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Ratio of torus and equivalent power to refractive cylinder and spherical equivalent in phakic lenses – a Monte-Carlo simulation study

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ABSTRACT.

Background: Spherical and astigmatic powers for phakic intraocular lenses are frequently calculated using fixed ratios of phakic lens refractive power to refractive spherical equivalent, and of phakic lens astigmatism to refractive cylinder. In this study, a Monte-Carlo simulation based on biometric data was used to investigate how variations in biometrics affect these ratios, in order to improve the calculation of implantable lens parameters.

Methods: A data set of over sixteen thousand biometric measurements including axial length, phakic anterior chamber depth, and corneal equivalent and astigmatic power was used to construct a multidimensional probability density distribution. From this, we determined the axial position of the implanted lens and estimated the refractive spherical equivalent and refractive cylinder. A generic data model resampled the density distributions and interactions between variables, and the implantable lens power was determined using vergence propagation.

Results: 50 000 artificial data sets were used to calculate the phakic lens spherical equivalent and astigmatism required for emmetropization, and to determine the corresponding ratios for these two values. The spherical ratio ranged from 1.0640 to 1.3723 and the astigmatic ratio from 1.0501 to 1.4340. Both ratios are unaffected by the corneal spherical / astigmatic powers, or the refractive cylinder, but show strong correlation with the refractive spherical equivalent, mild correlation with the lens axial position, and moderate negative correlation with axial length. As a simplification, these ratios could be modelled using a bi-variable linear regression based on the first two of these factors.

Conclusion: Fixed spherical and astigmatic ratios should not be used when selecting high refractive power phakic IOLs as their variation can result in refractive errors of up to ± 0.3 D for a 8 D lens. Both ratios can be estimated with clinically acceptable precision using a linear regression based on the refractive spherical equivalent and the axial position.

Key words: phakic lenses – refraction error – ametropia – astigmatism – Monte-Carlo simulation – IOL power calculation

Background

Over the last two decades, phakic IOLs (PIOLs) have proved to be an effective surgical procedure (Fechner et al. 1988; Fechner et al. 1989; Medical Advisory Board 2009). These IOLs are a very effective option for correcting spherical and cylindrical ametropia in young and middle aged eyes which still have a sufficient amount of physiological accommodation. Especially when other options such as LASIK are contraindicated (e.g. due to thin cornea and large amount of astigmatism or high ametropia), phakic lenses are a good surgical option for a permanent correction.

In contrast to refractive lens exchange where physiological accommodation is lost, accommodation is fully maintained with phakic lenses. Furthermore, phakic lenses have the benefit that re-alignment of the

astigmatism axis and IOL removal or exchange is possible even years after primary implantation, as they do not adhere to the capsular bag or sulcus tissue (Schrecker et al. 2013; Schrecker & Langenbucher 2016). On the other hand, if toric phakic lenses are implanted in the ciliary sulcus, rotation of the IOL axis might occur even years after surgery, reducing the effect of astigmatism correction and making re-alignment necessary.

During cataract surgery or refractive lens exchange, an optical element (the crystalline lens) is removed from the eye. Therefore, lens power calculation requires a full set of biometric data (Langenbucher et al. 2004a; Langenbucher et al. 2004b; Kohnen et al. 2005; Langenbucher et al. 2007a; Langenbucher et al. 2007b; Savini et al. 2013; Alpins et al. 2014; Abulafia et al. 2015; Kern et al. 2020) – as a minimum, target refraction, corneal curvature and axial length (Langenbucher et al. 2004a). In contrast, if an additional lens is implanted in front of the crystalline lens (phakic lens), all other optical elements stay in place. Hence, the calculation of such lenses requires only knowledge of the properties of all optical elements from the spectacle plane to the plane where the additional lens will be implanted (Langenbucher et al. 2007a; Langenbucher et al 2008; Eppig et al. 2011; Donaldson et al. 2018). This implies that the actual refraction (spherical equivalent (RSEQ) and refractive cylinder (RCYL)), target refraction, corneal equivalent power (CEQ) and astigmatism (CAST)) are mandatory for calculation. In the case of large ametropia with 6 dpt or more, the back vertex distance of the spectacle correction should also be measured. In addition, the axial position of the phakic lens has to be estimated from preoperative biometric data. As a phakic lens in front of the crystalline lens is more or less 'self-positioning', the distinct distance (vault) in front of the crystalline lens – if measured after surgery – shows typically much less variation (Alfonso et al. 2012; Schrecker et al. 2013; Schrecker & Langenbucher 2016; Hassenstein et al. 2017) compared to estimation of a capsular bag lens axial position in a cataract surgery or refractive lens exchange. With refractive IOL procedures, the correction of spherical and/or cylindrical errors (or some

portion of that) is shifted from the spectacle plane to the plane of the IOL (Langenbucher et al. 2007a; Langenbucher et al. 2008; Eppig et al. 2011).

Most of the calculation strategies are based on the classical 'van der Heijde vergence transformation formula' (Langenbucher et al. 2007a; Eppig et al. 2011), which is restricted to a spherical phakic lens. Meanwhile, some other calculations have been published which generalize the 'van der Heide formula' to spherocylindrical vergences or using 4x4 refraction and translation matrices to calculate the equivalent and astigmatic power of a toric phakic lens (Langenbucher et al. 2004b; Langenbucher et al. 2007b).

Most IOL manufacturers provide software tools or a calculation service to determine the power of spheric or toric phakic lenses. In addition, some independent online calculation tools are available. Some of these use a fixed ratio between lens torus (astigmatism) to corneal astigmatism / refractive cylinder and equivalent power of the lens to spherical equivalent at the spectacle plane. These ratios are empirically derived from (various) historical clinical results and do not consider the individual biometrical values or imaging properties of the eye. These simplifications in terms of 'one ratio fits all' may lead to large deviations in certain cases.

The **purpose of this paper** is to analyse the influence of biometric values on the ratio between the refractive power of the phakic lens and the manifest spectacle refraction.

Methods

We calculate the ratio of phakic lens refractive cylinder ($PIOL_{ast}$) to refractive cylinder (RCYL) and the ratio of the spherical equivalent of the phakic lens ($PIOL_{eq}$) to the refractive spherical equivalent (RSEQ) in a simulation model. A Monte-Carlo simulation is set-up based on a data set with 17 440 biometric data points, and the ratio for astigmatism ($ratio_{ast}$) and equivalent ($ratio_{eq}$) are evaluated for several biometric effect sizes in a linear regression model. Data processing, Monte-Carlo simulation and regression analysis were programmed using the engineering interpreter language MATLAB (Math-Works, Natick, USA, version 2019b). Clinical data sets were extracted from

IOLCon database (www.IOLCon.org) (Schwemm et al. 2017). This database provides a full set of technical specifications for intraocular lenses (IOL) on the market, together with delivery ranges and formula constants for ophthalmic surgeons to use for lens selection and lens power calculation prior to cataract surgery. An approval of this study by the local Ethics Committee was not required as this was a retrospective analysis involving biometric data only. All data had already been anonymized by the source before being transferred to us for evaluation meaning that no back-tracing to any personal patent data is possible.

Along with formula constants provided by the lens manufacturers, IOLCon optimizes formula constants for standard formulae such as Haigis, SRK/T, Hoffer-Q and Holladay 1 (Atchison & Smith 2000). For this constant optimization, ophthalmic surgeons, clinical centres and lens manufacturers upload data sets preferably from well-controlled clinical studies with preoperative biometric data, lens type and power, as well as stable postoperative refraction. For extracting the characteristic distributions of biometrical data, we used all data sets which were uploaded between June 2017 and March 2020 ($N = 17\ 440$). From these data sets, we used axial length (AL), phakic anterior chamber depth (PACD), and anterior corneal radii of the flat (Rf) and the steep (Rs) meridian. Corneal radii of curvature Rf and Rs were transformed to meridional powers using a keratometer index of $n = 1.332$ (according to the front vertex power of a cornea which re-samples the Gullstrand model eye) (Atchison & Smith, 2000). Corneal equivalent power (CEQ) and corneal astigmatism (CAST) were calculated from the average and difference of both corneal meridional power values Rf and Rs. Refractive spherical equivalent (RSEQ) and refractive cylinder (RCYL) before cataract surgery were available in 2240 data sets, and in all other data sets, we retrieved refraction from biometric data (AL, PACD, CAST) as well as implanted lens power, optimized formula constants and postoperative refraction (spherical equivalent and refractive cylinder).

In a first step, we recalculated postoperative refraction from preoperative biometry, implanted lens power and

formula constants (Haigis a0, a1 and a2). Comparing the measured subjective refraction with the recalculated refraction, we filtered out all data sets where the difference in spherical equivalent deviated more than 0.5 D. After this selection, $N = 16\,588$ data sets (master data) were retained. We used a thin lens model for the phakic lens. It was placed 0.5 mm in front of the crystalline lens at $ELP = PACD - 0.5\,mm$. This represents a typical vault between crystalline lens and phakic IOL (Alfonso et al. 2012).

In a second step, we analysed the probability density distributions and correlations between biometric values. All probability distributions were resampled and modelled by individual spline functions (kernel distribution). From the correlation matrix of the master data, we observed that there was a negligible correlation between RCYL/CAST with the other parameters such as RSEQ / CEQ / AL / ELP (Spearman's rho < 0.05 each). In contrast, there was a strong correlation between RCYL and CAST as well as between RSEQ, CEQ, ELP and AL. Kendall's rank correlation tau (Kendall 1970) as well as Spearman's rho (Best & Roberts 1975) are shown in Table 1. Therefore, we decided to simulate the interactions between the parameters (Kendall 1970; Best & Roberts 1975; Gibbons & Subhabrata 2011; Hollander et al. 2013) in the master data which are required for calculation of a toric phakic lens based on vergence transformation strategies using 2 separate copulas: one bi-variable copula (Aas et al. 2009) for refractive cylinder and corneal astigmatism and a second one (quadro-variable) for RSEQ, CEQ, ELP and AL. The AL is not directly required for

calculation of the phakic lens power, but gives some insight into the effect of ratio in the regression model.

In a third step, we built up both copulas. Copulas are normally used to build up generic data with specific distribution density characteristics and specific interactions between variables (Kendall 1970; Best & Roberts 1975). The number of samples in the new generic data was selected independently from the sample size of the basic data set (Bouyé et al. 2000; Aas et al. 2009; Sheldon 2012). For definition, we require the density distribution and the correlation characteristics between variables in the master data. From the matrix of Kendall's tau rank correlation coefficients for copula 1 and 2, we extract the correlation matrix with Spearman's rho. Then, a bi-variable and a quadro-variable copula with a sample size of $n = 50\,000$ are defined, which shows uniform distributions between [0 1] for the 2 or 4 variables. Correlation between the 2 and 4 variables in the data set is adopted from the correlation matrix of the basic data set. Next, each column of the generic data in the copula is transformed using the distribution density characteristics extracted from the master data while maintaining the interactions between variables. Fig. 1 shows the combined scatterplot and histogram for RCYL and CAST using copula 1 as an example. The interaction between both variables is fully maintained in the generic data and the Spearman rank correlation coefficient of RCYL and CAST is 0.8374 (instead of 0.8340 in the master data, see Table 1). Fig. 2 shows on the left side the cumulative density distribution of the master data alongside with the generic data from copula 1 for RCYL and CAST. The same procedure is used for copula 2: A copula with $n = 50\,000$ data sets and 4 variables is

created which shows a uniform distribution for all 4 variables, and the interaction of the variables resemble those of the master data for RSEQ, CEQ, ELP and AL. Transforming the 4 columns of this copula probability density distribution yields the respective data to be used for the Monte-Carlo simulation. Fig. 2 displays on the right side the cumulative density distribution of the 4 variables of the master data alongside the respective variables of the $n = 50\,000$ generic data. Spearman's rho and Kendall's tau for copula 2 are provided in Table 2. Combining the results of copula 1 and 2 together yields the generic data basis for our Monte-Carlo simulation.

In step 4, the Monte-Carlo simulation is performed: The back vertex distance of the spectacle correction is assumed to be 12 mm, and the toric phakic lens is targeted for postoperative emmetropia in all eyes. The axis of corneal astigmatism is set to 0° (with-the-rule astigmatism), and axis of the refractive cylinder (plus cylinder) is set to 90°. Two separate situations are considered:

a. a preoperative situation with spherocylindrical ametropia corrected with glasses at the spectacle plane, and

b. a postoperative situation with plano refraction and correction of preoperative refractive error with a phakic lens (Langenbucher et al. 2007a; Langenbucher et al. 2008; Eppig et al. 2011).

Using the generic data set, with (a) a plane wavefront in front of the glasses is transformed through the spherocylindrical spectacles to the corneal plane, and after considering corneal refraction to the ELP. With (b), a plane wavefront in front of the cornea is considered, and after adding up

Table 1. Spearman rho (upper right) and Kendall's tau (lower left) rank correlation coefficient based on the master data ($N = 16\,588$).

| tau | Rho | | | | | |
	Corneal astigmatism in D	Refractive cylinder in D	Corneal equivalent in D	Spherical equivalent in D	Axial position of the phakic lens in mm	Axial length in mm
Corneal astigmatism in D	1	0.8340	*	*	*	
Refractive cylinder in D	0.6279	1	*	*	*	
Corneal equivalent in D	*	*	1	0.0647	0.0627	0.6095
Spherical equivalent in D	*	*	−0.0412	1	−0.0541	0.5484
Axial position of the phakic/add-on lens	*	*	0.0399	−0.0344	1	0.4957
Axial length in mm	*	*	0.4173	0.3649	0.3302	1

The elements marked with a (*) showed a rank correlation coefficient rho lower than 0.02. The shaded block in the upper left part defines bi-variable copula 1 and the block in the lower right part quadro-variable copula 2.
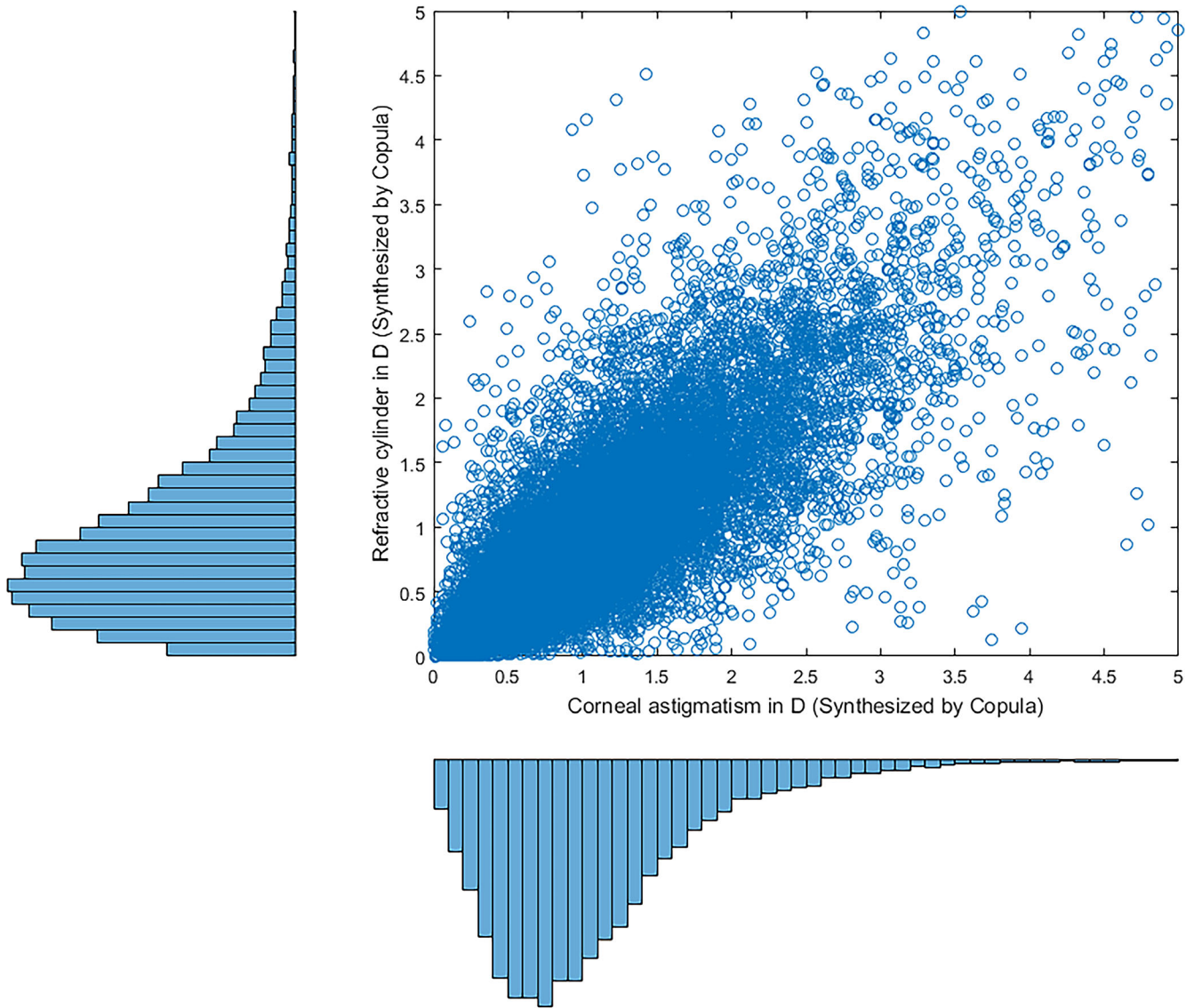
Fig. 1. Probability distribution (histograms) and interaction (scatterplot) [kann man nicht erkennen. Kannst du kleinere Sympbole nehmen oder farbkodiert die Dichte auftragen?] of corneal astigmatism and refractive cylinder in the generic data ($n = 50\,000$). Both variables show an excellent correlation with Spearman's rho = 0.8349, Kendall's tau = 0.6279.

corneal refraction the vergence is transformed to the ELP. Subtracting the vergence of situations (a) and (b) at the ELP plane yields the refractive power of the phakic lens implant. This procedure was performed for each data set. Finally, we calculated the ratio between the refractive power of the spectacles in situation (a) to the refractive power of the phakic IOL in situation (b).

Results

The ratio$_{eq}$ (spherical equivalent refractive power of the phakic lens to spherical equivalent of the preoperative refraction) ranges from 1.0640 to 1.3723 with an average of 1.2177 ± 0.0580 and a median of 1.2184. The ratio$_{ast}$ (astigmatism of the phakic lens to refractive cylinder of the preoperative refraction) ranges from 1.0501 to 1.4340 with an average of 1.2213 ± 0.0580 and a median of 1.2216.

The ratio$_{eq}$ and ratio$_{ast}$ are displayed in an overlay scatterplot in Figs 3, 4 and 5, alongside the linear regression lines in terms of minimizing the root mean squared error. Corneal equivalent power (Fig. 3A) or refractive cylinder (Fig. 3B) does not seem to have an impact on both ratios. The respective data from the linear regression (definition of the regression line, standard error, squared Spearman's rho and fit error) are shown in Table 3.

Figure 4 shows both ratios as a function of refractive spherical equivalent (Fig. 4A) and refractive cylinder (Fig. 4B). The RSEQ has a strong effect on both ratios: the more hyperopic the eye is before implantation of a phakic lens, the higher are the ratio$_{eq}$ and the ratio$_{ast}$. From Table 3, we can see that for each dioptre of spherical equivalent (RSEQ) towards hyperopia the ratio$_{eq}$ is increased by 0.013696, and the ratio$_{ast}$ is increased by 0.026982 (Spearman's rho = 0.682 and 0.841).

The axial position of the phakic lens derived from the distance between the corneal vertex to the anterior vertex of the crystalline lens and the standard vault has a mild effect on both ratios
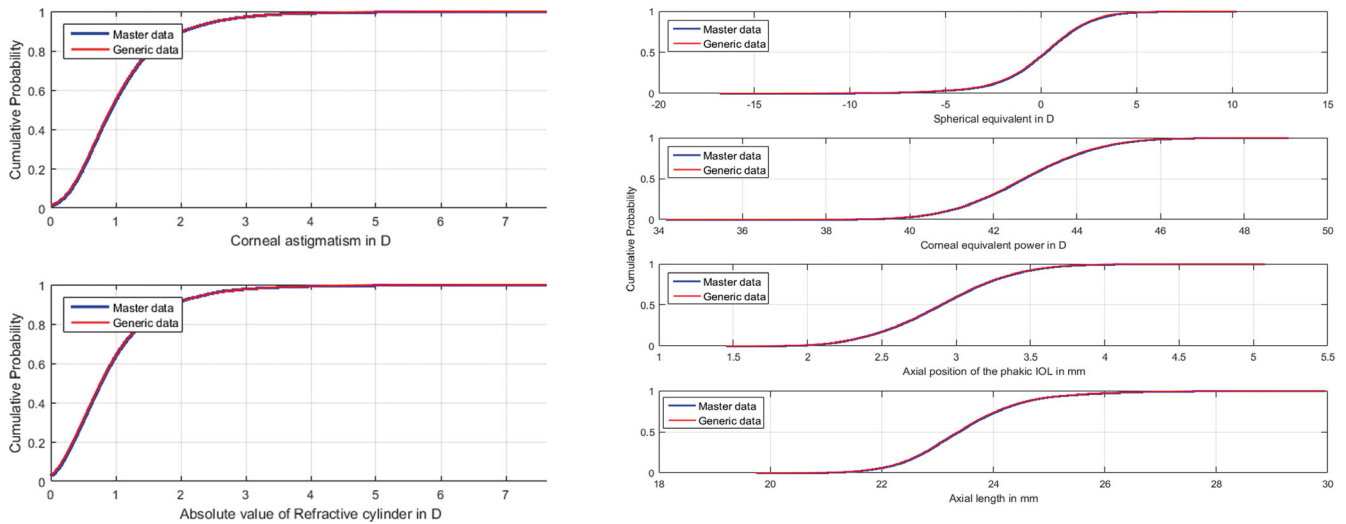
Fig. 2. Left side: Cumulative probability density distribution for refractive cylinder and corneal astigmatism. Right side: Cumulative probability density distribution for refractive spherical equivalent, corneal equivalent power, axial position of the phakic lens implant, and axial length. The blue lines indicate the distribution from the master data ($N = 16\ 588$), the red lines the respective distribution from the generic data set of copula 1 (left side) and copula 2 (right side) ($n = 50\ 000$).

Table 2. Spearman rho (upper right) and Kendall's tau (lower left) rank correlation coefficient based on the generic data ($n = 50\ 000$).

tau	Rho			
	Corneal equivalent in D	Corneal astigmatism in D	Axial position of the phakic lens in mm	Axial length in mm
Corneal equivalent in D	1	0.0647	0.0627	0.6095
Corneal astigmatism in D	−0.0454	1	−0.0541	−0.5484
Axial position of the phakic/add-on lens	−0.0489	−0.0394	1	0.4957
Axial length in mm	−0.4178	−0.3649	0.3429	1

The elements marked with a (*) showed a rank correlation coefficient rho lower than 0.05. The Spearman rho rank correlation coefficients are in good correspondence with the respective values from Table 1 (shaded lower right block, based on master data).

(Fig. 5A). From Table 3, we can see that for each millimetre that the distance of the phakic lens to the corneal vertex increases, the ratio$_{eq}$ is increased by 0.095097, and ratio$_{ast}$ is increased by 0.065063 (Spearman's rho = 0.041 and 0.388).

The ratio$_{eq}$ and the ratio$_{ast}$ are shown in Fig. 5B. There is a moderate inverse correlation for both ratios with axial length, which means that the larger the axial lengths, the lower are both ratios. From Table 3, we see that for each millimetre that the axial length increases, the ratio$_{eq}$ is decreased by 0.019164, and ratio$_{ast}$ is increased by 0.021777 (Spearman's rho = 0.439 and 0.333). For completeness, axial length is not explicitly required for calculation of a phakic lens implant.

In a bi-variable regression model using the spherical equivalent (RSEQ) and the axial position of the phakic lens (ELP) as independent variables, the ratio$_{eq}$ is defined by the regression

$y = 0.96639 + 0.018258 \cdot x1 + 0.086405 \cdot x2$ where x1 refers to the RSEQ in D and x2 to the ELP of the phakic lens with respect to the corneal front vertex in mm (mean squared fit error: 0.0150, Spearman's rho = 0.944). The respective bi-variable regression model for the ratio$_{ast}$ yields $y = 0.96888 + 0.02926 \cdot x1 + 0.08613 \cdot x2$ (mean squared fit error: 0.0127, Spearman's rho = 0.983).

Both ratios were modelled with a bi-variable regression (linear least squares fit with axial position of the phakic lens and preoperative spherical equivalent of refraction as co-variates) as shown above. The fit error of ratio$_{eq}$ (in blue) and ratio$_{ast}$ (red) is plotted in Fig. 6A for spherical equivalent and in Fig. 6B for the axial position of the phakic lens. In Fig. 6A, we can also see that for very small values of refractive spherical equivalent the ratio$_{eq}$ is numerically unstable due to the singularity around zero (Sheldon 2012). However, these conditions are clinically of minor

relevance as phakic lenses are typically implanted for larger values of spherical equivalent, larger refractive cylinder, or a combination of both. The equivalent power of the lens required for emmetropization of the eye is not affected by this numerical instability. Fig. 6B shows that the fit error does not behave in this way when plotted as a function of the axial position of the phakic lens.

Discussion

Phakic lenses are coming more and more into vogue to close the gap between corneo-refractive surgery and refractive lens exchange. In young patients with a sufficient amount of physiological accommodation, corneo-refractive surgery procedures such as LASIK, SMILE/FLEX or PRK are preferred in the case of low or moderate spherical and/or astigmatic refraction errors (Kohnen et al. 2005;
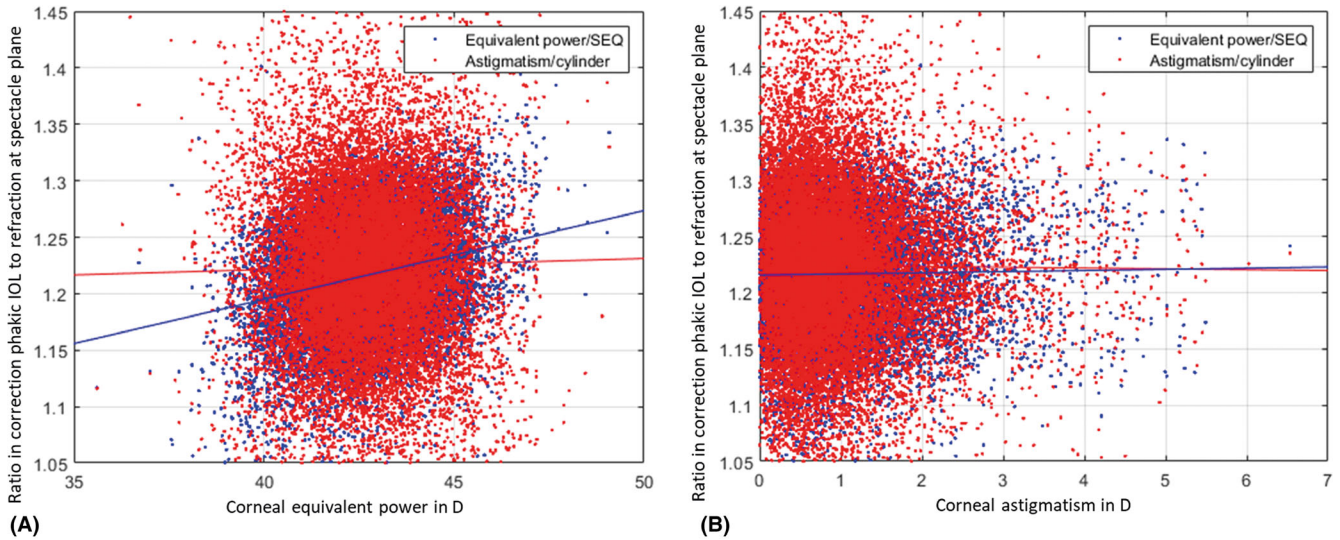
Fig. 3. (A) Ratio of phakic lens equivalent power to corneal equivalent power (blue) and ratio of phakic lens astigmatism to corneal astigmatism (red) as a function of corneal equivalent power. Data are based on the generic data ($n = 50\,000$). There is no systematic dependency of either ratio on the corneal power. (B) Ratio of phakic lens equivalent power to corneal equivalent power (blue) and ratio of phakic lens astigmatism to corneal astigmatism (red) as a function of corneal astigmatism. Data are based on the generic data ($n = 50\,000$). There is no systematic dependency of either ratio on the corneal power.
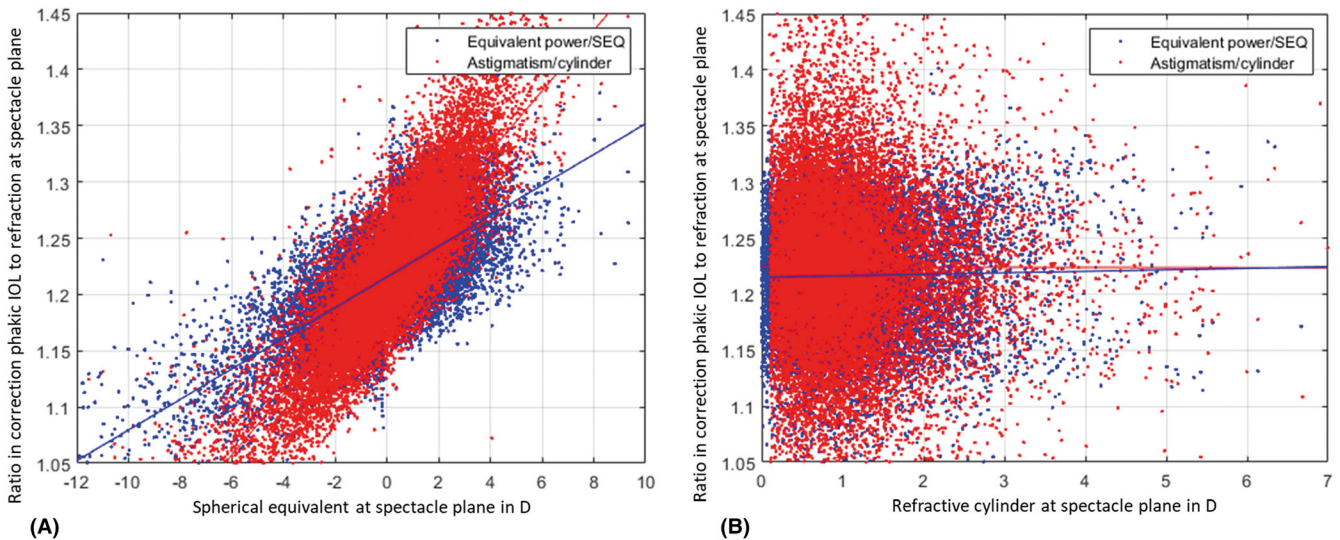

Fig. 4. (A) Ratio of phakic lens equivalent power to corneal equivalent power (blue) and ratio of phakic lens astigmatism to corneal astigmatism (red) as a function of preoperative refractive spherical equivalent. Data are based on the generic data ($n = 50\,000$). There is a strict positive correlation, which means that the more hyperopic the patient's eye is before surgery the higher both ratios, or vice versa, the more myopic the lower both ratios. (B) Ratio of phakic lens equivalent power to corneal equivalent power (blue) and ratio of phakic lens astigmatism to corneal astigmatism (red) as a function of preoperative refractive cylinder. Data are based on the generic data ($n = 50\,000$). There is no systematic dependency of either ratio on the refractive cylinder.

Donaldson et al. 2018). If presbyopia starts at an age of around 40 to 45, refractive lens exchange might be a good option in situations with large or excessive ametropia, but those surgical interventions are always accompanied with the loss of physiological accommodation. Before the onset of presbyopia, phakic lenses are particularly suitable in situations with large or excessive spherical or astigmatic refraction deficits and they maintain physiological accommodation. If cataract surgery subsequently becomes necessary due to opacification of the crystalline lens, the phakic lens can easily be removed before phacoemulsification.

For the calculation of phakic lenses, many surgeons rely on external calculation services provided by the manufacturers of those lenses. Some of them calculate the spherical and astigmatic lens power based on biometric values, others use empirical factors (fixed ratios), which translate the refraction deficit from the spectacle or contact lens plane to the correction plane of the phakic lens. In general, as phakic lenses are located closer to the corneal vertex than capsular bag lenses, these ratios for translating refractive spherical equivalent or refractive cylinder into
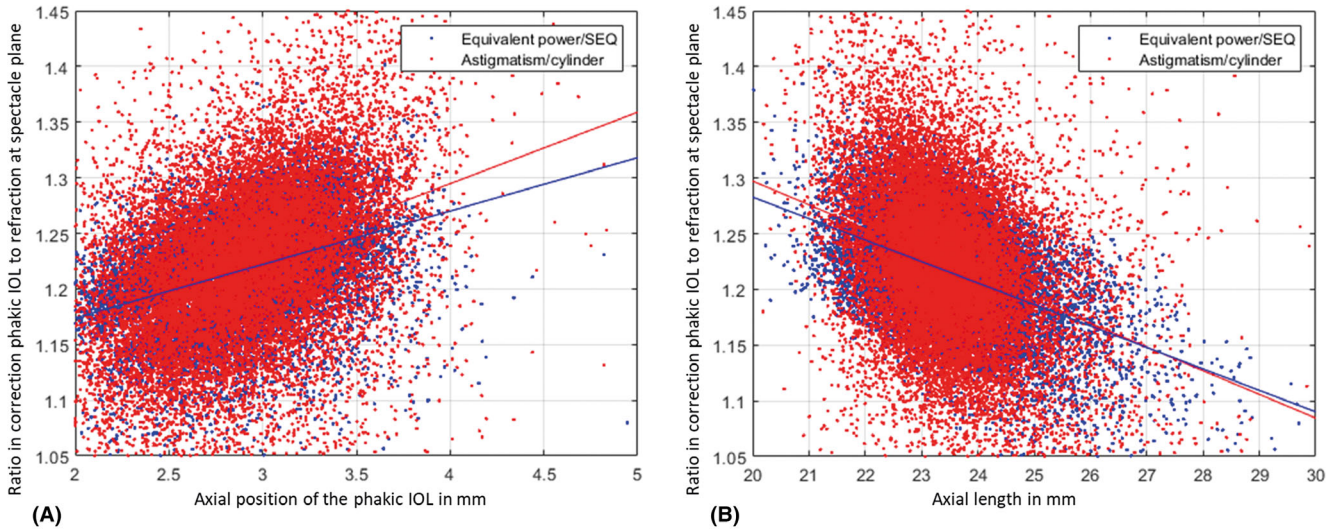
Fig. 5. (A) Ratio of phakic lens equivalent power to corneal equivalent power (blue) and ratio of phakic lens astigmatism to corneal astigmatism (red) as a function of axial position of the phakic lens (ELP). Data are based on the generic data (n = 50 000). There is a mild positive correlation, which means that the more distant the lens is positioned behind corneal vertex, the higher both ratios. (B) Ratio of phakic lens equivalent power to corneal equivalent power (blue) and ratio of phakic lens astigmatism to corneal astigmatism (red) as a function of axial length. Data are based on the generic data (n = 50 000). There is moderate negative correlation, which means that the larger the eye length, the lower both ratios.

an appropriate power are lower with phakic lenses than with capsular bag IOL (closer to 1). But – to the best of our knowledge – there is no publication which deals with those ratios for translation of spherical equivalent or refractive cylinder in a simulation model to overcome an empirical estimation using a fixed factor for all situations.

Monte-Carlo simulations are a valid strategy for estimating such values and obtaining more insight to the influencing factors (Hill & Potvin 2008). A very large number of data sets are used, and it is possible to investigate potential influencing factors separately or in combination using a multivariable regression. But most important is that the data reflect real-life conditions (Hill & Potvin 2008). The data set which is used should fulfil the following conditions: all biometric factors used for calculations should have probability distributions which match the clinical data sets of a population receiving phakic lenses. All the links between variables have to be considered, as the correlation between biometric values may change the results significantly. For example, if there is a strong correlation of axial length and equivalent power of the cornea and both values are used for calculation of a phakic lens, then this correlation has to be considered appropriately. In our Monte-Carlo simulation, we used data sets which were extracted from the

database on the IOLCon platform. In this comprehensive database, we store a large series of anonymized clinical data which have been used for formula constant optimization, and we derived the probability distributions and interactions of axial length, phakic anterior chamber depth, and corneal equivalent and astigmatic power from these data. With regard to refraction, there are only a limited number of spherical equivalent and refractive cylinder data prior to cataract surgery, and even those data might be unreliable as with increasing opacification of the crystalline lens, refraction measurements might be incorrect and a shift (e.g. myopic shift) might happen with dense cataracts. Therefore, we modelled refraction by axial length, corneal equivalent and astigmatic power, phakic anterior chamber depth, implanted lens type (formula constant) and power, as well as with postoperative refraction (Schrecker & Langenbucher 2016) and probability density distributions published in the literature (Fotouhi et al. 2011; Sanfilippo et al. 2015; Williams et al. 2015; Schuster et al. 2017; Sheeladevi et al. 2018; Irving et al. 2019, and our own unpublished data). This estimation of preoperative refraction may have some uncertainties. In addition, as we used data sets of eyes prior to cataract surgery, the characteristics of this population may deviate somehow from a

population of eyes before phakic lens implantation.

One advantage of our model is that we extracted the probability distributions of all variables and all interactions between parameters (Best & Roberts 1975; Sheldon 2012; Hollander et al. 2013). In the data set, refractive cylinder correlates significantly with corneal astigmatism, but neither of these variables interacts with other variables in the model. Refractive spherical equivalent, corneal equivalent power and axial position of the phakic lens show some correlation. Therefore, we decided to split our model into 2 parts: refractive cylinder and corneal astigmatism were modelled with one bivariable copula and refractive spherical equivalent, corneal equivalent power, axial position of the phakic lens and axial length were modelled with another quadro-variable copula. Copulas as an alternative to bootstraps are used to realize arbitrary model probability density distributions for variables and maintaining the interactions or correlations between variables (Kendall 1970; Best & Roberts 1975; Sheldon 2012). This means that the distribution probabilities of all variables such as axial length or corneal power could be analysed and resampled by individual kernel spline distributions and the interactions described by Kendall's tau or Spearman's rho rank correlation coefficients could be ensured. From

Table 3. Linear fit of ratio of equivalent power of the phakic lens to refractive spherical equivalent (upper part) and ratio of the astigmatism of the phakic lens to refractive cylinder (lower part) as a function of corneal equivalent power, corneal astigmatism, refractive spherical equivalent, refractive cylinder, axial position of the phakic lens, and axial length based on the generic data ($n = 50\,000$).

		Corneal equivalent power in D	Corneal astigmatism in D	Spherical equivalent in D	Refractive cylinder in D	Axial position of the phakic lens in mm	Axial length in mm
Ratio equivalent power of the phakic lens to refractive spherical equivalent	Regression line (least squares fit)	0.88185 / 0.0078325	1.2154 / 0.0012939	1.2158 / 0.013696	1.2152 / 0.0012815	1.0392 / 0.095097	1.6657 / −0.019164
	Standard error $\times 10^{-3}$	10.491 / 0.24534	0.56559 / 0.48023	0.26053 / 0.11471	062496 / 048798	47.476 / 16.212	7.1694 / 0.3056
	Correlation coefficient rho^2	0.0584	0.000442	0.465	0.00042	0.00172	0.193
	Root mean squared error	0.0443	0.0456	0.0334	0.0456	0.957	0.041
Ratio astigmatism of the phakic lens to refractive cylinder	Regression line (least squares fit)	1.1759 / 0.0011239	1.2235 / 0.00051767	1.2193 / 0.026982	1.2234 / 0.00047446	1.0351 / 0.065063	1.7343 / −0.021777
	Standard error $\times 10^{-3}$	14.301 / 0.33446	0.79327 / 0.6463	0.27076 / 0.12608	0.86975 / 0.646	3.2791 / 1.1187	10.496 / 044745
	Correlation coefficient rho^2	0.000593	3.37×10^{-5}	0.707	2.84×10^{-5}	0.151	0.111
	Root mean squared error	0.0687	0.0687	0.0372	0.0687	0.0633	0.0648

Regression line refers to: intercept / slope; standard error refers to standard error of intercept / slope, Correlation coefficient refers to squared Spearman's rho, and root mean squared error to the error of the linear fit. The regressions refer to Figs 3, 4, and 5.

the master data derived from IOLCon ($N = 16\,588$ validated data sets) or estimated (refraction data), we generated a new generic data with $n = 50\,000$ data sets.

However, there are some limitations of our study: first, we have made several assumptions to keep the results simple: for example, we have assumed a back vertex distance of the spectacle correction of 12 mm. Especially in high ametropia with 6 dpt or more, the back vertex distance might affect the transfer of spectacle correction to refractive correction at the corneal plane or the phakic lens plane. Therefore, a measurement of back vertex distance is highly recommended in cases with ametropia of 6 dpt or more. The axial position of the phakic lens was assumed to be 0.5 mm in front of the crystalline lens, which is in general a good estimate, but in some cases, the vault might be larger or smaller. Another limitation of our study was our modelling with copulas which does not consider heteroscedasticities for interaction of variables. But as we see, for example, from Fig. 6B, the fit error for both ratios shows a very homogeneous distribution over the entire range of axial position of the phakic lens. Another limitation is that both ratios ($ratio_{eq}$ and $ratio_{ast}$) are defined as a quotient of (equivalent or astigmatic) lens power of the phakic lens to the preoperative (equivalent or cylindric) refraction. Especially for small refractive corrections, the denominator might be close to zero, which causes numerical instabilities as shown in Fig. 6A for $ratio_{eq}$. However, these conditions are clinically of minor relevance as phakic lenses are typically implanted for larger values of spherical equivalent, larger refractive cylinder or combination of both. The equivalent power of the lens which is required for emmetropization of the eye is not affected by this numerical instability.

Calculation of the phakic lens power from the biometric data was performed using linear Gaussian optics. That means that we were restricted to the paraxial space. The difference compared to calculation of toric capsular bag lenses in a cataract surgery or refractive lens exchange is that axial length measurement is not required. But in principle, the calculation schemes are similar (Langenbucher &
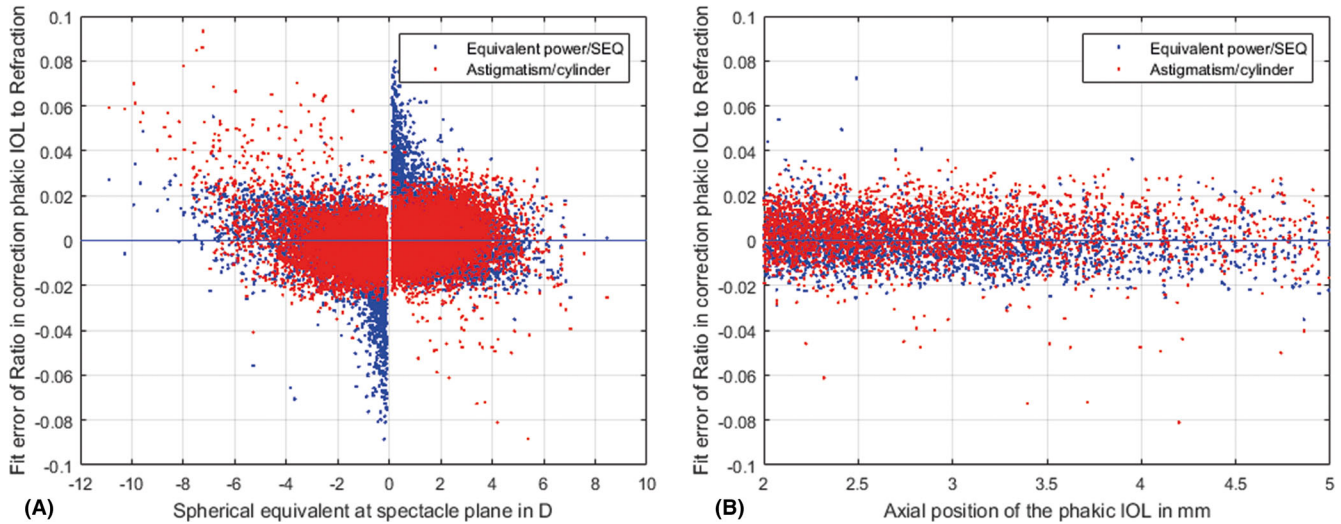
Fig. 6. (A) Fit error of the ratio of phakic lens equivalent power to corneal equivalent power (blue) and ratio of phakic lens astigmatism to corneal astigmatism (red) as a function of refractive spherical equivalent. The fit was performed using a bi-variate least squares model with spherical equivalent and axial position of the phakic lens as co-variates as shown in the Results section. Data are based on the generic data ($n = 50\,000$). There is a singularity for the ratio of phakic lens equivalent power to corneal equivalent power when refractive spherical equivalent values are small. (B) Fit error of the ratio of phakic lens equivalent power to corneal equivalent power (blue) and ratio of phakic lens astigmatism to corneal astigmatism (red) as a function of axial position of the phakic lens with respect to the corneal front vertex. The fit was performed using a bi-variate least squares model with spherical equivalent and axial position of the phakic lens as co-variates as shown in the Result section. Data are based on the generic data ($n = 50\,000$). There is a homogeneous distribution of the fit error over the entire range of phakic lens axial positions.

Seitz 2004; Langenbucher et al. 2004a; Langenbucher et al. 2004b; Langenbucher et al. 2007a; Langenbucher et al. 2007b; Langenbucher et al. 2008; Savini et al. 2013). There are different options for dealing with spherocylindrical refraction, for example using vergence propagation or matrix calculation (Langenbucher et al. 2004b), using 4×4 refraction and translation matrices. We decided to use step-by-step vergence propagation through the eye. The strategy behind this is already described in the literature. As an alternative, if full shape data were available from all surfaces and wavefront measurement data from the refraction error (instead of simple keratometry readings and spherical equivalent and refractive cylinder data), it would be possible to use full-aperture raytracing for our Monte-Carlo simulation. In general, we could see from our results that the ratio$_{eq}$ and the ratio$_{ast}$ show a large variation between 1.05 and 1.43.

The selection of high refractive power PIOLs is affected by the variability in this ratio. For example, let us assume an 8 D PIOL is implanted in an eye based on the average ratio$_{eq}$ (1.2177). The correct value for this ratio might also be one standard deviation larger (1.2757) or smaller (1.1597). This could result in an error of ±0.3 D. This difference is proportional to the power of the IOL. Therefore, a fixed factor should not be used for PIOL with high refractive power or large cylinder.

Direct calculation of the equivalent power or astigmatism of a phakic lens implant is not too mathematically complex (Langenbucher et al. 2007a; Eppig et al. 2011) and the axial position of the IOL could be estimated based on the measured phakic anterior chamber depth and the vault between IOL and crystalline lens with a sufficient precision (Alfonso et al. 2012). If in any situation a calculation of phakic lens power were not possible, the following simple linear regression formula could be used:

$$IOLP_{eq} = RSEQ \cdot (0.96639 + 0.018258 D^{-1} \cdot RSEQ + 0.086405 mm^{-1} \cdot ELP)$$

$$IOLP_{ast} = RCYL \cdot (0.96888 + 0.02926 D^{-1} \cdot RSEQ + 0.08613 mm^{-1} \cdot ELP)$$

where IOLPeq and IOLPast refer to the equivalent and astigmatic power of the phakic lens implant, RSEQ and RCYL to the spherical equivalent and refractive cylinder of the preoperative refraction to be corrected, and ELP to the estimated position of the phakic lens provided by the lens manufacturer.

Conclusions

In conclusion, we have set-up a Monte-Carlo simulation to investigate:

a the ratio of equivalent power of the phakic lens to spherical equivalent of the preoperative refraction and

b the ratio of astigmatism of the phakic lens to refractive cylinder of the preoperative refraction.

Both ratios were analysed as a function of refraction, corneal power, axial length and axial position of the phakic lens. Both ratios show a large range of variation. Therefore, for calculation of equivalent and toric power of a phakic lens implant, a fixed factor which just translates preoperative refractive values to lens power values should not be used. Such empirical assumptions may significantly over- or underestimate the lens power in an individual case. Whenever possible lens power should be calculated from individual biometric

values or through the use of a simple linear regression formula.

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