#### **Technical University of Denmark**



#### **Multidisciplinary Free Material Optimization of 2D and Laminate Structures**

**Weldeyesus, Alemseged Gebrehiwot; Stolpe, Mathias; Gaile, Stefanie**

Publication date: 2014

Document Version Peer reviewed version

[Link back to DTU Orbit](http://orbit.dtu.dk/en/publications/multidisciplinary-free-material-optimization-of-2d-and-laminate-structures(ca7045f8-6432-450c-9b3c-f843f23e9028).html)

Citation (APA):

Weldeyesus, A. G., Stolpe, M., & Gaile, S. (2014). Multidisciplinary Free Material Optimization of 2D and Laminate Structures [Sound/Visual production (digital)]. 10th World Congress on Structural and Multidisciplinary Optimization, Orlando, FL, United States, 19/05/2013

#### **DTU Library Technical Information Center of Denmark**

#### **General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.

- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

#### **Multidisciplinary Free Material Optimization of 2D and Laminate Structures**

Alemseged G Weldeyesus, PhD student Mathias Stolpe, Senior Scientist Stefanie Gaile





# **Free Material Optimization (FMO)**



FMO is a one of the different approaches to structural optimization.

In FMO,

- the design variable is the full material tensor,
- can vary freely at each point of the design domain,
- necessary condition for physical attainability is the only requirement.

M. P. Bendsøe, J. M. Guedes, R. B. Haber, P. Pedersen, and J. E. Taylor. An analytical model to predict optimal material properties in the context of optimal structural design. *J. Appl. Mech. Trans. ASME, 61:930–937, 1994.*

M. Kočvara and M. Stingl. Free material optimization for stress constraints. *Structural and Multidisciplinary Optimization*, 33:323-335, 2007.

M. Kočvara, M. Stingl, and J. Zowe. Free material optimization: Recent progress. *Optimization, 57(1):79–100, 2008.* 

# **Optimal solution**

- Solution to FMO yields optimal distribution of the material as well as optimal local material properties.
- The obtained design can be considered as an ultimately best structure.
- Conceptual, since it is difficult (or actually impossible) to manufacture a structure such that its property varies at each point of the design.
- FMO can be used to generate benchmarks and to propose novel ideas for new design situations.

# **The FMO problem formulation**



Mechanical assumptions

• static loads,

 $2D$ 

- linear elasticity,
- anisotropic material.

The basic minimum compliance problem(Discrete)

$$
\begin{array}{ll}\n\text{min} & \sum_{l} w_{l} f_{l}^{T} u_{l} \\
\text{subject to} & K(E) u_{l} = f_{l}, \qquad l = 1, \dots, L \\
\frac{\rho \leq Tr(E_{i}) \leq \rho}{\rho}, \qquad i = 1, \dots, m \\
\frac{E_{i} - \delta I \geq 0}{\rho}, \qquad i = 1, \dots, m \\
\sum_{i=1}^{m} Tr(E_{i}) \leq \overline{V}\n\end{array}
$$

Additional structural requirements can also be included through constraints

- on local stresses
- on local strains
- on displacement  $Cu \leq d$

 $\sum_{k=1}^N \bigl\| E_i B_{ik} u \bigr\|^2 \leq$ *G*  $E_i B_{ik} u \vert \vert^2 \leq s$ 1 2 σ  $\sum_{k=1}^{\infty} ||B_{ik}u||^2 \leq s$ *G*  $k=1$ σ

Adding such constraints destroys suitable problem properties, e.g. convexity.

*k*

## **Goals**

- Propose FMO model for laminate structures.
- Extend existing robust and efficient primal dual interior point method for nonlinear programming to FMO problems. ( second order method)



#### **FMO for laminate structures**

#### Based on FSDT

$$
\begin{bmatrix}\n\sigma_{11} \\
\sigma_{22} \\
\sigma_{23} \\
\sigma_{31} \\
\sigma_{12}\n\end{bmatrix} =\n\begin{bmatrix}\nC_{11} & C_{12} & 0 & 0 & C_{16} \\
C_{12} & C_{22} & 0 & 0 & C_{26} \\
0 & 0 & C_{44} & C_{45} & 0 \\
0 & 0 & C_{54} & C_{55} & 0 \\
C_{16} & C_{26} & 0 & 0 & C_{66}\n\end{bmatrix}\n\begin{bmatrix}\n\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{23} \\
\varepsilon_{31} \\
\varepsilon_{12}\n\end{bmatrix}
$$

$$
C = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{21} \\ C_{16} & C_{21} & C_{66} \end{bmatrix}, \quad D = \begin{bmatrix} C_{44} & C_{45} \\ C_{54} & C_{55} \end{bmatrix}
$$

#### Minimum compliance problem

 $t_k Tr(C_{ik}) + \frac{1}{2} t_k Tr(D_{ik}) \leq V$  $C_{ik} - \delta I \geq 0, D_{ik} - \delta I \geq 0,$   $i = 1,...,m, k = 1,...,N$  $\leq t_k Tr(C_{ik}) + \frac{1}{2} t_k Tr(D_{ik}) \leq \rho, \quad i = 1, ..., m, k = 1, ..., N$  $K(C, D)(u_1, \theta_1) = f_1,$   $l = 1, ..., L$  $w_l c(u)$  $u_l, \theta_l, C, D$  *l*  $u_l^{(l)}(u_l, o_l)$ *i k*  $\sum_{i,k} (t_k Tr(C_{ik}) + \frac{1}{2} t_k Tr(D_{ik})) \le$ subject to  $K(C, D)(u_1, \theta_1) = f_1$ *l l*  $_l$ ,  $v_l$  $\sum w_i c(u_i, \theta_i)$  $(t_k Tr(C_{ik}) + \frac{1}{2})$  $\rho \le t_k Tr(C_{ik}) + \frac{1}{2} t_k Tr(D_{ik}) \le \overline{\rho}, \quad i = 1, ..., m, k = 1, ...$  $, C,$ ,  $\theta$ θ min

**DTU Wind Energy, Technical University of Denmark** 6

WCSMO10, Orlando,USA, May 19-24, 2013

## **Optimization method**

- All FMO problems are SemiDefinite Programs (SDP), an optimization problem with many matrix inequalities.
- On the extension of primal-dual interior point methods for nonlinear programming to FMO problems (SDP).
- FMO problems can be represented by

 $X_i \geq 0,$   $i = 1, \cdots, m$ subject to  $g_j(X, u) \le 0,$   $j = 1, \dots, k$  (*P*)  $f$   $(X, u)$ X∈S,u∈R<sup>n</sup> min

 $f : S \times R^n \to R$ ,  $g : S \times R^n \to R^k$ , sufficiently smooth.

Introducing slack variables, the associated barrier problem is

$$
\begin{array}{ll}\n\text{min} & f(\mathsf{X}, u) - \mu \sum_{i} \ln(\det(\mathsf{X}_i)) - \mu \sum_{j} \ln(s_j) \\
\text{subject to} & g_j(\mathsf{X}, u) + s_j = 0, \quad j = 1, \cdots, k.\n\end{array} \tag{BP}
$$

We solve problem (BP) for a sequence of barrier parameter  $\mu_k \rightarrow 0$ .





Without stress constraints

- 28800 design variables,
- ~9600 nonlinear constraints,
- 4800 matrix inequalities

Design domain, bc, forces



Optimal density distribution



#### Optimal strain norms Optimal stress norms



- 43 iterations
- Higher stress concentration around the re-entrant corner

decreased by 30%)

• 28800 design variables

• 4800 matrix inequalities

(stress constraints)

 $\bullet$  ~9600 nonlinear constraints

With stress constraints (max stress is

• 4800 additional nonlinear constraints



Optimal density distribution



- 72 iterations
- Higher stress concentrations are distributed to negibhour regions







M. Kočvara and M. Stingl. Free material optimization for stress constraints. *Structural and Multidisciplinary Optimization*, 33:323-335, 2007.

WCSMO10, Orlando,USA, May 19-24, 2013



4 layers

Design domain, bc and forces



- No distinction between layers.
- Similar to 2D results.
- No out plane deformation.
- No material for D,

min(trace( $C$ ))/max(trace( $D$ ))=2.2413

Optimal density distribution



Optimal density distribution, Layer=1 Optimal density distribution, Layer=2



Optimal density distribution, Layer=3 Optimal density distribution, Layer=4





4 layers

- 129600 design variables
- 28800 matrix inequalities

Design domain, bc and forces



- 47 iterations
- Symmetric laminate
- High material distribution on top and bottom layers

Optimal density distribution, Layer=1 Optimal density distribution, Layer=2



Optimal density distribution, Layer=3 Optimal density distribution, Layer=4









In both cases, we have

- single layer,
- 90000 design variables,



#### **Conclusion**

- The extended primal dual interior point method solves FMO problems in a reasonable number of iterations.
- Number of iterations is almost independent to problem size.
- FMO has been extended to optimal design of laminate structures.

#### Thank you for your attention !