Technical University of Denmark

Benchmarking of optimization methods for topology optimization problems

Rojas Labanda, Susana; Stolpe, Mathias

Publication date:
2014

Document Version
Peer reviewed version

Link back to DTU Orbit

Citation (APA):
Rojas Labanda, S., & Stolpe, M. (2014). Benchmarking of optimization methods for topology optimization problems [Sound/Visual production (digital)]. 11th World Congress on Computational Mechanics, 5th European Conference on Computational Mechanics, 6th European Conference on Computational Fluid Dynamics, Barcelona, Spain, 20/07/2014

Benchmarking of optimization methods for topology optimization problems

Susana Rojas Labanda, PhD student
Mathias Stolpe, Senior researcher

11th World Congress on Computational Mechanics. Barcelona 2014

DTU Wind Energy
Department of Wind Energy

Why?

- Asses general purpose 2nd order optimization methods in topology optimization problems.

Why?

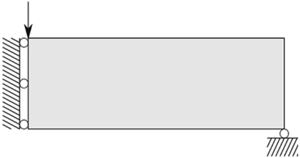
- Asses general purpose 2nd order optimization methods in topology optimization problems.

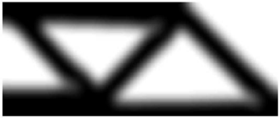
Main results from the Benchmarking

- GCMMA outperforms MMA.
- GCMMA and MMA tend to obtain a design with large KKT error.
- The performance of GCMMA and MMA do not highlight respect to other solvers.
- The interior-point solver IPOPT, when the exact Hessian is used (IPOPT SAND), produces the best designs using few number of iterations
- IPOPT SAND is the most robust solver in the study.
- The SAND formulation requires lot of memory and computational time.

Topology optimization problems

- **Goal**: Obtain optimal design of a structure with given loads.





- **Model as an optimization problem**

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g(x) \leq 0 \\ & h(x) = 0. \end{aligned}$$

Topology optimization formulations

SAND formulation:

- Minimum compliance

$$\begin{array}{ll} \underset{\mathbf{t},\mathbf{u}}{\text{minimize}} & \mathbf{f}^T\mathbf{u} \\ \text{subject to} & \mathbf{a}^T\mathbf{t} \leq V \\ & \mathbf{K}(\mathbf{t})\mathbf{u} - \mathbf{f} = \mathbf{0} \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

- Minimum volume

$$\begin{array}{ll} \underset{\mathbf{t},\mathbf{u}}{\text{minimize}} & \mathbf{a}^T\mathbf{t} \\ \text{subject to} & \mathbf{f}^T\mathbf{u} \leq C \\ & \mathbf{K}(\mathbf{t})\mathbf{u} - \mathbf{f} = \mathbf{0} \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

- Compliant mechanism design

$$\begin{array}{ll} \underset{\mathbf{t},\mathbf{u}}{\text{minimize}} & \mathbf{l}^T\mathbf{u} \\ \text{subject to} & \mathbf{a}^T\mathbf{t} \leq V \\ & \mathbf{K}(\mathbf{t})\mathbf{u} - \mathbf{f} = \mathbf{0} \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

- $\mathbf{f} \in \mathbb{R}^d$ the force vector.
- $\mathbf{a} \in \mathbb{R}^n$ the volume vector.
- $V > 0$ is the upper volume fraction.
- $C > 0$ the upper bound of the compliance.
- $\mathbf{l} \in \mathbb{R}^d$ vector that indicates the output displacement.

Topology optimization formulations

NESTED formulation:

- Minimum compliance

$$\begin{array}{ll} \underset{\mathbf{t}}{\text{minimize}} & \mathbf{u}^T(\mathbf{t})\mathbf{K}(\mathbf{t})\mathbf{u}(\mathbf{t}) \\ \text{subject to} & \mathbf{a}^T\mathbf{t} \le V \\ & \mathbf{0} \le \mathbf{t} \le \mathbf{1}. \end{array}$$

- Minimum volume

$$\begin{array}{ll} \underset{\mathbf{t}}{\text{minimize}} & \mathbf{a}^T\mathbf{t} \\ \text{subject to} & \mathbf{u}^T(\mathbf{t})\mathbf{K}(\mathbf{t})\mathbf{u}(\mathbf{t}) \le C \\ & \mathbf{0} \le \mathbf{t} \le \mathbf{1}. \end{array}$$

- Compliant mechanism design

$$\begin{array}{ll} \underset{\mathbf{t}}{\text{minimize}} & \mathbf{l}^T\mathbf{u}(\mathbf{t}) \\ \text{subject to} & \mathbf{a}^T\mathbf{t} \le V \\ & \mathbf{0} \le \mathbf{t} \le \mathbf{1}. \end{array}$$

- $\mathbf{u}(\mathbf{t}) = \mathbf{K}^{-1}(\mathbf{t})\mathbf{f}$.
- $\mathbf{f} \in \mathbb{R}^d$ the force vector.
- $\mathbf{a} \in \mathbb{R}^n$ the volume vector.
- $V > 0$ is the upper volume fraction.
- $C > 0$ the upper bound of the compliance.
- $\mathbf{l} \in \mathbb{R}^d$ vector that indicates the output displacement.

Considerations on the problem formulation

- Use only one external static load.
- Linear elasticity in the equilibrium equation.
- Assume $\mathbf{K}(\mathbf{t}) \succ 0$ to avoid ill-conditioning.
- Use continuous density variables.
- Use SIMP penalization and a density filter.

Bendsøe, M. P and Sigmund, O. Material interpolation schemes in topology optimization. *Archive of Applied Mechanics*,69:635–654,1999.

Bourdin, B. Filters in topology optimization. *International Journal for Numerical Methods in Engineering*,50(9):2143–2158, 2001.

Optimization methods

Topology optimization problem

- **OC**: Optimality criteria method.
- **MMA**: Sequential convex approximations.
- **GCMMA**: Global convergence MMA.

Andreassen, E and Clausen, A and Schevenels, M and Lazarov, B. S and Sigmund, O. Efficient topology optimization in MATLAB using 88 lines of code. *Structural and Multidisciplinary Optimization*, 43(1): 1–16, 2011.

Svanberg, K. The method of moving asymptotes a new method for structural optimization. *International Journal for Numerical Methods in Engineering*, 24(2): 359–373. 1987.

Svanberg, K. A class of globally convergent optimization methods based on conservative convex separable approximations. *SIAM Journal on Optimization*, 12(2): 555-573, 2002.

Optimization methods

DTU

Topology optimization problem

⇓

non-linear problem

- **OC**: Optimality criteria method.
- **MMA**: Sequential convex approximations.
- **GCMMA**: Global convergence MMA.

- **FMINCON**: Interior-point MATLAB. Use exact Hessian.
- **SNOPT**: Sequential quadratic programming. BFGS approximations.
- **IPOPT**: Interior-point software. Exact Hessian in the SAND formulation, BFGS in the NESTED formulation.

Gill, P. E and Murray, W and Saunders, M. A. SNOPT: An SQP Algorithm for Large -Scale Constrained Optimization. *SIAM Journal on Optimization*, 47(4):99–131, 2005.

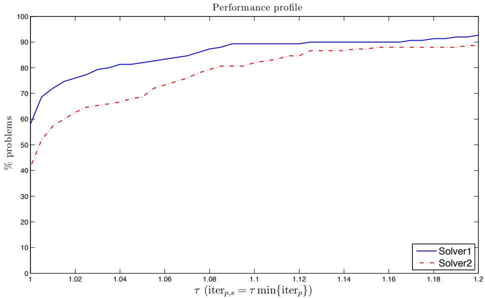
Wächter, A and Biegler, L. T. On the implementation of an interior point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106(1):25–57, 2006.

Benchmarking in topology optimization

- **How?** Using performance profiles.
  - Evaluate the cumulative ratio for a performance metric.
  - Represent for each solver, the percentage of instances that achieve a criterion for different ratio values.

$$\rho_s(\tau) = \frac{1}{n}\text{size}\{p \in P : r_{p,s} \leq \tau\},$$

$$r_{p,s} = \frac{\text{iter}_{p,s}}{\min\{\text{iter}_{p,s} : s \in S\}}.$$



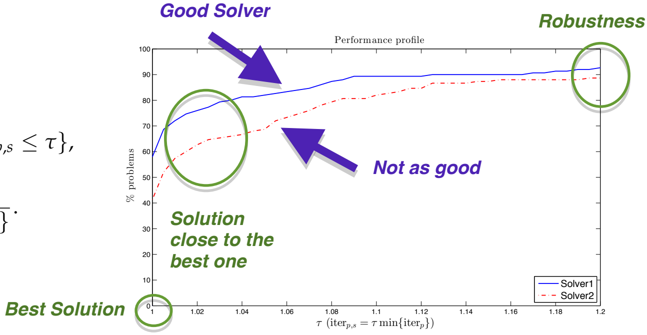
Dolan, E. D and Moré, J. J. Benchmarking optimization software with performance profiles. *Mathematical Programming*,91:201–213, 2002.

Benchmarking in topology optimization

- **How?** Using performance profiles.
  - Evaluate the cumulative ratio for a performance metric.
  - Represent for each solver, the percentage of instances that achieve a criterion for different ratio values.

$$\rho_s(\tau) = \frac{1}{n}\text{size}\{p \in P : r_{p,s} \leq \tau\},$$

$$r_{p,s} = \frac{\text{iter}_{p,s}}{\min\{\text{iter}_{p,s} : s \in S\}}.$$

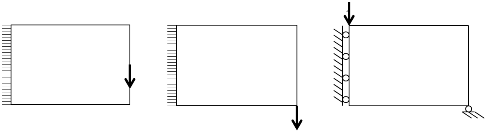


Dolan, E. D and Moré, J. J. Benchmarking optimization software with performance profiles. *Mathematical Programming*,91:201–213, 2002.

Benchmark set of topology optimization problems

Minimum compliance /minimum volume

- Michell, Cantilever and MBB domains, respectively.

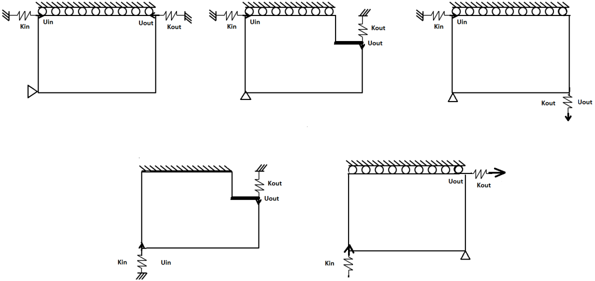


- Length ratio: Michell: 1×1, 2×1, and 3×1. Cantilever: 2×1, and 4×1. MBB: 1×2, 1×4, 2×1, and 4×1.
- Discretization: 20, 40, 60, 80, 100 elements per ratio.
- Volume constraint: $0.1 - 0.5$.
- Compliance constraint: 1, 1.25, $1.5 \times C$. Where $C = \mathbf{f}^T \mathbf{K}^{-1}(\mathbf{t}_0)\mathbf{f}$.
- Total Problems Compliance: 225.
- Total Problems Volume: 135.

Benchmark set of topology optimization problems

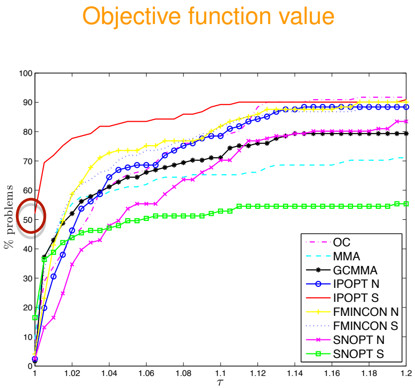
Compliant mechanism design

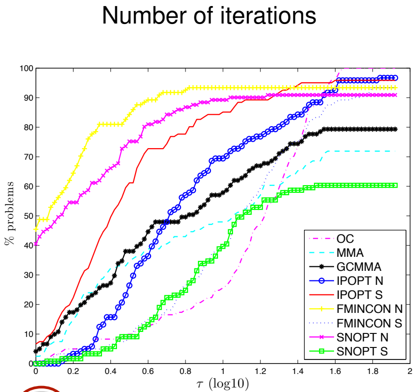
- Force inverter, Compliant gripper, Amplifier, Compliant lever, and Crimper domain examples, respectively.



- Length ratio: 1×1 and 2×1.
- Volume constraint: $0.2 - 0.4$
- Discretization: $20, 40, 60, 80, 100$ elements per ratio.
- Total Problems Mechanism Design: 150.

Performance profiles for minimum compliance problems



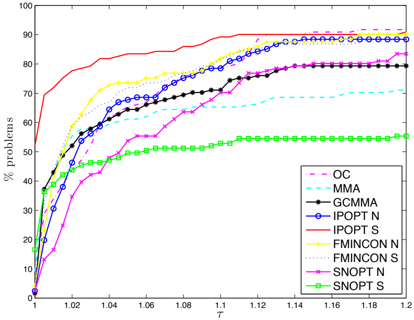


Performance profiles in a reduce test set of 121 instances.

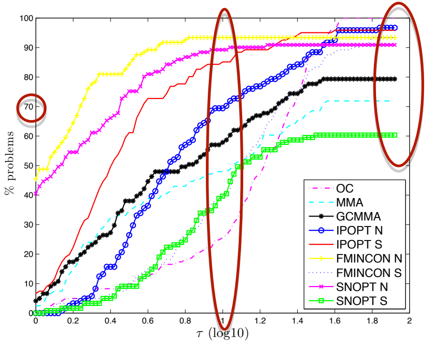
Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Performance profiles for minimum compliance problems

Objective function value



Number of iterations

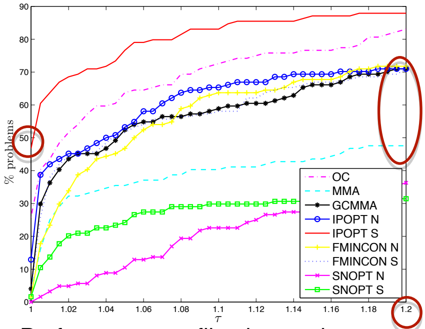


Performance profiles in a reduce test set of 121 instances.

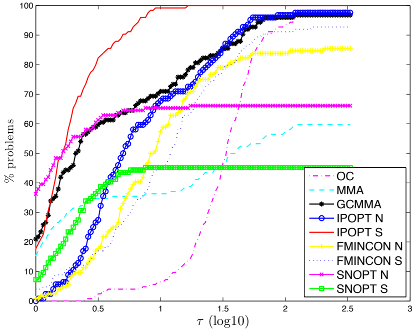
Penalization of problems with KKT error higher than $\omega = 1e-3$.

Performance profiles for compliant mechanism design problems

Objective function value



Number of iterations

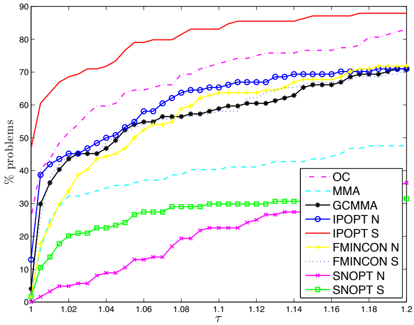


Performance profiles in a reduce test set of 124 instances.

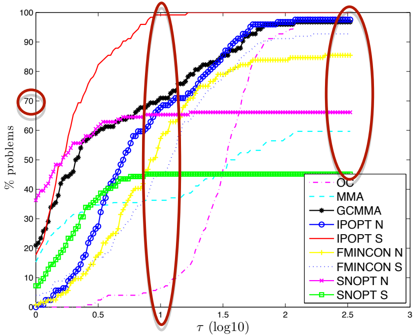
Penalization of problems with KKT error higher than $\omega = 1e-3$.

Performance profiles for compliant mechanism design problems

Objective function value



Number of iterations

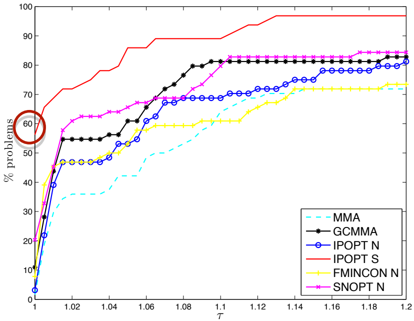


Performance profiles in a reduce test set of 124 instances.

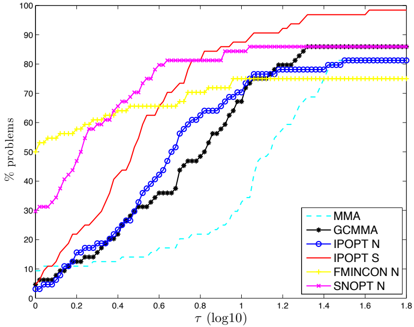
Penalization of problems with KKT error higher than $\omega = 1e-3$.

Performance profiles for minimum volume problems

Objective function value



Number of iterations

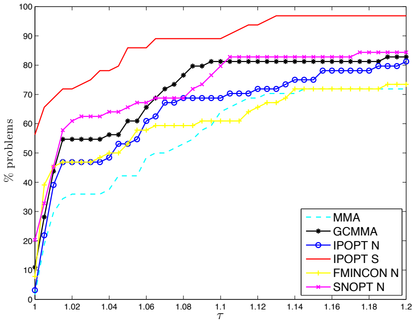


Performance profiles in a reduce test set of 64 instances.

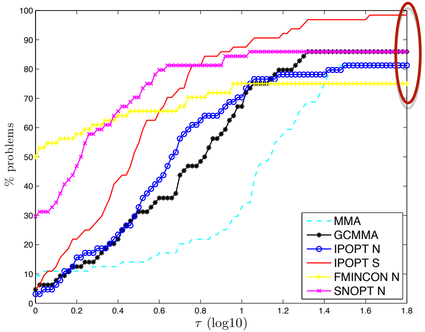
Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Performance profiles for minimum volume problems

Objective function value



Number of iterations



Performance profiles in a reduce test set of 64 instances.

Penalization of problems with KKT error higher than $\omega = 1e-3$.

Conclusions

- **Important contributions.**
  - Develop a large topology optimization test set.
  - Introduction to performance profiles in topology optimization.
  - First extensive comparative study of the performance of the state-of-art topology optimization methods with general non-linear optimization solvers.

Conclusions

- **Important contributions.**
  - Develop a large topology optimization test set.
  - Introduction to performance profiles in topology optimization.
  - First extensive comparative study of the performance of the state-of-art topology optimization methods with general non-linear optimization solvers.
- **What is missing?**
  - Large-scale problems, 3D domains, advance elements,...
  - Other regularization schemes.
  - Different formulations: Displacement constraint, stress constraints,...
  - More optimization solvers.
  - ...

Conclusions

- **Important contributions.**
  - Develop a large topology optimization test set.
  - Introduction to performance profiles in topology optimization.
  - First extensive comparative study of the performance of the state-of-art topology optimization methods with general non-linear optimization solvers.
- **What is missing?**
  - Large-scale problems, 3D domains, advance elements,...
  - Other regularization schemes.
  - Different formulations: Displacement constraint, stress constraints,...
  - More optimization solvers.
  - ...

- **What can we conclude from the performance profiles?**
  - GCMMA outperforms MMA.
  - GCMMA and MMA tend to obtain a design with large KKT error.
  - IPOPT-S produces better designs using few number of iterations
  - IPOPT-S is the most robust solver in the study.
  - The SAND formulation requires lot of memory and computational time.

THANK YOU !!!

This research is funded by the Villum Foundation through the research project Topology Optimization – the Next Generation (NextTop).