. (6) and (7). $H_{AMR}$ is the field, which the magnetocaloric effect is calculated from. It is important to distinguish the total magnetic field, **H**, found through solving Eqs. (3)-(5) and $H_{AMR}$. The former is represented by the definition of the $K$ field (Eq. (8)), which is then used in Eq. (9) to find the field needed in the AMR model.

### 3. RESULTS

As an example of calculating the internal magnetic field in a packed sphere structure the model was applied to the case of equal-sized spheres ordered in a regular simple packed structure. The resulting field (on the surface of the cube) is visualized in Fig. 1. The temperature and the applied field are constant. Notice that the color scale varies only a little over 0.1 T and so the resulting field is quite homogeneous for this case.
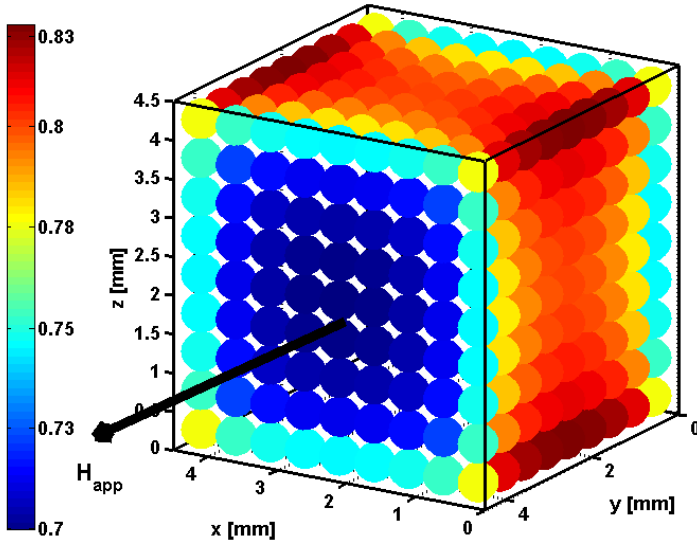


Fig. 1. An example of the magnetic field applied to packed spheres. The spheres are structured in a regular simple pack. The applied field is homogeneous and equal to one tesla (the direction is indicated by the arrow). The colorbar shows the internal field magnitude in the spheres (in units of tesla).

*Prototype modeling*

The model presented above has been applied in the design of the new magnetic refrigeration prototype at DTU presented elsewhere in these proceedings [9]. Following the procedure explained above this was used to find the internal field in the regenerator as the magnet rotates. Thus both the temporal and spatial variation of the applied field is taken into account when calculating the internal field. In combination with the flow profile this gives rise to some asymmetry in the internal field distribution in each regenerator bed. This is visualized in Fig. 2 half-way through the cold-to-hot blow period in one regenerator bed.

The internal field is greater at the hot end than at the cold end is merely due to the temperature gradient, i.e. the magnetocaloric material (here gadolinium) is more magnetic at the cold end. The internal field is also seen to peak roughly down through the center of the bed. This is caused by the fact that the applied field and the flow profile are synchronized so that half-way through a blow period the applied field peaks in the middle of the regenerator, i.e. the regenerator operation is balanced in terms of flow and applied field.

In Figure 3 an example is given of the importance of taking demagnetizing effects and the significant spatial variation of the applied field into account. The cooling power (at a temperature span of 30 K) is plotted as a function of the number of regenerators in the prototype. As this number increases the width of the individual bed decreases and the field applied to each individual regenerator is thus increasingly homogeneous resulting in a better performance.
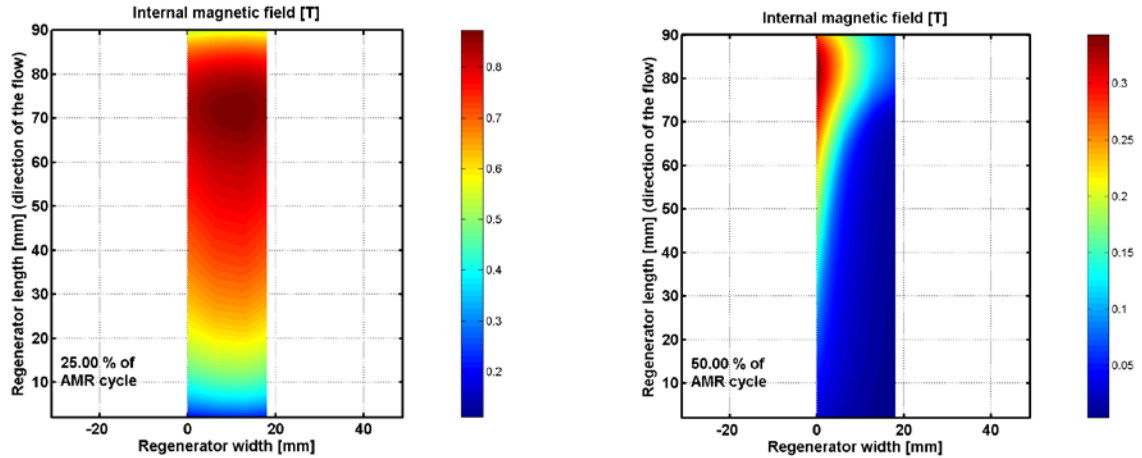
Fig. 2. The internal magnetic field modeled in the new prototype at DTU (Ref. [9]) as a function of flow direction (length of the magnet) and direction of rotation (azimuthal direction) for a specific regenerator (90x18 mm). The color bar shows the value of internal field in tesla. Note that the AMR is in cyclic steady-state and the cold side is at 0 mm while the hot side is at 90 mm in the flow direction. The temperature span is 30 K and the external field varies both in the flow direction and the direction of the regenerator width. Left: After 25 % of the AMR cycle (where the applied field is maximum). Right: After 50 % of the AMR cycle (where the applied field is half-way between maximum and minimum values). Note the color scales are not the same.

## 4.  CONCLUSION

The effect of demagnetization in active magnetic regenerators was presented in general. Specific models for handling parallel-plates and packed sphere geometries were discussed with emphasis on the internal magnetic field. The model for handling packed spheres in a demagnetization context was applied in the design of a new prototype built at DTU. The results indicate the importance of including both the effect of demagnetization as well as the effect of a spatially varying applied field (at any instant in time) for accurately capturing the relation between the spatially varying magnetocaloric effect and the fluid flow profile.
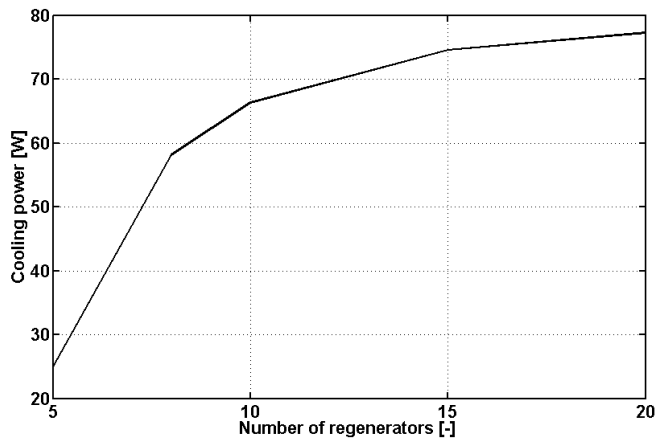


Fig. 3. Modeled cooling power as a function of the number regenerator beds in the DTU AMR prototype. As the number of beds is increased the width of each bed is smaller resulting in an increase in performance.

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