Multidisciplinary Free Material Optimization of 2D and Laminate Structures

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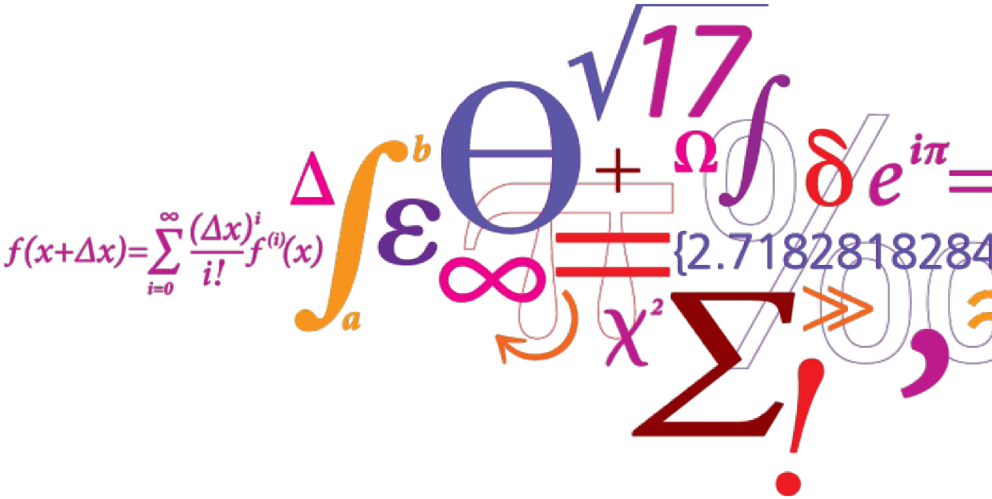

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Department of Wind Energy

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Free Material Optimization (FMO)

FMO is a one of the different approaches to structural optimization.

In FMO,

- the design variable is the full material tensor,
- can vary freely at each point of the design domain,
- necessary condition for physical attainability is the only requirement.

M. P. Bendsøe, J. M. Guedes, R. B. Haber, P. Pedersen, and J. E. Taylor. An analytical model to predict optimal material properties in the context of optimal structural design. *J. Appl. Mech. Trans. ASME, 61:930–937, 1994.*

M. Kočvara and M. Stingl. Free material optimization for stress constraints. *Structural and Multidisciplinary Optimization*, 33:323-335, 2007.

M. Kočvara, M. Stingl, and J. Zowe. Free material optimization: Recent progress. *Optimization, 57(1):79–100, 2008.*

Optimal solution

• Solution to FMO yields optimal distribution of the material as well as optimal local material properties.

• The obtained design can be considered as an ultimately best structure.

• Conceptual, since it is difficult (or actually impossible) to manufacture a structure such that its property varies at each point of the design.

• FMO can be used to generate benchmarks and to propose novel ideas for new design situations.

The FMO problem formulation

Mechanical assumptions

- static loads,
- linear elasticity,
- anisotropic material.

The basic minimum compliance problem(Discrete)

2D, 3D structures

$$\begin{aligned} \min_{u_l, E} \quad & \sum_l w_l f_l^T u_l \\ \text{subject to} \quad & K(E)u_l = f_l, && l = 1,\ldots,L \\ & \underline{\rho} \le Tr(E_i) \le \overline{\rho}, && i = 1,\ldots,m \\ & E_i - \delta I \succeq 0, && i = 1,\ldots,m \\ & \sum_{i=1}^{m} Tr(E_i) \le \overline{V} \end{aligned}$$

Additional structural requirements can also be included through constraints

- on local stresses $\sum_{k=1}^{G} \|E_i B_{ik} u\|^2 \le s_\sigma$
- on local strains $\sum_{k=1}^{G} \|B_{ik} u\|^2 \le s_\sigma$
- on displacement $Cu \le d$

Adding such constraints destroys suitable problem properties, e.g. convexity.

Goals

- Propose FMO model for laminate structures.
- Extend existing robust and efficient primal dual interior point method for nonlinear programming to FMO problems. (second order method)

FMO for laminate structures

Based on FSDT

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & 0 & 0 & C_{26} \\ 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & C_{54} & C_{55} & 0 \\ C_{16} & C_{26} & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{bmatrix}$$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{21} \\ C_{16} & C_{21} & C_{66} \end{bmatrix}, \quad D = \begin{bmatrix} C_{44} & C_{45} \\ C_{54} & C_{55} \end{bmatrix}$$

Minimum compliance problem

$$\min_{u_l, \theta_l, C, D} \quad \sum_l w_l c(u_l, \theta_l)$$

subject to

$$K(C,D)(u_l, \theta_l) = f_l, \qquad l = 1, \ldots, L$$

$$\underline{\rho} \le t_k Tr(C_{ik}) + \frac{1}{2} t_k Tr(D_{ik}) \le \overline{\rho}, \qquad i = 1, \ldots, m, \; k = 1, \ldots, N$$

$$C_{ik} - \delta I \succeq 0, \; D_{ik} - \delta I \succeq 0, \qquad i = 1, \ldots, m, \; k = 1, \ldots, N$$

$$\sum_{i,k} (t_k Tr(C_{ik}) + \frac{1}{2} t_k Tr(D_{ik})) \le \overline{V}$$

Optimization method

- All FMO problems are SemiDefinite Programs (SDP), an optimization problem with many matrix inequalities.
- On the extension of primal-dual interior point methods for nonlinear programming to FMO problems (SDP).
- FMO problems can be represented by

$$\begin{aligned} \min_{\mathrm{X}\in\mathrm{S}, u\in\mathrm{R}^n} \quad & f(\mathrm{X},u) & & \\ \text{subject to} \quad & g_j(\mathrm{X},u) \le 0, & j = 1,\cdots,k & \qquad (P) \\ & \mathrm{X}_i \succeq 0, & i = 1,\cdots,m & \end{aligned}$$

$f: \mathrm{S}\times\mathrm{R}^n \to \mathrm{R}$, $g: \mathrm{S}\times\mathrm{R}^n \to \mathrm{R}^k$, sufficiently smooth.

Introducing slack variables, the associated barrier problem is

$$\begin{aligned} \min_{\mathrm{X}\in\mathrm{S}, u\in\mathrm{R}^n, s\in\mathrm{R}^k} \quad & f(\mathrm{X},u) - \mu\sum_i \ln(\det(\mathrm{X}_i)) - \mu\sum_j \ln(s_j) & & \\ \text{subject to} \quad & g_j(\mathrm{X},u) + s_j = 0, & j = 1,\cdots,k. & \qquad (BP) \end{aligned}$$

We solve problem (BP) for a sequence of barrier parameter $\mu_k \to 0$.

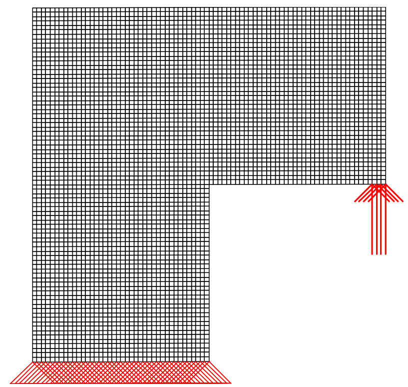
Numerical results

Without stress constraints

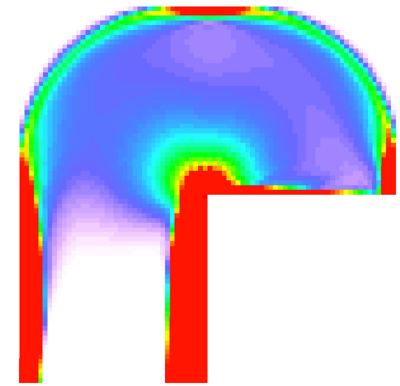
- 28800 design variables,
- ~9600 nonlinear constraints,
- 4800 matrix inequalities

- 43 iterations
- Higher stress concentration around the re-entrant corner

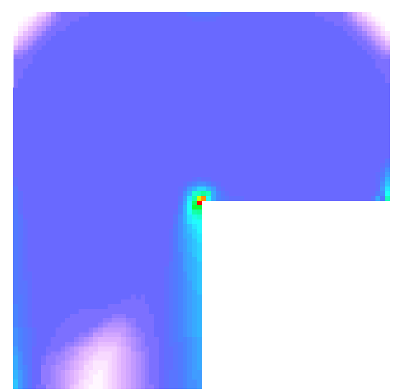
Design domain, bc, forces



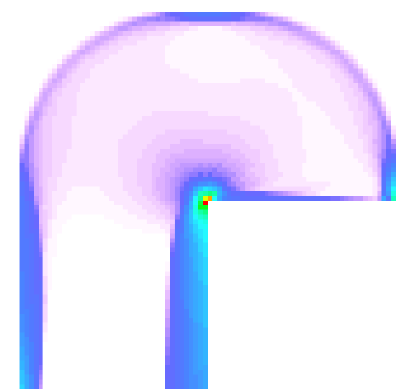
Optimal density distribution



Optimal strain norms

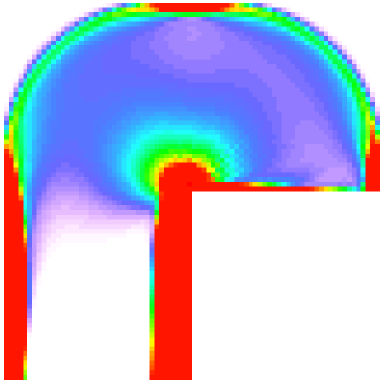


Optimal stress norms



Numerical results ...

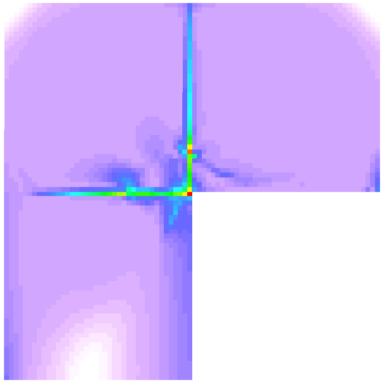
Optimal density distribution



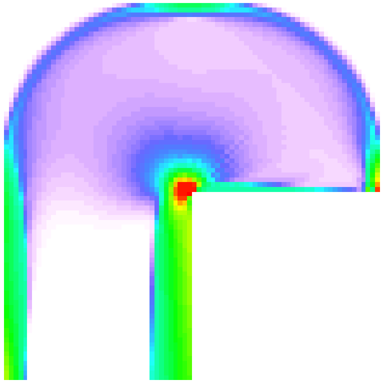
With stress constraints (max stress is decreased by 30%)

- 28800 design variables
- ~9600 nonlinear constraints
- 4800 matrix inequalities
- 4800 additional nonlinear constraints (stress constraints)

Optimal strain norms



Optimal stress norms



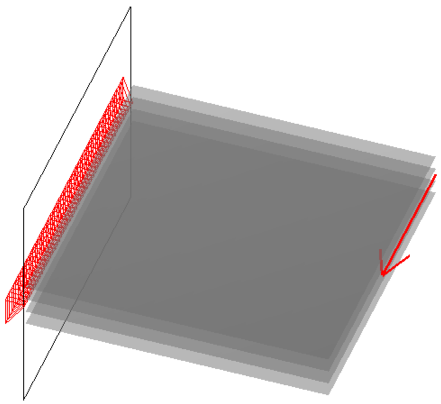
- 72 iterations
- Higher stress concentrations are distributed to negibhour regions

M. Kočvara and M. Stingl. Free material optimization for stress constraints. *Structural and Multidisciplinary Optimization*, 33:323-335, 2007.

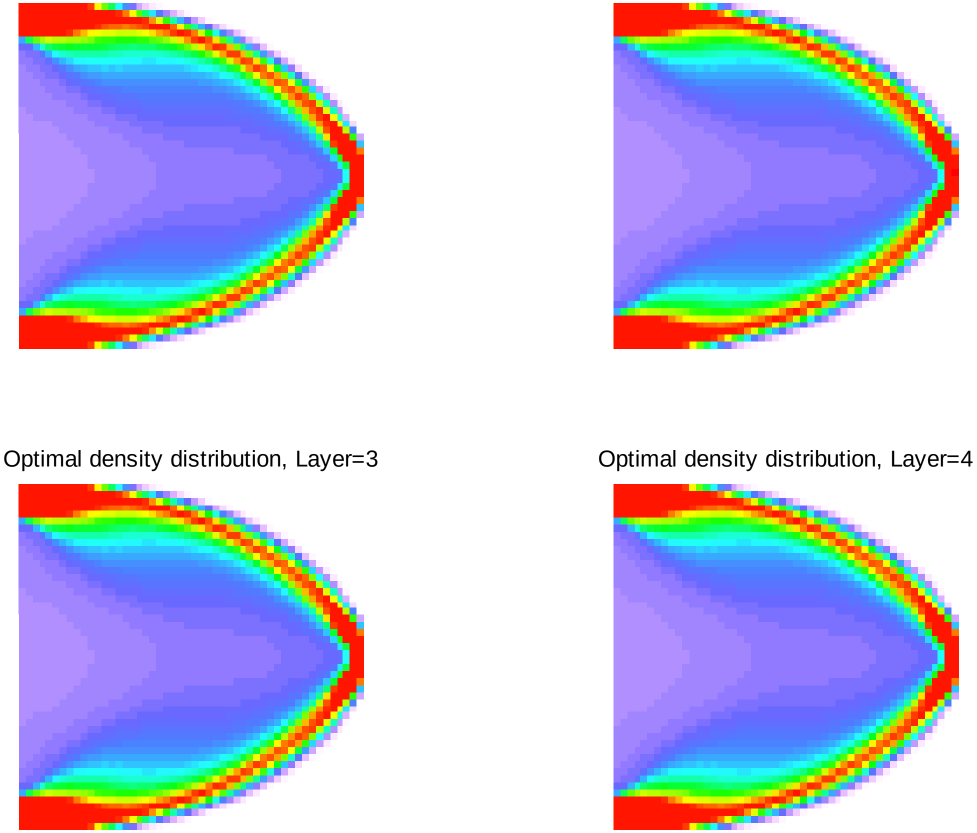
Numerical results …

4 layers

Design domain, bc and forces



Optimal density distribution



- No distinction between layers.
- Similar to 2D results.
- No out plane deformation.
- No material for D,

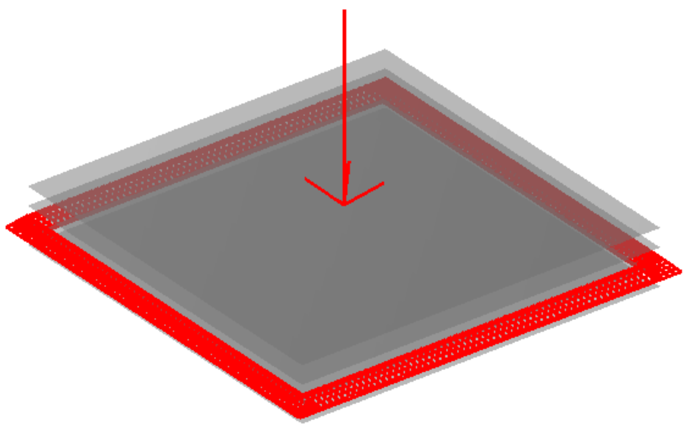
  min(trace(C))/max(trace(D))=2.2413

Numerical results ...

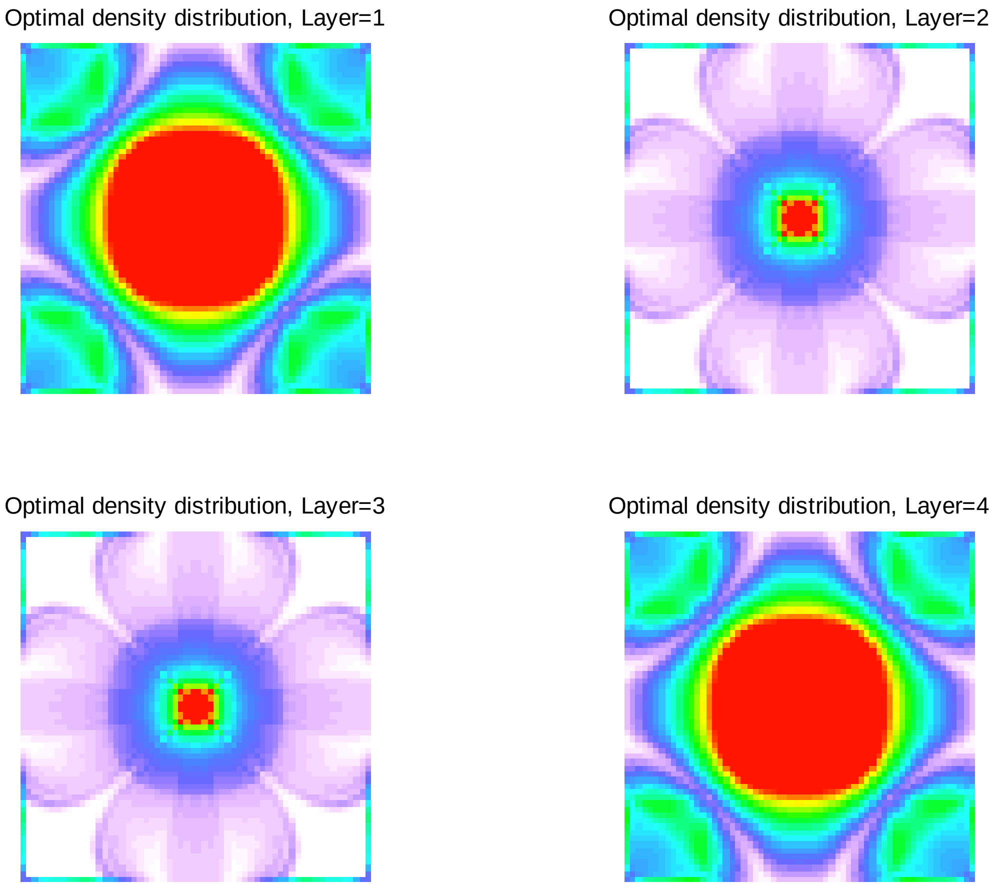
4 layers

- 129600 design variables
- 28800 matrix inequalities

Design domain, bc and forces



Optimal density distribution



- 47 iterations
- Symmetric laminate
- High material distribution on top and bottom layers

Numerical results ...

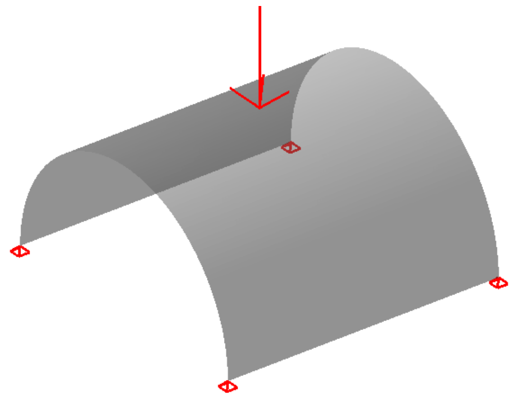
In both cases, we have

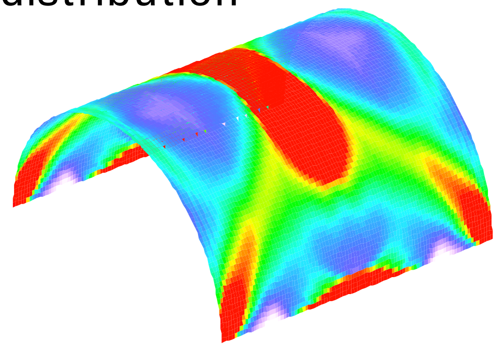
- single layer,
- 90000 design variables,
- 20000 matrix inequalities.

Design domain, bc and forces

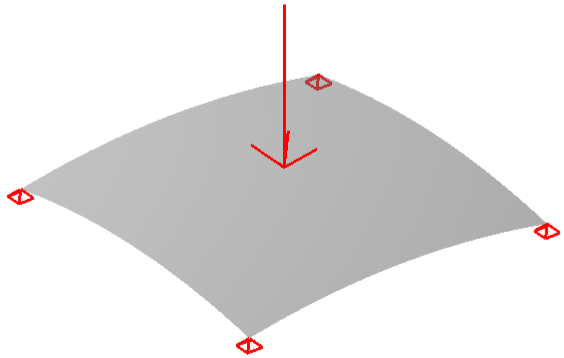
Optimal density distribution

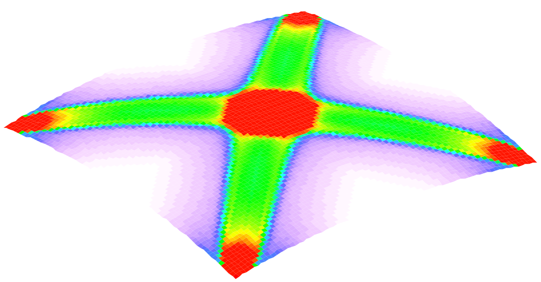
44 iterations





47 iterations





Conclusion

- The extended primal dual interior point method solves FMO problems in a reasonable number of iterations.
- Number of iterations is almost independent to problem size.
- FMO has been extended to optimal design of laminate structures.

Thank you for your attention !