Multidisciplinary Free Material Optimization for Laminated Plates and Shells

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Publication date:
2014

Document Version
Publisher's PDF, also known as Version of record

Multidisciplinary Free Material Optimization for Laminated Plates and Shells

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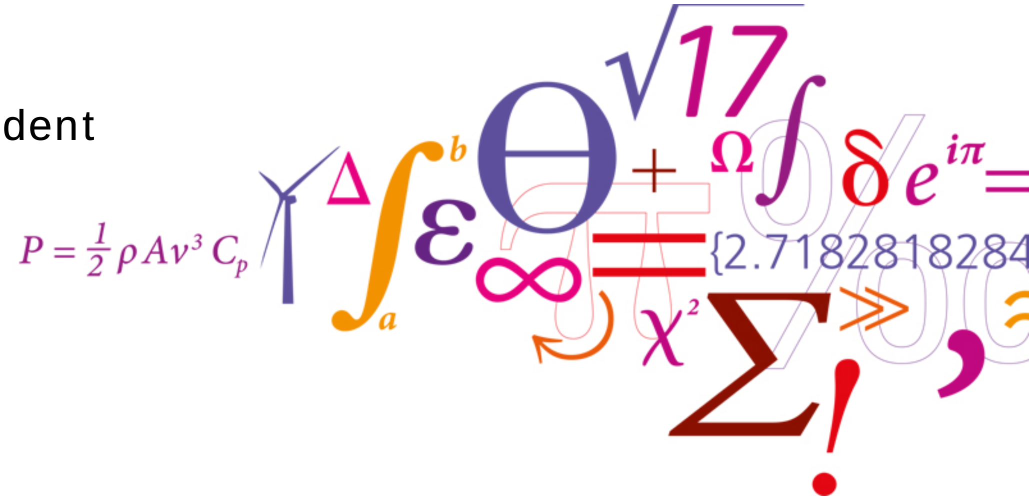

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Department of Wind Energy

Free Material Optimization (FMO)

FMO is a one of the different approaches to structural optimization.

In FMO,

- the design variable is the full material tensor,
- can vary freely at each point of the design domain,
- necessary condition for physical attainability is the only requirement.

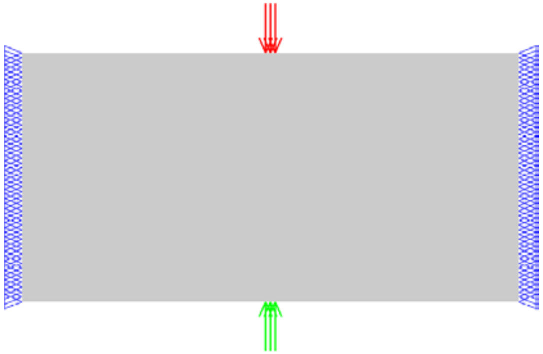
M. P. Bendsøe, J. M. Guedes, R. B. Haber, P. Pedersen, and J. E. Taylor. An analytical model to predict optimal material properties in the context of optimal structural design. *J. Appl. Mech. Trans. ASME, 61:930–937, 1994.*

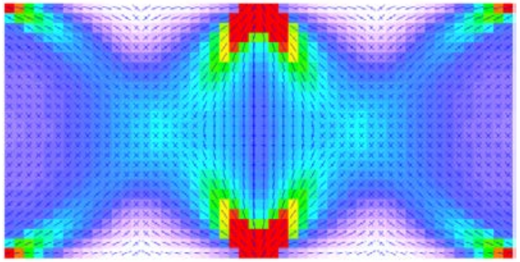
M. Kočvara and M. Stingl. Free material optimization for stress constraints. *Structural and Multidisciplinary Optimization*, 33:323-335, 2007.

M. Kočvara, M. Stingl, and J. Zowe. Free material optimization: Recent progress. *Optimization, 57(1):79–100, 2008.*

Optimal solution

- Solution to FMO yields optimal distribution of the material as well as optimal local material properties.
- The obtained design can be considered as an ultimately best structure.
- Conceptual, since it is difficult (or actually impossible) to manufacture a structure such that its property varies at each point of the design.
- FMO can be used to generate benchmarks and to propose novel ideas for new design situations.





The FMO problem formulation (solids)

Mechanical assumptions

- static loads,
- linear elasticity,
- anisotropic material.

The basic minimum compliance problem(Discrete)

$$\min_{u_l, E} \quad \sum_l w_l f_l^T u_l$$

$$\begin{aligned} \text{subject to} \quad & A(E)u_l = f_l, && l = 1,\dots,L \\ & \underline{\rho} \le Tr(E_i) \le \overline{\rho}, && i = 1,\dots,m \\ & E_i - \delta I \succeq 0, && i = 1,\dots,m \\ & \sum_{i=1}^{m} Tr(E_i) \le \overline{V} \end{aligned}$$

2D, 3D structures

Constraints on local stresses

- $$\|\sigma_{il}\|^2 := \int_{\Omega_i} \|\sigma_l\|^2 d\Omega \le s_\sigma, \qquad \sigma = E\varepsilon$$

- Highly nonlinear involving matrix variables.
- Adding such constraints destroys suitable problem properties, e.g. convexity.

Goals

- Propose FMO model for laminated plates and shells. (Such models are not available today)
- Develop special purpose optimization method that can efficiently solve FMO problems. (FMO problems lead to large-scale nonlinear SDP problems)

FMO for laminated plates and shells

Based on FSDT

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & 0 & 0 & C_{26} \\ 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & C_{54} & C_{55} & 0 \\ C_{16} & C_{26} & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{bmatrix}$$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{21} \\ C_{16} & C_{21} & C_{66} \end{bmatrix}, \quad D = \begin{bmatrix} C_{44} & C_{45} \\ C_{54} & C_{55} \end{bmatrix}$$

- (C,D), fixed within a layer in the thickness direction.
- Slightly violates the idea of FMO.

FMO for laminated plates and shells contd…

Minimum compliance problem

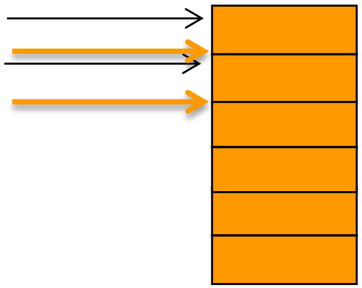
$$
\begin{aligned}
\min_{u_l,\theta_l,C,D} \quad & \sum_l w_l f_l^T (u_l,\theta_l) \\
\text{subject to} \quad & A(C,D)(u_l,\theta_l) = f_l, \qquad l=1,\dots,L \\
& \underline{\rho} \le t_k Tr(C_{ik}) + \frac{1}{2} t_k Tr(D_{ik}) \le \overline{\rho}, \quad i=1,\dots,m,\ k=1,\dots,N \\
& C_{ik} - \delta I \succeq 0,\ D_{ik} - \delta I \succeq 0, \qquad i=1,\dots,m,\ k=1,\dots,N \\
& \sum_{i,k} (t_k Tr(C_{ik}) + \frac{1}{2} t_k Tr(D_{ik})) \le \overline{V}
\end{aligned}
$$

Stress constraints

- Linear stress variation across the thickness within a layer.
- Two stress evaluations in each layer (at the top and lower surfaces) over each finite element to capture stress extremities.

$$\left\|\sigma^{a}_{ik,l}\right\|^2 := \int_{\omega_i} \left\|\sigma^{a}_{n,l}\right\|^2 d\omega \leq s_\sigma, \quad i=1,\ldots,m,\ k=1,\ldots,N$$

$$\left\|\sigma^{b}_{ik,l}\right\|^2 := \int_{\omega_i} \left\|\sigma^{b}_{n,l}\right\|^2 d\omega \leq s_\sigma, \quad i=1,\ldots,m,\ k=1,\ldots,N$$



Optimization method

- Primal-dual interior point method (second-order method).
- Combines standard known interior point methods for nonlinear programmings & linear SDPs.

Optimization method contd…

FMO problems can be represented by

$$\begin{aligned} \min_{X \in S, u \in R^n} \quad & f(X,u) \\ \text{subject to} \quad & g_j(X,u) \leq 0, \quad && j = 1,\cdots,k \qquad (P) \\ & X_i \succeq 0, \quad && i = 1,\cdots,m \end{aligned}$$

$f : S \times R^n \to R,\ g : S \times R^n \to R^k$, sufficiently smooth.

Introducing slack variables and barrier paramter μ, the associated barrier problem is

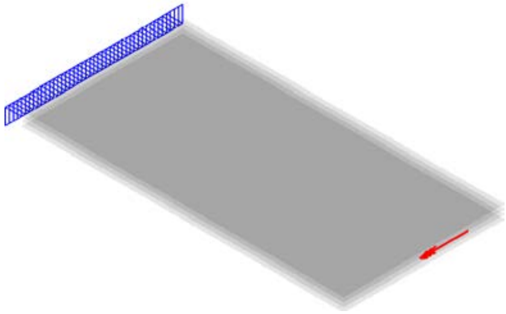
$$\begin{aligned} \min_{X \in S, u \in R^n, s \in R^k} \quad & f(X,u) - \mu \sum_i \ln(\det(X_i)) - \mu \sum_j \ln(s_j) \\ \text{subject to} \quad & g_j(X,u) + s_j = 0, \quad j = 1,\cdots,k. \qquad (BP) \end{aligned}$$

We solve problem (BP) for a sequence of barrier parameter $\mu_k \to 0$.

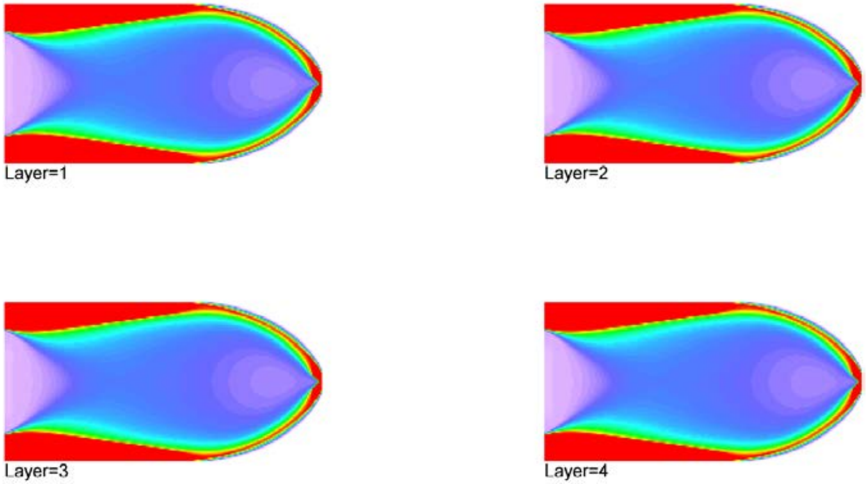
Numerical results

4 layers

Design domain, bc and forces



Optimal density distribution



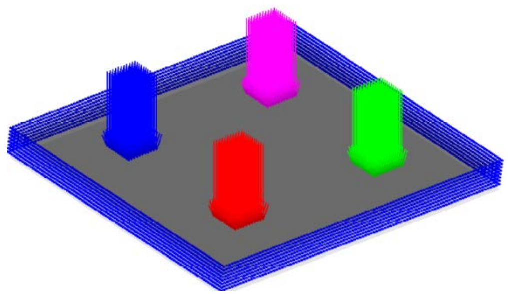
- # FEs 20,000.
- # 160,000 matrix inequalities.
- # 720,000 design variables.

- No distinction between layers.
- Similar to 2D results.
- No deformation out of the midsurface.
- No material for D.
- # 52 iterations.

Numerical results ...

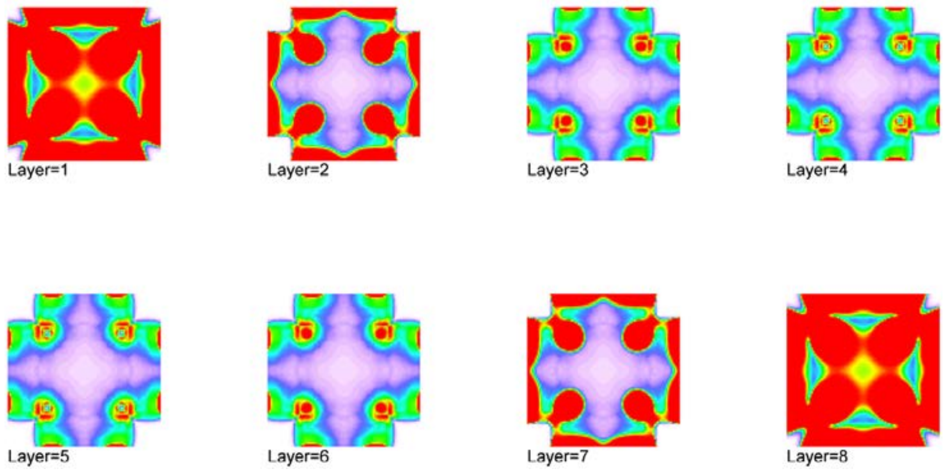
8 layers

Design domain, bc and forces



- 4 load-case.
- # FEs 10,000.
- # 160,000 matrix inequalities.
- # 720,000 design variables.

Optimal density distribution



- Symmetric laminate.
- High material distribution on top and bottom layers, implies sandwich structures. (similar results in DMO)
- # 25 iterations.

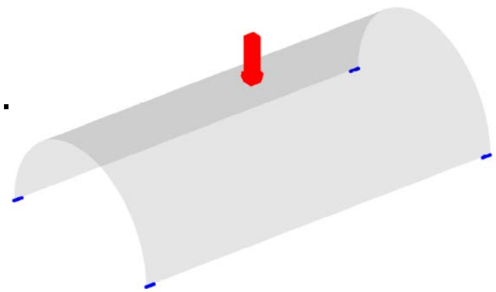
Numerical results contd…

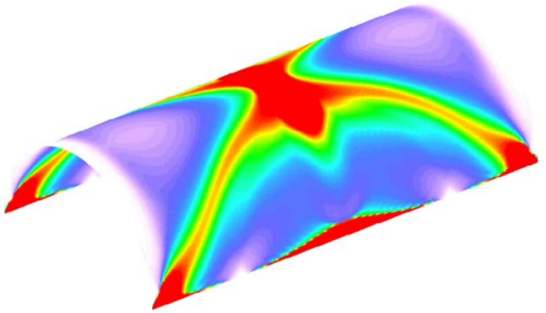
- Curved surfaces
- single layer

Design domain, bc and forces

Optimal density distribution

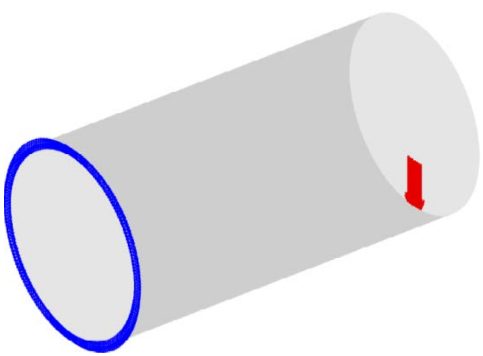
- # FEs 80,000.
- # 160,000 matrix inequalities.
- # 720,000 design variables.

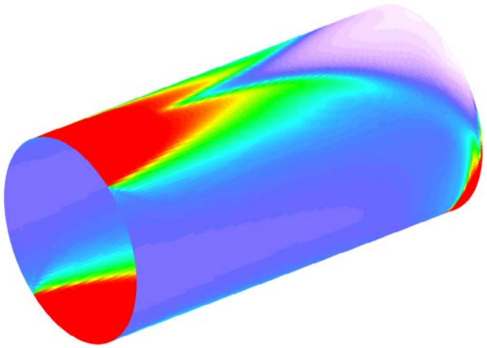




- # 54 iterations.

- # FEs 40,000.
- # 80,000 matrix inequalities.
- # 360,000 design variables.



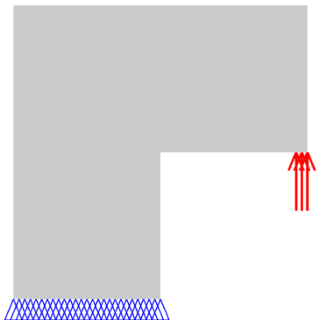


- # 51 iterations.

Numerical results contd…

Stress-constrained FMO problems

Design domain, bc and forces

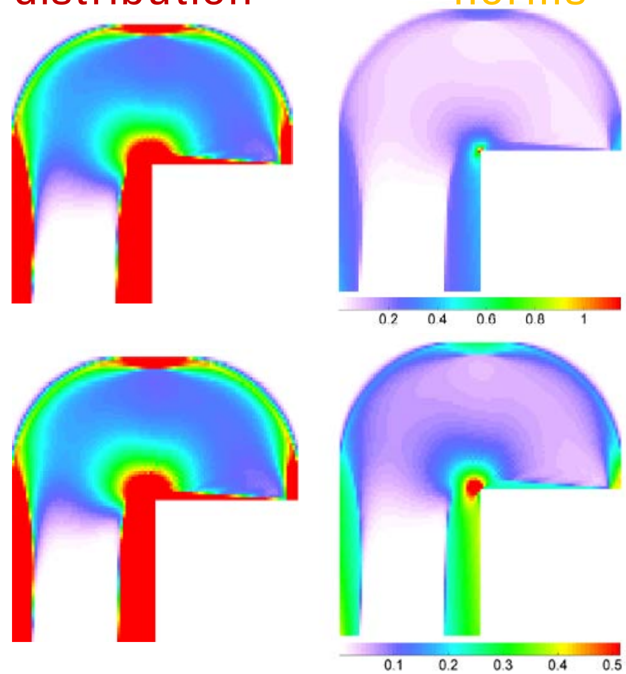


- # FEs 7,500.
- # 7,500 matrix inequalities.
- # 45,000 design variables.
- # 7,500 stress constraints.

Optimal density distribution | Optimal stress norms

Without stress constraints

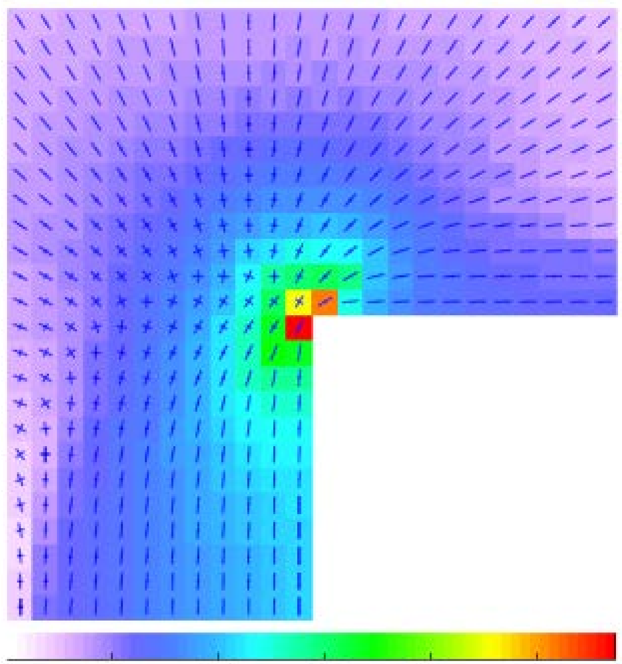
With stress constraints



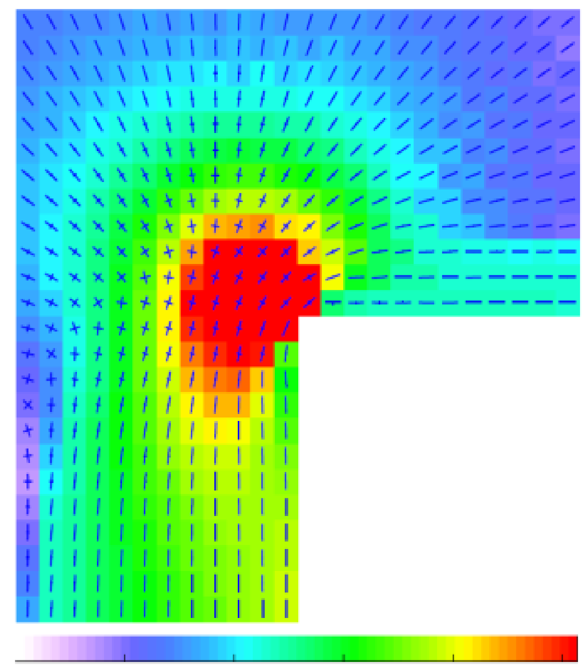
Numerical results contd…

Optimal principal stresses

Without stress constraints



With stress constraints

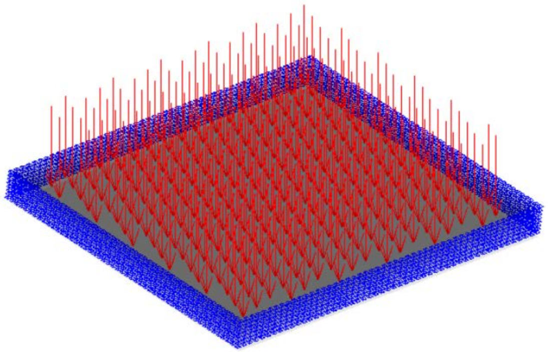


	Without stress constraints	With stress constraints , 60 %
# iterations	30	85
compliance	1.8951	1.9281

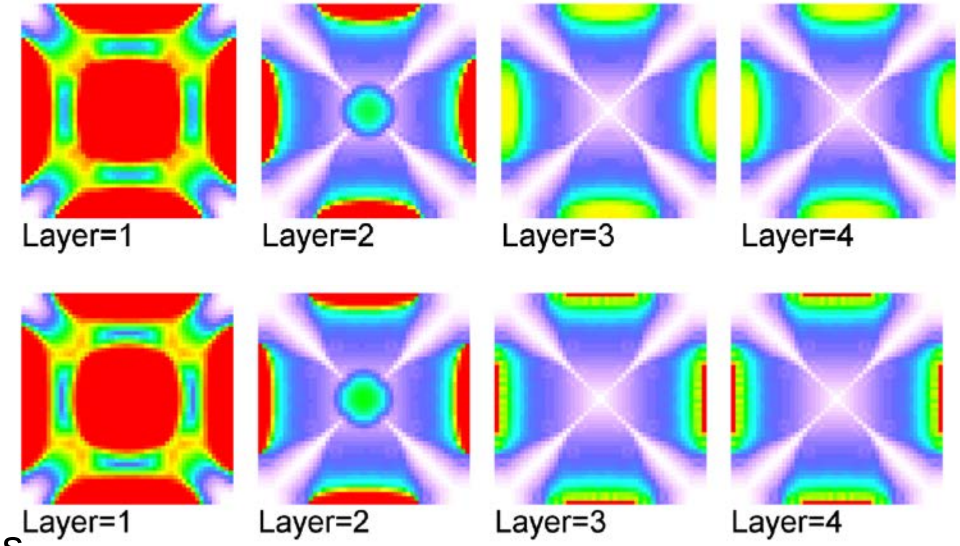
Numerical results contd…

8 layers

Design domain, bc and forces



Optimal density distribution of the top 4 layers



Without stress constraints

With stress constraints

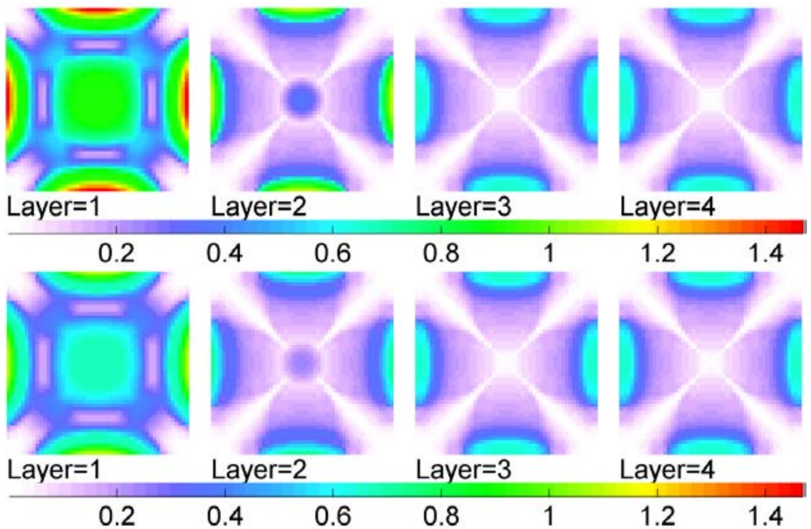
- # FEs 2,500.
- # 40,000 matrix inequalities.
- # 180,000 design variables.
- # 40,000 stress constraints.

Optimal density distribution – no such a significant difference

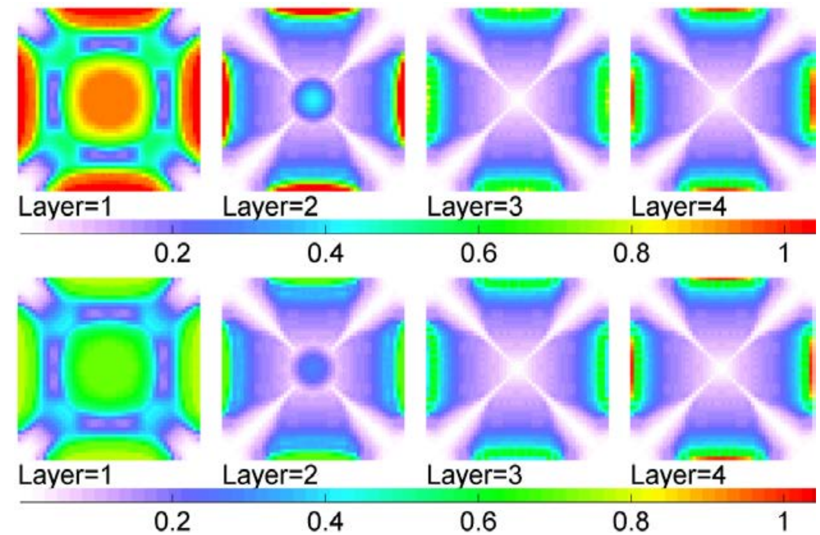
Numerical results contd…

Optimal stress norms at the upper and lower surfaces of the first top four layers

Without stress constraints



With stress constraints



Upper

Lower

	Without stress constraints	With stress constraints, 30%
# iterations	47	115
compliance	5.4771	5.5624

Conclusions

- The developed optimization method solves FMO problems in a reasonable number of iterations.
- Number of iterations is almost independent to problem size.
- More number of iterations is required for solving the stress constrained problems (but expected).
- FMO has been extended to optimal design of laminated structures.
- The change of material properties plays main role in reducing high stresses in FMO.
- Reduction of high stresses is achieved at a small increase in compliance.

Thank you for your attention !