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Combined shape and topology optimization for minimization of von Mises Stress

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The Deformable Simplicial Complex (DSC) method [4] has recently been used in structural topology optimization [1] where the objective was to minimize compliance subject to a volume constraint. We will apply the same scheme as in [1] but now we minimize the maximal von Mises stress. To generate a smooth cost function, we use the p-norm as an approximation to the max function. This, in combination with the finite element discretization, gives rise to a non-optimal jagged surface. To remedy this, we introduce a filtering of the surface nodes.

The DSC method discretizes the design domain into non-overlapping triangular elements (a simplicial complex). The elements are either labelled solid (filled with material) or void (filled with air). Consequently, the structural surface is represented explicitly by the edges which are sandwiched between a void and a solid element. The shape and topology of the structure is changed by relabeling elements and moving the surface nodes. The former is a discrete step which relabels the elements from solid to void based on values of their topological derivatives. The latter uses the values of the shape derivatives to find an improved shape which is within a small perturbation of the current shape. The surface is then deformed to this improved shape by the DSC method. Advantages of the DSC method include its ability to simultaneously accommodate both topological and shape changes while maintaining well-shaped triangular elements.

In our current application, we want to minimize the maximal von Mises stress subject to a volume constraint. Since the max function is not smooth, the differentiable p-norm

$$\max(\dots, \sigma_e^{vm}, \dots) \approx \left(\sum_e (\sigma_e^{vm})^p \right)^{\frac{1}{p}}$$

is used instead as also proposed by Duysinx and Sigmund [2]. The governing equations are solved with the finite element method. Furthermore, second order basis functions are used since they solve an issue with jagged surfaces [1]. However, using the p-norm reintroduces the jagged surfaces and we will therefore use a well-known remedy, a filtering scheme. We apply a Gaussian filter to the node coordinates which is an extension of the ideas proposed by Le et al. [3]. The filtered coordinate vector $\tilde{p}_i$ of node i is

$$\tilde{p}_i = \frac{\sum_{j=1}^{n} f(d_{ij}) p_j}{\sum_{j=1}^{n} f(d_{ij})}$$

where $f_r(d_{ij})$ is a function of the distance d_{ij} between node i and j and r is the filter radius. We have chosen to use a Gaussian function such that $f(d_{ij}) = e^{-3 \cdot d_{ij}^2 / r^2}$ if $d_{ij} < r$ and $f(d_{ij}) = 0$ otherwise. Note that the gradients and move limits are also filtered to make a mathematical consistent algorithm.

We have successfully minimized the maximal Von Mises stress of several 2D problems such as the Norwegian Sock problem and the sizing problem of a hole in a plate. The results show that stresses are uniformly distributed throughout the structures, wherefore the maximal Von Mises stress is minimized. Before the conference, we plan to apply the approach to more 2D problems and if time permits, extend the approach to 3D.

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